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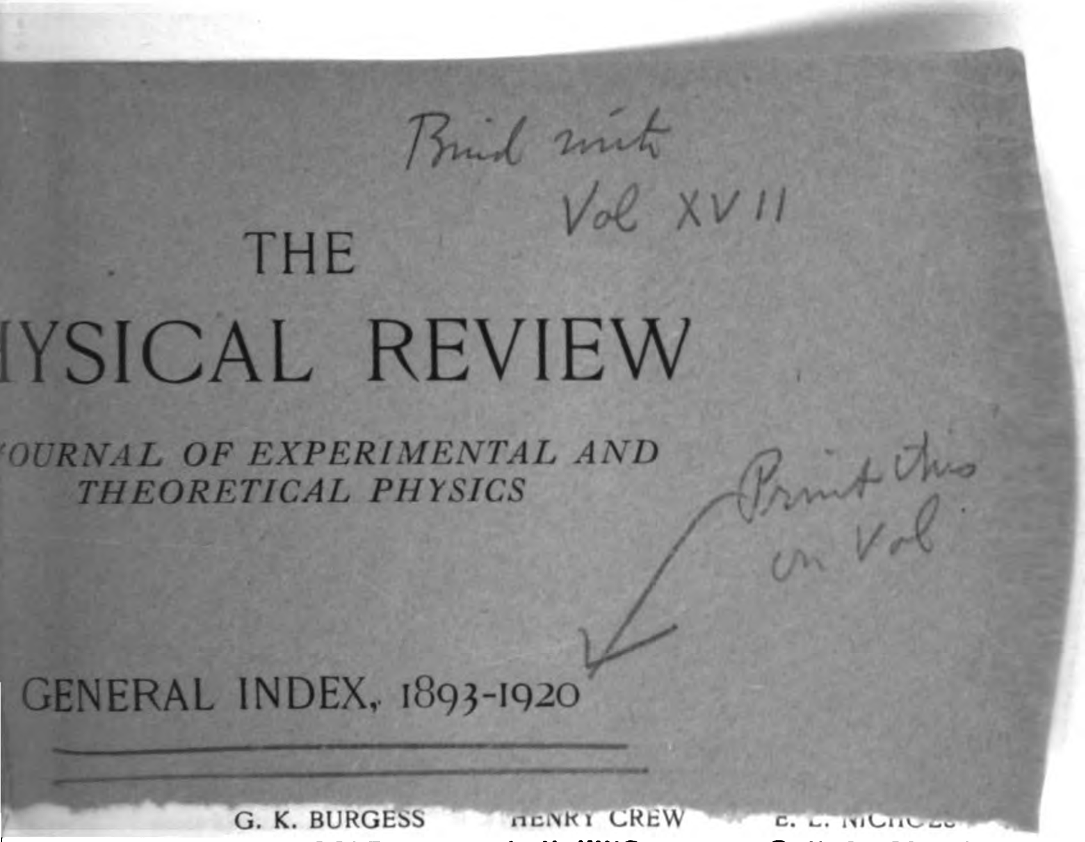

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THE PHYSICAL REVIEW.

CALCULATION OF THE AMPLIFICATION CONSTANT OF THE WEAGANT THERMIONIC VACUUM TUBE.

By Alexander Marcus.

Synopsis.

Amplification Constant of a Weagant Thermionic Vacuum Tube.—**This tube has the control electrode on the outside and because of the presence of the glass wall, amplification is not possible for constant grid potentials. By making some simplifying assumptions, the author is able to develop a *theoretical expression* for the amplification constant which involves the dimensions of the tube and the resistivity of the glass wall. When this resistivity is either very high or very low, the amplification is small; in the latter case it is shown to depend solely on the dimensions.**

So far as the writer is aware no one has hitherto published any theoretical investigation of that type of triode in which the control electrode is outside of the tube. Moreover, as the theory of this tube must take into account the physical properties of the glass wall the many theoretical investigations already published of the action of the control electrode in the usual type of internal grid triode do not apply to the Weagant Tube.

Before taking up the actual calculation of the amplification constant let us examine the physical phenomena. Let F, P, G and g in Fig. 1 stand for the filament, plate, glass wall of the tube and control electrode respectively. When F is hot and P is positive with respect to F there is a current between F and P. When a third electrode is wrapped around G and a voltage applied to it, the plate current is responsive only to changes in the latter. This tube therefore does not function as an amplifier for constant "grid" voltages. The cause of this peculiarity lies in the action of the glass. In this type of tube, as in the other varieties, the amplifying action is due to the variation in the space charge produced by the voltage on the control electrode. When there is no impressed po-

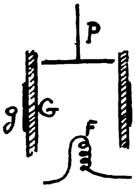

Fig. 1.

tential at the plate, there is a current due to diffusion outward from the filament in all directions until the space becomes saturated. Then the electron atmosphere is in equilibrium and a positive potential at the plate is required to maintain a current from filament to plate. Why a constant potential on the outside of the glass wall, if the resistivity of the glass be large compared with that of metals, is incapable of sustaining a radial electron flow is obvious from the following considerations.

The radial current toward the inner face of the wall of the tube is nev, where n is the average number of electrons in unit volume near the inner face, e, the charge on the electron, and v, the radial velocity. The current through the inner face of the glass due to a potential E is $\lambda E + C(dE/dt)$ where λ and C are the conductivity and specific inductive capacity respectively. We may write, $nev = \lambda E + C(dE/dt)$. For constant values of E the right-hand member of this equation vanishes, for, the conduction current term is small for ordinary glass and the displacement current $C(DE/dt)$ becomes 0 after the transient effect due to the impressed voltage has subsided. On the other hand, whenever E is variable $C(dE/dt)$ is not 0 and then the variations in the potential on the glass are communicated through the electron atmosphere to the stream between filament and plate. Amplification is then possible. In other words, a constant voltage on the glass is incapable of sustaining a radial flow and consequently incapable of varying the space charge between filament and plate.

The complete theory of the action of the control electrode in the Weagant tube would require that expressions be found for the density of space charge, the potential and current everywhere in the tube, in terms of the voltage distribution over an extremely complex boundary and in terms of the geometrical constants of the tube and the physical constants of the glass wall. Such a procedure would necessitate the introduction of too large a number of assumptions of uncertain validity. Moreover it would be unnecessarily laborious in view of the fact that there is an alternative method of calculating the amplification constant which is both physcially and mathematically simpler.

ELECTROSTATIC THEORY OF THE CONTROL ELECTRODE.

Given three bodies (1), (2), and (3) distributed at random and having charges Q_1, Q_2, and Q_3 and potentials V_1, V_2 and V_3, we wish to find how the charge on (2) is connected with the potentials of the three bodies.

It is proved in treatises on electrostatics that:

$$\begin{aligned} V_1 &= P_{11}Q_1 + P_{12}Q_2 + P_{13}Q_3, \\ V_2 &= P_{21}Q_1 + P_{22}Q_2 + P_{23}Q_3, \\ V_3 &= P_{31}Q_1 + P_{32}Q_2 + P_{33}Q_3, \end{aligned} \qquad (1)$$

where the p's are called the coefficients of induction. From these three equations we find

$$Q_2 = c_{12}V_1 + c_{22}V_2 + c_{23}V_3, \tag{2}$$

where the c's are the mutual capacitances and involve only the p's. The V's are absolute potentials. If (2) be taken as the body of standard potential, the difference in potential between (1) and (2) may be maintained by a battery of voltage e_1, and similarly the potential difference $V_3 - V_2$ may be maintained by a battery of voltage e_3. Equation (2) may now be written

$$Q_2 = c_{12}e_1 + c_{23}e_3 + (c_{12} + c_{22} + c_{23})V_2.$$

If V_2 be taken as zero, $Q_2 = c_{12}[e_1 + (c_{23}/C_{12})e]$ or $Q_2 = c_{12}(e_1 + ge_3)$, where g is the amplification constant, for, $\Delta Q_2/\Delta e_1 = c_{12}$ when e_3 is kept constant and $\Delta Q_2/\Delta e_3 = c_{12}g$ when e_1 is kept constant and $g = c_{23}/c_{12} = \Delta e_1/\Delta e_3$ when e_1 and e_3 are both variable.

We may now use this method of calculating g for our special case. We shall assume that the charges on the plate and on the glass wall are uniformly distributed and that the space charge is concentrated in the neighborhood of the filament. Before making use of equations (1) and (2), it is necessary to determine the value of Q_g, the charge on the cylindrical glass wall corresponding to a potential E on it.

Cut a section in the wall of thickness dr and height 1. The voltage at one face is E and at the neighboring face $E + (\partial E/\partial r)dr$. See Fig. 2. If R be the resistivity of the glass we may put:

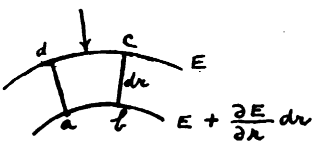

Fig. 2.

$$\frac{\partial E}{\partial r} dr = R\,dr\,I \quad (I \text{ being the current) or}$$

$$\frac{\partial E}{\partial r} = RI. \quad \text{(I.)}$$

The current flowing into the element dc will not equal the current flowing out, for two reasons, first the capacity Cdr will absorb a charge Cdr in the time dt and in the second place, the charge $KEdrdt$ will leak out in the time dt if K be the conductance of unit volume of the glass. Idt is the charge entering the section at dc and $dt[I + (\partial I/\partial r)dr]$ is the charge leaving at ab. Hence

$$C\frac{dE}{dt}drdt + KEdrdt = \left(I + \frac{\partial I}{\partial r}dr\right)dt - Idt;$$

$$\therefore \quad C\frac{dE}{dt} + KE = \frac{dI}{dr}. \tag{II.}$$

This equation together with (I.) determines the charge on the glass in terms of E.

Since $K = 1/R$, (II.) takes the form

$$C\frac{dE}{dt} + \frac{E}{R} = \frac{dI}{dr}$$

where C is of the order of $6/(9 \times 10^{11})$ farads, and R ranges in value from 10^{14} to 10^{10} ohms. If the impressed voltage be $E \sin pt$, p being of the order of $5 \cdot 10^{3}$, the term E/R is negligible compared with $C(dE/dt)$ for all values of E. From

$$C\frac{dE}{dt} = \frac{dI}{dr} \tag{III}$$

we get $CE = (dQ/dr)$ where Q is the charge per unit volume of the glass. From (I.) and (III.) we get $d^2E/dr^2 = CR(dE/dt)$, the solution of which is, $E = \epsilon^{-\alpha(a-r)} V_0 \sin \{pt - \alpha(a - r)\}$ where $\alpha = \sqrt{CRp/2}$, a, is the outer radius of the tube and $V_0 \sin pt$ is the voltage impressed upon the external electrode. $\alpha(a - r)$ is negligible compared with pt after a very short time, since αs, where s is the thickness of the tube, has a value less than 40 for $p = 5 \cdot 10^{3}$, $C = 6/(9 \times 10^{11})$, $R < 10^{14}$. We may therefore put $E = \epsilon^{-\alpha(a-r)} V_0 \sin pt$.

Designating the total charge on the cylinder by Q_g and the impressed voltage by V, we get:

$$V = \frac{\alpha Q g}{2\pi a h s C(1 - \epsilon^{-\alpha s})}.$$

We are now ready to resume the calculation of g. In pace of the subscripts 1, 2, and 3 in equations 1, we shall use p, f, and g to stand for plate, filament and glass respectively. Equations 1 may now be put into the form

$$V_p = P_p Q_p P_{pf} Q_f P_{pg} Q_g,$$
$$V_f = P_{fp} Q_p P_f Q_f P_{fg} Q_g,$$
$$V_g = P_{gp} Q_p P_{gf} Q_f P_g Q_g.$$

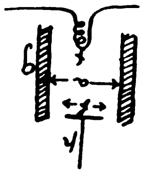

Fig. 3.

The axis of z passes through the filament and axis of the plate, as shown in Fig. 3.

$P_p Q_p = Q_p \frac{\pi}{b} \equiv$ Potential of the plate when all other conductors are at zero potentia'.

$$P_{pf} Q_f = \frac{Q_f}{\frac{\pi b^2}{4}} \int_0^{b/2} \frac{r\,dr\, 2\pi}{\sqrt{r^2 + h^2}}.$$

$$P_{pg} Q_g = \frac{Q_g}{2\pi a h} \int_0^{h} \frac{2\pi a\, dz}{\sqrt{(z - a)^2 + a}}.$$

This is based on the assumption that the charge on the plate occupies a small area compared with the area of the cylinder.

$$P_{fp}Q_p = 2\pi\int_0^{b/2}\frac{r\,dr}{\sqrt{r^2+h^2}}\,\frac{Q_p}{\dfrac{\pi b^2}{4}} = \frac{b^2}{8h}Q_p \text{ (approx.).}$$

$$P_{fg}Q_g = \int_0^h \frac{2\pi a\,dz}{\sqrt{z^2+a^2}}\,\frac{Q_g}{2\pi ah} = \frac{1}{h}\sinh^{-1}\frac{h}{a}\cdot Q_g.$$

$$P_{gp}Q_p = \int_0^h \frac{2\pi a\,dz}{\sqrt{(z-a)^2+a^2}}\,\frac{Q_p}{2\pi ah} = \frac{1}{h}\sinh^{-1}\frac{h}{a}\cdot Q_p.$$

$$V_p = \frac{\pi}{b}Q_p + \frac{b^2}{8h}Q_f + \frac{1}{h}\sinh^{-1}\frac{h}{a}Q_g.$$

$$V_f = \frac{b^2}{8h}Q_p + fQ_f + \frac{1}{h}\sinh^{-1}\frac{h}{a}Q_g\cdots.$$

(The value of f is not required for the purpose of calculating the amplification.)

$$V_g = \frac{1}{h}\sinh^{-1}\frac{h}{a}Q_p + \frac{\sinh^{-1}}{h}\frac{h}{a}Q_f + \frac{\alpha}{2\pi ahsC(1-\epsilon^{-\alpha t})}Q_g.$$

$$Q_f = A\begin{vmatrix} \dfrac{\pi}{b} & V_p & \dfrac{1}{h}\sinh^{-1}\dfrac{h}{a} \\ \dfrac{b^2}{8h} & V_f & \dfrac{1}{h}\sinh^{-1}\dfrac{h}{a} \\ \dfrac{1}{h}\sinh^{-1}\dfrac{h}{a} & V_g & \dfrac{\alpha}{2\pi ahsC(1-\epsilon^{-\alpha t})} \end{vmatrix},$$

where A is a determinant involving only the coefficients of the Q's.

$$\frac{Q_f}{A} = \left[\frac{b^2}{8h}\,\frac{\alpha}{2\pi ahsC(1-\epsilon^{-\alpha t})} - \left(\frac{\sinh^{-1}}{h}\frac{h}{a}\right)^2\right]V_p - [\;]V_f$$
$$+\left[\frac{\pi}{b}\,\frac{\sinh^{-1}}{h}\frac{h}{a} - \frac{b^2}{8h}\,\frac{\sinh^{-1}}{h}\frac{h}{a}\right]V_g.$$

Designating the amplification constant by g, we have

$$g = \frac{1}{\pi b^2}\sinh^{-1}\frac{h}{a}\,\frac{8\pi h - b^2}{\dfrac{\alpha}{2\pi ahsC(1-\epsilon^{-\alpha t})} - \dfrac{8h}{b^2}\left(\dfrac{\sinh^{-1}}{h}\dfrac{h}{a}\right)^2}.$$

In this expression for the amplification constant the physical properties of the glass are involved only in the term

$$\frac{\alpha}{2\pi ahsC(1-\epsilon^{-\alpha t})} \equiv T,$$

where

$$\alpha = \sqrt{\frac{CRp}{2}}.$$

For values of $R \geqq 10^{12}$ ohms/c.c., $\epsilon^{-\alpha s}$ is negligibly small compared with 1, therefore for large values of R, i. e., for non-conducting glass, T varies as $\sqrt{R}$ and g grows smaller as G grows larger. Using the values $a = 0.75$ cm., $b = 0.7$ cm., and $h = 1$ cm., gotten by a measurement of a tube, g is found to have the values 5.6 for $R = 10^{12}$, 0.9 for $R = 10^{13}$, and 0.3 for $R = 10^{14}$.

In making comparisons between calculated values and those determined experimentally it should be kept in mind that in our fundamental equations the capacity coefficients were calculated on the assumption that the tube had a height h extending from filament to plate. In the tube that was measured the total length of the tube was nine times the size of the portion designated as h in Fig. 4. It may be necessary in order to secure good agreement between theory and experiment to take as the effective length of the tube a value larger than the h indicated in the figure.

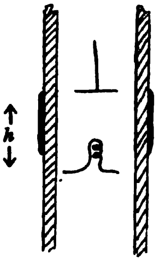

Fig. 4.

As might be expected from physical considerations, g does not continue to increase with a decrease in R. To make this fact apparent let us simplify the form of T for small values of R. For $p = 5.10^8$ and $C = 6(9 \times 10^{11})$ farads, $\alpha = 4 \cdot 10^{-6}\sqrt{R}$. Taking $s = 0.08$ cm., we find that when $R \leqq 10^{11}$ ohms/c.c. $\epsilon^{-\alpha s}$ is not negligible. Putting for $\epsilon^{-\alpha s}$ in T the expansion

$$e^{-\alpha s} = 1 - \alpha s + \frac{(\alpha s)^2}{\underline{|2}} - \cdots$$

and retaining only the lowest powers of α's we get

$$T = \frac{\alpha s}{2\pi a h s^2 \left(\alpha s + \frac{(\alpha s)^2}{\underline{|2}} \right)} = \frac{1}{2\pi a h s^2}$$

for small values of s.

For this condition T becomes large and is the predominating term in the denominator of the expression for g. Then g becomes less than 1, the actual value being dependent solely on the dimensions of the tube.

In conclusion the writer wishes to thank Professor Alfred N. Goldsmith for extending to him the facilities of the research laboratories of the Radio Corporation of America and Mr. Julius Weinberger of the Research Department for giving valuable information on the behavior of the tube in practice.

Research Department of the
Radio Corporation of America,
June 19, 1920.

STUDIES WITH THE IONIZATION GAUGE.

I. CONSTRUCTION AND METHOD OF CALIBRATION.

BY S. DUSHMAN AND C. G. FOUND.

SYNOPSIS.

An Ionization Gauge for Measuring Very Low Pressures.—This gauge is the result of an attempt to develop a more convenient instrument than those previously available. It is based upon the fact that the positive ionization produced by a given stream of electrons varies with the gas pressure. By adopting a *construction* similar to that used for low power pliotrons, erratic effects due to the charging up of the glass walls were avoided and, with a constant electron emission, a linear relation was obtained between ionization current and pressure down to the lowest pressure actually obtaiued, about 10^{-3} bar. For pressures above 10^{-1} bar the *calibration* was carried out by comparison with a McLeod gauge; but for lower pressures a method was used which involves an application of Knudsen's laws of flow. *Characteristic curves* showing the effect of varying the positive voltage and the electron emission on the ionization current are given. The *low limit of pressure* measurable with such a gauge is fixed only by the sensitivity of the galvanometer used to measure the ionization current. With an ordinary sensitive galvanometer, pressures as low as 10^{-4} bar can readily be determined.

INTRODUCTION.

DURING the past few years, as the technique of high vacuum production has improved, there have of necessity been developed a number of methods of measuring these extremely low gas pressures. Of these methods, the absolute gauge devised by M. Knudsen[1] and the molecular gauge suggested by I. Langmuir[2] have been used fairly extensively. In both types, however, the difficulties of construction and operation are such as to make their use very inconvenient.

Some time ago Dr. Hull suggested to the writers that a gauge might be devised based on the measurement of the amount of positive ionization produced by an electron stream. After some preliminary experiments had been carried out with a gauge based on this suggestion, a paper on the same subject appeared by O. E. Buckley.[3]

More recently Misamichi So[4] has also described the results of an investigation carried out with a gauge constructed on the same principle.

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[1] Ann. Phys., *32* (1910), and *44*, 525 (1914).

[2] PHYS. REV., 1, 337 (1913), and also S. Dushman, PHYS. REV., *5*, 212 (1915). This paper contains a brief review of the different types of gauges that have been used for measuring very low gas pressures.

[3] Proc. National Acad. of Sciences, *2*, 683 (1916).

[4] Proc. Physico-Mathem. Soc. Japan, I., 76 (1919).

The manometer consists of three electrodes sealed in a glass bulb which serve as cathode, anode and collector of positive ions respectively. The cathode may be either an incandescent tungsten filament or Wehnelt cathode. Under the influence of the accelerating field due to the anode the electrons ionize the gas molecules with which they collide and the positive ions thus produced are collected by the third electrode which is negatively charged with respect to the more negative terminal of the cathode.

According to Buckley, "The exact forms of the electrodes are not of great importance. The collector is preferrably situated between the other two electrodes and of such form as not to entirely block the electron current to the anode." Preliminary experiments were carried out with a simple gauge consisting of three V-shaped filaments placed in parallel planes. (A similar construction has also been used by So.) On trial, it was found, however, that with this construction it was not always possible to obtain a linear relation between ionization current and pressure. Also, there was trouble due to the charging up of the glass walls. In the present paper the authors describe a type of gauge which gives a linear relation between pressure and ionization current and which shows no erratic effects from charging up of the glass walls.

Description of Gauge.

Of the many types of construction tested, that shown in Fig. 1 has been found to have the best characteristics for measuring small pressures.

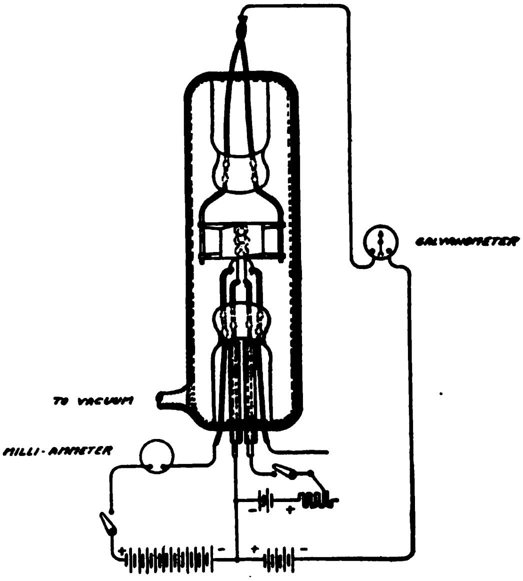

Fig. 1.

It consists of two tungsten filaments, each wound in the form of a double spiral and mounted co-axially on a four lead stem which is sealed into the lower end of a glass tube about 4 cms. in diameter and 12 cms. long· The inner spiral is made of 5 turns of 0.125 mm. tungsten wire wound on a 2.25 mm. mandrel. The outer spiral is made of 3 turns of 0.125 mm. tungsten wire wound on a 3.65 mm. mandrel. Surrounding the spirals is a molybdenum cylinder about 12 mm. in diameter and 12 mm. long, which is supported on a two-lead stem at the upper end of the tube.

PRELIMINARY TREATMENT OF GAUGE.

Before using the gauge for any measurements it is absolutely essential that all water vapor absorbed on the glass walls and gases occluded in the metal parts be completely eliminated. The usual practice adopted by the writers in the treatment of the gauge preliminary to calibration is as follows: After starting the exhaust with a Langmuir condensation pump, the gauge is heated in an oven to 360° C. for at least one hour. This removes practically all the water vapor. In order to free the molybdenum cylinder of gas, it is made anode with respect to the outer spiral which is used as hot cathode. The temperature of the latter is adjusted by varying the current through it until the electron emission is about 100 milliamps. With an anode voltage of 250 volts this corresponds to an energy input of 25 watts (5.5 watt/cm.^{2} approx. anode area) and is sufficient to raise the temperature of the cylinder to a red heat. The gas liberated in consequence of this electronic bombardment causes blue glow in the gauge, which disappears as the exhaust proceeds. The electron emission is then gradually increased until finally it reaches about 400 milliamps., corresponding to 100 watts energy input (22 watts/cm.^{2}). With this amount of energy the cylinder runs at a bright red heat (about 1900° K.) and the elimination of gas occurs very rapidly. Momentarily the energy input may be increased to 150 watts; but there is serious danger, under these conditions, of melting the cylinder locally. After this treatment the anode is usually quite bright and all traces of oxide on its surface have disappeared.

To clean the leads supporting the spirals, one spiral is used as hot cathode to bombard the other which is used as anode. With an energy input of 15 to 20 watts, the leads are heated to a bright red heat and gas is removed very rapidly. It is of the utmost importance to have the leads free from gas before taking any measurements with the gauge as otherwise there will be a continual evolution of gas during the experiment.

CHARACTERISTICS OF GAUGE.

The measurements which are discussed in this paper were carried out with pure argon, the gas being introduced into the gauge after it had been exhausted by the above method to a good vacuum. In all cases, the residual gas pressure (as shown by the ionization current) was less than 0.1 per cent. of the pressure of the gas introduced. As the ionization gauge reads *total gas pressure*, extra care was taken to eliminate mercury vapor and other condensible gases from the gauge when introducing the argon.

The positive ionization current will in general be a function of the gas pressure, electron current, anode voltage, and collector (negative) voltage. A number of characteristics were therefore taken to determine the effect of each of these variables. The arrangement of the electrodes for this set of measurements was as follows: The inner spiral was the cathode, the outer spiral was the anode and the cylinder was the collector of positive ions.

It was observed that varying the collector voltage (only) from 0 to − 22 volts (with respect to the negative end of the cathode spiral) caused only a slight increase (about 10 per cent.) in the ionization current. However, in order to eliminate absolutely the possibility of any electrons reaching the negative electrode, the potential of the latter has been maintained at − 22 in all the measurements with the gauge.

Fig. 2 shows the relation between anode potential and ionization cur-

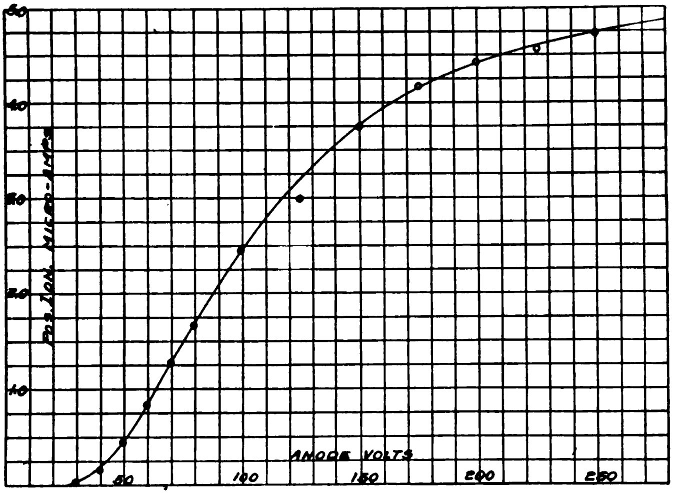

Fig. 2.

Argon at 1.85 bars.
Anode milli-amps = 0.5.
Collector volts = − 22.

rent at a pressure of 1.85 bars argon and with a constant electron current of 5×10^{-4} amps. It will be observed that just above 50 or 60 volts, the ionization current increases at first very rapidly and then more slowly, as the voltage is raised still higher. This portion of the curve is found to satisfy an empirical relation of the form

$$i = i_0 (1 - e^{-B(V-V_0)}),$$

where i = ionization current, V = anode volts i_0, B and V_0 are constants.

Fig. 3 shows the effect of varying the electron emission. The pressure

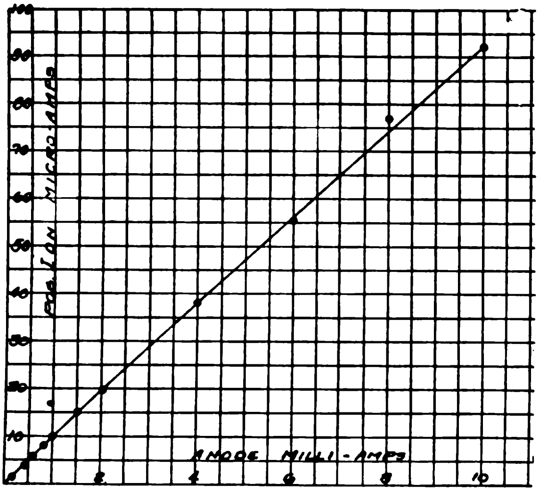

Fig. 3.

Argon at 1.85 bars.
Anode volts = 250.
Collector volts = − 22.

in this case was again 1.85 bar, and the anode potential, 250 volts. It will be observed that there is a deviation from the linear relation which might be expected to exist between ionization current and electron current. The reason for this is probably to be explained by the following considerations: With the arrangement of electrodes indicated above electrons passing from the inner spiral to the outer one do not take the shortest path. Most of them pass beyond the anode, into the space between the latter and the cylinder until the potential gradient due to the latter becomes negative and thus causes the return of the electrons towards the outer spiral. The amount of positive ionization depends both on the number of electrons and the length of path traversed by these electrons between cathode and anode. It is evident, however, that as the electron current increases the average length of this path is decreased because of the space charge of the electrons between the anode and collecting electrode, and consequently the relative ionization is also decreased.

From the characteristics just described, the following conditions were chosen: 250 volts on the anode, − 22 volts on the collecting cylinder and a maximum electron current of 20 milliamperes. The sensitivity of the gauge is controlled by the electron current but it is found that the two values 0.5 and 20 milli-amperes respectively, cover a range sufficiently broad for all practical purposes (50 to 0.0001 bar).

Calibration of Ionization Gauge.

The first calibration of the ionization gauge was made by checking directly against a fine McLeod gauge. Both gauges were attached to the same Langmuir condensation pump and the ionization gauge was separated from the McLeod gauge and mercury pump by a liquid air trap, which prevented mercury vapor and other condensible gases entering the ionization gauge. The entire system was evacuated and the metal parts of the ionization gauge thoroughly cleaned by bombardment as described above. Argon was then let into the apparatus and simultaneous readings taken on the McLeod and ionization gauges. The results of these observations are shown in the curves of Fig. 4. The three curves represent observations with three different electron currents. All show a straight line relation at the lower pressures and furthermore, this straight line when produced back passes through the origin. It, therefore, seemed justifiable to conclude that the ionization current under the above conditions, is directly proportional to the pressure for low pressures. As the pressure is increased the ionization current in-

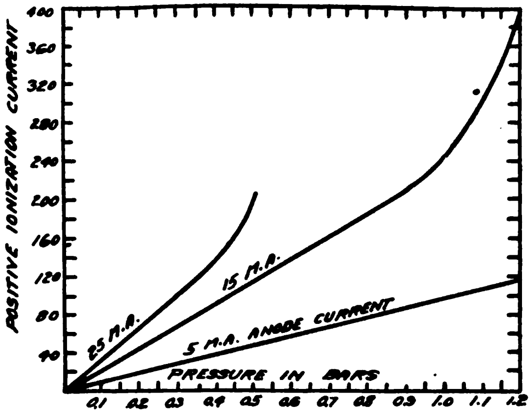

Fig. 4.

creases more rapidly, due, no doubt, to ionization by collision. This conclusion is in accord with the observation that at the pressure at which the curve begins to increase, a faint blue glow may be observed in the gauge. It will also be noted that the lower the electron current, the higher the pressure at which the linear relationship fails to hold. In accordance with this observation it was found that 0.5 milliamps and 125 volts on the anode gave a linear relation up to 50 bars, as shown in Fig. 5, so that under these conditions the ionization gauge may be used to read fairly high pressures, while with 20 milliamps and 250 volts, the linear relation is only valid for pressures below about 1 bar.

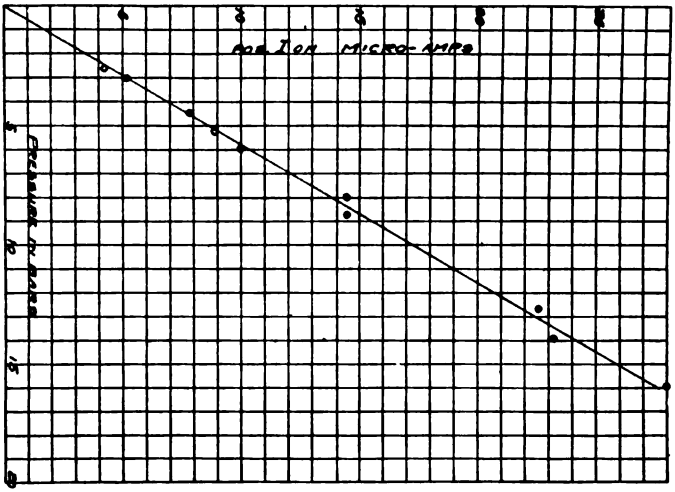

Fig. 5.

Argon.
Anode milliamps = 0.5.
Anode volts = 126.
Cylinder volts = − 22.
1 Bar = 1.72 × 10^{-4} amp.

Calibration of Gauge at Very Low Pressures.

In order to obtain a calibration of the gauge at pressures lower than those measurable on a McLeod gauge, a method was devised based on Knudsen's laws of flow.[1] A sketch of the apparatus is shown in Fig. 6. Two large bulbs *A* and *B*, are connected by a fine capillary tube, *C*; a McLeod gauge being connected to bulb *A* at *M*, and an ionization gauge *G* to bulb *B*.

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[1] M. Knudsen, Ann. der Phys., *28*, 81 (1909). These relations are also discussed by S. Dushman, Gen. Elec. Rev., June, 1920, p. 493.

Both bulbs are directly connected at P_1 and P_2 through mercury traps S_1 and S_3 to a Langmuir condensation pump in series with an oil pump. Between bulb A and the capillary tube is another mercury trap S_2 followed by a liquid air trap L_1 which condenses the mercury vapor and prevents its flow through the capillary. The capillary is joined to the bulb B, through a liquid air trap L_2, which serves to keep mercury vapor and other volatile gases out of the gauge. The complete system is evacuated to a high degree, special care being taken with bulb B, which is heated in an oven at 360° C. for an hour. After this treatment the ionization gauge is bombarded as described earlier in this paper, until the ionization current decreases to a very small value which does not increase when the gauge and bulb B are closed off by mercury traps S_2 and S_3 from the rest of the system. After the traps S_2 and S_3 are closed, bulb A is washed out with argon which is let in through I and when A contains from 10 to 20 bars of argon the trap S_1 is closed. When the trap S_2 between A and the capillary is opened the argon flows into the bulb B, and the ionization gauge. The rate of increase of pressure in B can be calculated from Knudsen's formula

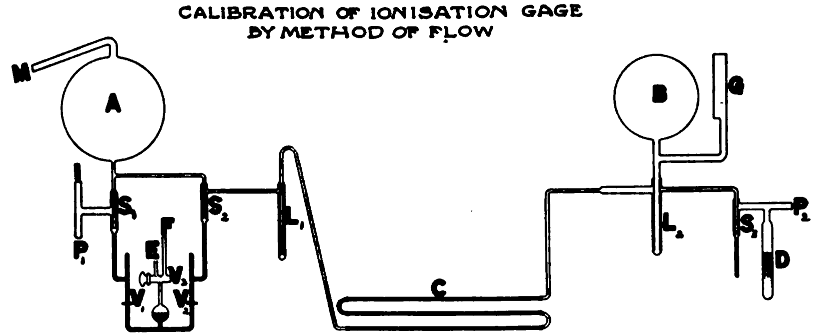

Fig. 6.

$$Q = V\frac{dp}{dt} = \frac{P_A - P_B}{W\sqrt{\rho}}, \tag{1}$$

where Q = quantity of gas flowing through the capillary per unit of time.

V = Volume of bulb B.

$\frac{dp}{dt}$ = rate of increase of pressure in B.

P_A = pressure in bulb A (read on McLeod gauge).

P_B = pressure in bulb B (ionization gauge).

W = "resistance" of capillary.

ρ = density of gas at unit pressure and temperature of capillary.

According to Knudsen,

$$W = \frac{2.394L}{D^3} + \frac{3.184}{D^2},$$

where L = length of capillary
D = diameter.

In the actual arrangement used for calibration, $L = 127.3$ cm., $D = 0.068$ cm.

Hence, $W = 9.68 \times 10^5 + 700$.

Furthermore

$$\rho = \frac{M}{83.15 \times 10^6\, T} \text{ (gms./cm.}^3 \text{ at 1 bar)}$$

$$= 1.6 \times 10^{-9} \text{ gms./cm.}^3 \text{ at 1 bar for argon at } 20^\circ \text{ C.}$$

Also $V = 3100$ cm.^{3}

Since P_B was always very small compared to P_A, we can write equation (1) in the form

$$\frac{dp}{dt} = \frac{P_A}{V \cdot W \cdot \sqrt{\rho}}. \tag{2}$$

The volume of the bulb A was so large, and the rate of flow of gas so

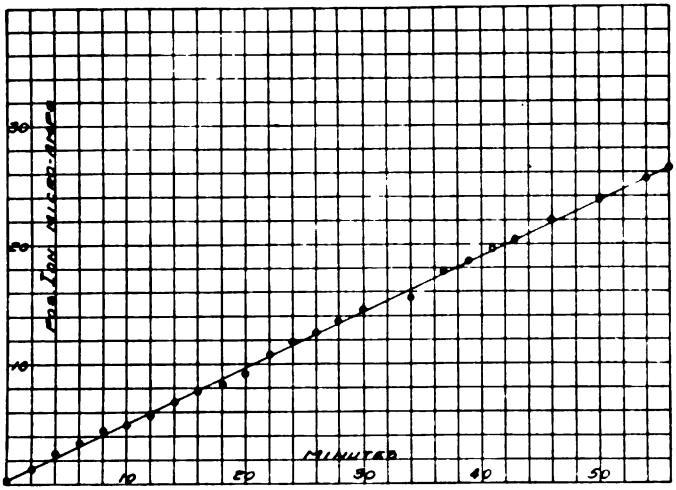

Fig. 7.

di/dt = .47 micro-amps. per min.
dp/dt = .006 bars per min.
$\therefore d/Li$ = .0128 bars per micro-amp.

Anode volts = 250.
Anode milli amps = 20.
Cylinder volts = − 22.
P_A = 12.6 bars.

small, that P_A remained practically constant during a calibration. Substituting in equation (2), the actual values for V, W and ρ, we obtain the relation:

$$\frac{1}{P_A} \cdot \frac{dp}{dt} = 4.98 \times 10^{-4} \text{ bars per minute per bar pressure in Bulb } A. \qquad (2a)$$

Now let di/dt denote the observed rate of increase in the positive ionization current as read on the gauge G. Assuming a linear relationship between i and p, the value of di/dt ought to be a constant for any one set of observations. That this is true is shown by the results plotted in Fig. 7 for a set of measurements in which $P_A = 12.6$ bars. Similar results were obtained at different initial values of P_A, so that we can write

$$\frac{dp}{dt} = k \cdot \frac{di}{dt} \qquad (3)$$

or

$$k = p/i. \qquad (3a)$$

The following table gives a summary of the results of a series of such calibrations for argon at different values of $dp/(dt)$, as calculated from equation (2a) with an anode current of 20 milliamps and anodes potential of 250 volts.

$\left(\frac{dp}{dt}\right) = \left(\frac{\text{Bars}}{\text{Min.}}\right)$ Calc.	$\left(\frac{di}{dt}\right) = \left(\frac{\text{Amps} \times 10^{-6}}{\text{Min.}}\right)$ Obs.	$k = \left(\frac{\text{Bars}}{\text{Amps} \times 10^{-6}}\right)$.
0.0116	0.85	0.0145
.0112	0.91	0.0124
.0081	0.58	0.0140
.0071	0.52	0.0133
.0060	0.47	0.0128
.0059	0.44	0.0132
.0057	0.39	0.0146
.0056	0.52	0.0108
		Average 0.0132

That is, with an anode current of 20×10^{-3} amps. 1 microampere positive ionization corresponds to 0.0132 bar argon. With an ordinary galvanometer it is possible to read currents as low as 10^{-8} amps., corresponding to about 1×10^{-4} bar, and with more sensitive galvanometers much lower pressures can evidently be measured. Actually, the lower limit of pressures observed in the above measurements was about 0.005 bar. Since the linear relation holds down to this pressure, it seems quite justifiable to assume that it would be valid down to the lowest attainable pressures.

"INTERNAL CONTROL" CONNECTION.

With the electrical connections used in the above calibration, it was found, as has already been mentioned, that the positive ionization current is not quite proportional to the electron current. In view of the explanation given above, it suggested itself that the length of path traversed by the electrons would be practically independent of the current density, if the outer spiral is used as cathode, the cylinder as anode, and the inner spiral as collector. Under these conditions it was anticipated that the relation between positive ionization current and anode current would be linear. Fig. 8 shows that this is actually the case. From analogy with the similar type of connection used in wireless work we have designated this as "internal control" connection. Fig. 9 shows the relation between anode voltage and ionization current. This curve shows a rapid increase in ionization current up to about 80 volts, after which the curve is practically flat up to 250 volts (the highest voltage used). There is also

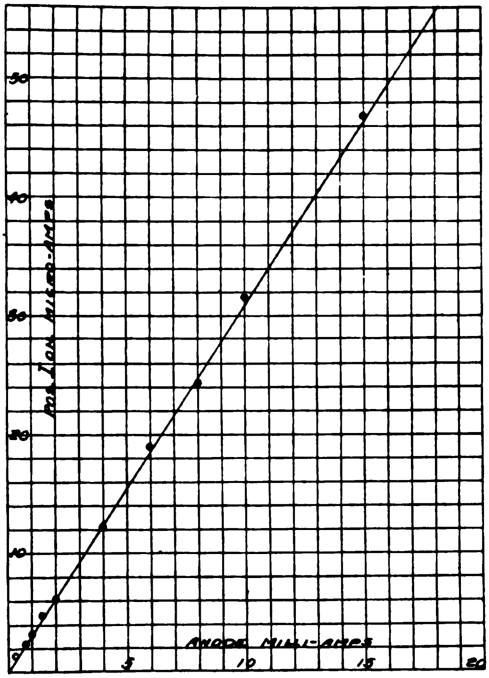

Fig. 8.

Argon at 1.85 bars.
"Internal control"
Cylinder volts = 250
Collector volts = − 22.

practically no change in ionization current for a variation in the potential of the inner spiral from 0 to 30 volts.

This method of connections has advantages over the first method, especially when it is to be used for measuring a considerable range of pressures. The sensitivity of the gauge can be changed by merely altering the electron current and if we have a calibration for one value, we have it for all values, since it is a linear relation. Moreover, changing the anode voltage from 250 to 125 does not sensibly affect the calibration. On the other hand, with the first method of connection, we must actually calibrate the gauge for each electron current at which we desire to work. There is, however, considerable difference in sensitivity for the two connections. The constant, *K*, of equation (3)

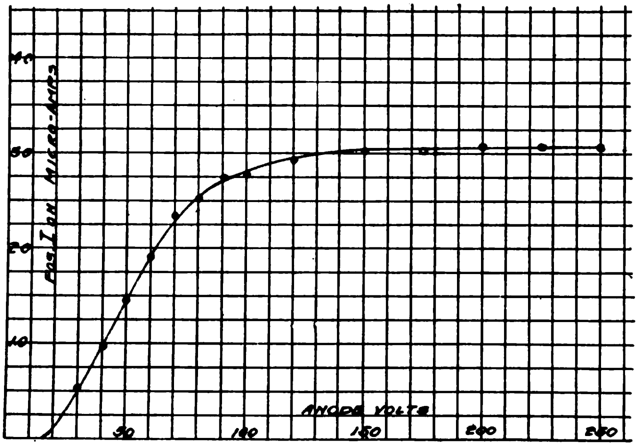

Fig. 9.

"Internal control"
Argon at 1.85 bars.
Anode milli-amps. = 10.
Collector volts = − 22.

$$k = p/i$$

when the anode voltage is 250, electron current is 20 milli-amps and collector voltage is − 22, is given below for the two methods of connections.

Cathode.	**Anode.**	**Collector.**	*K*
Inner spiral	Outer spiral	Cylinder	.013
Outer spiral	Cylinder	Inner spiral	.032

The sensitivity (as measured by microamps per bar) is evidently lower with the internal control connection. This is easily explained since with the first connection the electrons travel from the inner spiral almost to the cylinder and then back to the outer spiral, while in the second case they travel directly from the outer spiral to the cylinder. That is the sensitivity is in the ratio of the lengths of paths of the electrons. Thus, the second method has the advantage when a large range of pressures is to be measured, while the first method is more sensitive for measuring very low pressures, using the latter, pressures as low as 0.0001 bars have been measured. This corresponds to a positive ionization current of about 10^{-8} amperes which is about the limit for the ordinary galvanometer.

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THE CRYSTAL STRUCTURE OF ICE.

BY D. M. DENNISON.

SYNOPSIS.

Crystal Structure of Ice as Determined from X-ray Pattern.—A small sample of ice composed of minute crystals, gave by Hull's method a pattern of 12 clearly defined lines whose positions indicate that the lattice of an ice crystal corresponds to a hexagonal close-packed arrangement of molecules, consisting of two sets of interpenetrating triangular prisms with sides 4.52 A and height 7.32 A. Since for close-packed spheres the axial ratio should be 1.633 instead of 1.62, the ice molecules act like spheres which have become flattened by 0.8 per cent. in the direction of the hexagonal axis. This lattice is practically the same as that found for magnesium, but the differences in relative intensity between the lines of the two X-ray patterns suggest that ice molecules and magnesium atoms must differ considerably in shape.

Molecular Formula for Ice.—From the density of ice and the dimensions of the lattice it follows that the formula for an ice molecule is $(H_2O)_2$ or H_4O_2.

X-RAY photographs of ice were made in the same manner as the crystal powder photographs described by A. W. Hull.[1] A small quantity of distilled water was inclosed in a thin-walled capillary tube of lime glass. The tube was then plunged quickly into liquid air, freezing the water with such rapidity that only very minute crystals were formed. The tube was then kept at liquid air temperature in a specially constructed Dewar flask. The sample was rotated continuously during the 10-hour exposure to the X-rays from the molybdenum target of a water-cooled Coolidge tube which was running at 32,000 volts, 25 milliamperes direct current. The photographic film was bent in a semicircular arc of radius 19.8 cm. The tube of ice was placed in the axis of this arc. A good photograph was obtained which contained 12 lines, the data from which are recorded in the table.

The first column gives the angular deviation of each line, the second gives the intensities, which were only roughly estimated, and the third gives the intensities of the corresponding magnesium lines as estimated by Dr. Hull.[2] The fourth column gives the observed plane spacings, while the fifth gives the theoretical plane spacings which would correspond to a close packed hexagonal lattice, consisting of two sets of interpenetrating triangular prisms, the sides of whose bases are each

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[1] PHYS. REV., 9, 85, Jan., 1917.

[2] Pro. Nat. Acad. Sci., Vol. 3, pp. 470–473, July, 1917.

4.52 Ångstroms and whose height is 7.32 Ångstroms. The observed and theoretical spacings agree within the limit of experimental error.

Angular Deviation of Line 2θ (Degrees).	Intensity of Line (Estimated).	Intensity of Magnesium Line for Comparison.	Spacing of Planes in Ångstroms.		Indices of Form.	No. of Coöperating Planes.
			Obs.	Cal.		
10.44	1	4.7	3.92	3.915	$10\bar{1}0$	3
11.16	10	—	3.67	3.671	0001	1
11.88	2	10.0	3.44	3.453	$10\bar{1}1$	6
15.30	1.5	3.3	2.68	2.675	$10\bar{1}2$	6
18.12	1	4.7	2.26	2.260	$11\bar{2}0$	3
19.86	5	4.0	2.065	2.065	$10\bar{1}3$	6
21.38	1	4.0	1.92	1.925	$11\bar{2}2$	6
27.16	1.5	1.0	1.516	1.528	$20\bar{2}3$	6
30.20	2	1.3	1.368	1.372	$10\bar{1}5$	6
			1.368	1.372	$12\bar{3}2$	12
31.76	.25	0.2	1.30	1.305	$10\bar{1}0$	3
33.08	.25	1.0	1.25	1.268	$12\bar{3}3$	12
35.54	.5	0.3	1.167	1.165	$20\bar{2}5$	6

The above values give an axial ratio of 1.62 in good agreement with the crystallographer's value of 1.617.[1] An hexagonal lattice of close packed spheres has an axial ratio of 1.633. The data from the photograph seem to indicate that 1.62 is a more probable value than 1.633 for the structure of ice. This ratio and the dimensions of the elementary prism were determined by means of a set of graphs, made by Dr. Hull and Dr. Davey, which is to be published shortly.

Estimating ρ the density of ice at liquid air temperature as $\rho = .944$, the number N of molecules of H_2O per cubic centimeter may be calculated.

$$N = \frac{\rho(6.023) \times 10^{23}}{18} = 3.154 \times 10^{22}.$$

The volume V of the elementary prism may also be found, knowing that a the base and h the height of the prism are 4.52 Å. and 7.32 Å. respectively.

$$V = \frac{\sqrt{3}a^2h}{4} = 0.6478 \times 10^{-22} \text{ c.c.}$$

The number n of H_2O molecules per unit prism is then equal to the product VN.

$$n = VN = 2.04.$$

Ice belongs then very nearly to one of the two alternate arrangements which inelastic uniform spheres may assume if packed as closely as possible. This is the same form that magnesium atoms take. If the molecules of ice and the atoms of magnesium were identical in arrangement and form, not only would the plane spacings be proportional but also the intensities of the lines would be similar. From columns 2 and 3 of the table it may be seen that the intensities are not proportional. With increased knowledge of crystal structure these data may furnish information regarding the shape of the molecule of water. With the hexagonal close packed type of lattice each prism contains on the average one molecule. The result calculated above indicates that in ice crystals the molecules of water are of the form $(H_2O)_2$ or H_4O_2.

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[1] Gmelin Kraut, Handbuch der Anorganischen Chemie, Heidelberg, Vol. I, 1, p. 107 (1907).

Research Laboratory,
General Electric Company,
Schenectady, N. Y.,
August 14, 1920.

THE HALL EFFECT AND THE NERNST EFFECT IN MAGNETIC ALLOYS.

By Alpheus W. Smith.

Synopsis.

Hall Effect in Magnetic Alloys.—In the *iron-copper series* a maximum effect was obtained with 1.5 per cent. copper. The variation with composition is very similar to the variation of electric resistance. In the *nickel-copper series* the effect reaches a maximum with about 26 per cent. copper when the composition corresponds to the formula $CuNi_3$, and then drops suddenly to a small fraction of its maximum value. In the *iron-nickel series,* although the effect is positive for iron and negative for nickel, the effect increases linearly with the proportion of nickel added to iron until for 13 per cent. nickel it is six times as great as for pure iron. In certain alloys the *variation of the effect with the field strength* is anomalous, the effect being negative for weak fields and positive for strong ones. An *explanation based on the theory of Borelius* is suggested which assumes that in addition to the positive part of the effect which is proportional to the field strength there is a negative part which reaches a limiting value for strong fields.

Nernst Effect in Magnetic Alloys.—In the *nickel-copper series* the effect varies with composition in nearly the same way as the Hall effect does, reaching a maximum for the composition $CuNi_3$. In the *iron-nickel series,* the effect decreases with the addition of nickel, becoming zero for about 2.2 per cent. nickel and positive for higher concentrations. With 13 per cent. nickel the effect is about five times as great as in pure iron.

Relation of the Hall and Nernst Effects to Crystal Structure.—It is *suggested* that the amount and direction of these effects are determined by the crystal lattices and the fields of force in the intramolecular spaces.

THE Hall effect and the Nernst effect in magnetic alloys are of peculiar interest because of the fact that any theory of the reversa of these effects considers the molecular magnetic fields about the molecules as a large factor in determining the direction of these effects. That these molecular fields can not offer a complete explanation of these reversals is evident from the fact that the Hall effect is positive in iron and cobalt and negative in nickel,—the other magnetic element. Yet these molecular fields must have a part either directly or indirectly in determining the direction of these effects. Whether these molecular fields produce their effect by determining the polarization as the theory of Borelius[1] indicates or by producing a deflection of the free electrons opposite to that caused by the impressed magnetic field as the theory of Thomson requires,[2] is question for which there is now no answer.

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[1] Borelius, Ann. der Phys., 58, p. 489, 1919.

[2] Thomson, The Corpuscular Theory of Matter, p. 70.

In view of the importance of the direction of the Hall effect and the Nernst effect in magnetic substances it seemed desirable to make some additional observations on these effects in magnetic alloys.

THE ALLOYS.

The magnetic, optical and thermal properties of a number of magnetic alloys prepared by Burgess and Aston[1] from very pure electrolytic iron have been studied in the Electrochemical Laboratory and in the Physical Laboratory[2] of the University of Wisconsin. Since these alloys were exceptionally pure and made with great care it was desirable to make the observations on the Hall effect and the Nernst effect in these same alloys as far as possible. This avoids the uncertainty which arises out of composition and methods of preparation. Professor Kowalke, of the Department of Chemical Engineering of the University of Wisconsin, kindly supplied me with the specimens of iron-copper and iron-nickel alloys used in this investigation. For this assistance I am greatly indebted to Professor Kowalke. These specimens were taken from alloys prepared by Burgess and Aston. Concerning their preparation and purity reference is made to the work of Burgess and Aston[3] and to that of Ingersoll.[4] From these specimens were forged plates of suitable dimensions for my observations.

The nickel entering into the nickel-copper series was kindly furnished me by Mr. John F. Thomson, of the International Nickel Co. It was electrolytic nickel giving according to Mr. Thomson the following analysis: nickel, 99.840 per cent.; copper, 0.108 per cent.; and iron, 0.130 per cent. The copper used in the nickel-copper series was obtained from the Baker Chemical Co. The analysis supplied with it showed only negligible traces of impurities.

The nickel-copper alloys were prepared by melting weighed amounts of nickel in a magnesia crucible heated in a gas blast furnace. When the nickel had melted weighed amounts of copper were dropped into the crucible and the mixture thoroughly stirred with a graphite rod. The alloy was then allowed to cool slowly until it had solidified and reached room temperature. This cooling took place without removing the crucible from the furnace and required about four hours. From the ingots thus prepared was cut a plate of suitable thickness and area for observations on the Hall effect and the Nernst effect. The compositions indicated in the following tables were determined from the weighed amount of nickel and copper entering into the alloys.

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[1] Burgess and Aston, Met. & Chem. Eng., 8, 23 and 79, 1910.

[2] Littleton, PHYS. REV., (33. p. 453. 1911) Ingersoll, PHYS. REV., N.S., 16, 126, 1920.

[3] Burgess and Aston, ibid.

ngersoll, ibid.

THE HALL EFFECT.

The observations on the Hall effect were made at room temperature by the method described by the author[1] in previous papers. The value of the Hall constant R was calculated by means of the familiar equation,

$$E = R\frac{iH}{d},$$

where

E = the Hall electromotive force in absolute units,

H = the magnetic field in C.G.S. units,

i = the current in the plate in C.G.S. units,

d = the thickness of the plate in centimeters.

The observations on the iron-copper series have been plotted in Fig. 1;

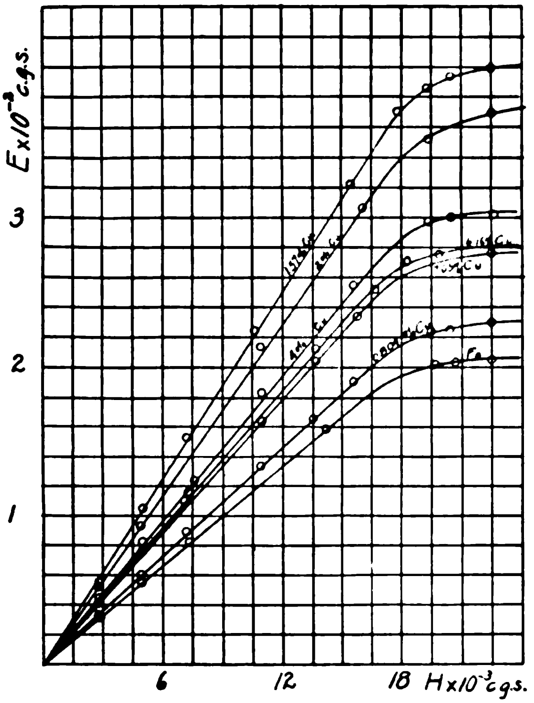

Fig. 1.

those on the nickel-copper series in Fig. 2 and those on the iron-nickel series in Fig. 3. In these figures the Hall electromotive forces in a plate 1 cm. thick, carrying a current of one absolute unit have been plotted as ordinates and the magnetic fields as abscissæ. These curves showing

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[1] Smith, Phys. Rev., p. 30, 1, 1910.

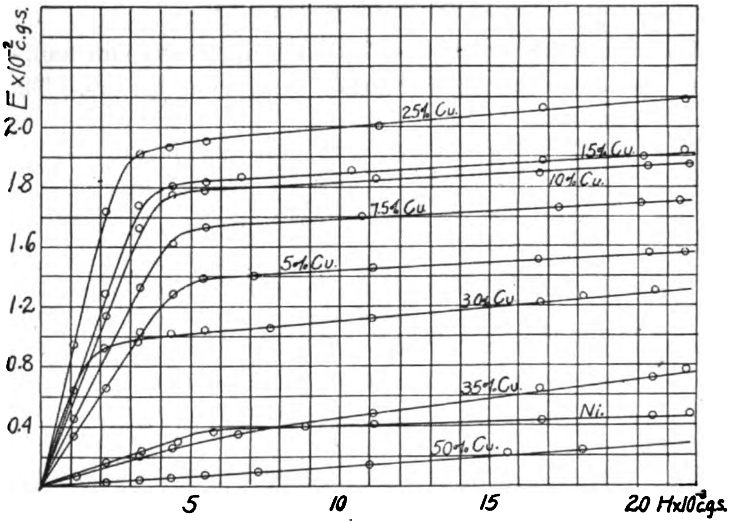
E×10⁻² c.g.s.
2.0
1.8
1.6
1.2
0.8
0.4
25% Cu.
15% Cu.
10% Cu.
7.5% Cu
5% Cu.
30% Cu
35% Cu.
Ni.
50% Cu.
5
10
15
20 H×10⁻³ c.g.s.

Fig. 2.

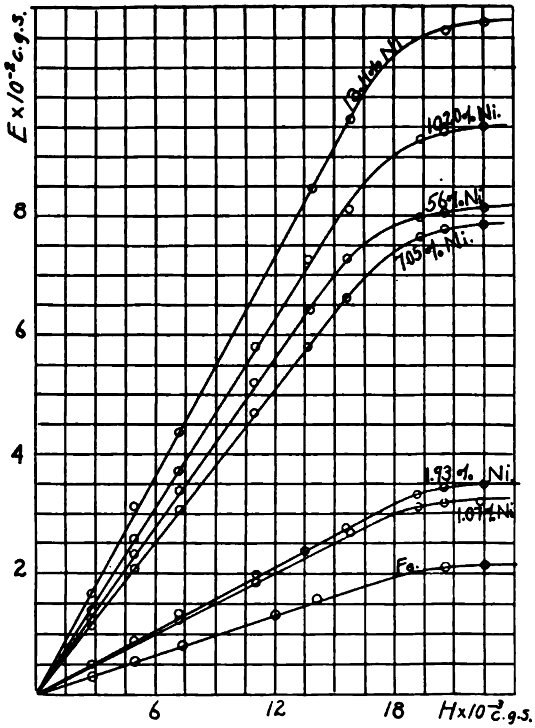
E×10⁻² c.g.s.
8
6
4
2
18.4% Ni
10.20% Ni.
5.6% Ni
7.05% Ni.
1.93% Ni
1.07% Ni
Fe.
6
12
18
H×10⁻³ c.g.s.

Fig. 3.

the relation between the Hall electromotive force and the magnetic field which produced it, consist of a part in which the electromotive force is proportional to the magnetic field followed by a part showing the electromotive force increasing slowly and approaching saturation for large magnetic fields.

The Hall constants for these alloys have also been plotted as functions of the composition in Figs. 4 and 5. In these figures the Hall constants have been used for ordinates. In Fig. 4 the abscissæ are the percentage of copper in the alloys and in Fig. 5 they denote the percentages of nickel in the iron-nickel series. The values of the Hall constants plotted in these curves and also given in Tables I., II. and III. have been calculated for values of the magnetic field at which there is still proportionality between the magnetic field and the Hall electromotive force so that for these and smaller fields the Hall constant is independent of the magnetic field. Curve *A* (Fig. 4) showing the relation between the Hall constants and the percentages of copper in the alloys for the iron-copper series calls attention to the rapid rise in the Hall constant produced by adding a small amount of copper to iron. Although the Hall effect is positive in iron and negative in copper the addition of copper to iron causes an

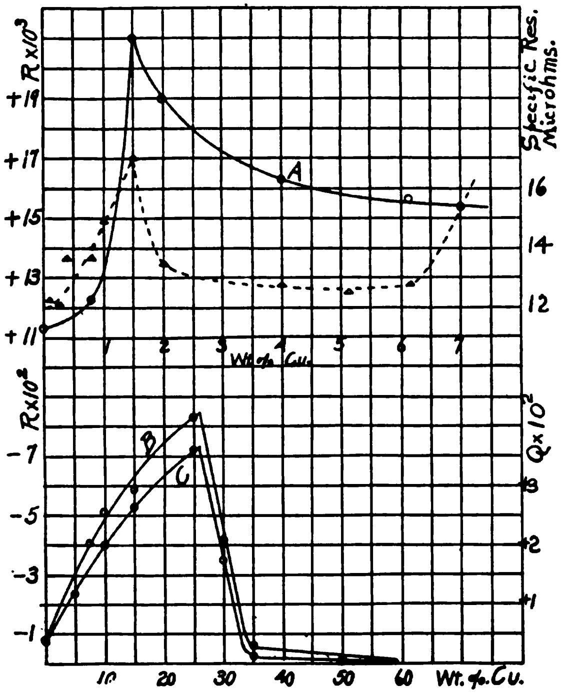

Fig. 4.

increase in the Hall effect. After the alloy contains 1.5 per cent. of copper a further increase in the percentage of copper produces a sudden drop in the Hall effect after which the effect decreases slowly with further increase in the content of copper. The form of this curve is quite similar to the dotted curve of Fig. 4 showing the dependence of the specific resistance on the percentage of copper in the alloys. This curve is from the work of Burgess and Aston[1] on these same alloys. The peaks in the curves occur at the same concentration and with the exception of the point representing the alloy containing 7 per cent. of copper the general characteristics of the curves are the same. Evidently the factors which determine the electrical resistance in this case must in large measure determine the Hall effect.

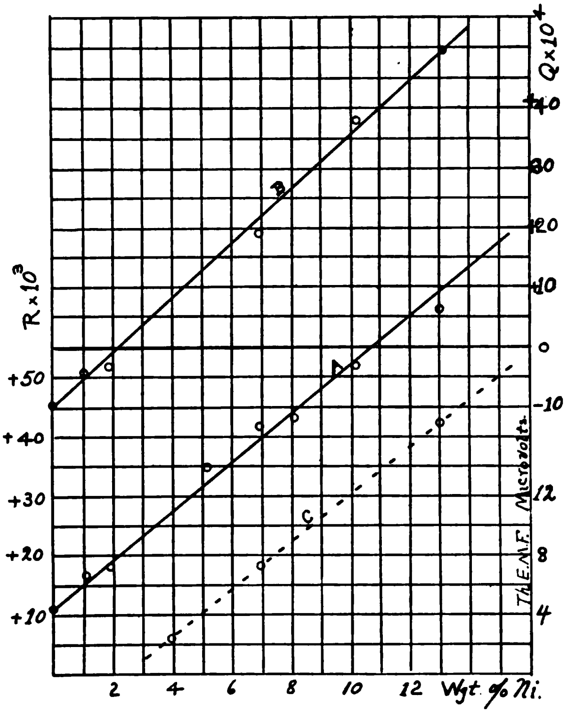

Fig. 5.

Curve *B* (Fig. 4) shows the influence of copper on the Hall effect in nickel. There is at first a rapid rise in the effect with the addition of copper. This rise continues until the alloy contains a little more than 26 per cent. of copper where the Hall effect has its largest value which is about eleven times its value in pure nickel. As the percentage of copper

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[1] Burgess and Aston, Met. & Chem. Eng., 8, 79, 1910.

Table I.

The Hall Effect in Iron-copper Alloys.

Per Cent. Fe.	Per Cent. Cu.	R
100	0	$+11.2 \times 10^{-3}$
99.196	.804	$+12.2$
98.49	1.51	$+21.0$
98.0	2.0	$+19.0$
96.0	4.0	$+16.3$
93.84	6.16	$+15.6$
92.95	7.05	$+15.2$

is still further increased the effect decreases rapidly reaching a value which is about eighty per cent. of the value in pure nickel for the alloy containing 35 per cent. of copper. As the percentage of copper is increased more and more the Hall effect continues to decrease evidently approaching the value in pure copper. The break in the curve showing the Hall constant as a function of the concentration of nickel indicates clearly the formation of the compound $CuNi_3$ when the proper amounts of nickel and copper are present in the alloy. It is worthy of note that although the direction of the Hall effect in nickel is opposite to that in iron, the effect produced by introducing small quantities of copper into iron or nickel is the same—a very large increase in the Hall effect. This increase is more rapid in iron than in nickel but continues to higher concentrations of copper in the case of nickel than in the case of iron.

Table II.

The Hall Effect and the Nernst Effect in Nickel-copper Alloys.

Per Cent. Ni.	Per Cent. Cu.	R	Q
100	0	-7.65×10^{-3}	$+3.48 \times 10^{-3}$
95	5	-29.8	$+11.8$
92.5	7.5	-40.9	$+14.7$
90	10.	-52.0	$+20.7$
85	15.	-59.1	$+25.6$
75	25	-83.7	$+35.0$
70	30	-41.9	$+17.4$
65	35	-6.31	$+0.97$
50	50	-1.33	$+0.17$

The relation between the Hall constant and the percentage of nickel in the iron-nickel series is given in Curve A (Fig. 5) for alloys containing not more than 13.11 per cent. of nickel. It is seen from this curve that there is a proportionality between the Hall constants and the percentages of nickel in the alloy for these alloys. Although the effect is positive in iron and negative in nickel, the addition of nickel to iron causes a marked increase in the Hall effect so that an alloy containing 13.11 per cent. of nickel has a Hall effect which has the same direction as the effect in pure iron and about five times its magnitude. This increase does not of course continue indefinitely as is seen by noting the observations on an alloy containing 56 per cent. of nickel. The Hall effect in this alloy (Table III.) is considerably less than it is in an alloy containing 13.11 per cent. of nickel. Consequently somewhere between these two concentrations the Hall effect must cease to increase and begin to decrease with increasing concentration of nickel. The increase in the Hall effect with increasing concentration of nickel probably continues until the alloy has a nickel content of about 35 per cent. corresponding to the compound Fe_2Ni where these alloys have been found to show a number of unusual properties. As soon as this concentration is passed there is a decrease in the effect with further increase in the concentration of nickel giving values of the Hall constant which finally approach the value in pure nickel as the percentage of nickel is increased indefinitely. In Curve *C* (Fig. 5) are represented the thermoelectric heights of these alloys against copper as a function of the concentration of nickel for concentrations lying between 4 per cent. and 13.11 per cent. The data for this curve are from the work of Ingersoll. Curve *A* and Curve *C* (Fig. 5) are nearly parallel to each other. Hence in this case as in a number of other cases the thermoelectric heights of the metals or alloys are nearly proportional to the Hall constants.

TABLE III.

The Hall Effect and the Nernst Effect in Iron-nickel Alloys.

Per Cent. Fe.	Per Cent. Ni.	*R.*	*Q.*
100	0	$+ 11.2 \times 10^{-3}$	$- 9.75 \times 10^{4}$
98.93	1.07	+ 16.9	− 4.21
98.07	1.93	+ 17.8	− 3.06
92.95	7.05	+ 42.5	+ 19.03
91.83	8.17	+ 43.1	——
89.80	10.20	+ 52.0	+ 38.0
86.89	13.11	+ 61.5	+ 49.5
44.0	56.0	+ 46.9	+ 1.33

THE NERNST EFFECT.

The Nernst effect was measured in the manner described by the author[1] in preceding papers on this effect in metals and alloys. The temperature gradients in the plates were measured by means of advance-copper thermal couples used with a Wolff potentiometer. Observations were made at only one strength of magnetic field and this field was so chosen that it was not great enough to produce the beginning of saturation which always occurs when the Nernst effect or the Hall effect is studied in magnetic metals or alloys. With fields less than that at which saturation begins to manifest itself, the Nernst electromotive force is proportional to the magnetic field and the constant which describes the effect is independent of the magnetic field. This constant which is usually designated as Q was calculated from the following familiar equation,

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[1] Smith, PHYS. REV., 32, p. 192, 1911.

$$E = Q\beta H\frac{\partial T}{\partial X},$$

where E = the Nernst electromotive force in absolute units.
β = the width of the plate in centimeters.
H = the intensity of the magnetic field in C.G.S. units.
$\frac{\partial T}{\partial X}$ = the temperature gradient in the plate in degrees Centigrade per centimeter.

The temperature at which Q was determined was found by taking the mean of the temperatures indicated by the thermal couples which were used to determine the temperature gradient in the plate. This mean temperature was between 55° and 56° C. The values of Q recorded in Table II. and III. are, therefore, the values of the Nernst constant at 55° or 56° C.

In Fig. 4 (Curve C) the values of the Nernst constant Q have been plotted against the percentages of copper in the nickel-copper series of alloys. The data from which this curve was plotted have also been recorded in Table II. The form of the curve obtained in this way is very similar to the corresponding curve for the Hall effect in these alloys (Curve B, Fig. 4). The break in each of the curves occurs at the same place indicating the formation of the compound $CuNi_3$. The other characteristics of the curves are quite similar and it is evident that the Hall effect and the Nernst effect in this case are intimately associated, since the presence of the copper in the nickel changes the two effects in the same way.

The relation between the Nernst effect and the concentration of nickel in the iron-nickel series is represented in Curve B of Fig. 5. The data from which this curve was obtained are recorded in Table III. The Nernst effect is regarded as negative in iron and positive in nickel. The introduction of nickel into the iron causes the Nernst effect to decrease in magnitude and become zero when the alloy contains about

2.2 per cent. of nickel. When the concentration of nickel becomes higher than about 2.2 per cent. the direction of the effect reverses and becomes that in nickel. When the concentration of the nickel is still further increased the Nernst effect increases and reaches in an alloy containing 13.11 per cent. of nickel a value which is about five times its value in pure iron. For these lower concentrations of nickel the relation between the Nernst effect and the concentration of nickel is represented by a straight line. In this respect the behavior of the Nernst effect for this series of alloys is like the behavior of the Hall effect and the thermoelectric heights. From Fig. 5 it is evident that the curve in each case is a straight line and that the lines are nearly parallel to each other. From the observations on the Nernst effect in an alloy containing 56 per cent. of nickel (Table III.) it is clear that the increase in the Nernst effect with increasing concentration of nickel ceases at some concentration lying between 13.11 per cent. and 56 per cent. of nickel. The concentration at which this increase in the Nernst effect ceases and a decrease begins is probably the concentration at which the compound Fe_2Ni is formed. This is also the concentration at which the Hall effect ceases to increase and begins to decrease with further increase in the concentration of nickel. The behavior of the Nernst effect at this point is very similar to the behavior of the Hall effect and some of the physical properties of the alloys. This relationship between the Hall effect, the Nernst effect and the other physical properties of these alloys suggests that these effects are intimately associated with the structure of the alloys. The similarity of the behavior of the Hall effect, the Nernst effect and the thermoelectric heights in these alloys for the lower concentrations of nickel is of considerable interest. It suggests clearly an intimate relation between these effects and the possibility that this relation may not be very complicated. The nickel produces the same kind of change in each of these cases.

POSITIVE AND NEGATIVE HALL EFFECTS.

The existence of positive and negative Hall effects has been the subject of much difficulty in the electronic theory of metals. Recently Borelius[1] has pointed out that his conception of magnetism leads to an explanation of positive as well as negative Hall effects. In this theory of the electrical and magnetic properties of metals use is made of the space lattice theory of the solid state. The space lattice is assumed to consist of alternate positive metal ions and negative electrons. Each atom of the metal has lost a single electron. Under the action of repulsive force arising from the fields of force surrounding the metal ions, these electrons are equally spaced between the atoms at positions of equilibrium. When an atom in a crystal lattice swings out from its position of equilibrium, it pushes the electron in front of it in the same direction. The electron moves forward in a curved path to occupy the free space behind the atom. When the atom swings back in the opposite direction, an electron moves forth now in the opposite direction. Assuming that all the electrons move on paths of equal radius a magnetic field perpendicular to $X-Y$ plane in the direction of the positive Z-axis will exert a force on the electrons revolving in this plane causing a change in their magnetic moment. This change in magnetic moment may be regarded as made up of two parts—that arising from the change in the radius of the path of the electron and that arising from the change in the angular velocity of the electron in its path. From these changes in magnetic moment—one radial, the other circular—Borelius calculates the radial and circular diamagnetic susceptibilities.

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[1] Borelius, Ann. der Phys., 58, 489, 1919.

The action of the magnetic field on the electrons revolving in the $X-Z$ and $Y-Z$ planes is to cause a sort of polarization by which the electronic paths lying in these planes are rotated in such a way that they tend to set their planes of rotation perpendicular to the magnetic field—that is, perpendicular to the Z-axis. With this polarization is associated a change in magnetic moment in the direction of the Z-axis. This change will be an increase in magnetic moment and is, therefore, a paramagnetic effect.

Suppose that in addition to the magnetic field which points in the direction of the positive Z-axis there is applied to the metal an impressed electromotive force so that the negative or electron current flows in the direction of the negative X-axis. The current consists of an excess of those electronic paths which convey the current in the direction of the negative X-axis. Assume that one half of these electronic paths lie in the $X-Z$ plane and the other half in the $X-Y$ plane. Consider first the $X-Y$ plane. The cause of the transverse Hall current when the primary current is closed or the magnetic field is established, is thought of as an electric polarization which exists as the combined action of the magnetic field and the primary current. This polarization is from considerations of symmetry parallel to the Y-axis. Since change in the angular velocity of the electrons produces no polarization, this polarization arises only in connection with radial diamagnetism, in which there is a displacement of the electrons in the $X-Y$ plane due to the action of the magnetic field. This displacement is along the radius of the path of the electron. The direction of the resulting polarization is such as would be set up by an electric force acting in the direction of the negative Y-axis. Such a force would shove an electron moving in a curved path along the negative X-axis in the direction of the positive Y-axis. The Hall effect arising from such a displacement of electrons is in the customary notation called positive. It is important to notice that the displacement of the electrons obtained on the basis of this theory is opposite to the displacement of electrons obtained under corresponding conditions on the basis of the gas free electron theory of Drude and others.

The value of the Hall constant R_d for electrons revolving in the $X-Y$ plane is found to be

$$R_d = \frac{3}{4en}.$$

It is, therefore, independent of the magnetic field, the current in the plate and the thickness of the plate. Moreover, it is always positive.

Consider now the action of the magnetic field on the electrons revolving in the $X-Z$ plane. The effect here will be the sort of polarization which gives rise to paramagnetism. The nature of the polarization in this case may be made clearer by comparing it with the polarization which arises in the doublet theory of electrical and thermal conductivity which has been proposed by J. J. Thomson.[1] Let AB be a doublet lying in the $X-Z$ plane with a magnetic field acting in the direction of the positive Z-axis and an electric field in the direction of the positive X-axis. Assume that the doublet is free to turn about the positive charge A. The electric field gives a force tending to bring the axis of the doublet in line with the X-axis. As soon as the negative charge at B begins to move, it will be acted on by a downward force by making B dip below the $X-Z$ plane, and thus making the negative end of the doublet below the positive. This has the effect of making more doublets have their negative ends below their positive ends than above them. This is equivalent to the action of an electric force in the direction of the positive Y-axis. Such a force would drive electrons down the negative Y-axis; that is, in the opposite direction to that in which they were driven in the diamagnetic part of the Hall effect. Borelius gives for this part of the Hall constant which is associated with paramagnetism,

$$R_p = -\frac{1}{ne}F(\psi),$$

where $F(\psi)$ is a function of the magnetic field and is always positive. For this reason this part of the Hall constant is always negative.

The total Hall constant is the sum of its positive and negative parts and is therefore given by the equation,

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[1] Thomson, Corpuscular Theory of Matter, p. 100.

$$R = \left[\frac{3}{4ne} - \frac{1}{ne} F(\psi) \right].$$

For the case of unit current flowing in a plate of unit thickness, the Hall electromotive force becomes,[1]

$$E = \left[\frac{3}{4ne} - \frac{1}{ne} F(\psi) \right] H,$$

where H is the magnetic field; n, the number of electrons per cubic centimeter and c, the electronic charge.

This electromotive force may then be regarded as made up of two parts—a positive part which is represented by a straight line through the origin and a negative part represented by the function $F(\psi)H$ about which we know little. For values of ψ lying between $\psi = 104°$ and $\psi = 157°$ Borelius states that the function $F(\psi)$ decreases. Hence $F(\psi)H$ which measures the negative part of the Hall electromotive force would increase less rapidly than if proportional to the magnetic field.

Reversal of the Hall Effect in Alloys.

It seems to me that the reversal of the Hall effect in alloys offers some evidence for the correctness of this theory. In the study of the Hall effect in alloys of bismuth and tin von Ettingshausen and Nernst[1] found that the effect in these alloys is either positive or negative according to the intensity of the magnetic field. With small magnetic fields the effect is negative. It increases to a maximum value as the magnetic field is increased, then decreases and reverses its direction when sufficiently large magnetic fields are used. The magnetic field necessary to produce this reversal is larger for alloys containing a small amount of tin than for alloys containing a large amount of tin. More recently Becquerel[2] has studied this reversal of the Hall electromotive force in a plate cut from a crystal of bismuth. In such plates the magnetic field at which the reversal takes place depends on the direction of the crystalline axis with respect to the magnetic field and the plane of the plate. A similar reversal was found by the author[3] in alloys of bismuth and lead. A curve showing the relation between the Hall electromotive force and the magnetic field for these alloys is given in Fig. 6. Now as Becquerel pointed out, such curves can be split up into two parts—Curve A, a straight line passing through the origin, and Curve B, which rises to a fixed value and then remains parallel to the horizontal axis. The sum of the ordinates of these two curves gives the ordinates of the observed curve. This analysis regards the Hall electromotive force as made up of two parts which are opposite in sign. That part which is positive is a linear function of the magnetic field. The negative part increases rapidly at lower fields but reaches a limiting value at higher fields beyond which it does not increase. Because the negative part reaches a limiting value while the positive part increases indefinitely, it is obvious that at sufficiently high magnetic fields the positive part will predominate. As the percentage of lead in the alloy is increased the form of the curve is not changed but the slope of the curve showing that part of the Hall electromotive force which is proportional to the magnetic field is smaller the greater the percentage of lead or tin in the alloy.

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[1] von Ettinghausen and Nernst, Ann. der Phys., 33, p. 474, 1888.

[2] Becquerel, Comp.-Rendu., 154, pp. 1795–1798, 1912.

[3] Smith, Phys. Rev., N.S., 10, p. 358, 1917.

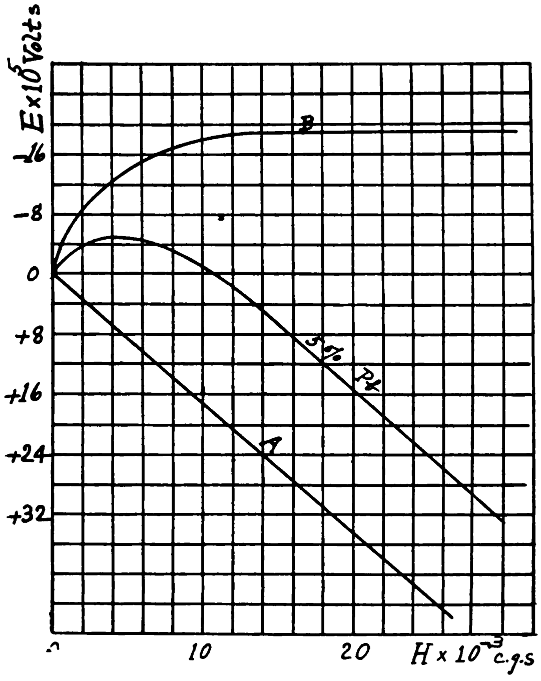

Fig. 6.

Now the splitting up of a Hall electromotive force into two such parts as those shown in Curve *A* and Curve *B* of Fig. 6 is precisely what the theory of Borelius would require. The theory requires that one of these parts be a linear function of the magnetic field and that this part be positive. Curve *A* in Fig. 6 represents then both the theoretical and the observed relation between this part of the Hall electromotive force and the magnetic field. The qualitative agreement between the theory and observations for this part of the Hall effect seems satisfactory. The gas free electron also requires that the Hall electromotive force be a linear function of the magnetic field but the slope of the curve is opposite to that obtained from the theory of Borelius and, therefore, opposite to the slope of Curve A (Fig. 6). The gas free electron theory can not, therefore, offer an explanation of that part of the Hall electromotive force represented by Curve A (Fig. 6). With respect to the second part of the Hall electromotive force the agreement between the theory of Borelius and the observations is less certain. This uncertainty arises out of the fact that we do not know how the function $F(\psi)$ depends on the magnetic field. For certain values of the angle ψ Borelius states that this function $F(\psi)$ decreases. If we are at liberty to assume that $F(\psi)H$ at first increases with the magnetic field and at higher fields reaches a limiting value beyond which it does not increase, then curves of the form of Curve B would represent the requirements of the theory as well as the observations. In view of the dependence of $F(\psi)$ on the magnetic field such as assumption is reasonable. If such an assumption can be accepted the reversal of the Hall effect in alloys agrees qualitatively with the theory proposed by Borelius.

CONCLUSION.

From this study of the Hall effect and the Nernst effect in these magnetic alloys it is seen that these effects are intimately associated. In some cases as in the iron-copper series there is also an intimate relationship between the Hall constant and the electrical resistance and in other cases as in the iron-nickel series an intimate relationship between the Hall constants and the thermoelectric heights of the alloys. All of these phenomena are evidently intimately related to each other and to the other physical properties of the alloys. In view of the dependence of these effects on the mechanical properties of the metals and alloys, the most inviting suggestion for a theory by which to explain them is that made by Borelius who undertakes to use the conception of the crystal lattice and the intermolecular fields of force as the basis of his theory. In the vibration of the atoms in the crystal lattice and in the variation of the fields of force about them rather than in the action of the magnetic field on the free electrons in the interstices between the atoms, the explanation of these rather complicated effects will probably be found.

I wish to express my thanks to the Rumford Committee of the American Academy of Arts and Sciences for financial assistance in aid of this investigation.

Physical Laboratory,
Ohio State University.

THE ABSORPTION OF GAMMA RAYS BY MAGNETIZED IRON.

BY A. H. COMPTON.

SYNOPSIS.

Ring Electron Theory of Magnetism; a Deduction Regarding the Absorption of Gamma Rays by Iron.—We should expect a ring electron to absorb more energy if its axis is parallel to a transversing beam of gamma rays than otherwise; therefore, if by magnetization the axes of the ring electrons are turned so as to be more nearly parallel to the gamma rays, the part of the absorption due to the transfer of energy to such electrons would be increased. The negative result given by the experiment carried out to test this deduction, however, does not disprove the theory but indicates merely that the absorption due to this cause is small.

Absorption of Gamma Rays by Iron.—*The effect of magnetization* was to change the absorption by less than 0.03 per cent. whether the magnetic field was parallel or perpendicular to the beam of hard gamma rays used.

THE recent experiments of Rognley and the writer[1] have indicated that the ultimate magnetic particle which is responsible for the property of ferromagnetism is probably the individual electron. If this is the case, it is natural to suppose that the electron may have the form of a ring, such as that proposed by Parson[2] in his "magneton" theory of atomic structure. It is clear, however, that the conceptions of a magnetic electron and a ring electron are not synonymous, since any form of electric charge in rapid rotation will exhibit magnetic polarity. The present investigation of the effect of magnetization on the absorption coefficient of gamma rays in iron has been carried out with the hope of detecting an effect due to the orientation by a magnetic field of anisotropic, magnetically polarized electrons.

Let us suppose that a ring electron has its axis parallel with an incident beam of gamma rays. The phase of the electromagnetic wave will in this case be the same over all parts of the ring. The acceleration of the electron produced by the electric vector of the incident wave will correspondingly be greater than if the plane of the ring is so inclined that the phase of the incident wave differs at different points on the ring. Because of its greater acceleration, the energy transferred to the motion of the electron as a whole will be greater when its axis is parallel with the incident beam than when oriented in any other manner. If by magnetization the axes of some of the electrons in iron are turned until they are parallel with the transmitted gamma rays, any absorption which is due to motion imparted to the electron as a whole should be increased.[1] On the other hand, any energy absorbed in setting up internal or nutational oscillations of the electrons may or may not be affected by changing the electron's orientation, depending upon the manner in which the transfer of energy is effected.

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[1] A. H. Compton and Oswald Rognley, PHYS. REV., 16, 464 (1920).

[2] A. L. Parson, Smithsonian Inst. Pub. No. 2371 (1915).

When this experiment was undertaken, it was supposed on the basis of the work of Florance,[2] Ishino[3] and others that a large part of the total absorption of gamma rays is due to scattering, the magnitude of which depends upon the acceleration of the whole electron. Recent experiments by the writer,[4] however, have shown that if there is any true scattering of gamma rays, the energy thus dissipated is only a small fraction of that transformed into fluorescent radiation. These experiments have indicated further that the principal part of the fluorescent radiation produced is of a kind which is not characteristic of the particular absorbing element, thus differing from the fluorescent radiation excited by X-rays of ordinary hardness. While for X-rays most of the absorption due both to scattering and fluorescence is probably due to energy imparted to the motion of the electron as whole,[5] it is therefore not improbable that in the case of the much shorter gamma rays the principal part of the absorption is due to the transfer of energy to nutational and elastic oscillations of the electron itself. Because of this uncertainty as to the mechanism of absorption, while a positive effect on the absorption coefficient of gamma rays due to magnetization would presumably mean an anisotropic electron, the interpretation of no effect at all is ambiguous.

The Experiments.—In view of this ambiguity in the interpretation of the results it hardly seems worth while to record the details of the experimental procedure. It may suffice to say that extended series of observations were made by an accurate balance method, and all possible precautions, including check measurements with other metals substituted for iron, were taken to eliminate any errors due to such causes as the mechanical effects of the strong magnetic field. Though in the experiments on parallel magnetization the size of the beam of gamma rays was necessarily determined by the holes in the pole pieces of the magnet, a sufficiently large amount of radium emanation (about 75 millicuries) was employed to secure relatively intense ionization.

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[1] The theory of this effect has been discussed in a preliminary manner by the writer in Journ. Wash. Acad. Sci., 8, 1 (1918).

[2] D. C. H. Florance, Phil. Mag., 27, 225 (1914).

[3] M. Ishino, Phil. Mag., 33, 140 (1917).

[4] A. H. Compton, Phil. Mag., in printer's hands.

[5] Cf. A. H. Compton, Phys. Rev., 14, 253 (1919).

The results of the experiment may be expressed as follows: When the iron was magnetized to saturation parallel with the transmitted beam of gamma rays, the observed increase in its absorption coefficient was (0.004 ± 0.019) per cent. For magnetization perpendicular to the gamma ray beam the absorption coefficient was increased by (0.023 ± 0.018) per cent. That is, for parallel magnetization the effect on the absorption coefficient is probably less than 1 part in 5,000, and for perpendicular magnetization the effect is probably less than 1 part in 3,000.

Interpretation of Results.—If the ratio of the wave-length of the gamma rays λ to the radius of the electron a is approximately 2.5, as was estimated by the writer on the basis of Ishino's experiments on the total scattering of gamma rays,[1] and if the scattering by a ring electron is calculated on the basis of the assumptions previously employed,[2] it can be shown that the part of the absorption coefficient due to scattering for (1) random orientation, (2) the axes of the ring electrons parallel and (3) the axes perpendicular to the transmitted gamma rays are in the ratio of about 1.0 to 1.5 to 0.8 respectively. Similarly the part of the fluorescent absorption which is due to energy transmitted to the motion of the electron as a whole may be calculated in the same general manner as that used by the writer in calculating the absorption of very hard X-rays.[2] The acceleration of a ring electron is so altered by its orientation that if $\lambda/a = 2.5$ this type of absorption in the three cases should be in the ratio 1.0 to 7.2 to 0.14 respectively. Since the writer's more recent experiments have indicated that these types of absorption do not play an important part in the total absorption of gamma rays, one does not feel justified in presenting the details of these calculations.

There is good reason to believe that when iron is saturated the elementary magnets of which it is composed are in nearly perfect alignment with the external magnetic field.[3] Thus if the elementary magnet is the ring electron, and if either of the two types of absorption just considered constitute any appreciable fraction of the total absorption of gamma rays, it is clear that magnetization of the iron should have produced a measurable effect upon the absorption coefficient. For if all the absorption were to be accounted for in this manner, the orientation of 1 electron in 20,000 by the applied magnetic field would hardly have escaped detection. If we accept the idea of nearly complete alignment of the elementary magnets with the external field when saturation occurs, it would therefore seem necessary to conclude either (1) that the ultimate magnetic particle is not a ring electron, or (2) that energy transmitted to the motion of the electron as a whole is not responsible for any considerable part of the total absorption of gamma rays.

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[1] A. H. Compton, PHYS. REV., 14, 26 (1919).

[2] A. H. Compton, PHYS. REV., 14, 253 (1919).

[3] Cf., e.g., Compton and Rognley, loc. cit.

If only one ring electron in each atom is oriented by the magnetic field, an effect of one part in 5,000 should have been produced on parallel magnetization if but 1 per cent. of the total absorption is due to scattering. Although experiment shows that by far the greater part of the absorption of gamma rays is due to fluorescence, the fact that for very hard X-rays scattering is still prominent makes it appear improbable that scattering should be as unimportant as this result would imply. Thus while it is not possible to draw any definite conclusion from this experiment until more information is available with regard to the mechanism of gamma ray absorption, the evidence seems rather opposed to the hypothesis that a ring electron is the ultimate magnetic particle.

It will be remembered, on the other hand, that A. H. Forman described in this journal several years ago[1] an experiment on the absorption of X-rays by magnetized iron which showed an increase in the absorption coefficient of about 10 times the probable error of measurement when iron was magnetized parallel with the transmitted rays. He found no effect for perpendicular magnetization. The writer has pointed out elsewhere[2] that the effect observed by Forman is of the order of magnitude to be expected if the electrons are rings of electricity which are oriented by the magnetic field. It is however rather surprising that while Forman found the change in the absorption coefficient due to magnetization to increase rapidly for shorter wave-lengths, the present experiment shows no effect for the still shorter wave-length gamma rays. In view of the apparently conflicting evidence, the writer does not feel justified in drawing any conclusions concerning the form of the electron from these experiments.

This work has been performed under the auspices of the National Research Council. I desire to thank Professor Rutherford for placing at my disposal the facilities of Cavendish Laboratory for carrying out the experiment.

Cavendish Laboratory,
Cambridge University,
August 20, 1920.

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[1] A. H. Forman, Phys. Rev., 7, 119 (1916).

[2] A. H. Compton, Journ Wash. Acad. Sci., 8, 1 (1918).

THE CRYSTAL STRUCTURE OF CALCIUM.

BY A. W. HULL.

SYNOPSIS.

Crystal Structure of Calcium.—The X-ray pattern obtained with powdered calcium shows that the atoms are arranged in a face-centered cubic lattice. The side of the elementary cube is 5.56 Å. The Ca atoms in Ca have therefore the same arrangement as in CaO and CaF_2; but in the fluoride and oxide the atoms are respectively 2 and 14 per cent. closer together than without the fluorine and oxygen.

CALCIUM has generally been considered hexagonal, partly, perhaps, from analogy with magnesium and zinc. The only reference given by Groth[1] is an observation of Moissan,[2] that it forms hexagonal plates or rhombohedra, and shows hexagonal growth forms.

The X-ray analysis shows it to be face-centered cubic.

The analysis was made by the X-ray powder photograph method previously described.[3] Pure Electrolytic Calcium was ground in a ball mill to very fine powder, and exposed to Monochromatic rays from a molybdenum tube. The specimen was mounted in a glass tube about 3 mm. in diameter. The lines on the film were very wide (nearly 5 mm.) and some of them quite faint. In spite of this, the independent measurements on either side of the center (columns 2 and 4 respectively, table 1) agree with each other within ½ per cent., and the planar spacings calculated from their average agree with the theoretical spacings within ⅓ per cent. in every case except two, where the deviation is 1 per cent. The full data is given in the table.

Since calcium oxidizes rapidly in air, it seemed possible that the very fine powder used in this experiment was entirely oxidized, and that the above values were due to CaO. A second sample was therefore prepared, consisting of coarse filings from a cast rod of electrolytic Ca, kept in rotation during exposure. This gave lines identical with those in Table I.

As a final check, a photograph was taken, using a tube whose lower half was filled with fine Ca powder, and the upper half with dry CaO. A septum of thin brass prevented overlapping of the scattered rays. This photograph is reproduced in Fig. 1. It shows that the patterns of

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[1] Groth, Chemische Krystallographie, Vol. 1.

[2] Moissan, C. R., 127, 585, 1898; Ann. Chim. Phys., 18, 303, 1899.

[3] PHYS. REV., 10, 661, 1917.

Ca and CaO are similar, but on a very different scale. The strongest CaO line is faintly visible on the Ca negative, corresponding to about 2 per cent. CaO present.

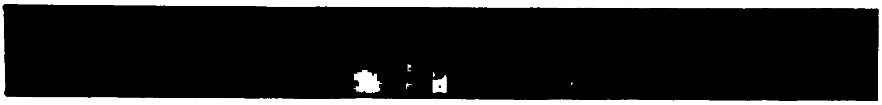

Fig. 1.

Lime above; calcium below.

The theoretical spacings in column 7 are those of a face centered cubic lattice, the side of whose elementary cube is 5.56 Å. The number of atoms per elementary cube is

$$n = \frac{\rho a^3}{M} = 4.00,$$

ρ = density = 1.56,

a = side of elementary cube in cm.,

M = mass of 1 atom in grams.

This is the correct number for a face-centered cubic lattice.

Table I.

Calcium.

Left.		Right.		Angle of Reflection (Degrees).	Spacing of Planes.		Indices of Planes.
Intensity of Line (Estimated).	Distance of Line from Center (Cm.×2.5).	Intensity of Line (Estimated).	Distance of Line from Center (Cm.×2.5).		Experimental.	Theoretical.	
100	1.76	100	1.76	6.34	3.21	3.21	111
40	2.02	40	2.02	7.27	2.81	2.78	100 (2)
40	2.89	50	2.89	10.40	1.966	1.966	110 (2)
50	3.40	60	3.40	12.25	1.672	1.672	311
10	3.55	8	3.56	12.80	1.600	1.605	111 (2)
1	4.12	½	4.14	14.90	1.378	1.390	100 (4)
10	4.49	8	4.47	16.13	1.278	1.275	331
8	4.61	6	4.61	16.60	1.242	1.246	210 (2)
7	5.07	5	5.08	18.28	1.131	1.135	211 (2)
6	5.39	6	5.38	19.40	1.068	1.070	511 111 (3)
1	5.91	½	5.93	21.3	.975	.982	110 (4)
5	6.16	1	6.16	22.2	.940	.940	531
1	6.25	1	6.22	22.5	.927	.928	100 (6) 221 (2)
½	6.62	—	—	23.8	.878	.878	310 (2)
1	6.87	¼	6.85	24.7	.850	.847	533
1	6.98	¼	6.95	25.1	.838	.838	311 (2)

From the lines in the upper half of Fig. 1, it is evident that the Ca atoms in CaO are also in face centered cubic arrangement, but that the cube is *smaller* than that of pure Ca. The oxygen atoms *not only take up no room, but actually pull the Ca atoms* 14 *per cent. closer together, without changing their arrangement.*

The Ca atom in CaO is a Ca^{++} ion, with the same number of electrons, and presumably the same shape, as an atom of argon. It is probable that the Ca atoms in metallic calcium are also Ca^{++} ions, the two extra electrons going into the lattice as free electrons in the same positions as the F^- ions in CaF_2. This is made plausible by the fact that the distance between Ca atoms in CaF_2, where they are also in face-centered arrangement, differs by only 2 per cent. from that in metallic Ca (side of elementary cube in $CaF_2 = 5.43$ Å.). The free electrons are slightly less effective than the F^- ions in pulling the Ca atoms together, due probably to their great mobility. The F^- ions, like the O^{--} ions in CaO, fit into the Ca lattice without increasing its dimensions. Their dimensions appear to be smaller than those of the holes left in the Ca lattice by closest possible packing of Ca atoms.

GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

NOTE ON THE RADIATING AND IONIZING POTENTIALS OF HYDROGEN.

By K. T. Compton and P. S. Olmstead.

Synopsis.

Radiating and Ionizing Potentials of Hydrogen.—To detect and distinguish between radiation and ionization, the Lenard method, modified by the introduction of a receiving electrode of variable area, was used. Both radiation and ionization were obtained at about 10.8 volts, probable radiation at about 13.4 volts, and strong ionization at 15.9 volts. These *experimental results* are compared with those of other investigators; and as an *explanation of apparent discrepancies* it is pointed out that the following effects may occur together: radiation from free atoms near 10.8 volts and ionization near 13.5 volts; ionization without dissociation of molecules near 10.8 volts, dissociation plus radiation from one of the atoms near 13.4 volts and dissociation plus ionization of one atom near 15.9 volts.

A SURVEY of the results of experimental investigations of the radiating and ionizing potentials of hydrogen reveals such lack of agreement as can scarcely be attributed to experimental errors or to uncertainties in the interpretation of observations made by the several methods which have been employed to detect and distinguish between radiation and ionization produced by electron impacts. Early investigators agree in locating a critical potential between 10.5 and 11.5 volts, but did not distinguish between effects of radiation and of ionization. Bishop[1] discovered two critical potentials at 11.0 and 15.7 volts. The following table gives the data found by the recent investigators who

Table I.

Experimenter.	Radiation.		Ionization.			
Davis & Goucher[2]	11.0	13.6	11.0,		15.8	
Mohler & Foote[3]	1.06			13.1,	16.4	
Horton & Davies[4]	10.5	13.9		14.4,	16.9	
Frank, Knipping & Krüger[5]		13.6	11.5,		17.1,	30.45
Found[6]	not tested		(15.1 or 15.8)			

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[1] Phys. Rev., 10, p. 244, 1917.

[2] Phys. Rev., 10, p. 101, 1917.

[3] Jour. Optical Soc. of America, 4, p. 49, 1920.

[4] Roy. Soc. Proc., A, 97, p. 23, 1920.

[5] Ber. d. D. Phys. Ges., 21, p. 728, 1920.

[6] Phys. Rev., 16, p. 41, 1920.

have employed means to distinguish between effects due to radiation and to ionization. Unless some explanation of the differences observed in this table can be found, it can scarcely be said that the experimental results as a whole support the recent theories of the relations between critical potentials and spectral series which have been so well supported by experiments on monatomic gases and vapors.

The present paper presents the results of some preliminary experiments made by the method which one of us used to distinguish between ionization and radiation and to estimate the proportion of either in the presence of both.[1] In the course of these experiments certain observations suggested a possible explanation of the above discrepancies. This explanation is briefly presented near the close of the paper for the consideration of any who may now be at work on the same problem. We are modifying our apparatus to permit of more refined observations to test some of the considerations involved in the problem.

The experimental apparatus has already been fully described. A glass tube contained a tungsten wire filament as the source of the bombarding electron stream, a platinum gauze and a platinum electrode which was connected to an electrometer whose sensitiveness could be varied over a considerable range. A variable accelerating field V_A was applied between the filament and the gauze, and a larger retarding field V_R was maintained between the gauze and the collecting electrode. A galvanometer introduced in the connection to the gauze served to measure the total electron current G. The distinguishing feature of the apparatus was the collecting electrode, which was cylindrical in form, with one end closed by platinum foil and the other end by platinum gauze. The cylinder could be rotated so as to present either end to the oncoming stream of ions or radiation. If the effect observed is due only to ionization of the gas, it is immaterial which end is turned toward the filament, and the ratio R between the electrometer deflections E_f and E_g with the foil and gauze respectively presented is unity. If the observed effect is due to radiation, E_f is larger than E_g, so that R is greater than unity by a definite amount depending on the relative areas presented to the radiation by the two ends. If radiation and ionization are both present, R has some intermediate value, from which it is easy to calculate what proportion of the observed effect is due to radiation and what to ionization.

After thoroughly "baking out" the tube and glowing the filament, hydrogen was admitted. It was obtained by electrolysis of dilute sulfuric acid and was purified by passing through a drying tube, over copper oxide and metallic copper heated to a dull red, through another drying tube and through a liquid air trap. The hydrogen was found to still contain dectable traces of impurities with critical potentials below that of hydrogen, and it disappeared rapidly because of the "clean up" due to the hot filament. These difficulties were overcome by introducing *U* tubes filled with coconut charcoal and immersed in liquid air. After originally filling the apparatus with hydrogen to about 15 mm. pressure, the application of liquid air to the charcoal tubes removed all traces of impurities and reduced the hydrogen pressure to about 0.01 mm. The equilibrium between the absorbed and free hydrogen served to maintain the free hydrogen at the desired pressure during the time necessary for experimental observations, hydrogen being liberated by the charcoal as fast as it was removed by the filament.

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[1] K. T. Compton, Phil. Mag.,

Examples of the results obtained at various gas pressures and current densities are given in the accompanying figures. In Figs. 1*a*, 2*a* and 3*a* are shown the variation of the electrometer deflection *E* with accelerating voltage, the gauze end of the cylinder being used in each case. In Figs. 1*b*, 2*b* and 3*b* are shown the derivatives *D* of curves 1*a*, 2*a* and 3*a*, respectively, and also the ratios *R* of corresponding readings with the foil and gauze ends of the cylinder turned toward the filament. The derivative curves were obtained by plotting differences between successive readings

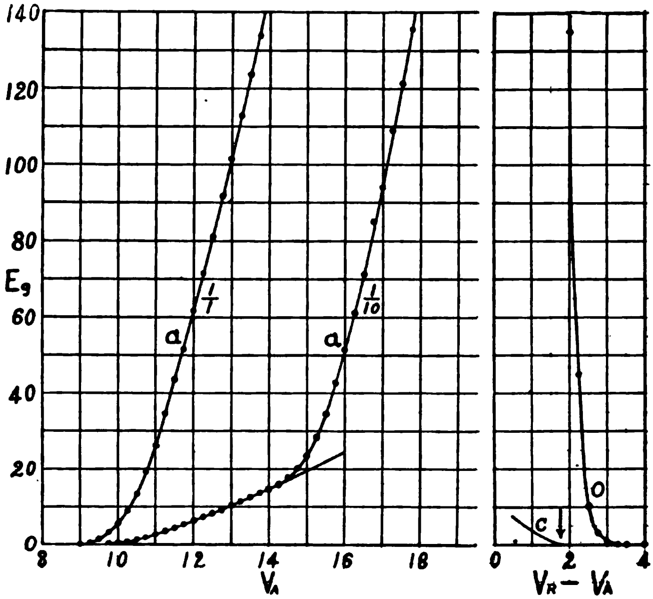

Fig. 1*a*.

$p = 0.008$ mm., $G = 0.125$ microamperes.

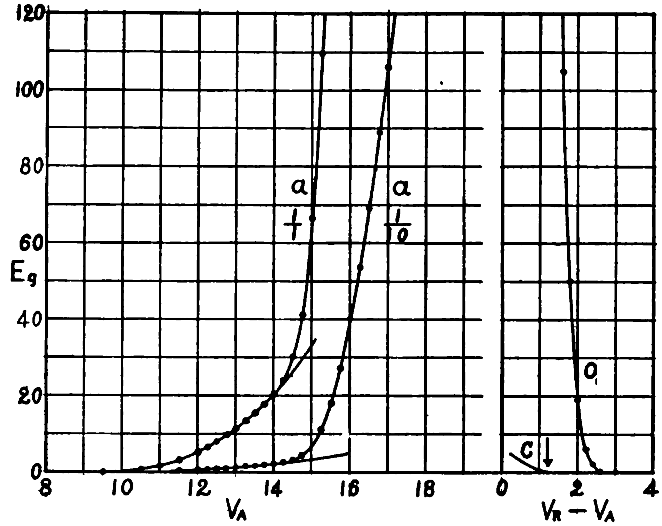

Fig. 2*a*.

p = 0.018 mm., *G* = 0.125 microamperes.

at the mid-points, and are seen to accentuate the sharpness with which the critical potentials are indicated. Fig. 4 illustrates the case of a very intense electron current, in which the electrometer currents were too large to be measured beyond the voltage indicated.

The corrections for velocity distribution were made by Smyth's method,[1] and at the right of Figs. 1*a*, 2*a* and 4 are shown the observed

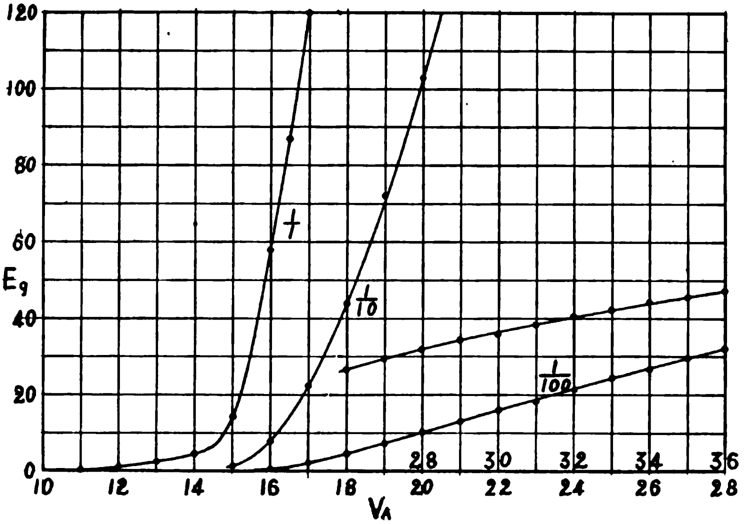

Fig. 3*a*.

p = 0.018 mm., *G* = 0.025 microamperes.

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[1] PHYS. REV., Vol. 14, p. 409, 1919.

and corrected velocity distribution curves. Arrows mark the points taken as giving the appropriate velocity distribution corrections.

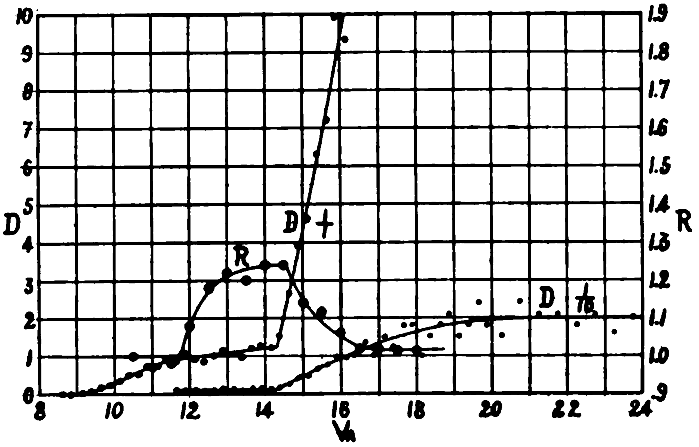

Fig. 1b,

After making this correction to each curve and averaging all the results, we obtained 10.8 volts and 15.9 volts as the two critical potentials which were definitely observed. The ratio curves suggest an additional critical potential at about 13.4 volts, although this cannot be seen by inspection of the current or difference curves. It should be borne in mind that the present apparatus, containing only three electrodes, does not permit the emphasizing of separate weak intermediate effects of radiation or ionization, so that the failure definitely to observe these is not surprising.

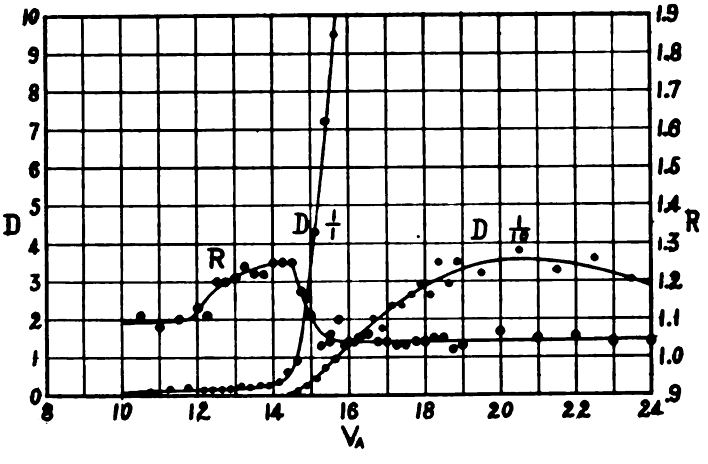

Fig. 2b.

The critical potential of 15.9 volts marks the setting in of very intense ionization, as is evident by the rapid fall of the ratio curves to nearly unity just above this voltage. What indication there is of a break at 13.4 volts in the ratio curves, points toward the setting in of radiation at about this voltage. The interpretation of the lower critical potential is more uncertain. In all cases where the electron current *G* was relatively small, the ratio curve starts from a value near unity at about 10.8 volts and increases up to the point where strong ionization sets in at 15.8 volts. This indicates a large proportion of ionization at the lower voltages, but with an increasing proportion of radiation as the voltage is raised from 10.8 to 15.9 volts. On the other hand, with intense electron

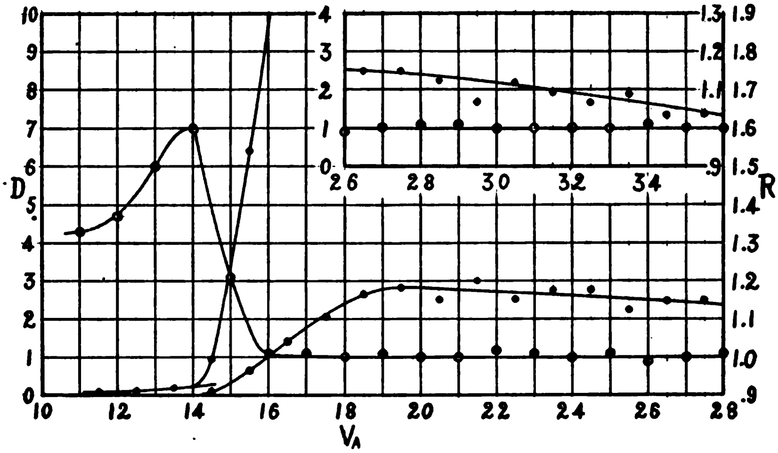

Fig. 3*b*.

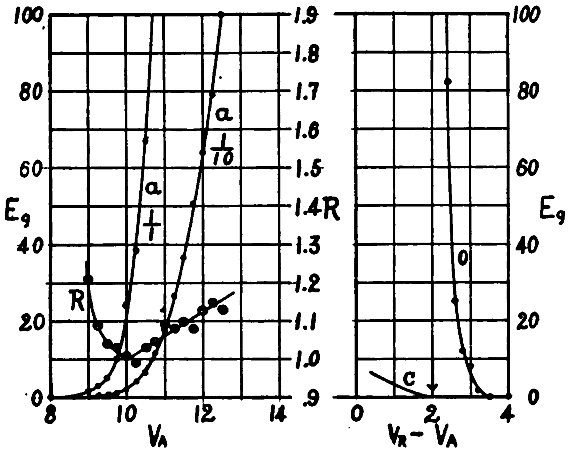

Fig. 4.

p = 0.018 mm., G = 12.5 microamperes.

currents, the first effect seems to be largely one of radiation, with an increasing amount of ionization as the voltage is raised; then increasing radiation and finally strong ionization at 15.8 volts. These characteristics are illustrated by the accompanying curves, and were exhibited by numerous additional curves, not shown.

One difficulty was encountered in this work with hydrogen which did not appear in the work with helium. It was an effect somewhat similar to photoelectric fatigue, by which the sensitivities of the foil and gauze ends of the cylinder to radiation varied slowly with the time during which they were turned toward the filament. Thus the value of the ratio R for pure radiation could be considerably altered by leaving one end of the cylinder turned toward the filament for a long time. This introduced irregularities which make it impossible to compare one curve quantitatively with another, or to calculate the exact proportion of the effect due to radiation in any given case. However, this does not affect the interpretation of the curves as regards the existence of critical voltages or the type of effect which sets in at any critical voltage. Possibly this "fatigue" was due to action of atomic hydrogen coming from the filament.

These results agree with those of Davis and Goucher in proving the presence of *both* radiation and of ionization at about 10.8 volts and of strong ionization at about 15.9 volts. The results also show that one effect or the other may predominate at the lower voltage, depending on conditions, and thus explain the apparently contradictory results which investigators have obtained.

The most obvious explanation of these results is to suppose that there is radiation from the atom at about 10.8 volts and also ionization without dissociation of the molecule at about 10.8 volts, dissociation with radiation from one atom at about 13.4 volts and dissociation with ionization of one of the atoms at about 15.8 volts. The atoms present at the lower voltages are produced by the action of the hot filament and probably also by low speed impacts. The existence of charged hydrogen molecules, which are not stable on Bohr's theory, is indicated by several investigations, among them the recent published and unpublished work of Frank, Knipping and Krüger.[1] If this explanation is correct, it is evident that radiation at the lower voltages will be shown by an apparatus so designed as to cause considerable dissociation of the hydrogen and to be sensitive to the detection of radiation. Furthermore a given apparatus will emphasize the radiation at 10.8 volts by using a very hot filament and large electron current, thus promoting dissociation.

It will be noticed in Table I. that two investigators have found an additional critical potential for ionization between 10.8 and 15.9 volts. This is to be ascribed to ionization of free atoms. It is suggestive that Mohler and Foote, who probably used the largest electron currents of any investigators, detected only radiation at 10.6 volts and ionization at 13.1 volts, both ascribed to collisions with free atoms.

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[1] *Loc. cit.*

Attention may here be called to the recent results of Found, shown in Table I., which appear to be at variance with those of other investigators and with the demands of probable spectral relationships by indicating ionization beginning at 15.1 volts. A reëxamination of his published data has shown, however, that there is strong ionization beginning at 15.8 volts, while below this point there is indication of ionization of another type which is too weak to permit a determination of the lower critical voltage. Mr. Found writes that his unpublished data show these same characteristics, so that his results are actually in good agreement with ours.

As regards the actual values of the critical potentials for radiation and ionization near 10.8 volts, a consideration of the velocity distribution correction for the two cases is interesting. As pointed out by Smyth,[1] the appropriate correction is the equivalent voltage not of the fastest or the average or the most probable velocity electrons escaping from the filament, but of electrons of such velocity that on the average one ionizing or radiating impact is produced by each group of electrons of this or higher speed—the number of electrons in this group depending in a calculable manner on the mean free path, gas pressure, dimensions of apparatus, etc. Now the larger is the gas pressure, or the greater the probability of a collision, the less is the difference between the calculated and observed velocity distribution curves. If, therefore, the radiation observed near 10.8 volts is due to atoms, which are present in relatively small proportion, the velocity distribution correction should be less than that appropriate to ionization of the molecules. The corrections in the present case were calculated on the basis of the partial pressure of the molecules. Thus it is probable that the critical radiating potential is considerably below 10.8 volts, and it may well be 10.1 volts as required by Bohr's theory to fit the spectral data. This is experimentally supported by the fact that the corrected critical voltages were lowest in those cases in which the filament was hottest, as in the case illustrated by Fig. 4.

It would be highly desirable to test for the presence of two critical voltages, very near together, near 10.8 volts, and especially to investigate the influence of very varying degrees of molecular dissociation. Unless this can be done, comparison of ionization with spectral data will be more or less uncertain. The experimental difficulties involved are, unfortunately, serious.

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[1] *Loc. cit.*

Finally, it may be mentioned that an effort was made to locate a critical potential at about 30.45 volts, reported by Frank, Knipping and Krüger and ascribed to dissociation of molecules and ionization of both atoms by single impacts. Fig. 3 shows that we detected no such effect. Our apparatus may not have been suited to give it sufficient prominence.

Palmer Physical Laboratory,
Princeton, N. J.

AN ELECTROMAGNETIC THEORY OF GRAVITATION.

By H. A. Wilson.

Synopsis.

An Electromagnetic Theory of Gravitation.—An electric system in a medium whose specific inductive capacity k varies from point to point tends to move in the direction of increasing k. It is suggested that if we *assume the specific inductive capacity of the ether to vary near matter*, gravitation may be explained as a result of this tendency. In a medium in which at a distance r from a mass m, $k = 1 + m/r$, it is shown that a rigid electrostatic system would be acted on by a force directed toward m and equal to mm'/r^2, where m' is the electromagnetic mass of the system. But in order to *explain the observed deflection of light by the sun* we must have $k = 1 + 2m/r$; and this will not give the force mm'/r^2 unless the system contracts in the ratio of $1 : 1 - m/r$. A physical explanation of this assumed contraction is suggested. If the system with the mass m' is also supposed to modify k, it is necessary to take into account the energy changes in m and in the ether. The *effect of gravitation on the frequency of the light emitted by an atom*, which was predicted by Einstein, can be easily deduced from the present theory.

THE recent discovery of the deflection of light by the gravitational field of the sun shows for the first time a connection between electromagnetic phenomena and gravitation. This deflection shows that the velocity of light and therefore the specific inductive capacity and magnetic permeability of the ether vary with the gravitational potential.

In an electrostatic field an insulator tends to move to regions where the electric field is stronger and in a medium of varying specific inductive capacity an electrostatic system will tend to move towards places where the specific inductive capacity is greater. The theory proposed in this paper is that matter which is believed to be composed of electrical charges tends to move through the ether in the direction in which the specific inductive capacity and permeability of the ether increase most rapidly and that this tendency is the cause of gravitation.

Consider the case of a small rigid system of massless electrically charged bodies immersed in a medium of specific inductive capacity K. If the charges remain constant the electrostatic energy (E) of the system varies inversely as K. Hence $E = E_1/K$ where E_1 denotes the value of E when $K = 1$. Hence if K is not constant throughout the medium but varies we have

$$F_s = -\frac{\partial E}{\partial s} = \frac{E_1}{K^2}\frac{\partial K}{\partial s} = E\frac{\partial}{\partial s}(\log K),$$

where F_s denotes the force on the system, tending to move it in the direction s due to the variation of the electrostatic energy E inside the system.

Suppose that in the space around a body of large mass m,

$$K = 1 + Gm/rc^2$$

where c denotes the velocity of light in the ether in the absence of a gravitational field, r the distance from the mass m, and G is the constant of gravitation. Then except very close to the mass m, Gm/rc^2 will be extremely small compared to unity[1] so that $\log K = Gm/rc^2$ and

$$F_r = -GEm/r^2c^2.$$

The small mass (m') of the electrical system if due entirely to its electrical energy is given by $m' = E/c^2$ so that

$$F_r = -Gmm'/r^2,$$

which expresses Newton's law of gravitation for the electrical system and the mass m.

Thus if we regard matter as an electrostatic system, having purely electromagnetic mass, the observed gravitational attraction can be explained by supposing that $K = 1 + Gm/rc^2$. If the matter also contains magnetic energy then if we suppose that the magnetic permeability of the ether in the space around a body of mass m is given by

$$\mu = c^2(1 + Gm/rc^2)$$

we get the same force per unit mass on the magnetic energy as on the electrostatic energy so that $F_r = -Gmm'/r^2$ where m' now denotes the total mass due to the electric and magnetic energies.

Taking $c = 1$ and $G = 1$ we have[2] for the refractive index (v) of the ether $v = \sqrt{\mu K} = 1 + m/r$. This value of v gives only half the observed deflection of light by the sun. The deflection found requires that $v = 1 + 2m/r$.[3]

In the above simple theory it is assumed that the size of the electrostatic system remains unchanged when it moves into a region where the specific inductive capacity has a different value; but since the forces between the parts of the system depend on K we should expect the size of the system to vary with K.

The electromagnetic forces alone are not sufficient to determine the size of a system composed of positive and negative electrons so that it is easy to see that the size must be determined by the internal forces which hold the parts of the electrons together. If l is a quantity proportional to the linear dimensions of the system and equal to unity when $K = 1$, then the electrostatic energy is inversely proportional to Kl so that $E = E_1/Kl$.

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[1] The greatest value of Gm/rc^2 in the solar system is about 2×10^{-6} at the surface of the sun.

[2] Taking $c = 1$ is equivalent to adopting 3×10^{10} cms. as the unit of length if the second is taking as the unit of time. Taking $G = 1$ is equivalent to adopting for the unit of mass, a mass which gives unit gravitational acceleration at unit distance from it. This unit is about 4×10^{28} grams when the unit of time is one second and the unit of length 3×10^{10} cms.

[3] Report on The Relativity Theory of Gravitation, Eddington, page 54.

For the energy of an electrostatic system is equal to $\frac{1}{2}\Sigma EV$ where V denotes the potential of a charge E. Also

$$V = \frac{1}{4\pi}\int \frac{\rho dv}{Kr},$$

where ρ denotes the density of electricity in the element of volume dv and r the distance of the element dv from the point at which the potential is V. If the linear dimensions of the whole system change uniformly then ρdv will remain constant so that V varies inversely as Kl and hence the energy also varies inversely as Kl when the charges remain constant. In this case the force on the system, due to the variation of the energy inside it, will be given by

$$F_s = + E\frac{\partial}{\partial s}(\log Kl).$$

Thus to get the correct value of F_r we must have $Kl = K^{1/2}$ or $l = K^{-1/2}$. If $K = 1 + 2m/r$ then l must be equal to $1 - m/r$ so that

$$F_r = + E\frac{\partial}{\partial r}(\log(1 + m/r)) = -\frac{mm'}{r^2}.$$

If $\mu = 1 + 2m/r$ there will be an equal force per unit mass on the magnetic energy also.

The tension inside the Lorentz electron is supposed not to vary when the shape of the electron changes, due to its motion through the ether, but when the specific inductive capacity of the ether changes it may be supposed to change.

The internal tension in the electron may be regarded as a sort of elastic reaction against the electric displacement or polarization at its surface. We should therefore expect the tension to be proportional to the displacement. This gives

$$\frac{1}{Ka^4} \propto \frac{1}{a^2},$$

where a is the radius of the electron, so that $aK^{1/2}$ is constant. It is easy to see that the linear dimensions of an electrostatic system, entirely composed of positive and negative electrons, will be proportional to a so that if $aK^{1/2}$ is constant then $l = K^{-1/2}$ which is the value required to give the observed gravitational attraction. For in an electrostatic system we have $\Delta V = -\rho/K$ and if we put $x = lx'$, $y = ly'$, $z = lz'$, $V = V'/Kl$ and $\rho dxdydy = \rho' dx'dy'dz'$, this becomes $\Delta V' = -\rho'$ which is the equation giving the potential in a system in which $K = 1$. Thus a system having any value of K can be transformed into a corresponding system in which $K = 1$ by changing the linear dimensions by a factor l having any value. We conclude that the actual change of dimensions will be determined by the change in the radii of the electrons.

The internal energy of the electron is inversely proportional to Kl like the external electrostatic energy so that there will be the same force per unit mass on the internal energy as on the electrostatic energy.

It appears therefore that if $K = \mu = 1 + 2m/r$ and the system contracts in the ratio $1 : K^{-1/2}$ or $1 : 1 - m/r$ then $F_r = -mm'/r^2$ where m' now denotes the mass of the total energy of the system including the electrostatic, magnetic and internal energies.

The contraction in the ratio $1 : K^{-1/2}$ agrees with that indicated by Einstein's theory, to the order of approximation used here, for if in the expression for the line element in the four dimensional manifold

$$ds^2 = -\gamma^{-1}dr^2 - r^2d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \gamma dt^2,$$

where $\gamma = 1 - 2m/r$, we put $r = r_1 + m$ and neglect squares of m/r_1 it becomes

$$ds^2 = -\gamma^{-1}(dr_1^2 + r_1^2d\theta^2 + r_1^2 \sin^2 \theta d\phi^2) + \gamma dt^2.$$

This indicates a contraction in the ratio $1 : \gamma^{1/2}$ or $1 : 1 - m/r$ in the element of length $\sqrt{dr_1^2 + r_1^2d\theta^2 + r_1^2 \sin^2 \theta d\phi^2}$.

According to the theory outlined above both gravitation and the deflection of light are due to variations in the specific inductive capacity and permeability of the ether. How these variations are produced by the presence of matter in the ether, at distant points, remains to be explained.

The attracting mass m has been supposed to be surrounded by a region in which $K = \mu = 1 + 2m/r$ and the attracted mass m' has been supposed to be an electric system and it has been shown that the electric system will be acted on by a force very approximately equal to the Newtonian gravitational attraction and also it is known that light will be deflected in consequence of the assumed variations in K and μ, in the way observed.

The attracted mass m' however must also be supposed to modify K and μ so that there will be changes produced in the electrical energy in the mass m by the motion of the mass m'. These changes have so far been left out of account.

We have seen that the energy in the mass m' is given by

$$E = E_1/\sqrt{K} = E_1(1 - m/r)$$

and in the same way the energy (F) in the mass m will be given by $F = F_1(1 - m'/r)$.

We have $m_1' = E_1$ and $m_1 = F_1$ approximately so that the total energy in the two masses is approximately equal to

$$m_1'(1 - m/r) + m_1(1 - m'/r)$$

or $m_1' + m_1 - 2mm'/r$ very nearly. The stress tending to diminish r might therefore be expected to be equal to $2mm'/r^2$ instead of mm'/r^2.

We might suppose that $Kl = K^{1/4}$ instead of $Kl = K^{1/2}$ and so diminish the calculated attraction again by one half but I think it is better to adopt a different plan for $Kl = K^{1/2}$ seems a reasonable assumption and it agrees approximately with Einstein's theory.

The modification of the ether represented by the change of K and μ from unity to $1 + 2m/r$ must require some energy for its production so that besides the energy variations inside the attracting masses we ought also to consider the energy variations in the surrounding ether.

Suppose that there is in the ether an amount of energy per unit volume equal to

$$\frac{1}{32\pi}\left\{\left(\frac{\partial K}{\partial x}\right)^2 + \left(\frac{\partial K}{\partial y}\right)^2 + \left(\frac{\partial K}{\partial z}\right)^2\right\}.$$

This expression need not include μ because $K = \mu$ everywhere; it has been chosen so as to give the desired energy variation and is otherwise a pure assumption.

Then the energy outside a mass m of radius a is equal to

$$\int_a^\infty \frac{1}{32\pi}\left(\frac{\partial K}{\partial r}\right)^2 4\pi r^2 dr,$$

so that since $K = 1 + 2m/r$ it is equal to $\frac{1}{2}m^2/a$. This is $\frac{1}{2}m$ multiplied by the gravitational potential m/a.

If we have two masses m and m' with radii a and b the outside energy will be

$$\tfrac{1}{2}m\left(\frac{m}{a} + \frac{m'}{r}\right) + \tfrac{1}{2}m'\left(\frac{m'}{b} + \frac{m}{r}\right) = \frac{m^2}{2a} + \frac{m'^2}{2b} + \frac{mm'}{r}.$$

Thus the total energy inside and outside is equal to

$$m_1 + m_1' + \frac{m^2}{2a} + \frac{m'^2}{2b} - \frac{mm'}{r},$$

so that the stress tending to diminish r is approximately equal to mm'/r^2, as it should be, for the variation of $m^2/2a$ and $m'^2/2b$ is negligible compared with that of mm'/r.

If m is very large compared with m' we may regard m as fixed. In this case when m' moves towards m the electrical energy in m' diminishes by an amount corresponding to the gravitational attraction and also the energy in m diminishes by an equal amount and the energy outside in the ether increases by an equal amount. Thus the loss of energy in the large mass is compensated by an equal gain in the ethereal energy. The total energy of the small mass remains constant and since the ethereal energy is nearly all near the large mass the total energy attracting the small mass also remains almost constant. The effective masses therefore remain constant unless the masses come near each other. This theory makes the energy in the ether positive which is satisfactory. It has always been difficult to believe that the ether in a gravitational field contains less energy per unit volume than the ether at a great distance from matter.

It will be observed that on this theory the gravitational force on either of the two attracting masses is equal to that due to its tendency to move in the direction of increasing specific inductive capacity due to the energy variation inside itself. Thus the force on each body can be attributed to a change of potential energy inside the body itself when it moves and so to an action between the body and the ether inside it.

The change in the frequency of the light emitted by an atom, due to gravitation, predicted by Einstein can be easily deduced from the present theory. We have seen that the electrical energy of any electrical system is equal to $E_1K^{-1/2}$ where E_1 denotes its energy when $K = 1$. According to Bohr's theory of the emission of light the frequency is determined by the energy (ϵ) in the quantum emitted by means of the equation $\epsilon = hn$ where n is the frequency and h is Planck's constant. If the emitting atom is in ether of specific inductive capacity K instead of ether for which $K = 1$ its energy in each of its possible stable states will be diminished by the factor $K^{-1/2}$ so that the energy of any quantum emitted will also be diminished in the same ratio. The frequency will therefore also be diminished by the factor $K^{-1/2}$ or $1 - m/r$ as predicted by Einstein.

In conclusion it may be said that since matter is certainly partly electrical and since the refractive index of the ether certainly varies near large masses, it seems certain that part at least of the observed gravitational forces must be due to an action of the kind considered in this paper, so that it is satisfactory to find that it is possible to explain the whole attraction by means of this kind of action.

The Rice Institute,
Houston, Texas,
September 13, 1920.

THE LUMINESCENCE OF SAMARIUM.

BY HORACE L. HOWES.

SYNOPSIS.

Luminescent Spectrum of Samarium Oxide in Calcium Oxide.—The effect of heat treatment was studied by comparing the cathodo-luminescent spectra given by fresh preparations which had been carefully heated at various temperatures ranging from 200° to 3000°. The intensity of the light emitted increased with the temperature of heating, but no variations in the wave-lengths of the very narrow bands was observed. Thirteen *new bands* were discovered and measured which help fill out the eight series of constant frequency intervals.

Thermo-luminescence of samarium oxide in calcium oxide.—The *minimum temperature* was found to be about 200°.

THE rare earth elements, in composition with lime or other diluents have been made to phosphoresce by Sir William Crookes,[1] Leecoq de Boisbaudran,[2] Urbain[3] and others. The most favored mode of excitation is by the cathode rays. The long controversy between Urbain and Crookes over the phosphorescent spectra of the rare earths has resulted in the control of those exacting chemical processes whereby the rare earths can be prepared in definite grades of purity. Such beautiful analyses have been made that the physicist is now able to begin his study of the rare earths with an exact knowledge of the composition. The method of making samples for luminescence (using the term "luminescence" to include the older terms "phosphorescence," as well as "fluorescence") can be varied systematically, the heat treatment well controlled, and the variation of the spectra, if present, noted. As has always been found true, heat treatment is necessary to render the specimen in a luminescent condition. The molecular aggregates are no doubt so changed by the heat treatment that luminescent centers are produced in highly localized positions scattered through the general structure.

One of the chief reasons why the rare earths in solid solution offer an excellent field for attacking the problem of luminescent radiation is because of the highly resolved bands which constitute the luminescence spectrum. Hitherto, with the exception of such solids as the uranyl compounds and the ruby and such gases as iodine, sodium, mercury, sulphur, etc. studies have been made with substances, which, although brilliant, exhibit only broad bands. The difficulties in the path when analysis of such very broad bands is attempted are considerable because of the extensive overlapping of the components, but the rare earth compounds present—like the uranyl compounds[1] where very narrow bands are viewed against a dark background—a very convenient field. The present work on samarium must be considered a prelude to a rather complete analysis of rare earth luminescence.

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[1] Crookes, Comptes Rendus, Vol. 88, p. 283 (1879).

[2] Leecoq de Boisbaudran, Comptes Rendus, Vol. 101, p. 552 (1885).

[3] Urbain, Annales des Chimie et de Physique, Series 8, Vol. 18, p. 222 (1909).

The present problem was the outcome of earlier work done in collaboration with Professor E. L. Nichols and the investigation was made possible by a grant from the Rumford Fund. The specimens of samarium oxide were obtained from Professor C. James, of New Hampshire State College, and the solid solutions were prepared by Dr. D. T. Wilber of Cornell University. The samarium oxide was weighed and dissolved in hydrochloric acid and diluted to 1/100. This solution was added to pure calcium carbonate which had previously been tested for non-luminescence in a proportion which gave one atom of rare earth to 125 atoms of calcium. The mixture was evaporated to dryness with stirring. The heat treatment was carried on in the following manner: limited portions were placed in uncovered glazed crucibles and treated at several temperatures between 200° and 1200° C. in an electrical resistance furnace. The duration of heat treatment at one temperature was also varied between one and three hours. It was found that it was necessary to heat to a critical temperature of approximately 600° C. to render the luminescence sufficiently bright for spectrum analysis when the specimen was subsequently cooled to + 20° C. and excited by the cathode rays. The 200° C. specimens gave no luminescence at all and the 400° C. specimens exhibited but few regions of luminescence. As regards the duration of heat treatment one hour served as well as three hours—probably because only thin layers were treated. About twenty specimens from the same stock were thus heat treated, and finally one specimen was heated five minutes in the core of an arc carbon. Although no quantitative measurements of intensity were taken it was observed that specimens previously heated to 1200° C., or above that temperature, were the most brilliant under cathode rays at + 20° C.

The luminescence spectrum consists of more than thirty bands of varying intensities, which plotted to a frequency scale can be resolved into several series: *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, of constant frequency intervals as shown in Fig. 1. The intervals, although constant for a given series

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[1] Nichols and Howes, "The Fluorescence of the Uranyl Salts," Publications of the Carnegie Institution of Washington, No. 198.

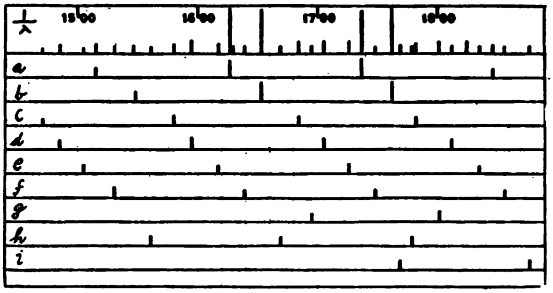

Fig. 1.

vary slightly from one series to another; series *a* for example of 110 units interval, series *b* of 108 units interval. In Table I. will be found the wave-length and reciprocal of wave-length of each band together with the relative intensity and series letter. The bands not previously discovered are marked, but the object of the present investigation was to study the conditions under which the spectrum is produced and to analyze it—rather than to discover new bands. The times required for one exposure averaged approximately ten minutes.

Table I.

Luminescence Bands of Samarium Oxide in Calcium Oxide.

Nature.	λ.	1/λ.	Series.	Nature.	λ.	1/λ.	Series.
v.d.	6801	1470	*c*	v.d.v.n.	5898[1]	1695	*g*
d.	6734	1485	*d*	m. br.	5868	1705	*d*
v.v.d.	6643[1]	1505	*e*	m.	5793	1726	*e*
d.	6600	1515	*a*	v. str.	5757	1737	*a*
d. br.	6536	1530	*f*	v.v.d.n.	5721	1748	*f*
d.	6470	1546	*b*	v. str.	5675	1762	*b*
d. Vag.	6405	1561	*h*	d.	5653[1]	1769	*i*
d. br.	6329	1580	*c*	v.d.	5621[1]	1779	*h*
m.br.	6270	1595	*d*	d.Vag.	5581[1]	1792	*c*
m.Va.	6183[1]	1617	*e*	m.	5553	1801	*g*
V.*str*.	6146	1627	*a*	v.d.n.	5520[1]	1812	*d*
v.d.n.	6140[1]	1629	*a'*	m.	5481	1824	–
v.d.	6103	1639	*f*	v.d.n.	5451[1]	1835	*e*
v.str.	6050	1653	*b*	v.d.n.	5416[1]	1846	*a*
v.v.d.n.	5990[1]	1669	*h*	v.d.br.	5387[1]	1856	*f*
m.	5940	1684	*c*	v.d.br.	5328[1]	1877	*i*

The most startling behavior of the samarium oxide rests in the fact that *no shift in the positions of the bands could be measured, whether the*

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[1] Wave-lengths of bands hitherto undiscovered.

specimen was heat-treated at 600° *C. or* 3000° *C.* This remarkable stability is *not* generally associated with a luminescent substance and hence is of great importance. It may be due in part to the greater than ordinary purity of the materials employed as well as to consistency in the conditions of excitation, exposure, etc.

The specimens must be kept in a desiccator or sealed in glass since both the samarium and calcium oxides upon taking water from the air become non-luminescent as a compound. The specimen was mounted on an aluminum shelf which tended to keep the temperature from rising much above room temperature by conducting the heat due to impact to a hollow aluminum cylinder of large area. That this precaution served the purpose of maintaining the temperature of the very thin lamina of the specimen near room temperature was observed in a unique manner. After an exposure it was observed that a small portion of the cathode ray bundle did not strike the specimen but passed on to excite the luminescence particles of the specimen which had fallen to the bottom of the glass tube. When the cathode stream was stopped the particles on the glass surface exhibited brilliant thermo-luminescence due to heating by the cathode rays while the sample on the aluminum shelf showed no thermo-luminescence.

Subsequently it was found that the temperature required for thermo-luminescence was approximately 200° C.

Summary.

1. The spectrum of the cathode-ray luminescence of samarium oxide in calcium oxide is unchanged by heat treatment at several temperatures ranging from 600° to 3000° C.

2. The spectrum consists of eight short series of constant frequency intervals.

New Hampshire State Colleg
June 20, 1920.

ELECTRICITY AND GRAVITATION.

BY H. BATEMAN.

SYNOPSIS.

Electric Doublet Theory of Gravitation.—It is suggested that gravitation may be an effect arising from fluctuations of the electric charges associated with the electrons and positive nuclei of atoms. It is assumed (1) that the ether contains an enormous number of electric doublets moving in all directions with the speed of light; (2) that each charged particle is continually both absorbing and emitting doublets at a rate proportional to its mass; and (3) that during the absorption and emission of each doublet the charge on the particle fluctuates. If these fluctuations exist, it is shown that the mean value of the force exerted by one charged particle on another includes, in addition to the ordinary electrostatic force, an attraction proportional to the product of the masses.

Electric Doublet Theory of Radiation.—If we suppose that the doublets emitted by a particle possess available energy only when the energy of the particle changes, and that the effect of changing the energy is to produce periodic gaps in the emission of doublets with a frequency proportional to the amount of energy lost, we have a theory of radiation which is said to be compatible with the theories of Bohr, Planck and Einstein.

1. *The Field of an Electric Pole with a Fluctuating Electric Charge.*—We shall consider a type of field in which the electric charge associated with a pole varies on account of the emission of electrified light-particles. Expressions for the components of the field vectors when the pole moves with a velocity less than c and the direction of projection of the light particles varies in an arbitrary manner have already been given.[1] It will be sufficient to give here the components of the field vectors E and H for the simple case in which the pole remains stationary at the origin of coördinates and the light-particles are emitted in the direction of the negative portion of the axis of z. We then have

$$E_x = \frac{x}{r^3} f\left(t - \frac{r}{c}\right) + \frac{xz}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right), \quad H_x = -\frac{yr}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right),$$

$$E_y = \frac{y}{r^3} f\left(t - \frac{r}{c}\right) + \frac{yz}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right), \quad H_y = \frac{xr}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right),$$

$$E_z = \frac{z}{r^3} f\left(t - \frac{r}{c}\right) - \frac{r-z}{cr^2} f'\left(t - \frac{r}{c}\right), \quad H_z = 0,$$

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[1] Electrical and Optical Wave Motion, Cambridge University Press (1915), p. 128, Messenger of Mathematics, May (1915), p. 1.

where $f[t - (r/c)]$ is an arbitrary function which determines the instantaneous value of the electric charge associated with the pole O.

A line of electric force can be regarded as the locus of light-particles projected in directions specified by the successive values of a unit vector s which is a function of $\tau = t - (r/c)$. Assuming that the vector E is proportional to the vector $rds - csd\tau$, we see that if (l, m, n) are the 3 components of s they must be supposed to vary with τ according to the equations

$$\frac{dl}{d\tau} = -\frac{ln}{1 + n}\frac{f'(\tau)}{f(\tau)}, \quad \frac{dm}{d\tau} = -\frac{mn}{1 + n}\frac{f'(\tau)}{f(\tau)}, \quad \frac{dn}{d\tau} = (1 - n)\frac{f'(\tau)}{f(\tau)}.$$

Solving these equations we obtain

$$(1 - n)f(\tau) = (1 - n_0)f_0 = a,$$

say,

$$\frac{lf}{\sqrt{2f - a}} = \frac{l_0 f_0}{\sqrt{f_0(1 + n_0)}}, \quad \frac{mf}{\sqrt{2f - a}} = \frac{m_0 f_0}{\sqrt{f_0(1 + n_0)}},$$

where l_0, m_0, n_0, f_0, denote initial values of l, m, n, f, at some instant τ_0. These equations indicate that l, m and $1 + n$ all vanish when $2f = a$. This means that a line of electric force may end at some position of an electrified light-particle. If $f(\tau)$ is initially zero and $f'(\tau)$ has always the same sign, the lines of force all start from O and end at some position of an electrified light-particle, but if $f'(\tau)$ changes sign so that $f(\tau)$ fluctuates in value some lines of force go from one electrified light particle to another, while some of the lines of force issuing from O bend outwards to avoid the region occupied by the lines of force just mentioned. It is easy to see in fact that if f fluctuates, l and m do so also, and a line of force assumes a wavy form.

The fact that lines of force seem to avoid the region occupied by an emitted electric doublet indicates that the presence of doublets emitted in different directions at different times, so as to cover nearly every direction, may give the lines of force the characteristics of separate physical entities. The unit tube of force may thus be more than a mathematical fiction.

When $f(\tau)$ is not initially zero some of the lines of force issuing from O may go to infinity instead of ending at an electrified light-particle. The total amount of electricity that has been projected from O may, in fact, not be sufficient to compensate the charge at O at the time under consideration and so it is natural to expect that some lines of force will go to infinity.

The properties of these moving lines of force seem to be perfectly analogous to those of the stationary lines of force used in electrostatics. When $f(\tau)$ fluctuates for a short time and then remains constant a pulse travels outwards and leaves an electrostatic field behind. The flow of energy in the field is naturally of some interest. A simple calculation indicates that

$$E_yH_z - E_zH_y = -\frac{xz}{cr^4(r+z)}ff' + \frac{x}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2,$$

$$E_zH_x - E_xH_z = -\frac{yz}{cr^4(r+z)}ff' + \frac{y}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2,$$

$$E_xH_y - E_yH_x = \frac{x^2+y^2}{cr^4(r+z)}ff' + \frac{z}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2.$$

In addition to the radial flow of energy specified by the second terms there is a tangential flow whose direction depends upon whether $[f(\tau)]^2$ is increasing or decreasing. If $f(\tau)$ fluctuates and returns to its original value, so that the emitted light-particles form an electric doublet, energy may be supposed to flow out tangentially from one constituent of the doublet and flow in tangentially to the other so that on the whole the energy associated with the doublet remains constant.

2. *Theory of Gravitation.*—We shall now suppose that an electrified particle is continually absorbing and emitting doublets which cause its charge to fluctuate. The reason why the charge appears to fluctuate is that when an electric charge travels with velocity c there are no terms in its field of order r^{-2} at an ordinary point of space, but when the charge is stopped for a short interval of time these terms appear as soon as a point is reached by a pulse which travels outwards from the point where the charge is stopped or deflected. If now the constituents of a doublet reach a charged particle at slightly different times the charge on the particle will appear to fluctuate. Similarly if the constituents of a doublet are emitted from a charged particle at slightly different times the charge on the particle will appear to fluctuate.

We shall now suppose that a charged particle absorbs doublets one by one and emits them one by one, the emission and absorption of individual doublets being regarded as separate events separated by an interval of time. The direction in which a doublet is emitted is supposed to vary at random so that on the whole doublets are emitted equally in all directions.

The number of doublets emitted per second is supposed to be enormous being large, perhaps, in comparison with the number of electrons in the universe. We shall suppose, however, that this number is proportional to the mass of the particle.

Let us now consider two particles A and B whose charges and masses are normally e, m and e', m' respectively. The number of doublets absorbed by B per second is supposed to be proportional to m. Of these doublets a certain percentage may be supposed to have been emitted by A. Now any particular doublet may have come from any one of the charged particles in the universe or it may have come from the ether. The number which comes from the ether may, perhaps, be large in comparison with the number which come directly from charged particles, for the doublet from the ether may have really come from a charged particle by a zigzag path. It seems reasonable, however, to assume that the percentage of doublets which have come directly from A to B in unit time is proportional to the number emitted from A per unit time, *i.e.*, to the mass of A. The number of doublets which B receives from A per unit time, when A and B are stationary, may thus be supposed to be proportional to the product of the masses of A and B. It is possible that this statement may need alteration when there are material particles between A and B but for simplicity we shall disregard this case. Let us now consider the very short interval of time during which the charge on B fluctuates on account of the arrival of a doublet emitted from A. If at an instant t of this interval the charge on B is $e' - F(t)$ the charge on A at the corresponding instant $t - 1/c(AB)$ may be supposed to be $e + F(t)$. The mean value of the force exerted by A on B is thus

$$\int_{t_0}^{t_0+T} [e' - F(t)]E\,dt,$$

where

$$f(\tau) = e + F\left(\tau + \frac{r}{c}\right).$$

Now if F returns to its initial value at the end of the interval T and if the mean value of F is zero, the mean value of the force is

$$\frac{ee'}{r^2} - \frac{1}{r^2T}\int_{t_0}^{t_0+T} [F(t)]^2 dt.$$

Taking the mean value for a long period of time such as a second, the number of terms of the second type is proportional to the product of the masses of A and B, so in addition to the ordinary electrostatic force ee'/r^2 we obtain an attraction proportional to the product of the masses of A and B and inversely proportional to the square of the distance between A and B. It is thought that this force represents the gravitational attraction of A on B.

If in a short interval of time T the charge on B fluctuates on account of the absorption of a doublet, but the charge on A does not fluctuate during the corresponding interval $t - (r/c)$ to $t + T - (r/c)$ the mean value of the force exerted by A on B is simply ee'/r^2.

Again, if the charge on A fluctuates during the interval τ to $\tau + T$ on account of the emission of a doublet, while the charge on B does not fluctuate at all during the corresponding interval $\tau + (r/c)$ to $\tau + T + (r/c)$, the mean value of the force exerted by A on B is again ee'/r^2. The emission by A of a doublet during the interval τ to $\tau + T$ and the absorption by B of a different doublet during the corresponding interval $\tau + (r/c)$ to $\tau + T + (r/c)$ is supposed to be a rare event which results only in a small correction to the ordinary force of gravitation. As shown in § 1 the emission of doublets is accompanied by the emission of pulses but when the mass of the source remains constant these pulses follow one another so rapidly that the equivalent frequency of the radiation is much higher than that of light or X-rays and so the radiation remains ordinarily undetected.

3. *Theory of Radiation.*—When the mass of a charged particle is altered by the emission or absorption of a doublet, the doublet may be supposed to carry available energy equal to the amount of energy gained or lost by the charged particle. If the charged particle loses energy the doublet carries it away and may give the energy up to some other charged particle. Since the energy of the other charged particle is less than before, the number of doublets emitted per second becomes smaller than before but the number emitted in a very short interval of time does not change immediately. The result is that sooner or later there is a gap in the emission of doublets and this gap may be supposed to occur periodically with a frequency proportional to the amount of energy lost. This makes the number of doublets emitted per second proportional to the new mass.

This state of affairs may be supposed to continue for some time until a new steady state is reached. The light emitted by the charged particle thus consists of a long train of waves.

This theory of radiation seems to be quite compatible with Bohr's theory for it depends on the assumption that the amount of energy lost by an electron in a discontinuous change is proportional to the frequency of the light which is emitted after the sudden change of energy. The energy of the electron may be supposed to remain practically constant in spite of the emission of light. See, however, the note at the end.

When a particle absorbs energy from a doublet possessing available energy the number of doublets emitted per second increases but the number emitted in a short interval of time does not increase immediately. The result is that more doublets than usual are emitted at certain intervals and we again get light of a frequency proportional to the amount of energy gained. In this case, however, no doublet carrying available energy is emitted from the charged particle.

It should be mentioned that the present theory of radiation agrees in some respects with that which has been proposed by W. H. Bragg It also agrees with the theories of Planck and Einstein.

August 11, 1920.

NOTE ADDED NOVEMBER 18, 1920.

When an electron is adjusting itself to a new steady state of motion it may be supposed to oscillate with the frequency at which gaps occur in the shower of doublets. The lines of force of the electron at this time may end at one or more of the doublets that have been emitted and it may be that the available energy, of which we have spoken, is associated with undulations produced by the electron on a doublet's lines of force after the doublet has been emitted. The final transfer of the energy of these undulations to the doublet may be a result of the tangential flow of energy and of the change in form of the lines of force when the doublet is being absorbed

The number of doublets which hit a charged particle of mass m every second may, perhaps, be given by the formula $h\nu = mc^2$, where h is Planck's constant; at any rate it would seem, if the above ideas are correct, that ν must be a lower limit to the number of doublets. This number ν is of order 10^{20} in the case of an electron.

NEW BOOKS.

A Kinetic Theory of Gases and Liquids. By **Richard D. Kleeman.** New York: John Wiley and Sons, Inc., 1920. Pp. xvi + 272. Price, $3.00.

It is the purpose in this book to develop a general kinetic theory applicable to matter in any state, in so far as it is possible to do so without knowing the exact nature and immediate result of molecular interaction. It is shown that the subject may thus be developed along sound mathematical lines and a number of important formulæ derived which furnish a structure by means of which further advances may be made.

The Nature of Animal Light. By **E. Newton Harvey.** Philadelphia: J. B. Lippincott Co., 1920. Pp. x + 182. Price, $1.50.

The phosphorescence of living organisms from the point of view of the biological chemist is the theme of this volume. After chapters on Luminescence and Incandescence and the Physical Nature of the Light emitted by Animals, the Structure of the Luminous Organs is taken up. The book concludes with the establishment of the general proposition that the light emitted by animals is due to the action of an oxidizing enzyme "lucifrase" upon a photogenic substance termed "luciferin," and with a short chapter on the dynamics of luminescence.

Aëronautics. By **Edwin Bidwell Wilson.** First Edition. New York: John Wiley and Sons, Inc., 1920. Pp. vii + 265. Price, $4.00.

Contains the substance of courses of lectures on topics in dynamics, rigid and fluid, which are especially applicable to the problems of flight. Of particular interest are the chapters on Stability of the Airplane; on Planar Motion; on the Forces on an Airplane and on Stream Function and Velocity Potential.

Simplified Method of Tracing Rays through any Optical System of Lenses, Prisms, and Mirrors. By **Ludwik Silberstein.** New York: Longmans, Green and Co., 1918. Pp. ix + 37.

The problem discussed in this brief manual is:

"*Given the ray incident upon any system of lenses, mirrors and prisms, find the emergent ray.*" The treatment which is vectorial is comprised under nine heads:

(1) The Fundamental Reflection and Refraction Laws. (2) Nature of the Data defining an Incident Ray. (3) The Transfer Formula for Spherical Surfaces. (4) The Tracing Process for Systems of Spherical Surfaces. (5) Hints and Examples. (6) Prismatic Systems. (7) System of Plane Mirrors.

(8) Dyadic representing the most General Reflector. (9) Differential Properties of Reflected or Refracted Pencils.

System of Physical Chemistry. Vol. III. Quantum Theory. By WILLIAM C. McC. LEWIS. New York: Longmans, Green and Co., 1919.

This third volume of Lewis's Physical Chemistry deals "with some physicochemical applications of the principles of statistical mechanics."

After two introductory chapters on Probability, Entropy, Equipartition of Energy and like topics and on Planck's Concept of Quanta, the remainder of the volume contains the relations of the Quantum theory to: Atomic Heats of Solids; Molecular Heats of Gases; Structures of the Atom; Photo-electric Effects; Thermal Reactions; Reaction Velocities; Mass Reaction and the Heat of Reaction.

Researches in Physical Optics. Part II. Resonance Radiation and Resonance Spectra. By R. W. WOOD. New York: Columbia University Press, 1919. Pp. viii + 184.

This sumptuous volume contains fifteen papers by R. W. Wood and various co-workers (Kimura, Speas, Ribaud, Hemsalech, Meyer, Dunoyer and Mohler).

Ten of the titles are on resonance spectra of iodine and sodium vapors. Two deal with the fluorescence of gases excited by very short ultra-violet waves and the remainder are on Spectrographs of Long Focus; The Scattering and Regular Reflection of Light and the Separation of Close Spectrum Lines.

Report on the Quantum Theory of Spectra. By L. SILBERSTEIN. London: Adam Hilger, 1920. Pp. 1 + 42.

A very brief statement of Bohr's theory; Rydberg's constant; Orbits in Space; Sommerfeld's theory of Fine Structure; Theory of Roentgen Spectra, of the Stark effect, Zeeman effect, photo-electric effect, etc.

Mirrors, Prisms and Lenses. By JAMES P. C. SOUTHALL. New York: The Macmillan Company, 1918. Pp. xix + 579. Price, $3.25.

This is a simplified version of the author's Principles and Methods of Geometrical Optics. A considerable amount of new material has however been introduced. In the earlier chapters the well-known phenomena of the passage of light through prisms and lenses are simply dealt with and the treatment throughout has been adapted to the mathematical limitations of the average undergraduate.

The volume contains however many topics normally found only in the more advanced treatises on Applied Optics.

Colloid Chemistry. By JEROME ALEXANDER. New York: D. Van Nostrand Company, 1919. Pp. v + 90.

A brief elementary manual giving a classification of colloids and their general properties and enumerating various applications. A few pages are devoted to a description of the ultra-microscope.

Notes on Magnetism. By C. G. LAMB. Cambridge: The University Press, 1919. Pp. viii + 94. Price, $1.50.

An eminently practical set of notes for students of engineering. There are seventy topics regarded by the author as essential to enable students "to read the ordinary technical textbooks with intelligence."

These are treated with brevity and simplicity and there is no pretence of of novelty either of matter or manner.

Annuaire Pour L'An 1920, Publie Par Le Bureau des Longitudes. Paris: Gauthier-Villars et Cie, 1920. Pp. viii + 708.

The Annual of the French Bureau of Longitudes for 1920 contains the usual surprising amount of usefu linformation, tabular and descriptive, on Astronomy, Navigation, Metrology, Physics, Chemistry and Engineering.

Unified Mathematics. By LOUIS C. KARPINSKI, HARRY Y. BENEDICT and JOHN W. CALHOUN. Boston: D. C. Heath and Co., 1918. Pp. viii + 522.

A course in elementary mathematics intended for first year students in colleges and technical schools. The characteristic feature is the relinquishment of the usual divisions of the subject and the development of the essential unity of the two great natural divisions, *analysis* and *geometry* in the treatment of such general topics as:

Linear and quadratic functions; Trigonometric functions; Series; Triangles and the Conic Sections.

Leçons de Physique Générale. Électricité et Magnétisme. By JAMES CHAPPUIS AND ALPHONSE BERGET. Paris: Gauthier-Villars et Cie, 1920. Pp. I + 623.

The second volume of the third edition of the well known compendium of Chappuis and Berget, has been entirely rewritten by Chappuis and Lamotte. It deals with Electricity and Magnetism and covers the ground in the manner characteristic of the best class of French textbooks.

The Indian Association for the Cultivation of Science. Bulletin No. 15; Calcutta: Printed at the Baptist Mission Press and Published by the Indian Association for the Cultivation of Science, 1918, iii + 158. Price, Rs. 2/8.

This Bulletin contains Part I. (158 pp.) of a monograph by Professor C. V. Raman on the "Mechanical Theory of the Vibrations of Bowed Strings and of Musical Instruments of the Violin Family, with Experimental Verification of the Results."

There are many plates of velocity diagrams, displacement diagrams and vibration curves illustrating the nine types of vibration discussed by the author.

The effects of pressure and velocity of bowing; of finite width of the bow; of the yielding of the bridge; of the thickness of the string; of damping and of the body of the instrument are considered at some length in this preliminary portion of Professor Raman's treatise.

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THE

PHYSICAL REVIEW.

THE ABSOLUTE SIGNIFICANCE OF MAXWELL'S EQUATIONS.

By F. D. Murnaghan.

Synopsis.

Elementary Proof of a Generalized Stokes' Theorem for Hyper-space.—After giving precise definitions of the ideas of integrals extended over lines, surfaces and hyper-surfaces in a space of arbitrary metrical character, the author presents an elementary proof of a generalized Stokes' theorem which reduces the integral over an open hyper-surface to an integral over the closed surfaces which bound it.

Statement of Maxwell's Equations as Integrals over Hyper-surfaces, brings out their physical significance more clearly as relations between surface integrals and hyper-surface integrals. It is explained how this form of statement lends itself at once to the methods of the absolute differential calculus.

Electromagnetic Potential.—The *existence* of this potential, which is a hyper-space combination of vector and scalar potentials, is proved and the normal form of its *line integral over any open curve* is given (1) for an electromagnetic field where $(\overline{E}.\overline{B}) = 0$, and (2) for any other field.

THE fundamental importance of Maxwell's electromagnetic equations has long been recognized. In recent years, however, attention has been directed to the *absolute* character of physical equations and the aim has been to express physical laws in a manner independent of the particular coördinates used to give measures of the quantities involved. It may be well, then, to point out explicitly the absolute content of Maxwell's equations, or in other words to answer the query "what absolute relation connecting the vectors of electromagnetism with space and time is expressed by these fundamental equations?" To explain further what is attempted here let us recall the *induction vector* $\overline{B}$ which is so important to the electrical engineer; to tell a student of engineering that the induction vector $\overline{B}$ is more important or fundamental than the magnetic force vector $\overline{H}$, for example, because its divergence $(\partial B_x/\partial x) + (\partial B_y/\partial y) + (\partial B_z/\partial z)$ is zero is not so satisfactory or illuminating as to express the same fact by saying that the surface integral $\int B_n dS$ over any closed surface is zero. This absolute statement gives at once a useful picture of what is taking place and leads to the conception of induction lines and tubes which is so fruitful. Now it so happens that the content of Maxwell's equations can be stated in an analogous absolute form and we can apply to them the methods of the Absolute Calculus explained in Einstein's paper on General Relativity in the Annalen der Physik (1916). However we shall have to speak of four dimensional space and as it is desirable to allow gravitational, *i.e.*, non-euclidean spaces it may be permitted to recall the mathematical definition of space and to show that the results obtained in no way depend on the metrical properties of the space.[1]

§ 1.

For definiteness we shall speak of a four-dimensional space although the argument is identical for a space of any number of dimensions. By the term *point* is meant an ordered set of four numbers (x_1, x_2, x_3, x_4), which set is for brevity spoken of as the point x—the numbers being called the four coördinates of the point. By the term space we mean simply the aggregate of all the points x but we shall suppose that our space is real and continuous *i.e.*, the numbers (x_1, x_2, x_3, x_4) can each take up all real values in a continuous manner. If the coördinates of a variable point x are expressible as continuous functions of a single independent variable u, say,

$$x_i = x_i(u), \qquad i = 1, 2, 3, 4,$$

the point x is said to trace a *curve* in the space, as the independent variable u varies; we shall suppose that our curves are *smooth* *i.e.*, the functions $x_i(u)$ have continuous derivatives with respect to u.[2] But if the coördinates of a variable point x have more freedom and are expressible as functions of two independent variables (u_1, u_2), say,

$$x_i = x_i(u_1, u_2), \qquad i = 1, 2, 3, 4,$$

the point is said to trace out a surface or variety of two dimensions as the independent variables (u_1, u_2) vary. Again we make the hypothesis of smoothness. The functions $x_i(u_1, u_2)$ must be *distinct*, *i.e.*, not all the Jacobian determinants

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[1] Most of the results stated in this note are familiar to several writers on Mathematical Physics and the reader is referred to a paper by Bateman, Proc. Lond. Math. Soc., Ser. 2, Vol. (8), 1910, p. 223, for further developments. It is hoped that the elementary presentation here given will enable physicists to become more easily familiar with the important results of this and similar papers.

[2] It will not affect our argument if any of our curves have several "corners" *i.e.*, the derivatives may be discontinuous at a finite number of values of u.

$$\frac{\partial(x_i x_j)}{\partial(u_1 u_2)} \equiv \begin{vmatrix} \dfrac{\partial x_i}{\partial u_1} & \dfrac{\partial x_i}{\partial u_2} \\ \dfrac{\partial x_j}{\partial u_1} & \dfrac{\partial x_j}{\partial u_2} \end{vmatrix} \qquad i, j = 1, 2, 3, 4,$$

can vanish for all values of u_1, u_2. If this were the case our point x would not "have more freedom" and would really trace a curve. Similarly if $x_i = x_i(u_1, u_2, u_3)$, where the x_i are distinct functions of three independent variables (u_1, u_2, u_3), the point is said to trace a hypersurface or variety of three dimensions as the independent variables vary. Once more we make the hypothesis of smoothness of our variety. It remains to define the terms line integral, surface integral, hypersurface integral and as we are not using any metrical properties of space we shall avoid terms such as "element of arc," "surface element," "volume element" so familiar in this connection.

Line Integral.

By

$$\int_C (A_1 dx_1 + A_2 dx_2 + A_3 dx_3 + A_4 dx_4) \equiv \int_C \sum_{s=1}^{4} (A_s dx_s)$$

over a curve C whose equations are $x_i = x_i(u)$, $i = 1, 2, 3, 4$, and where the A's are given functions of the variables (x_1, x_2, x_3, x_4), we mean that we write in for each x_i its value $x_i(u)$ and for dx_i its value $(dx_i/du)du$ and then evaluate the definite integral

$$\int_{u^0}^{u'} \left\{ \sum_s \left(A_s \frac{dx_s}{du} \right) \right\} du$$

u^0 and u' being the values of the independent variable, or parameter, u corresponding to the end points of the curve C. It is important to notice that this line integral is really *attached to the curve, i.e.*, its value is independent of the particular parameter u used to define the curve; for suppose we change the parameter u to another v by an equation $u = u(v)$, our line integral is

$$\int_{v^0}^{v'} \left\{ \sum_s A_s \frac{dx_s}{dv} \right\} dv$$

and by the well-known rule for changing the variable of integration the previous value is

$$\int_{v^0}^{v'} \left(\sum A_s \frac{dx_s}{du} \right) \frac{du}{dv} dv,$$

which is identical with that just written since

$$\frac{dx_s}{dv} = \frac{dx_s}{du} \cdot \frac{du}{dv}.$$

Surface Integral.

Consider a set $A_{s_1 s_2}$, $s_1, s_2 = 1, 2, 3, 4$, of functions of (x_1, x_2, x_3, x_4); since s_1, s_2 may each take four values there are 16 functions; but we shall make the hypothesis that the $A_{s_1 s_2}$ form a skew symmetric set, *i.e.*, a permutation of the suffixes s_1, s_2 merely alters the sign of the function. This makes the four functions $A_{11}, \cdots, A_{44}$ identically zero and the remaining twelve are, in pairs, equal in magnitude but opposite in sign. There are, then, really only six essentially distinct functions $A_{s_1 s_2}$. Now when we are given any surface

$$x_i = x_i(u_1, u_2), \qquad i = 1, 2, 3, 4,$$

we may write down the double integral

$$I_2 \equiv \int \left\{ \sum_{s_1 s_2}^{1 \ldots 4} A_{s_1 s_2} \frac{\partial x_{s_1}}{\partial u_1} \frac{\partial x_{s_2}}{\partial u_2} \right\} du_1 du_2$$

extended over that range of values of (u_1, u_2) which corresponds to the surface [each x being replaced by $x(u_1, u_2)$]. The double summation under the integral sign is extended over all permutations of s_1, s_2 but we can, on account of the skew-symmetric relations $A_{s_1 s_2} = - A_{s_2 s_1}$, group the terms where the same combination $s_1 s_2$ occurs and we obtain

$$I_2 = \int \left\{ \sum_{s_1 s_2}{}' A_{s_1 s_2} \frac{\partial(x_{s_1}, x_{s_2})}{\partial(u_1, u_2)} \right\} du_1 du_2$$

where the prime attached to the sign of summation indicates that $s_1 < s_2$. As before we may satisfy ourselves that this integral is really *attached to the surface* and in no way depends for its value on the accidental choice of parameters (u_1, u_2) used to specify the surface. For if parameters (v_1, v_2) are used we have as equations connecting the parameter pairs $u_1 = u_1(v_1, v_2)$; $u_2 = u_2(v_1, v_2)$ and the well-known formula for change of variables in a double integral gives us

$$I_2 = \int \left\{ \sum_{s_1 s_2}{}' A_{s_1 s_2} \frac{\partial(x_{s_1}, x_{s_2})}{\partial(u_1, u_2)} \right\} \frac{\partial(u_1, u_2)}{\partial(v_1, v_2)} dv_1 dv_2$$

—the integral being extended over the region of values (v_1, v_2) corresponding to the previous region of values (u_1, u_2)—and this by the elementary theorem as to the multiplication of Jacobians is

$$\int \left\{ \sum_{s_1 s_2}{}' A_{s_1 s_2} \frac{\partial(x_{s_1}, x_{s_2})}{\partial(v_1, v_2)} \right\} dv_1 dv_2.$$

In order to bring into prominence this independence of the choice of parameter I_2 is usually written in the abbreviated form $\int \sum' A_{s_1 s_2} dx_{s_1} dx_{s_2}$ which form is, then, merely a *convenient way of writing* the surface integral defined in this paragraph.

Hyper-Surface Integral.

After the previous details this can be explained briefly. Consider the set of $4^3 = 64$ functions $A_{s_1s_2s_3} \cdots s_1, s_2, s_3 = 1, 2, 3, 4$, and agree that a mere interchange of two suffixes simply changes the sign of the function. Forty of the functions become identically zero and the remaining 24 arrange themselves in four groups of six functions, each group consisting of the same function three times with a positive and three times with a negative sign. We define

$$I_3 \equiv \int \left\{ \sum_{s_1s_2s_3}^{1\ldots4} A_{s_1s_2s_3} \frac{\partial x_{s_1}}{\partial u_1} \frac{\partial x_{s_2}}{\partial u_2} \frac{\partial x_{s_3}}{\partial u_3} \right\} du_1du_2du_3,$$

where each x_i is replaced by $x_i(u_1, u_2, u_3)$, $i = 1, 2, 3, 4$, from the equation of the hypersurface and the triple integral is extended over the range of values (u_1, u_2, u_3) corresponding to the hypersurface. Grouping the terms as before we have

$$I_3 = \int \left\{ \sum_{s_1s_2s_3}{}' A_{s_1s_2s_3} \frac{\partial(x_{s_1}, x_{s_2}, x_{s_3})}{\partial(u_1, u_2, u_3)} \right\} du_1du_2du_3,$$

where the prime indicates that $s_1 < s_2 < s_3$. This form shows that I_3 is really attached to the hypersurface and to indicate this we abbreviate it to

$$I_3 = \int \sum_{s_1s_2s_3}{}' A_{s_1s_2s_3}dx_{s_1}dx_{s_2}dx_{s_3}$$

there being but four terms in the summation.

By the term closed variety, *i.e.*, curve, surface, hypersurface we mean one which has no boundaries. (By the boundary of a curve we mean its endpoints; of a surface its bounding curve and of a hypersurface its bounding surface.) A variety which has boundaries is said to be open and it is clear[1] that any open variety lying at a finite distance (*i.e.*, all of whose points have finite coördinates) has a *closed* variety of one less dimension as its boundary. We have not defined points with infinite coördinates so we shall not discuss them here. There is an important relation known as Stokes' generalized lemma connecting an integral over any open variety with an integral over its closed boundary. It will be seen that this yields as special cases those fundamental theorems of Mathematical Physics known as Stokes' Theorem, Green's Lemma, Gauss' Theorem. From a pedagogical point of view it seems preferable to discuss these important theorems in this connected way and not to give more or less complicated proofs of each theorem separately; proofs which, moreover, give no hint of the close relation between the theorems.

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[1] Bearing in mind that the coördinates of points on any variety are *continuous* functions of the parameters used in specifying the variety.

It will be best to start with the simplest case although the method is the same for all.

Case I. *Open Surface Bounded by One or More Closed Curves.*

There is no lack of generality in supposing that there is but one closed curve for boundary for we can connect all the separate curves by lines drawn on the surface in such a way that in tracing the boundary these lines will each be traversed twice—in opposite directions. Hence any integral extended over the new boundary will be the sum of the values of the same integral extended over the original closed curves—the contributions from the new connecting lines cancelling out. The boundary will be given by a relation between the parameters $\phi(u_1, u_2) = 0$. It will be convenient to avail ourselves of the arbitrariness as to the choice of parameters and we write $v_2 = \phi(u_1, u_2)$, $v_1 = f(u_1, u_2)$, where f is any convenient function distinct from ϕ. The advantage of the new parameters is that the boundary is given by $v_2 = 0$ and so is one of the curves $v_2 = \text{const.}$ All these curves are closed, since they cannot intersect the boundary;[1] and we obtain the entire surface as traced out by the parameter curves $v_2 = \text{constant}$, the constant varying for each curve from zero to some fixed value. Consider now a line integral

$$I_1 = \int \sum_{s_1}^{1\ldots 4} (A_{s_1} dx_{s_1})$$

attached to one of these level curves; v_2 is to be constant in the integration and I_1 is a function of v_2 which merely says that I_1 will depend on the particular curve v_2 along which the integral is extended. Writing out I_1 in full,

$$I_1 = \int \left\{ \sum_{s_1}^{1\ldots 4} A_{s_1} \frac{\partial x_{s_1}}{\partial v_1} \right\} dv_1,$$

we get on differentiating

$$\frac{dI_1}{dv_2} = \int \sum_{s_1}^{1\ldots 4} \left(A_{s_1} \frac{\partial^2 x_{s_1}}{\partial v_1 \partial v_2} + \sum_{s_2}^{1\ldots 4} \frac{\partial A_{s_1}}{\partial x_{s_2}} \frac{\partial x_{s_2}}{\partial v_2} \frac{\partial x_{s_1}}{\partial v_1} \right) dv_1,$$

where we have remembered that A_{s_1} is not directly a function of the parameters v but a function of the x's. Now along any *closed* curve

$$\int \frac{\partial F}{\partial v_1} dv_1 = 0,$$

where F is any *one-valued* function of position because the integral is simply the difference of the values of F at the end points of the curve.

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[1] The range of values of the parameters of a surface is always supposed to be so restricted that the same point x on the surface cannot correspond to different values of the parameters.

Take

$$F = \sum_{s_1}^{1\ldots4} A_{s_1}\frac{\partial x_{s_1}}{\partial v_2}$$

and we have

$$\int\left\{\sum_{s_1}\left(A_{s_1}\frac{\partial^2 x_{s_1}}{\partial v_1\partial v_2} + \sum_{s_2}^{1\ldots4}\frac{\partial A_{s_1}}{\partial x_{s_2}}\frac{\partial x_{s_2}}{\partial v_1}\frac{\partial x_{s_1}}{\partial v_2}\right)\right\}dv_1 = 0.$$

From this we have

$$\frac{dI_1}{dv_2} = \int\left\{\sum_{s_1,s_2}^{1\ldots4}\left(\frac{\partial A_{s_1}}{\partial x_{s_2}} - \frac{\partial A_{s_2}}{\partial x_{s_1}}\right)\frac{\partial x_{s_1}}{\partial v_1}\frac{\partial x_{s_2}}{\partial v_2}\right\}dv_1,$$

where we have interchanged in the latter term the symbols of summation s_1 and s_2. Writing now

$$A_{s_1s_2} \equiv \frac{\partial A_{s_1}}{\partial x_{s_2}} - \frac{\partial A_{s_2}}{\partial x_{s_1}}$$

we see that it is skew symmetric and we obtain

$$I_1 = \int\sum{}' A_{s_1s_2}dx_{s_1}dx_{s_2}$$

(there is no constant of integration to be added since the curves $v_2 =$ const. must close to a point[1] and so I_1 at one of the limits for v_2 is zero). (It might be argued that we should write $-I_1$ on the left-hand side of this last equation but there is an indeterminateness in the sign of $\int\sum' A_{s_1s_2}dx_{s_1}dx_{s_2}$ anyhow. When we said that "the integral" was independent of the parameters it would have been more accurate to say its "absolute value." When the parameter transformation is direct the sign is unchanged, when inverse it is changed. The Jacobian of the transformation—$\partial(u_1u_2)/\partial(v_1v_2)$—is not supposed to vanish and so it is definitely positive or negative; when positive the transformation is direct, otherwise inverse. This change in sign is expressed geometrically by referring to the *two sides of a surface* and when we write

$$\int\sum_{s_1}^{1\ldots4} A_{s_1}dx_{s_1} = +\int\sum{}' A_{s_1s_2}dx_{s_1}dx_{s_2}$$

the surface integral is said to be extended over that side of the surface which is positively related to the direction of integration along the boundary). Attention is called to the fact that when all $A_{s_1s_2} \equiv 0$ the integral $I_1 = \int\sum A_{s_1}dx_{s_1}$ attached to *any closed curve* is zero. In this case I_1 is said to be the *integral of an exact differential*. The conditions $A_{s_1s_2} \equiv 0$ are necessary as well as sufficient for this. To see this, take the surface $x_{s_1} = u_1$, $x_{s_2} = u_2$, whilst the other x's are constant; the surface integral reduces to $\int A_{s_1s_2}du_1du_2 = 0$ and this is to be zero for any portion of the surface. If $A_{s_1s_2} \neq 0$ we take a point where it is positive and on account of continuity it will be definitely positive in the neighborhood of this point and the integral could not vanish over a small portion of the surface $x_{s_1} = u_1$; $x_{s_2} = u_2$ lying in this neighborhood.

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[1] Or else a segment of a curve traced twice in opposite directions; for example the curves might be confocal ellipses.

Case 2. Open Hyper-surface Bounded by One or More Closed Surfaces.

Just as in Case 1 there is no lack of generality in supposing the boundary to consist of a *single* closed surface which will be given by a relation $\phi(u_1, u_2, u_3) = 0$ on the parameters (u_1, u_2, u_3) of the hypersurface. Make a change of parameters by writing v_1, v_2, v_3 as distinct functions of (u_1, u_2, u_3) and such that $v_3 \equiv \phi(u_1, u_2, u_3)$. Then the surfaces $v_3 =$ const. trace out the hypersurface as the constant varies; they are *closed* surfaces and the curves got on any one of them by putting $v_1 =$ const. or $v_2 =$ const. are *closed* curves. Consider now the integral

$$I_2 = \int \sum_{s_1 s_2}^{1\ldots4} A_{s_1s_2} dx_{s_1} dx_{s_2}$$

attached to one of the surfaces $v_3 =$ const. On writing I_2 out explicitly and differentiating we have

$$\frac{dI_2}{dv_3} = \int \sum_{s_1 s_2}^{1\ldots4} A_{s_1s_2}\left(\frac{\partial^2 x_{s_1}}{\partial v_1 \partial v_3}\frac{\partial x_{s_2}}{\partial v_2} + \frac{\partial x_{s_1}}{\partial v_1}\frac{\partial^2 x_{s_2}}{\partial v_2 \partial v_3}\right.$$
$$\left. + \sum_{s_3}\frac{\partial A_{s_1s_2}}{\partial x_{s_3}}\frac{\partial x_{s_1}}{\partial v_1}\frac{\partial x_{s_2}}{\partial v_2}\frac{\partial x_{s_3}}{\partial v_3}\right) dv_1 dv_2.$$

Now if F is a single-valued function of the x's the integral $\int (\partial F/\partial v_1) dv_1 dv_2$ extended over any of the surfaces $v_3 =$ const., is zero since the curves $v_2 =$ const. on that surface are closed. Take

$$F \equiv \sum_{s_1 s_2}^{1\ldots4} A_{s_1s_2}\frac{\partial x_{s_1}}{\partial v_3}\frac{\partial x_{s_2}}{\partial v_2}$$

and we get

$$\int \sum_{s_1 s_2}^{1\ldots4}\left\{A_{s_1s_2}\left(\frac{\partial^2 x_{s_1}}{\partial v_1 \partial v_3}\frac{\partial x_{s_2}}{\partial v_2} + \frac{\partial x_{s_1}}{\partial v_3}\frac{\partial^2 x_{s_2}}{\partial v_2 \partial v_1}\right)\right.$$
$$\left. + \sum_{s_3}\frac{\partial A_{s_1s_2}}{\partial x_{s_3}}\frac{\partial x_{s_1}}{\partial v_3}\frac{\partial x_{s_2}}{\partial v_2}\frac{\partial x_{s_3}}{\partial v_1}\right\} dv_1 dv_2 = 0.$$

Similarly

$$\int \sum_{s_1 s_2}^{1\ldots4}\left\{A_{s_1s_2}\left(\frac{\partial^2 x_{s_1}}{\partial v_1 \partial v_2}\frac{\partial x_{s_2}}{\partial v_3} + \frac{\partial x_{s_1}}{\partial v_1}\frac{\partial^2 x_{s_2}}{\partial v_2 \partial v_3}\right)\right.$$
$$\left. + \sum_{s_3}\frac{\partial A_{s_1s_2}}{\partial x_{s_3}}\frac{\partial x_{s_1}}{\partial v_1}\frac{\partial x_{s_2}}{\partial v_3}\frac{\partial x_{s_3}}{\partial v_2}\right\} dv_1 dv_2 = 0.$$

Add together these two equations and notice that

$$\sum_{s_1 s_2}^{1\ldots 4} A_{s_1 s_2}\left(\frac{\partial x_{s_1}}{\partial v_3}\frac{\partial^2 x_{s_2}}{\partial v_2 \partial v_1}+\frac{\partial x_{s_2}}{\partial v_3}\frac{\partial^2 x_{s_1}}{\partial v_1 \partial v_2}\right)=0$$

because an interchange of the symbols $s_1 s_2$ of summation should not change its value whilst this interchange does in effect change its sign on account of the skew symmetric character of $A_{s_1 s_2}$. On making some obvious interchanges in the symbols of summation s_1, s_2, s_3 we accordingly obtain

$$\frac{dI_2}{dv_3}=\int\sum_{s_1 s_2 s_3}^{1\ldots 4}\left\{\left(\frac{\partial A_{s_1 s_2}}{\partial x_{s_3}}+\frac{\partial A_{s_2 s_3}}{\partial x_{s_1}}+\frac{\partial A_{s_3 s_1}}{\partial x_{s_2}}\right)\frac{\partial x_{s_1}}{\partial v_1}\frac{\partial x_{s_2}}{\partial v_2}\frac{\partial x_{s_3}}{\partial v_3}\right\}dv_1 dv_2.$$

Writing now

$$A_{s_1 s_2 s_3}\equiv\frac{\partial A_{s_1 s_2}}{\partial x_{s_3}}+\frac{\partial A_{s_2 s_3}}{\partial x_{s_1}}+\frac{\partial A_{s_3 s_1}}{\partial x_{s_2}}$$

we notice that it is skew-symmetric and we can write, on integrating with respect to v_3,

$$I_3=\int\sum_{s_1 s_2 s_3}{}' A_{s_1 s_2 s_3} dx_{s_1} dx_{s_2} dx_{s_3},$$

the integral being extended over the hypersurface. Attention is called to the fact that, if all $A_{s_1 s_2 s_3}\equiv 0$, I_2 extended over any *closed* surface is zero and in this case I_2 is said to be the *integral of an exact differential.* Just as in Case I the vanishing of all $A_{s_1 s_2 s_3}$ may be shown to be a necessary as well as a sufficient condition for

$$I_2\equiv\sum_{s_1 s_2}^{1\ldots 4} A_{s_1 s_2} dx_{s_1} dx_{s_2}$$

to be the integral of an exact differential.

Some of our readers may be interested in the general results for any number of dimensions. The proofs go along exactly as those given; there are two cases, according as p is even or odd (I_p is over the closed variety of p dimensions bounding the variety of $p+1$ dimensions over which I_{p+1} is extended).

A. p odd.

$$A_{s_1\cdots s_{p+1}}=\frac{\partial A_{s_1\cdots s_p}}{\partial x_{s_{p+1}}}-\frac{\partial A_{s_2\cdots s_{p+1}}}{\partial x_{s_1}}+\cdots-\frac{\partial A_{s_{p+1}s_1\cdots s_{p-1}}}{\partial x_{s_p}}.$$

B. p even.

$$A_{s_1\cdots s_{p+1}}=\frac{\partial A_{s_1\cdots s_p}}{\partial x_{s_{p+1}}}+\cdots+\frac{\partial A_{s_{p+1}\cdots s_{p-1}}}{\partial x_{s_p}}.$$

In each case

$$I_p\equiv\int\sum{}' A_{s_1\cdots s_p} dx_{s_1}\cdots dx_{s_p}$$

is equal to

$$I_{p+1}\equiv\int\sum{}' A_{s_1\cdots s_{p+1}} dx_{s_1}\cdots dx_{s_{p+1}}.$$

§ 2. APPLICATION TO MAXWELL'S EQUATIONS.

Consider that half of Maxwell's equations which in the usual notation and units are written

$$\operatorname{curl} \overline{E} + \frac{1}{c}\frac{d\overline{B}}{dt} = 0; \qquad \operatorname{div} \overline{B} = 0$$

and for the sake symmetry write (x_1, x_2, x_3, x_4) for (x, y, z, t). Then if we write

$$\begin{aligned} A_{12} &= -A_{21} = B_z = B_3; & A_{34} &= -A_{43} = cE_z = cE_3 \\ A_{23} &= -A_{32} = B_x = B_1; & A_{14} &= -A_{41} = cE_x = cE_1 \\ A_{31} &= -A_{13} = B_y = B_2; & A_{24} &= -A_{42} = cE_y = cE_2 \end{aligned}$$

the first component

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{1}{c}\frac{dB_x}{dt} = 0$$

of the vector equation may be written

$$\frac{\partial A_{34}}{\partial x_2} + \frac{\partial A_{42}}{\partial x_3} + \frac{\partial A_{23}}{\partial x_4} = 0$$

or in our previous notation $A_{234} = 0$. The other equations express the vanishing of A_{314}, A_{124}, A_{123} and so this group of Maxwell's equations simply says that

$$I_2 = \int \sum_{s_1 s_2}^{1\ldots 4} A_{s_1 s_2} dx_{s_1} dx_{s_2} \equiv \int (B_x dydz + B_y dzdx + B_z dxdy) + c(E_x dxdt + E_y dydt + E_z dzdt)$$

is the integral of an exact differential—its value extended over any closed surface is zero. Just as in the static problem (which is a special case when the surface lies in the three-dimensional variety $x_4 = t =$ const.) the surface integral $\int (B_x dydz + B_y dzdx + B_z dxdy)$ is zero when extended over any closed surface.

The remaining half of Maxwell's equations may be written

$$\operatorname{curl} \overline{H} - \frac{1}{c}\frac{d\overline{D}}{dt} = \overline{j}; \qquad \operatorname{div} \overline{D} = \rho,$$

where $\overline{D}$, the displacement vector, coincides with the electric force vector $\overline{E}$ in empty space, $\overline{j}$ is the current density, ρ the volume density of charge and $\overline{H}$ is the magnetic force vector. With the same notation as before for the integrals write

$$\begin{aligned} A_{12} &= -A_{21} = -D_z; & A_{14} &= -A_{41} = c\overline{H}_x; \\ A_{23} &= -A_{32} = -D_x; & A_{24} &= -A_{42} = c\overline{H}_y; \\ A_{31} &= -A_{13} = -D_y; & A_{34} &= -A_{43} = c\overline{H}_z; \end{aligned}$$

and the first component—

$$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}-\frac{1}{c}\,\frac{dD_x}{dt}=j_x$$

of the vector equation may be written

$$\frac{\partial A_{34}}{\partial x_2}+\frac{\partial A_{42}}{\partial x_3}+\frac{\partial A_{23}}{\partial x_4}=cj_x$$

or

$$A_{234}=cj_x.$$

The other equations may be similarly written $A_{314}=cj_y$; $A_{124}=cj_z$; $A_{123}=-\rho$ and so this group of Maxwell's equations expresses that

$$\int c(H_xdxdt+H_ydydt+H_zdzdt)-(D_xdydz+D_ydzdx+D_zdxdy)$$
$$=\int c(j_xdydzdt+j_ydzdxdt+j_zdxdydt)-\rho dxdydz,$$

the surface integral being extended over the boundary of the *arbitrary* variety of three dimensions over which the triple integral is extended. In the static problem we have Gauss' Theorem which is usually expressed in the form $\int D_ndS=\int\rho dxdydz$; if we are careful to leave the metrical properties of the space undefined the symbol $\int D_ndS$ is simply an abbreviation for $\int(D_xdydz+D_ydzdx+D_zdxdy)$. Incidentally for any *closed* variety of three dimensions we have

$$\int c(j_xdydzdt+j_ydzdxdt+j_zdxdydt)-\rho dxdydz=\text{zero}.$$

For example the variety may be considered as traced out by a moving surface which may change its form continuously as it moves, and the vanishing of this integral simply says that the rate of accumulation of charge inside the surface is equal to the rate of flux of current through the surface. As a matter of fact from the equations

$$cj_x=A_{234}\equiv\frac{\partial A_{23}}{\partial x_4}+\frac{\partial A_{34}}{\partial x_2}+\frac{\partial A_{42}}{\partial x_3},\qquad\text{etc.},$$

we obtain at once $c\operatorname{div}\bar{j}=\partial\rho/\partial t$ which is the analytical statement of what has just been said.

The Vector Potential.

The fact that $\int B_xdydz+B_ydzdx+B_zdxdy+c(E_xdxdt+E_ydydt+E_zdzdt)$ is the integral of an exact differential leads to the conclusion that its value, when extended over any open surface, depends merely on the boundary and it should, therefore, be equivalent to a certain line integral extended over that boundary. In fact let us write the surface integral equal to $\int(c\phi dt-(A_xdx+A_ydy+A_zdz))$—the minus sign being introduced merely for the purpose of maintaining a well-established notation. Here $A_1 = -A_x$; $A_2 = -A_y$; $A_3 = -A_z$; $A_4 = c\phi$ and so, using Case 1 of Stokes' Theorem

$$B_x = A_{23} \equiv \frac{\partial A_2}{\partial x_3} - \frac{\partial A_3}{\partial x_2} = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}.$$

There are two similar equations for B_y, B_z and all three may be grouped in the vector equation $\overline{B} = \operatorname{curl} \overline{A}$. Further

$$cE_x = A_{14} \equiv \frac{\partial A_1}{\partial x_4} - \frac{\partial A_4}{\partial x_1} = -\frac{\partial A_x}{\partial t} - c\frac{\partial \phi}{\partial x};$$

there are two similar equations for E_y, E_z and we again group the three equations in the vector equation

$$\overline{E} = -\operatorname{grad} \phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}.$$

We have then six partial differential equations for the four unknown quantities A_x, A_y, A_z, ϕ. They are *consistent* owing to the fact that

$$\int (B_x dydz + B_y dzdx + B_z dxdy + c(E_x dxdt + E_y dydt + E_z dzdt))$$

is the integral of an exact differential. To make this clear we shall actually construct an integral as follows. Let us try for an integral where $A_x \equiv 0$. The equations

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}; \qquad B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}; \qquad E_x = -\frac{\partial \phi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}$$

yield

$$A_z = -\int_{x_0}^{x} B_y(xyzt)dx + A_z'(y, z, t),$$

$$A_y = \int_{x_0}^{x} B_z(xyzt)dx + A_y'(y, z, t),$$

$$\phi = -\int_{x_0}^{x} E_x(xyzt)dx + \Phi(y, z, t),$$

where x_0 is a constant of integration and A_z', A_y', Φ are, so far, arbitrary functions of y, z, t. On substituting in the three remaining equations and availing ourselves of the conditions for an exact differential—Maxwell's equations in fact—we obtain three equations of the same form as the six we started out with but involving merely *three* variables y, z, t instead of four x, y, z, t. For example from

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

we have

$$B_x(x, y, z, t) = \frac{\partial A_z'}{\partial y} - \frac{\partial A_y'}{\partial z} - \int_{x_0}^{x} \left(\frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) dx$$

$$= \frac{\partial A_z'}{\partial y} - \frac{\partial A_y'}{\partial z} + \int_{x_0}^{x} \frac{\partial B_x}{\partial x} dx - \text{since } \operatorname{div} \overline{B} = 0,$$

whence

$$B_x(x_0yzt) = \frac{\partial A_z'}{\partial y} - \frac{\partial A_y'}{\partial z}.$$

The remaining equations are

$$E_y(x_0yzt) = -\frac{\partial \Phi}{\partial y} - \frac{1}{c}\frac{\partial A_y'}{\partial t}; \qquad E_z(x_0yzt) = -\frac{\partial \Phi}{\partial z} - \frac{1}{c}\frac{\partial A_z'}{\partial t}.$$

As before we seek a solution of this simpler set such that $A_y' \equiv 0$ say—and so on. The solution obtained in this way is

$$A_x = 0; \qquad A_y = \int_{x_0}^{x} B_z(xyzt)dx;$$

$$A_z = -\int_{x_0}^{x} B_y(xyzt)dx + \int_{y_0}^{y} B_x(x_0yzt)dy;$$

$$\phi = -\int_{x_0}^{x} E_x(xyzt)dx - \int_{y_0}^{y} E_y(x_0yzt)dy - \int_{z_0}^{z} E_z(x_0y_0zt)dz,$$

where x_0, y_0, z_0 are constants of integration. The three quantities (A_x, A_y, A_z) form Maxwell's vector potential whilst ϕ is his scalar potential; from our point of view they are intimately bound together to form what is called the *electromagnetic potential*. It may be remarked that the explicit formula obtained is of little interest to the physicist; the electromagnetic potential is useful to him mainly when it can be calculated independently thus enabling him to derive the values of the electric and induction vectors. However it is necessary to show that there actually are functions A_x, A_y, A_z, ϕ with the characteristic property of potential functions—*i.e.*, such that the required vectors can be obtained from them by differentiation. It may be of interest to say a few words more about the line integral $\int c\phi dt - (A_x dx + A_y dy + A_z dz)$; linear differential forms—$\sum_{s_1}^{1\ldots 4} A_{s_1} dx_{s_1}$—have important applications in many branches of theoretical physics, analytical dynamics and hydrodynamics for instance; they have been extensively studied by the mathematicians under the name of Pfaffians. Suppose we are interested in the value of $\int \sum_{s_1}^{1\ldots 4} A_{s_1} dx_{s_1}$ over an open curve and ask are there any extremal curves?; in other words given two points does there exist a joining curve such that the integral over it is a maximum or minimum. By the application of the first principles of the calculus of variations we find as necessary conditions

$$\sum_{s_2}^{1\ldots 4} A_{s_1 s_2} dx_{s_2} = 0, \qquad s_1 = 1, 2, 3, 4,$$

where, as before,

$$A_{s_1 s_2} = \frac{\partial A_{s_1}}{\partial x_{s_2}} - \frac{\partial A_{s_2}}{\partial x_{s_1}}.$$

We have here four homogeneous linear equations for $dx_1 \cdots dx_4$ and they are inconsistent unless the determinant of the coefficients vanishes. This determinant is skew symmetric and is, accordingly a perfect square, it being $(A_{12}A_{34} + A_{13}A_{42} + A_{14}A_{23})^2$. In the problem of the electromagnetic potential line integral, then, the condition for extremals is the vanishing of $(\overline{E} \cdot \overline{B})$. Fields of this type are quite special yet are of frequent occurrence especially in radiation problems. Now it is possible by a change of variables from $x_1 \cdots x_4$ to $(y_1 \cdots y_4)$, say, to reduce the linear differential form to a simpler one which is called the *standard* or *normal* form. There are two cases according to whether extremals do or do not exist:

(*a*) *Extremals exist.* $(\overline{E} \cdot \overline{B}) = 0$. Electromagnetic field is special. Standard form is $\sum_1^p L_s dy_s + dU$, p = zero or 2. The L's and U are functions of $y_1 \cdots y_4$ and therefore indirectly of the original space time variables x, y, z, t.

(*b*) *Non-existence of Extremals.* Standard form is $L_1 dy_1 + L_2 dy_2$.[1]

§ 3. CONNECTION WITH THE ABSOLUTE DIFFERENTIAL CALCULUS.

The statement of Maxwell's equations in the form of relations between integrals lends itself at once to the methods of the absolute calculus, and it may not be out of place to mention here that Bateman, in his paper already quoted, had in this way obtained the general mode of transformation of the electric and magnetic force vectors prior to the publication of Einstein's researches on the *General Relativity Principle.* The differentials dx_s, $s = 1 \cdots 4$, form a contravariant tensor of rank 1; the products $dx_r dx_s$ form, from their definition a skew-symmetric contravariant tensor of rank 2; (it is the product of the difference of the two tensors

$$\frac{\partial x_r}{\partial u_1} \cdot \frac{\partial x_s}{\partial u_2} = \phi^{rs}$$

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[1] It is impossible to give without unduly enlarging this paper proofs of the statements made with reference to linear differential forms. The reader is referred to a delightfully simple yet thorough exposition of the underlying mathematical theory by E. Goursat, Bull. de la Soc. Math. de France, t. 44 (1916), "Sur quelques remarques relatives au problème de Pfaff."

and

$$\frac{\partial x_r}{\partial u_2}\cdot\frac{\partial x_s}{\partial u_1} = \phi'^{rs}$$

by du_1, du_2, and this latter product of the differentials of the independent variables is invariant). Similarly $dx_r dx_s dx_t \equiv \phi^{rst}$ is a skew-symmetric contravariant tensor of rank 3. Consider then the integral

$$\int c(j_x dydzdt + j_y dzdxdt + j_z dxdydt) - \rho dxdydz$$

and let us make the relativistic demand of the invariance of form of physical equations, *i.e.*, when we change our coördinates from (x, y, z, t) to (x', y', z', t') what we shall have for current density in the new system if the integral is to keep its form is

$$j'_x = j_x\frac{\partial(yzt)}{\partial(y'z't')} + \text{etc.} - \frac{\rho}{c}\frac{\partial(xyz)}{\partial(y'z't')},$$

since

$$dydzdt = \frac{\partial(y, z, t)}{\partial(u_1, u_2, u_3)}du_1du_2du_3 = \frac{\partial(y, z, t)}{\partial(x', y', z')}dx'dy'dz'$$
$$+ \frac{\partial(y, z, t)}{\partial(y', z', t')}dy'dz'dt' + \text{etc.}$$

This gives the four quantities j_x, j_y, j_z, $(-\rho/c)$ as the four distinct functions of a covariant, skew-symmetric tensor of rank 3. It is only in the special relativity theory where the transformations are restricted to linear orthogonal transformations that this is equivalent to saying that these 4 quantities form a four vector.

Again from the equivalent surface integral we have that cH_x, cH_y, cH_z, $-D_x$, $-D_y$, $-D_z$ are the six distinct functions of a skew-symmetric covariant tensor of rank 2.

From the surface integral which is zero over every closed surface we get $(B_x, B_y, B_z, cE_x, cE_y, cE_z)$ as the six distinct functions of a skew-symmetric covariant tensor of rank 2 and then the invariance of the line integral gives $(A_x, A_y, A_z, -c\phi)$ as the four components of a covariant tensor of rank 1. That Maxwell's equations are tensor equations follows at once. For from any covariant tensor ϕ_{rs}, of rank two we get by differentiation a covariant tensor of rank 3

$$\phi_{rs,t} \equiv \frac{\partial\phi_{rs}}{\partial x_t} - \sum_g\begin{Bmatrix} rt \\ g\end{Bmatrix}\phi_{gs} - \sum_g\begin{Bmatrix} st \\ g\end{Bmatrix}\phi_{rg},$$

where we have used the ordinary notation for the Christoffel three index symbols of the second kind. By cyclic permutation of the suffixes, we get two other covariant tensors of rank 3

$$\phi'_{rst} \equiv \phi_{str}, \qquad \phi''_{rst} = \phi_{trs},$$

and adding up these three tensors we get a new covariant tensor of rank 3. In particular if ϕ_{rs} is skew symmetric this new tensor turns out to be

$$\frac{\partial\phi_{rs}}{\partial x_t}+\frac{\partial\phi_{st}}{\partial x_r}+\frac{\partial\phi_{tr}}{\partial x_s}.$$

Generalizing this, we see the tensor character of the conditions for an exact differential. Finally, one set of Maxwell's equations express the vanishing of the skew-symmetric covariant tensor of rank 3 formed in this way from the skew-symmetric covariant tensor of rank 2 $(\bar{B},\ c\bar{E})$; and the other set, the equivalence of a skew-symmetric covariant tensor of rank 3 formed in this way from the skew-symmetric covariant tensor of rank 2 $(c\bar{H},\ -D)$ to the current skew-symmetric covariant tensor of rank 3.

Johns Hopkins University,
July 1, 1920.

THE FORMATION OF NEGATIVE IONS IN AIR.

By Leonard B. Loeb.

Synopsis.

Mobility of Negative Ions in Air at Low Pressures.—The purpose of the experiments was to decide between the two theories of the formation of negative ions, proposed by Wellisch and by J. J. Thomson. The carriers were generated as photo-electrons by focusing ultra-violet light on one plate of a condenser at grazing incidence, thus eliminating stray light effects. Great care was used to secure pure air. The *curves* obtained agree in general with the results of previous investigators; they show anomalous mobilities below a critical pressure which was found to vary with the frequency of commutation of the E.M.F. No "free electrons" of the Wellisch type were observed. A repetition of the Wellisch experiments with photo-electrons showed that the "free electrons" were really a type of carrier which, started as electrons but attached themselves to molecules in the measuring field, as the Thomson theory demands. To *test the Thomson theory*, curves were computed assuming certain values for the constants. Theoretical and observed curves are similar in shape; in particular, the inflections near the low voltage end fall fairly closely together; It is also shown that the Wellisch *results* are in accord with this theory. The *chance of negative ion formation constant*, $1/n$, comes out about $1/2.5 \times 10^5$ *for air*. As $1/n$ is 0 for *nitrogen*, and as $1/n$ in *oxygen* is about 1/50,000, we conclude that it is to the oxygen molecule in air that electrons attach to form ions.

Introduction.

It has long been known that the velocity in unit electric field (*i.e.*, mobility) of normal negative ions in air ceased to be strictly inversely proportional to the pressure, for pressures below 10 cm. of air. In fact for pressures lower than this the mobility has been found to increase more rapidly than the pressure decreased. This behavior has been taken as powerful evidence for the existence of so-called "cluster" ions, but the work of Wellisch (1), Loeb (2), and Yen (3) has shown that the assumption of a fairly stable cluster ion must be given up. One must accordingly look elsewhere for an explanation of the foregoing phenomenon.

In most if not in all ionization processes in gases the first step consists in the liberation of an electron from a neutral atom or molecule. As all evidence points to the fact that the normal negative ion in a gas is a body of molecular dimensions, the electrons formed by the ionizing agent must first attach themselves to neutral gas molecules in order to form the type of carriers that are usually observed. It seems quite likely that in this process may lie the explanation of the abnormal mobilities of gas ions observed in air at low pressures.

In fact Sir J. J. Thomson (4) proposed a theory to account for these abnormal, or better "anomalous," mobilities, on the basis of a definite mode of formation of ions from electrons and neutral molecules. This theory assumes that out of n collisions of an electron with a given type of neutral molecule on the average only one will result in the attachment of the electron to the molecule: n being a constant characteristic of the type of gas molecules considered. Such an assumption might be interpreted as meaning that a certain region of the molecule must be struck by the electron under the proper conditions (*e.g.*, such as the velocity of the electron or the state of the molecule) before it can be incorporated in the molecule. It is obvious that if n be great enough, particularly at low pressures, the electron might cover considerable distances as an electron before attaching itself to a molecule to form an ion. Thus since the mobility of the electron is of the order of a hundred fold that of the ion it would be expected that below certain pressures the mobility of the negative ions would show abnormalities. On the basis of this theory J. J. Thomson derived the equations governing the behavior of carriers which change their nature while being measured as a function of the pressure. His computations are applicable to the Langevin method of mobility determination.

In two very able papers Wellisch (1) published the results of an investigation of the mobilities of positive and negative ions at low pressures in air as determined by the Franck (5) modification of the Rutherford alternating current method. Down to 8 cm. he observed the single class of carriers of slightly abnormal mobilities observed by the earlier workers (5). Below 8 cm. pressure his curves were of such a form that it seemed only possible to explain them on the basis of the existence of *two types of carriers*. These Wellisch interpreted as being carriers of mobilities approaching electronic magnitudes, which he termed "free electrons," and normal ions whose mobility was inversely proportional to the pressure down to about .15 mm. The interpretation of the process of ion formation given by Wellisch, as a result of his experiments (which yielded two types of carriers at pressures where other observers had found but a single type), differs radically from that offered by Thomson. Assuming that in air the carriers are of two distinct kinds, "free electrons" and ions, the former increasing in numbers relatively to the latter as the pressure decreases, Wellisch proposed the following theory. In order that an electron may attach to a molecule to form a negative ion he assumed that the electron must strike the molecule with a relative velocity, corresponding to a certain energy-value greater than E, which he styles the "potential energy of ion formation." If the energy at impact is greater than E the electron and molecule will unite to form an ion. If the energy of the impact be less than E the electron can never attach to the molecule. It then becomes a "free electron," which it remains unless it can acquire the energy E in the electric field. Now this energy E he further assumes is generally acquired in the act of ionization. In receding from the positive remainder of the molecule from which it came the electron transfers some of its initial kinetic energy to potential energy of separation. Accordingly if before striking its first molecule it has traveled so far that its residual energy is less than E it will remain a "free electron." The number of electrons which have a chance of thus remaining free will obviously be the greater the lower the pressure, for the mean free path of an electron is inversely proportional to the pressure.[1]

One has therefore two widely different theories of ion formation. The theory of Thomson assumes that out of a great many collisions the electron may strike a molecule in such a manner that it can attach, an event which if one is given enough impacts must eventually take place *for all electrons*. The theory of Wellisch postulates that it is the first impact that determines whether an electron shall form an ion or whether it shall pursue its subsequent career as a *permanently* "free electron." These theories represent different interpretations of the results of two slightly different methods of experimentation, which find reconciliation in the light of the results which follow.

The Problem.

In studying this question it became evident to the writer that no decision as to the validity of the two theories could be made without a further experimental investigation. Accordingly in October, 1919, a series of experiments were undertaken in order to attempt to solve this problem. What was needed was a method of distinguishing sharply between electrons and ions in experiments of the type of the earlier investigators. It was thought possible so to modify the general method used by Kovarik (6) as to make it serve this purpose. Kovarik measured the mobilities of carriers produced by photo electrons generated at the surface of a plate by light incident normally upon it, for various pressures in air, using the Rutherford alternating current method. In again taking up experiments of this sort an attempt was made to fulfil the following conditions.

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[1] It is to be remembered that the force between the parent atom and the electron escaping from it is inversely proportional to the square of the distance and not to a relatively high power of the distance as in cohesion.

1. To use a detecting system having a high sensibility in order to detect the presence of small numbers of free electrons.
2. To use carriers all starting from the same plane as electrons.
3. To use fields as uniform as possible.
4. To control the upper limit of the velocity of emission of the photo-electrons, by filters if necessary.
5. To eliminate the effect of "stray light" which Kovarik claims to have been bothered by.
6. To eliminate all *organic vapors* and *impurities*.
7. To use a constant source of ultra-violet light.

Apparatus.

The apparatus finally evolved is indicated in Fig. 1. In an all metal case *C* are housed the two condenser plates *P* and *E* of 10 cm. diameter.

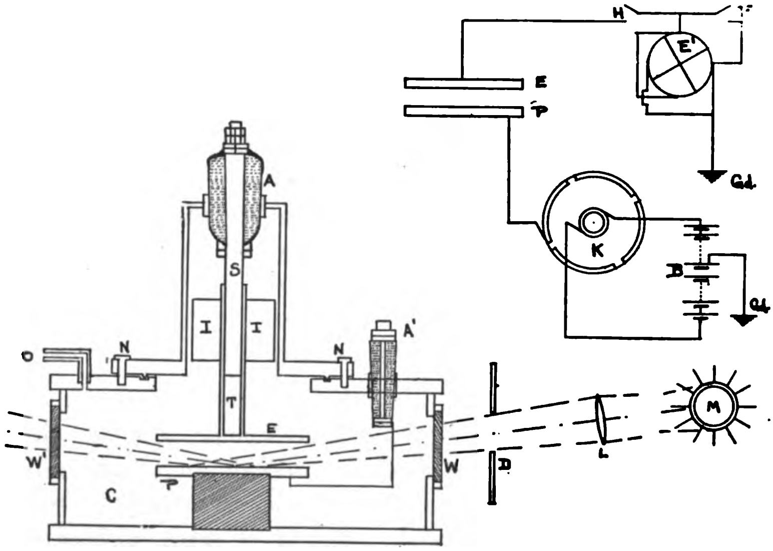

Fig. 1.

P was a plate of highly polished speculum metal which seemed to have quite suitable qualities as a constant source of photo-electrons. *E* was a plate of oxidized copper suspended by means of the threaded tube *T* so as to be approximately parallel to *P*. *E* could be screwed up or down on the threaded support *S*, by means of an electromagnet operated from outside the housing which acted on the iron mass *I* attached to *T*. *S* was insulated by an amber plug *A* ground into the top of the housing case *C*, and was attached to the electrometer system. The image of a quartz mercury arc *M* was focused upon *P* through a quartz window *W*, 4.5 cm.

in diameter, by means of the quartz lens *L*. From *P* most of the light was reflected out through a glass window *W* through which also the distance *PE* could be accurately measured by means of a cathetometer. The interior of *C* was oxidized so that the stray light falling on the walls of the housing emitted practically no electrons. The beam of light was still further cut down by the diaphragms *D*. Access into *C* was gained through the removal of the top of the case *C*, by unscrewing the screws *N*, which insured the tightness of the joint. This joint was made gas tight through the use of an unvulcanized rubber gasket, similar to those used by Professor Millikan in the attainment of high vacuua on his vacuum spectrometer.

In sealing in the windows of the chamber *C* as well as in securing gas tight joints at the amber insulating plug A^1, which carried the leads from the plate *P* to the source of the alternating current, great care was taken to make the possibility of the diffusion of any vapors from these points into the case negligible.

The electrometer E^1 was one of the Dolezalek type having with its system of switches and leads an electrostatic capacity of 260 E.S.U. and a sensibility of 3,700 mm. per volt on a scale 2 m. distant. The alternating potential came from a bank of dry cells *B* the center of which was grounded at *Gd*, the other poles of which were connected to the two brushes of a commutator *K*. Two commutators were used, one having twenty segments, and the other one having two segments. They were driven by a motor through sets of gears having the ratios 5 : 1, 2 : 1, and 1 : 1. The speed of the motor was 2,100 R.P.M. so that by changing gears and commutators a fairly complete control of frequency of alternation lying between 15 and 750 cycles per second was obtained. The speed of the commutator was measured directly by means of a revolution-counter and stopwatch. The speed was kept constant by regulating the field current of the motor. Small variations in the speed of the motor were registered by variations in the E.M.F. given by a small dynamo driven by the shaft of the driving motor. This indicator was so sensitive that any desired degree of constancy could be obtained.

The air used in the measurements was purified in the following manner. It passed through a tube of copper oxide heated to dull redness to destroy any organic vapors present. The air then passed through long tubes of NaOH, $CaCl_2$, and P_2O_5, finally passing through a trap cooled to from − 125° to − 90° C. by frozen alcohol. Before any series of measurements on air was made, the apparatus was exhausted to about 2 cm. and filled to atmospheric pressure with the purified air at least three times, generally more.

THE MEASUREMENTS.

Measurements were conducted for a given pressure and a given frequency of commutation by determining the variation of the current to the electrometer plate as a function of the value of the alternating potential applied to P. This was accomplished by disconnecting the lead to the electrometer at switch H, leaving the latter grounded through F and allowing the alternating potential to act on plate P for a given time (from 5 to 30 seconds). The electrometer ground F was then disconnected, the plate E connected to the electrometer, and the deflection noted. Since the capacity of the leads to the electrometer was appreciable compared with the capacity of the condenser formed by the plates P and E, a correction of about 7.5 per cent. had to be subtracted from the value of the P.D. given by the commutator, in order to give the true value of the P.D. acting across P and E.[1] In general the potentials taken from the cells were so arranged that the value of the positive or retarding, potential applied to the plate P by the commutator, was always from 2 to 20 per cent. higher than the corresponding negative, or accelerating potential. The purpose of this was to prevent any possibility of carriers *working* their way across from P to E by stages covering several cycles. Control experiments showed that the magnitude of this difference in the two sides of the alternating P.D. did not play any rôle as long as the retarding potential was a trifle greater than the accelerating potential. The values of the deflections for various potentials were generally taken starting at the higher values and going down to the lowest, then working back again over the curve taking values of the potentials at intermediate points between those first chosen. The readings were always quite consistent, unless by some accident the value of the potential across the mercury arc had changed during the measurement. For a series of readings on a given sample of air the curve at atmospheric pressure would be taken first, and then the values for the lower pressures were obtained, the pressure being reduced step by step, by means of a water aspirator. The range of pressures over which satisfactory work could be done in air, lay between 760 mm. and 30 mm. Below this pressure it was impossible to go because the frequencies attainable by the commutator were not high enough for the type of curves obtained. The frequencies used varied from 15 cycles per second to 750 cycles while the distances between P and E varied from 1 cm. to 1.7 cm. At some point in each series of determinations, for a given sample of air a "control" set of readings was taken in which the deflection of the electrometer was measured when different steady negative potentials were applied to the plate *P*, for a given time. This gave the saturation photo-electric current from the plate *P* under the influence of the arc at the time of experiment. Knowledge of this current served as a means of comparing data taken on successive days. It was also of use in applying the theory of Thomson.

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[1] The total fall of potential given by the commutator was distributed across the condenser formed by P and E, and the condenser formed by the leads and the ground, these two representing condensers coupled in series. The true field acting on the carriers was then less than that produced by the commutator. Measurements made by the method of mixtures gave this correction as 7.5 per cent.

When the electrometer deflections for a given series of accelerating potentials were plotted against the value of the accelerating potential, curves of the type shown in Fig. 2, curves I., II., III., IV., and V., were

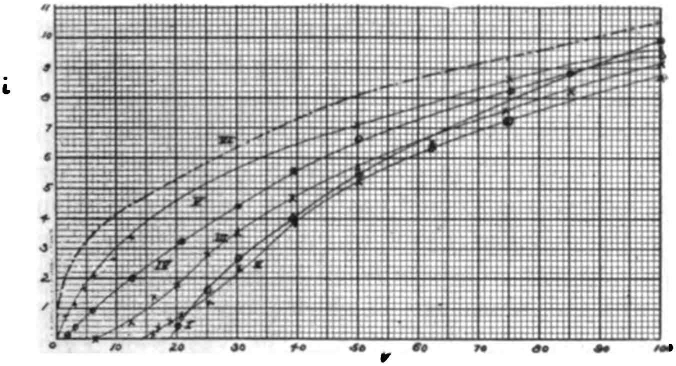

Fig. 2.

June 2, 1920. $d = 1.05$. Electrometer deflections in cm. scale reading plotted against the alternating P.D. on *P*.

I. $p = 747.0$ $N = 35.7$ Mobility $K' = 2.27$
II. $p = 98.0$ $N = 359.0$ " $K' = 2.97$
III. $p = 75.5$ $N = 705.0$ " $K' = 7.92$
IV. $p = 53.5$ $N = 705.0$ " $K' = 26.0$
V. $p = 38.5$ $N = 705.0$ " $K' = 39.4$
Observed x. . . x. . .
VI. Saturation —. —. —.

obtained. These correspond to different pressures and to different speeds of commutation. The curve numbered VI. in the figure is the control or saturation curve taken with a series of steady negative potentials applied to plate *P*. The curves in general have the characteristic form of mobility curves. They are nearly parallel to the saturation curves at the higher potentials, and then drop sharply towards the potential axis tending to cut it at some fairly well defined point. The voltage value of this point one may designate as V_0. In the curves obtained at the higher pressures this point is quite sharply defined.

If the value of V_0, obtained from a given curve and corrected for the potential drop across the electrometer leads by being reduced by 7.5 per cent. of its value, be inserted as V_0 in the equation $U = Nd^2/V_0$ (where N is the frequency of commutation in half cycles, and d is the distance between the plates), the average value of U the mobility of the carriers between the plates is obtained. If the U thus obtained for a given pressure, be multiplied by the factor $p/760$ (where p is the pressure in mm.), the value of K the *mobility constant* for normal ions in air should be obtained. The values of K for the curves depicted in Fig. 2 may be computed from the data given in the legend. They are 2.27, 2.97, 7.92, 26 and 39.4. cm. per second respectively for curves I., II., III., IV. and V., Fig. 2.

RESULTS.

The results of twenty-five different series of determinations, taken under all imaginable conditions in air, may be summed up as follows:

1. No evidence of the existence of the *two* types of carriers described by Wellisch could be found in these results, although they were carefully looked for; nor was there any evidence of the "free electrons" of Wellisch. Such carriers would have been indicated by complex curves of the types shown in Figs. 14 and 15, taken under different circumstances.

2. The curves were consistently of the types shown in Fig. 2 having intercepts with the voltage axis V_0 which above 100 mm. pressure yielded approximately normal values of K. The values of V_0 obtained at pressures below this gave anomalous values for K which were in general accord with the values found by Kovarik.

3. The value of K for carriers below 100 mm. pressure was found to be a conspicuous function of the frequency of commutation. This is again in agreement with the experimental results of Kovarik.

4. The anomalous values of K for air were not always the same under similar conditions but varied somewhat with the purification which the air had undergone.

5. As may be seen in the figures, the apparently sharp intercepts of the curves with the potential axis begin to vanish below about 100 mm. and the feet of the curves tend to approach the axis more and more asymptotically the lower the pressure (see especially II. and III.). The appearance of such feet in the curves was also noted by Kovarik who ascribed them to "stray light" effects. A careful study of these portions of the curves which appear in all curves when the pressures become low enough, under all conditions of illumination and plate distance, is sufficient to show that *they are intimately connected with the phenomenon of the anomalous mobilities and are not due to stray light.*

It may be concluded from the above summary that these results verify the experimental results of the early observers. They however go a step further than those results indefinitely connecting the asymptotic portions of the curves near the potential axis with the phenomenon of the anomalous mobility. The work was carried out in such a manner that it permits an attempt at the quantitative application of the results to the theory of J. J. Thomson.

Application of Results to Thomson Theory.

For carriers liberated at one plate P of a parallel plate condenser, and accelerated for a time T to the other plate E distant d, by a potential difference of value V only those carriers will reach E which have for a velocity in unit field a value U given by

$$U \geqq d^2/VT(1). \qquad (1)$$

Now the theory of J. J. Thomson assumes that an electron starting from P and moving towards E in the electric field may on its career in one out of n impacts collide with a gas molecule in such a manner as to attach itself to the molecule, and thus continue its way to E as an ion. A carrier which reached E after undergoing such a change must have had an average, "hybrid" mobility U which is determined by the condition laid down above. The value of U for such a carrier would then lie between that of an electron and that of an ion. In other words it would have a "fictitious" abnormal or an anomalous mobility such as is observed for carriers in air at pressures below 100 mm. Assume that such a carrier travels x cm. as an electron and $(d - x)$ cm. as an ion. If K' be the mobility of an electron and K be that of an ion the time T taken to travel d in unit field will be

$$T = x/K' + (d - x)/K.$$

Since however K' is of the order of 100 times as great as K, one may for all practical purposes neglect the first term and write

$$T = (d - x)/K.$$

Since $U = d/T$ one may write

$$U = \frac{Kd}{(d - x)},$$

which expresses the fictitious hybrid mobility, in terms of the mobility of the ion and the ratio of the whole distance to the distance traversed as an ion. In order to get a current from P to E with an alternating current whose time of alternation is T one must impose the condition (1), which gives:

$$U = K\frac{d}{(d - x)} \geqq d^2/VT.$$

This implies that in order to reach E at all the carrier must have covered a distance x as an electron, x being governed by the equation

$$d - KVT/d \leqq x. \tag{2}$$

This condition can only hold up to $(KVT)/d = d$, for negative values of x have no meaning.

On the basis of the assumptions quoted, and further assuming that *the velocity of drift of the electrons in the electric field is small compared to their velocity of kinetic agitation,* Thomson shows that out of I_0 electrons starting from P the number I which go a distance x without uniting to form ions, is given by the equation

$$I = I_0 e^{-Wx/(nK'\lambda V/d)} \tag{3}$$

(where W is the mean velocity of kinetic agitation of the electrons, d the distance between the plates, K' the mobility of the electron, λ the mean free path of the electron, and $1/n$ the chance that any collision of an electron with a molecule will result in the formation of an ion). Now the carriers that have traveled a distance x without combining to form an ion can only be detected when they contribute to the current from P to E, so that in order that the above equation (3) may apply to the carriers detected, one must include the condition imposed by (2), viz., that

$$x = d - KVT/d,$$

where x can have only positive values. Putting this condition into (3) one obtains

$$I/I_0 = e^{\frac{-dW}{nK'V\lambda}(d - KVT/d)}. \tag{4}$$

Which gives the fraction of the initial electron current starting from P that reaches the plate E as a function of the experimental variables d, V and T. Furthermore since K is inversely proportional to the pressure, and assuming K' and λ to be so (*i.e.*, to be proportional to $760/p$, where p is the pressure in mm. at which the measurements are made), and finally writing $T = 1/N$ the equation (4) becomes

$$I/I_0 = e^{\frac{-W}{nK'\lambda}\left(\frac{d^2(p/760)^2}{V} - \frac{K(p/760)}{N}\right)}. \tag{5}$$

If one assumes that the electrons are in thermal equilibrium with the gas molecules he may evaluate W. Taking λ for the electrons as $4\sqrt{2}$ times that of the molecules, and assuming that K' for electrons is about 200 (1, 7) while K for ions is about 2, the equation becomes applicable to experimental verification and the determination of the constant n, namely it takes the form

$$I/I_0 = e^{\frac{-9.9 \times 10^8}{n}\left(\frac{d^2(7/760)^2}{V} - \frac{(2p/760)}{N}\right)}. \tag{6}$$

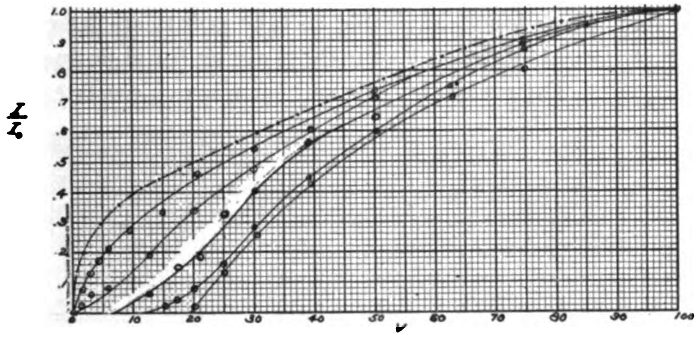

Fig. 3.

June 2, 1920. $d = 1.05$. Curves of Fig. 2 reduced to fraction of saturation.
I. $p = 747.0$ $N = 35.7$
II. $p = 98.0$ $N = 359.0$
III. $p = 75.5$ $N = 705.0$
IV. $p = 53.5$ $N = 705.0$
V. $p = 38.5$ $N = 705.0$
Observed x...x...
VI. Saturation —. —. —.

On the basis of the experimental curves obtained one may solve for n from any curve by taking the values of I/I_0,[1] d, p, V and N under which the curve was determined, and substituting them in the equation above.

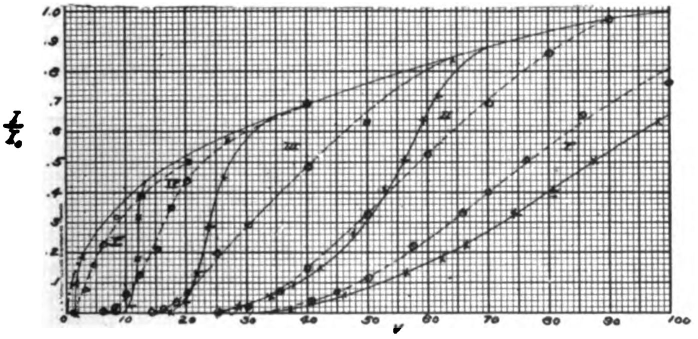

Fig. 4.

April 23, 1920. $d = 1.66$, $K = 2.1$, $p = 95$, $n = 2.5 \times 10^5$.
I. $N = 710$ III. $N = 362$
II. $N = 147$ IV. $N = 72$
V. $N = 14.7$
Observed o...o...
Computed -x———x-
Saturation —. —. —.

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[1] I/I_0 was taken as the ratio of the electrometer deflection at a given voltage V taken with alternating potentials to the deflection obtained by letting the same steady voltage act for half the time. This gives the ratio of the true I/I_0 corrected for the decrease in the saturation current at the lower potentials.

In doing this it was found advisable to take points on the curves lying near the feet (*i.e.*, at values of V near those giving the abnormal mobilities for the curves). The reason for this will become evident later on. Solutions of this equation for some fifteen or more different curves were made at one time or another. The curves so studied covered a range of pressures from 130 mm. down to 20 mm. in pressure, and frequencies ranging

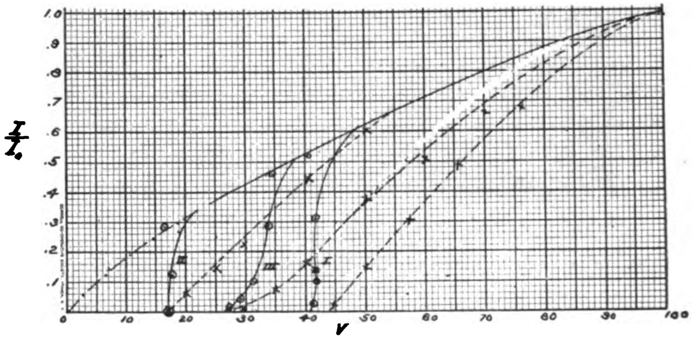

Fig. 5.

April 27, 1920. $d = 1.66$, $K = 2.45$, $n = 2.5 \times 10^5$.
I. $p = 738$ $N = 37.2$
II. $p = 738$ $N = 15.2$
III. $p = 148$ $N = 150.0$
Observed x...x...
Computed –o——o–
Saturation –. –. –. –.

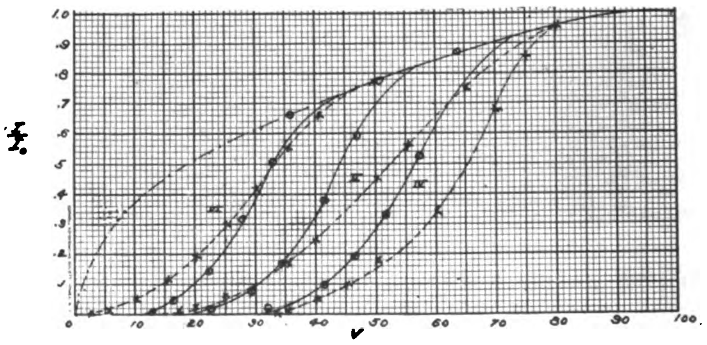

Fig. 6.

April 27, 1920. $d = 1.66$, $K = 2.45$, $n = 2.50 \times 10^5$.
IV. $p = 105$ $N = 372$
V. $p = 84$ $N = 378$
VI. $p = 59.2$ $N = 378$
Observed x....x...
Computed –o——o–
Saturation –. –. –. –

from 150 cycles to 750 cycles. The determinations also utilized three different values of d ranging from 1.05 cm. to 1.66 cm. and were taken on at least five different samples of air purified under somewhat different conditions. The values of n for air thus obtained ranged from 9×10^5 to 1.5×10^5, the greater portion of the values lying around 2.5×10^5. The reason for this variation will be considered in connection with a study of the curves. Measurements were made in pure oxygen, and in

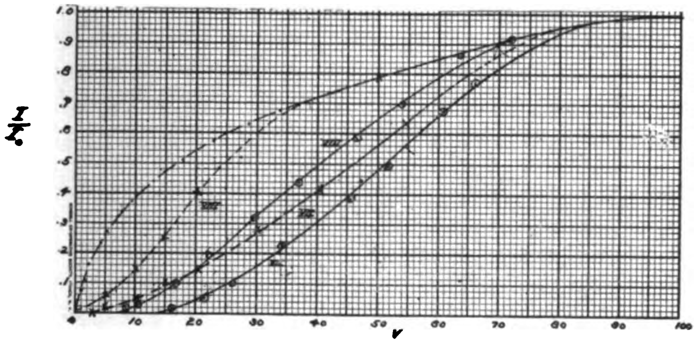

Fig. 7.

April 27, 1920. $d = 1.66$, $K = 2.45$, $n = 2.5 \times 10^5$.
VII. $p = 59.2$ $N = 742$
VIII. $p = 41.5$ $N = 742$
Observed x. . . x. . .
Computed -o———o-
Saturation —. —. —.

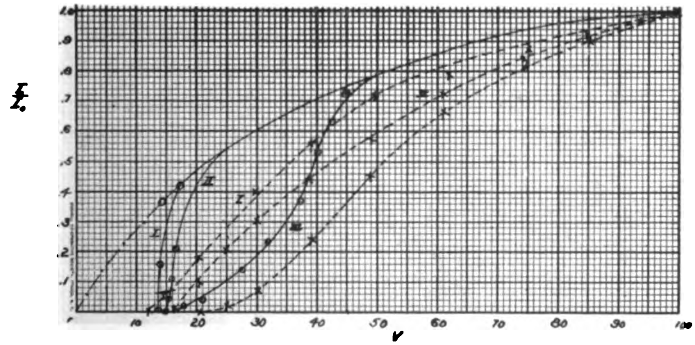

Fig. 8.

June 24, 1920. $d = 1.50$, $K = 2.55$, $n = 4 \times 10^5$.
I. $p = 752$ $N = 14.9$
II. $p = 198.5$ $N = 73.0$
III. $p = 99.5$ $N = 373.0$
Observed x. . . x. . .
Computed -o———o-
Saturation —. —. —. —.

nitrogen containing small percentages of oxygen, and will form the basis of a later paper. They covered pressures ranging from 760 mm. pressure to 15 mm. as well as frequencies from 37 cycles to 750 cycles and yielded

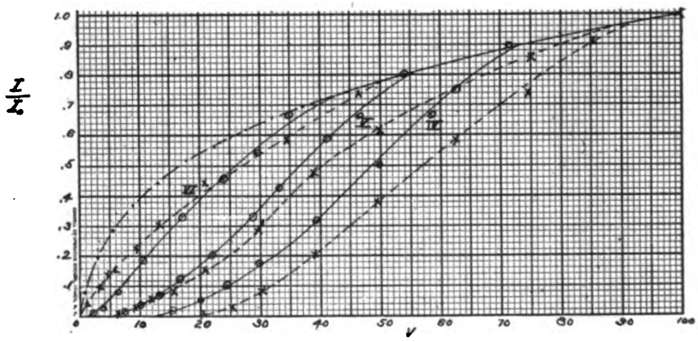

Fig. 9.

June 24, 1920, $d = 1.50$, $K = 2.55$, $n = 4 \times 10^5$.

IV. $p = 80.5$ $N = 727$
V. $p = 59.5$ $N = 740$
VII. $p = 32.5$ $N = 735$
Observed x...x...
Computed -o———o-
Saturation —. —. —. —.

values of n for air, computed back to air on the basis of the oxygen content, which agreed well with those already cited.

Using values of $n = 2.5 \times 10^5$ and 4×10^5 and using values of K, the ionic mobility, as calculated from the curve taken at 760 mm. (which

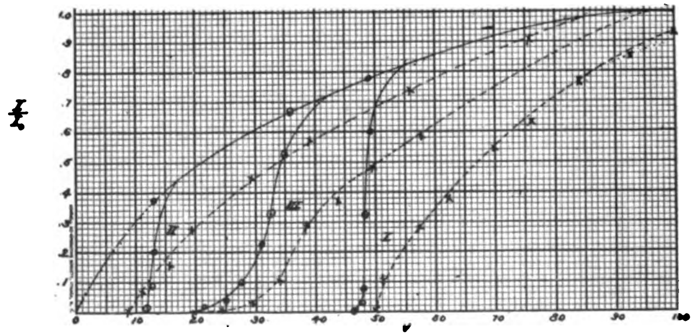

Fig. 10.

April 6, 1920. $d = 1.66$, $K = 2.16$, $n = 4 \times 10^5$.

I. $p = 744$ $N = 37.2$
II. $p = 207$ $N = 37.7$
III. $p = 134$ $N = 151.0$
Observed x...x...
Computed -o———o-
Saturation —. —. —. —.

ranged from 2.55 to 2.16 cm./sec.) the values of I/I_0 were computed as a function of V from the equation (6) for pressures and values of N under which some of the experimental determinations were made. The values of I/I_0 plotted as ordinates against the voltage V as abscissæ yielded a series of curves. These computed curves when corrected for the value of the saturation photo-electric current, and for the value of the reduction of potential between the plates due to the induction effect, are reproduced in Figs. 4 to 12 as full curves with circles. In the same figures are given, as dotted curves with crosses, the corresponding values of I/I_0 plotted against the voltage V as actually determined experimentally.[1] The curves represented by the dash dot lines, with no points marked, are in each case the actual saturation photo-electric current curves taken at the time of the experiment.

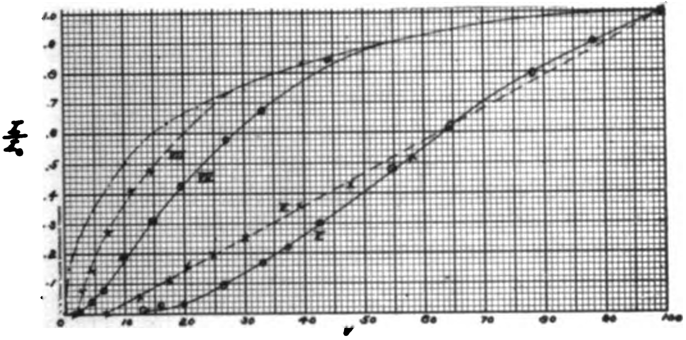

Fig. 11.

April 6, 1920. $d = 1.66$, $K = 2.16$, $n = 4 \times 10^5$.
V. $p = 76.5$ $N = 726$
VII. $p = 33.0$ $N = 742$
Observed x...x...
Computed –o———o–
Saturation –. –. –. –.

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[1] The values of I/I_0 for the experimental curves were determined from the experimental curves by reducing the electrometer deflections actually measured to fractions of the apparent "saturation" value of the current. For measurements where the curves struck the axis below 40 volts this saturation value of the current was generally taken as the deflection at 100 volts accelerating potential. In some cases however it was taken from the half value of the "control" deflection at 100 volts. The two are closely the same for at 100 volts under most conditions of measurement the number of the carriers reaching the plate E in a given time of exposure to the alternating potential is practically equal to the number that reach E at the same voltage when the direct accelerating potential is on for half the time. The reason for reducing the readings with the direct potential to half is that with the alternating accelerating potential the field is on for but half the time of exposure. A typical transformation of the curves actually obtained may be seen in the case of the curves of Fig. 3 which are the curves of Fig. 2 as experimentally obtained reduced in the manner above indicated. In any case the changes produced near the feet of the curves by such a transformation are small and it is these parts of the curves which are of greatest importance.

The curves shown in Fig. 4 are the observed and the computed curves obtained at a pressure of 95 mm. with different frequencies. Figs. 5, 6 and 7, give the observed and computed curves taken on another sample of air with varying pressures and frequencies. The value of n used in the computation of both these sets of curves was 2.5×10^5. In the Figs. 8 and 9, as well as in the Figs. 10, 11 and 12, are given the observed and computed curves for two sets of determinations in air at varying frequencies and pressures, whose computed curves were obtained assuming a value of $n = 4 \times 10^5$. In all cases the legend gives the conditions under which the determinations were made, and the points illustrated are the actual points obtained. A number of other sets of determinations have been computed and compared with the experimental results with the same degree of success.

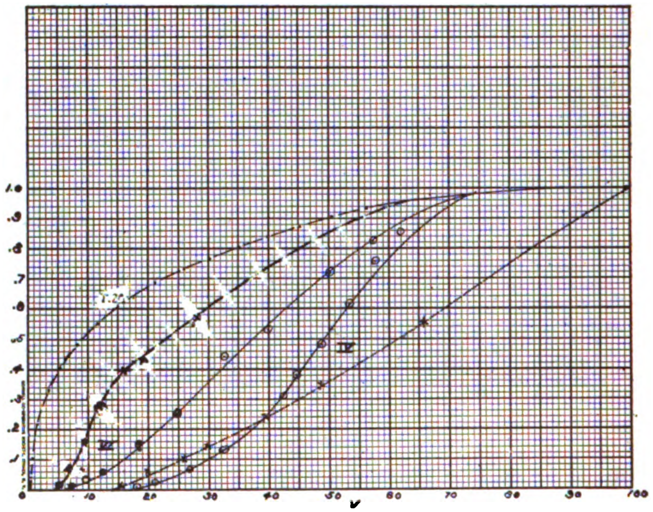

Fig. 12.

April 6, 1920. $d = 1.66$, $K = 2.16$, $n = 4 \times 10^5$.
IV. $p = 97$ $N = 376$
VI. $p = 55$ $N = 742$
Observed x. . . x. . .
Computed -o———o-
Saturation —. —. —.

In examining the curves two points stand out prominently. First barring the upper portions of the curves taken at or near atmospheric pressure (where for reasons to be given later the curves should not agree) there is fair agreement between the general forms of the observed and computed curves: That is to say there is sufficient similarity in the form of the curves and in their variations with r and N to justify the assumption that they correspond to an equation of the type used. Secondly it can be observed that the values of the voltage at which the ratio I/I_0 reaches some definite low value (*e.g.*, $I/I_0 = .02$ near the limit of measurement of the electrometer), are quite closely the same for the observed and the corresponding computed curves. *Since it is these portions of the experimental curves* (which one may term the feet of the curves), ***which yield the values of V_0 corresponding to abnormal mobilities, it is obvious that the curves deduced on the basis of the results of the theory of J. J. Thomson yield values of V_0 quite well in accord with those actually observed.*** The theory of Thomson then is in a certain measure well able to predict the anomalous mobilities found for the negative ions for low pressures, which have been the cause of so much discussion.

The reason for the failure of the curves to fit at their upper portions is as follows: At first sight one would expect the curves which are obtained at atmospheric pressure (where the carriers are for the most part normal ions from the beginning), to follow the saturation current curve with decreasing voltage until a value of V is reached, at which in the time of an alternation the carriers can just cross from the plate P to E. At this voltage, V_0, the current should abruptly fall to 0. This condition is implied in the application of the Thomson theory for in the development of that theory it is assumed for simplicity that as soon as V is such that $V = d^2/uT$ *the ions of mobility u all get across.* This is not the case in practice. Starting at a value of the alternating potential equal to V_0 in the negative phase on P, only those carriers will get across to E that were liberated in the first instants of that phase. For carriers liberated in the later portions of that phase will not have time enough to reach E under the existing field. On the reversal of the field such later ions will be all dragged back to the plate P and lost. As V is increased carriers that are liberated later, and later in the negative phase of the potential will succeed in crossing. So that the current to E will approach a sort of saturation value when V_0 has been considerably exceeded. The experimental curve would then not be expected to rise nearly vertically at V_0 as assumed, but would be expected to *begin* to rise at this value and then to gradually increase reaching a maximum at values of V a good deal higher than V_0. Such, as is well known, is actually the case.

In the case of the Franck modification of this method the curves are more like the theory, for only a part of the ions which are driven back to the plane of the gauze are absorbed by it while a plate, as distinguished from a gauze, will absorb them all.

Because of the presence of carriers which are electronic for a considerable portion of their path between the plates as the pressures are reduced below 200 mm. the curves obtained on the basis of the Thomson theory with its simplifying assumption become themselves increasingly inclined to the vertical below this pressure. This has for a result that the differences in shape of the computed and the observed curves becomes much less marked at the lower pressures, as is readily seen.

There is however another point in which the curves do not conform. This is noticeable in Figs. 5, 6, and 7, where the constant n was taken as 2.5×10^5. While the feet of the curves at pressures above 59 mm. coincide quite well for the theoretical and the observed results, it is seen that the feet of the experimental curves below 59 mm. lie at voltages well *below* those for the computed curves. In other words the "anomalous" portions of the experimental curves actually seem to develop more rapidly with decreasing pressures than the theory would predict. This tendency is seen in the curves represented in Figs. 8 to 12, where the value of n chosen was 4×10^5. In this case the feet of the curves at 59 mm. coincide reasonably well, while the feet of the experimental curves taken for 80 and 100 mm. lie at *higher* voltages than the corresponding theoretical curves. In other words the *higher* value of n causes a closer agreement between the curves at the *lower* pressures, than at the higher ones. Accordingly n does not seem to be constant but appears to increase with decreasing pressure. This was the result obtained throughout in computing n. The curves at the lower pressures from 20 mm. to 60 mm. yielded values of n from 9 to 4×10^5, while the curves for pressures between 60 mm. and 130 mm. yielded values of n ranging from 4.0 to 1.5×10^5. The value of n was computed from the theory, and is subject to the condition that the assumptions of the theory are fulfilled. Now J. J. Thomson assumed that the velocity of the electron produced by the field between the plates was *negligible* compared to the velocity w which it possessed in virtue of its kinetic energy of agitation. Now w at 20° C. is about 1.2×10^7 cm./sec., while the velocity acquired between impacts in a field of 10 volts per cm., at 30 mm. pressure lies in the neighborhood of 7.1×10^6 cm./sec., *i.e.*, it is about half of w. If in addition it be considered possible that the velocity thus acquired in the field is not completely wiped out at each impact (*e.g.*, due to quasi-elastic impacts between electrons and nitrogen molecules) one sees how at considerably higher pressures the conditions assumed by the theory may not be met. Such elastic impacts were observed between electrons and hydrogen molecules by Franck (8) and Hertz, and Compton (9) states that they may also be possible in the case of impacts with oxygen molecules though to a much less degree than in hydrogen. In fact Wellisch (1) uses this concept as a possible explanation of some peculiar results obtained in studying the mobilities of electrons. The writer in assuming the value of K' to be a constant of about 200 cm./sec. in value introduced any variation in K', due to the causes above considered, into n. One must then ascribe the deviations of these results from the theoretical ones not to variations in n but to the causes above outlined. In choosing a value for n for future reference it is best, in view of this increase of n with lower pressures, to choose the value of n which was determined at the lower range of pressures (*i.e.*, $n = 2.5 \times 10^5$), where less errors are introduced into the assumptions.

From the above curves one is justified in concluding that *these results furnish a strong qualitative verification* of the J. J. Thomson theory of ion formation.

Correlation of Thomson Theory with Results of Wellisch.

Having now obtained evidence for the correctness of the Thomson theory it remains to correlate these results and the theory with the apparently contradictory results of Wellisch. The essential difference between the present experiments and those of Wellisch lies in the fact that Wellisch using the *Franck modification* of the Rutherford alternating current method generated his ions by means of alpha particles in a sort of auxiliary chamber *PG*, Fig. 13. Furthermore Wellisch in general worked at pressures well below those of the other workers.

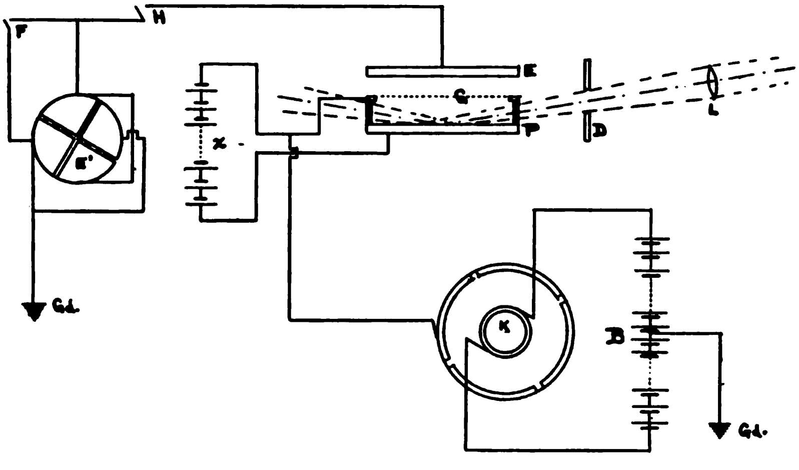

Fig. 13.

In an auxiliary chamber such as *PG* electrons liberated in the process of ionization have to travel distances as great as 2 cm. in air in fields of about 8 volts per cm. They thus make many collisions with neutral molecules before passing through the meshes of the gauze G into the measuring field GE. Accordingly on the Thomson theory there must be quite a proportion of carriers which though starting as electrons form attachments by virtue of their many encounters and therefore enter the space GE as ions. These, in measurements of the Wellisch type, give mobilities which are perfectly normal, *i.e.*, the measured mobility should be inversely proportional to the pressure. Besides such carriers there enter the field GE through the gauze electrons, which increase in relative numbers as the pressures decrease. These should behave just as did the photo-electrons in the experiments of the writer in giving anomalous mobilities as the Thomson theory demands. The result to be expected on applying the Thomson theory to the method employed by Wellisch would then be that the current-voltage curves obtained consist of the two parts i and E of curves II. and III., Figs. 14, 15, 16, and 17. The part due to the ions (i) should yield at its lower extremity (*i.e.*, at the point of inflection) mobilities inversely proportional to the pressure; while the other portion (E) due to electrons uniting to form ions in going

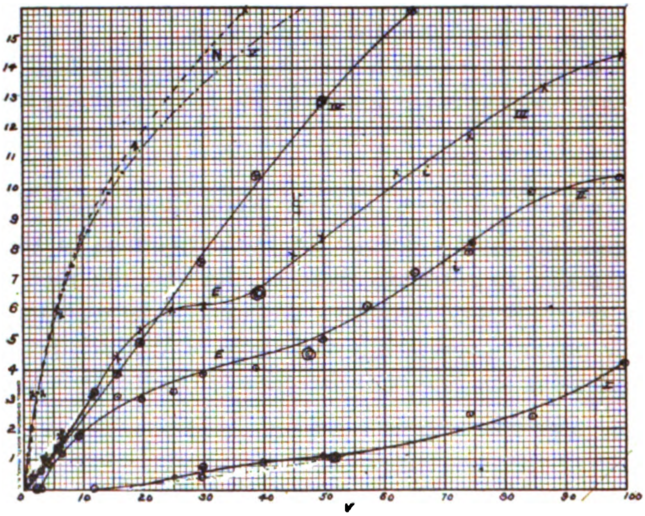

Fig. 14.

May 21, 1920. Using auxiliary gauze G 1.5 cm. above P, $d = 1.33$, $p = 49$ mm., $N = 710$.

I. $X = 1.5$ volts Mobility $K = 1.65$
II. $X = 4.5$ " " $K = 1.80$
III. $X = 9.0$ " " $K = 2.20$
IV. $X = 45.0$ " " ....
V. $X = 45.0$ " Saturation —. —. —. —.

Anomalous mobility K' for all about 33.0. N curve ...... for free electrons in N_2 gas at atmospheric pressure. $N = 360$.

from G to E should yield the anomalous mobilities previously observed.

Now Wellisch in his experiments found curves consisting of two distinct parts which were quite similar to the curves represented above. He showed that the upper portions of his curves, corresponding to the portion of the curves marked i in Fig. 14, yielded values for the mobility-constants of the carriers which they represented equal to those of normal ions. These mobilities moreover were inversely proportional to the pressure. Wellisch however did not find that the feet of the E portions of the curves were due to carriers yielding the fictitious abnormal mobilities as required by the Thomson theory. In fact in most of his published curves the feet of his E curves cut the axis so close to the O value that he ascribed them to electronic carriers which remained *permanently free*. It was this interpretation of his curves that lead Wellisch to the theory of ion formation proposed by him.

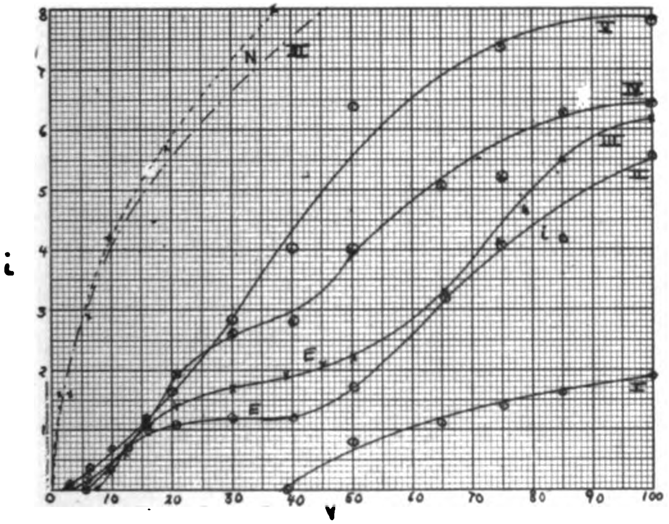

Fig. 15.

May 22, 1920. Using auxiliary gauze G 1.5 cm. above P, $d = 1.33$, $p = 56$ mm.

I. $X = 4.5$ volts $N = 710$ Mobility $K = 2.05$
II. $X = 9.0$ " $N = 710$ " $K = 1.90$
III. $X = 13.5$ " $N = 710$ " $K = 1.70$
IV. $X = 22.5$ " $N = 704$ " $K = 2.05$
V. $X = 45.0$ " $N = 704$ " $K = 2.20$
VI. $X = 45.0$ " $N = 0$ Saturation —. —. —. —.

Anomalous mobility K' for all curves is about 19. N curve ...... for free electrons in pure N_2 gas at atmospheric pressure and $N = 360$.

It is then clear that to correlate the Wellisch experiments with the Thomson theory one must show that the E portions of Wellisch curves are not due to ***permanently free electrons*** as Wellisch was led to believe but are due rather to the attachment, ***while in the measuring field of electrons to molecules to form ions.*** In consequence of the low pressures employed by Wellisch it is impossible to estimate from the intercepts of his curves with the voltage axis whether the values of the mobility-constant K' for the E type of carrier were of an electronic order of magnitude (*e.g.*, above 200 cm./sec.) (1, 7) or whether they had values of the order of magnitude which the Thomson carriers yield at those pressures.

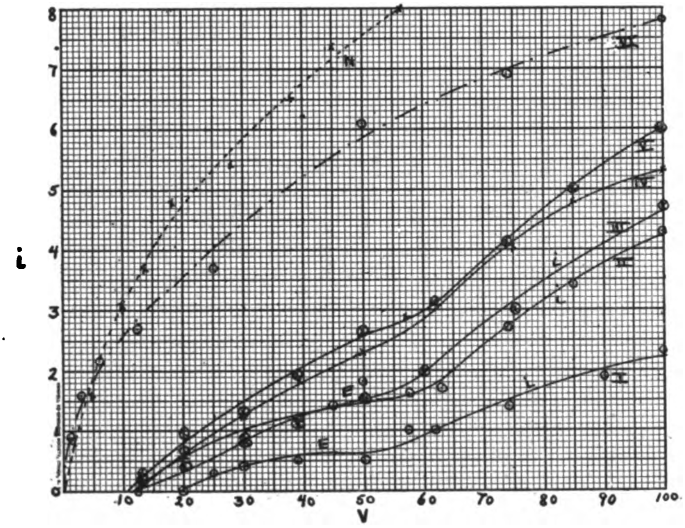

Fig. 16.

Oct. 15, 1920. Using auxiliary gauze G 1.5 cm. above P, $d = 1.28$, $p = 75$ mm.

I. $X = 4.5$ volts $N = 658$ Mobility $K = 1.98$
II. $X = 9.0$ " $N = 658$ " $K = 1.98$
III. $X = 18.0$ " $N = 658$ " $K = 1.98$
IV. $X = 45.0$ " $N = 658$ " $K = 1.98$
V. $X = 85.5$ " $N = 658$ " $K = 1.98$
VI. $X = 13.5$ " $N = 0$ Saturation —. —. —. —.

Anomalous mobility K' for all curves about 9.3. Curve labeled N free electron curve for N_2 gas at atmospheric pressure with $N = 147$. Curve ......

The experiments of Wellisch were accordingly repeated by the writer using pressures ranging from 34 mm. to 84 mm. (*i.e.*, where the anomalous mobilities lie between 100 and 5 cm./sec.) with a view determining from the intercepts of the E portions of the curves with the voltage axis whether the carriers yielded mobility constants K' of electronic or ionic magnitudes.

The apparatus of the writer was accordingly converted into one of the Wellisch type using photo-electrons as a source of ions. This was accomplished by interposing the gauze G, Fig. 13, between the plate P and the plate E. The distance PG was 1.5 cm. and the distance GE was 1.30 cm. A steady potential difference X was maintained between P and G by means of a battery of small flashlight cells, x, in such a sense as to drive negative carriers liberated at P through the meshes of the gauze G. If for a fixed value of the field X a series of measurements was made of the current to the electrometer plate E, for various values of the alternating potential applied to the gauze, curves were obtained of the type found by Wellisch. A series of such curves using different values of the field X varying from 1.5 to 85.5 volts was obtained for a number of pressures ranging from 34 mm. to 84 mm. Four typical sets of curves are represented in Figs. 14, 15, 16, and 17 in which electrometer deflections in cm. are plotted against the alternating potential. These figures include as well the saturation photoelectric curves at those pressures, and the characteristic curve (N) yielded by purely electronic carriers (permanently free electrons), obtained in nitrogen at atmospheric pressure. The values of X as well as other data concerning the curves are given in the legend.

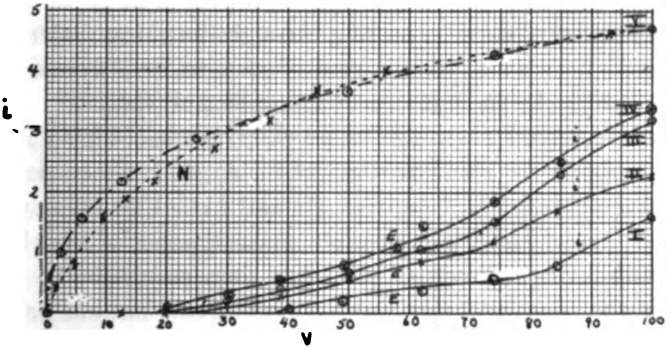

Fig. 17.

Oct. 16, 1920. Using auxiliary gauze G 1.5 cm. above P, $d = 1.28$, $p = 84$ mm.

I. $X = 4.5$ volts $N = 660$ Mobility $K = 1.89$

II. $X = 13.5$ " $N = 660$ " $K = 1.89$

III. $X = 27.0$ " $N = 655$ " $K = 1.89$

IV. $X = 85.5$ " $N = 650$ " $K = 1.89$

V. $X = 45.0$ " $N = 0$ Saturation –. –. –. –.

N. Curve for free electrons in N_2 gas at atmospheric pressure with $N = 146$. Curve dotted .... Anomalous mobility K' for all curves about 6.4.

It is obvious at once that as the pressure increases in Figs. 14, 15, 16, and 17 the asymptotic feet of the E portions of all curves have their points of inflection (or intercepts with the V axis where points of inflection are poorly defined) at higher and higher values of the voltage, which is a characteristic of the Thomson type of carrier. It is also patent that in no case do these portions of the curves follow the free electron curve obtained in nitrogen. If the average value of the fictitious abnormal mobility of these curves be estimated from the points of inflection, or intercepts, they yield the values 33, 19, 9.3, and 6.4 cm./sec. which are similar to those obtained for carriers in the absence of the gauze at these pressures and frequencies.

The variation in form of the Wellisch type of curves with the field X, which is obvious when one regards the set of curves for any one pressure, may be simply explained on the Thomson theory. If the values of X are low the ions spend considerable time in the auxiliary field PG before reaching the gauze G. In this time they make many impacts with neutral molecules. On the Thomson theory it is to be expected that the greater proportion of carriers passing through the gauze under these conditions would be normal ions. This is seen to be the case in curves numbered I, Figs. 14 and 15. As X increases and the time spent in PG is therefore decreased more and more electrons succeed in reaching G without attachment. The E portions of the curves consequently increase in relative importance as in curves II. and III., Figs. 14, 15, 16. On still further increasing X it is possible to cause most of the electrons to reach G without attachments. One has then precisely the conditions obtained in the absence of the gauze, curves having but one part E being observed. These are found in curves IV., Fig. 14, and V., Fig. 15. If the pressure be reduced the effect on the form of the curve is the same as is produced by increasing X, for the time spent in PG is therefore reduced.

From the foregoing one must conclude that not only is there no contradiction between the results of Wellisch, and those of the other workers, but the results of Wellisch are actually in accord with the theory of Thomson. The apparent discrepancy lay therefore only in the *interpretation* of the E portions of the Wellisch curves. Having then shown that the process of ion formation in air probably consists as Thomson postulated in the attachment of an electron to a neutral molecule which happens in one out of about 2.5×10^6 collisions, one may be permitted to speculate on the conditions determining this attachment. It has been shown by Franck (7), and also recently by the writer (10), that the electrons do not attach to form ions in pure nitrogen gas. And since the curves obtained in pure oxygen, as well as those in nitrogen contaminated with small quantities of oxygen, yield values of n of the same order of magnitude as those in air (when they are calculated back to air on the basis of their oxygen content), one may safely conclude that it is to the oxygen molecule in pure dry air that the electron attaches itself to form the negative ion. One must then picture the process of ion formation as resulting from the particular conditions surrounding the impact of an electron with an oxygen molecule, these conditions being fulfilled on the average in only one out of approximately 50,000 impacts. The nature of these conditions furnishes a problem for the future. These conditions may consist in the electrons striking a certain electronic ring in the molecule under just the right conditions of velocity. They may consist in the molecule struck being in a certain chemical or physical state, *e.g.*, ozone, or in some particular state as regards radiation, as is suggested by Perrin (11) for chemical phenomena. It is hoped that some data may shortly be available on the effect of the energy of the electron on this process. Some light will also be thrown on this process from the investigation of the behavior of other gases in this respect. Work is now under way in an attempt to determine *n* for different gases, and to correlate this with their chemical nature.

The writer in concluding desires to express to the National Research Fellowship Board his sincere thanks for the opportunity given him, as National Research Fellow, to clear up a problem which has fascinated him for a number of years. He also wishes to express his gratitude to Professor R. A. Millikan for the excellent facilities he placed at the writer's command for pursuing this work in the Ryerson Laboratory as well as for his kind advice and criticism in the writing of this paper.

Note Added Dec. 30, 1920.

Recent results obtained in this laboratory by Mr. Wahlin show definitely that the energy imparted to the electron is not a factor in determining its attachment to a neutral molecule to form the negative ion in air. Although Mr. Wahlin subjected electrons to fields varying from very low values up to values such that they acquired energies causing ionization by collision no noticeable increase in the formation of ions could be observed. These results definitely indicate that the theory of ion formation proposed by Wellisch is untenable.

Summary.

1. Experimental work was undertaken to attempt to decide between the two theories of negative ion formation, from electrons and neutral molecules, proposed by J. J. Thomson and by Wellisch.

2. Mobilities of the carriers formed by photo-electrons liberated from one plate of a parallel plate condenser by a beam of ultraviolet light, focussed on it at a glancing angle by a quartz lens, were determined at different pressures for air using the Rutherford alternating current method.

3. The results in general confirmed the results of previous observers, yielding a single class of carriers whose mobilities became abnormal below pressures of 150 mm. The value of these mobilities was also found to be a function of the frequency of commutation in agreement with earlier results.

4. The manner of introduction of the ultra-violet light into the chamber having reduced the "stray light" effects, it was found that the asymptotic feet of the curves observed below 200 mm. pressure were a real and important feature of the phenomenon.

5. The mathematical theory of J. J. Thomson was adapted to fit these measurements; and on the basis of the equation so deduced the chance of ion formation, n, was determined from experiment.

6. Within the limits of accuracy of the method n was found to be equal to about 2.5×10^5 for pure dry air.

7. The current-voltage curves computed on the basis of the Thomson theory were compared with the observed curves and marked general similarities were noticed below 200 mm. pressures.

8. The asymptotic feet of the computed and observed curves lie close together; which is significant inasmuch as it is these portions of the observed curves that yield the abnormal values of the mobility.

9. Deviations of the observed curves from those computed at the higher and the lower pressures are explained.

10. Repetition of the Wellisch experiments shows that what he termed "free electrons" are the carriers of abnormally high mobilities observed by the earlier workers.

11. It was also shown that the identity of Wellisch "free electrons" with the carriers changing from electrons to ions in the measuring field once established, the other results of Wellisch are to be expected on the Thomson theory.

12. It is shown that as the electrons do not attach to N_2 molecules, and that as the values of n obtained in pure O_2 and in N_2 with small quantities of O_2 in it agree with the values found for air on the basis of its oxygen content, one must conclude that it is to the O_2 molecules in air that the electrons attach. The value of n for O_2 molecules is then 50,000.

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THEORY AND CALCULATION OF VARIABLE ELECTRICAL SYSTEMS.

By John R. Carson.

Synopsis.

Variable Electrical Systems are defined as those in which either the circuit elements (resistance, inductance or capacity) are explicit time functions or those in which the relation between current and applied E.M.F. is non-linear. In the present paper, the *theoretical methods of solution* of "invariable systems" are extended to include variable systems by integral equations of the Volterra type. A number of representative problems are worked through to indicate the appropriate mathematical procedure and the applicability of the method to both transient and steady state phenomena.

THE symbolic or operational method of solution of problems in electric circuit theory is a highly developed and very serviceable mathematical tool which is responsible in considerable degree for the rapid developments of the more abstract side of electrical engineering where the problems encountered are of such character as to render essential an adequate theoretical guide in predicting and interpreting phenomena. The method, however, is explicitly limited in its application to those physical systems or networks which may be mathematically described by a set of linear differential equations in which the coefficients of the differential operators are constants. Physically this means two things; first the currents are proportional to the applied forces, and secondly the circuit elements of the system (resistance, inductance and capacity) are invariable.[1] A system or network in which these restrictions hold will be termed invariable. In the great majority of problems these restrictions are not serious from a practical standpoint, since the departures from the requirements of the ideal invariable systems are usually small, and can usually be taken into account by indirect methods. Theoretically, however, these restrictions are undesirable and furthermore an increasing number of problems is being encountered in which we are directly concerned with departures from the ideal requirements. Such systems, for example, are the vacuum tube where the current is not proportional to the applied electromotive force, and where the operation of the device as a detector or modulator is strictly conditioned by the fact that the characteristic is non-linear; the induction generator where the mutual inductance between primary and secondary is variable; and the microphone transmitter circuit where the resistance is varied by external means. The differential equations which describe variable systems of which the foregoing examples are typical, have, of course, been extensively studied from the standpoint of pure mathematics. At the same time curiously little application of general methods of solution appears to have been made to such physical problems, in spite of their great and increasing technical importance. In the present paper it will be shown that the differential equations of variable electrical systems can be thrown at once into the form of integral equations, by aid of which formulation the solution is quite simply expressible in terms of the solution of the corresponding invariable system. For a discussion of the application of integral equations to circuit theory the reader is referred to my paper on Transient Oscillations (March, 1919, Proc. A.I.E.E.).

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[1] For an interesting discussion of variable electrical systems from the standpoint of dynamics, see "Theory of Variable Electrical Systems." Phys. Rev., Aug., 1917, by H. W. Nichols.

The theoretical analysis of the present paper is based on the following fundamental theorem, which is derived and discussed in the paper referred to above:[1]

If the current flowing in any branch (or mesh) of a network in response to a "unit E.M.F." (zero before, unity after, time $t = 0$) is denoted by $A(t)$, then the current $I(t)$ which flows in response to the arbitrary applied E.M.F. $f(t)$ is given by the formula:

$$I(t) = \frac{d}{dt}\int_0^t f(y)A(t-y)dy. \tag{1}$$

The function $A(t)$ which characterizes the network will be termed the *indicial admittance.*[2]

It is important to observe that formula (1) is restricted in its direct applicability to *invariable systems* as hereinbefore defined; nevertheless, as will be shown, it enables us to deal successfully with systems which are not so restricted; that is ones which contain variable circuit elements, and ones in which the relation between current and voltage is non-linear.

Since the present paper is concerned exclusively with the extension of circuit theory to systems which include variable circuit elements it is assumed in the following that the indicial admittance $A(t)$ of the invariable or unvaried system is known. For a full discussion of the theory and calculation of invariable systems and of methods for determining the indicial admittance the reader is referred to Fry's paper, PHYSICAL REVIEW, August, 1919, and my previous papers, PHYSICAL REVIEW, September, 1917, and Proc. A.I.E.E., March, 1919.

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[1] See also a paper by T. C. Fry on the Solution of Problems in Circuit Theory, which appeared in the Physical Review, August, 1919.

[2] This terminology is suggested by the physical and mathematical significance of the function.

Before developing the theory in more general terms we shall consider the simplest possible example which may be termed:

THE MICROPHONE TRANSMITTER PROBLEM.

Consider a circuit containing an invariable impedance denoted operationally by Z in series with a variable resistance $rf(t)$, to which is applied the E.M.F. $E(t)$. The equation for the resultant current $I(t)$ may be written in operational notation as:

$$(Z + rf(t))I(t) = E(t)$$

or

$$ZI(t) = E(t) - rf(t)I(t). \tag{2}$$

Inspection of equation (3) shows at once that $I(t)$ is equal to the current flowing in an *invariable circuit* of impedance Z in response to the applied E.M.F. $E(t) - rf(t)I(t)$; consequently if the *indicial admittance* of the *invariable* or *unvaried circuit* ($r = 0$) be denoted by $A(t)$ it follows at once from (1) that the current in the actual circuit may be written as:

$$I(t) = \frac{d}{dt}\int_0^t E(y)A(t-y)dy - r\frac{d}{dt}\int_0^t A(t-y)f(y)I(y)dy. \tag{3}$$

The first term on the right-hand side of (3) is by comparison with formula (1) simply the current which would flow in the unvaried circuit ($r = 0$) in response to the applied E.M.F. $E(t)$; denoting this current by $I_0(t)$ we have:

$$I(t) = I_0(t) - r\frac{d}{dt}\int_0^t A(t-y)f(y)I(y)dy. \tag{4}$$

Equation (4) is an integral equation of the Volterra type, methods for the formal solution of which are well known. The following series solution recommends itself by reason of the direct physical significance of each term of the series:

$$I(t) = I_0(t) - I_1(t) + I_2(t) - I_3(t) + \cdots + (-1)^n I_n(t), \tag{5}$$

where the successive terms of the series are defined by the relations:

$$\begin{aligned} I_1(t) &= r\frac{d}{dt}\int_0^t A(t-y)f(y)I_0(y)dy, \\ I_{n+1}(t) &= r\frac{d}{dt}\int_0^t A(t-y)f(y)I_n(y)dy. \end{aligned} \tag{6}$$

Referring to the fundamental formula (1) it will be observed that the successive terms of the series (5) as defined by (6) admit of direct physical interpretation as follows: $I_1(t)$ is equal to the current which would flow in the unvaried circuit of impedance Z in response to the fictitious applied E.M.F. $rf(t)I_0(t)$; $I_2(t)$ is equal to the current in the same circuit in response to the fictitious applied E.M.F. $rf(t)I_1(t)$; etc. That is to say the product of the variable resistance $rf(t)$ into each component current of the series (5) acts like an additional component E.M.F. in the unvaried circuit to produce an additional component current.

The solution is of course complete in that it formulates the resultant current for all types of applied forces and all possible forms of resistance variations. In particular if the impressed E.M.F. and the resistance variation are both periodic or sinusoidal the solution includes both transient as well as steady states. In this case, if we are concerned only with the ultimate steady state of the network it is not necessary to evaluate the definite integrals of (6). All that is necessary in order to write down the steady state solution corresponding to the series solution (5) is to express the product of $rf(t)$ into each component current (starting with $I_0(t)$) as a periodic time function, and then to evaluate the succeeding component current by operating on the periodic function with the impedance Z in accordance with usual operational rules. This is considered in greater detail below.

To illustrate the solution (5) and (6) the simplest possible case will be dealt with: into a circuit of unit resistance r and inductance $L = 1/a$ in which a steady current I_0 is flowing a resistance r is suddenly inserted at time $t = 0$; required the resultant current $I(t)$. In this case we have:

$$\begin{aligned} A(t) &= \text{indicial admittance of unvaried circuit} \\ &= 1 - e^{-at}, \\ f(t) &= 1, \end{aligned}$$

and the integral equation of the problem is:

$$\begin{aligned} I(t) &= I_0 - r\frac{d}{dt}\int_0^t (1 - e^{-ay})I(t - y)dy \\ &= I_0 - ra\int_0^t I(t - y)e^{-ay}dy. \end{aligned} \tag{7}$$

If the solution is carried out as indicated in (5) and (6), and if the notation $at = x$ is introduced, we get without difficulty

$$I(t) = I_0 \left\{ \begin{aligned} 1 - r(1 - e_1(x)e^{-x}) + r^2(1 - e_2(x)e^{-x}) \\ - r^3(1 - e_3(x)e^{-x}) + \end{aligned} \right. \tag{8}$$

Where the function $e_n(x)$ is defined as:

$$e_n(x) = 1 + x/1\,! + x^2/2\,! + x^3/3\,! + \cdots + x^{n-1}/(n-1)\,!$$
$$= \text{first } n \text{ terms of the exponential series.}$$

For all finite values of the resistance increment r the series (8) can be summed by aid of the identity

$$1 - e_n(x)e^{-x} = \int_0^x dx e^{-x}x^{n-1}/(n-1)\,!$$

Substitution of this identity in (8) gives

$$I(t) = I_0\left(1 - r\int_0^x e^{-(1+r)x}dx\right)$$
$$= I_0\frac{1 + re^{-(1+r)x}}{1+r}.$$

A more interesting example than the former is presented when the applied E.M.F. and the resistance variation are both sinusoidal time functions. In this case if the frequency of the applied E.M.F. be denoted by $F = q/2\pi$ and that of the resistance variation by $f = p/2\pi$, it is easy to show that the current $I_0(t)$ in the unvaried circuit is ultimately[1] a steady state current of frequency F. This follows from the fact that the definite integral of (3) which defines the current $I_0(t)$ is resolvable into the ultimate steady state current corresponding to an applied force of frequency F, and the accompanying transient oscillations which ultimately die away. The fictitious E.M.F. which may be regarded as producing the component current $I_1(t)$ is $rf(t)I_0(t)$; this is ultimately the product of the two frequencies F and f, and therefore resolvable into two terms of frequency $F + f$ and $F - f$ respectively. Carrying through this analysis it is easy to show that each component current is ultimately a steady-state but poly-periodic oscillation, as indicated in the following table.

Component Current	Frequency	
I_0	F	
I_1	$F+f,\ F-f$	
I_2	$F+2f,\ F,\ F-2f$	(10)
I_3	$F+3f,\ F+f,\ F-f,\ F-3f$	
I_4	$F+4f,\ F+2f,\ F,\ F-2f,\ F-4f$	

It is of importance to observe that the component currents involve, from a mathematical standpoint, multiple integrals of successively higher orders, the nth component, $I_n(t)$, involving a multiple integral of the nth order with respect to $I_0(t)$. Consequently the successive currents require longer and longer intervals of time to build up to their proximate steady-state values, so that the time required for the resultant steady-state to be arrived at cannot be inferred from the time constant of the unvaried circuit.

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[1] It hardly seems necessary to remark that the reference time $t = 0$ is purely arbitrary and that the resistance variation may start at such a time thereafter that $I_0(t)$ may be regarded as steady state during the entire time interval in which we are interested. Going farther, if we confine our attention to sufficiently large values of t, the whole process may be treated as steady state.

From table (10) it will be seen that the ultimate steady-state current is obtained by rearranging the series $I_0 + I_1 + I_2$ and is of the form

$$\sum_{n=-\infty}^{+\infty} A_n \cos (q + np)t + B_n \sin (q + np)t.$$

It is interesting to note that this series comes within the definition of a Fourier series only when $q = 0$ or an exact multiple of p. The steady-state solution is of very considerable importance and is considered in more detail in a succeeding section.

So far nothing has been said regarding the convergence of the formal series solution (5). For the case of variable resistance, however, it can be shown that the sufficient conditions for the absolute convergence of the series correspond to the physical restrictions imposed in a large and important class of problems. In the first place the series is absolutely convergent when $A(0) = 0$, since in this case the successive terms are related by the equation

$$I_{n+1}(t) = r\int^t A'(t - y)f(y)I_n(y)dy,$$

where

$$A'(t - y) = \frac{d}{dt}A(t - y).$$

In physical terms this restriction means that the branch of the network in which the variable resistance is located contains inductance also. In a large number of problems this condition is satisfied.

In the second place the solution is a power series in the parameter r, which fixes the size of the resistance variation. In general therefore the series will be absolutely convergent for some restricted range of values of r, which will, however, depend on the particular network under consideration. For the very important case of periodic resistance variations physical considerations restrict the maximum value of the variable resistance to a value less than the invariable resistance in the same branch. It is easy to see from physical considerations that in this case series (5) is absolutely convergent.

However a perfectly general restriction, imposed by physical conditions, is that $[1 + rA_{11}(0)f(t)] > 0$, since otherwise the branch would contain negative resistance. In view of this restriction an absolutely convergent series solution of equation (4) is obtainable by aid of the transformation.

$$J(t) = I(t)[(1 + rA(0)f(t)],$$

$$\phi(t) = f(t)/(1 + rA(0)f(t)).$$

In terms of J and ϕ the integral equation (4) becomes

$$J(t) = I_0(t) - r\int_0^t A'(t-y)\phi(y)J(y)dy.$$

This integral equation has the absolutely convergent solution:

$$J(t) = J_0(t) - J_1(t) + J_2(t) - \cdots,$$

where the terms of the series are defined by the relations:

$$J_0(t) = I_0(t)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$J_{n+1}(t) = r\int_0^t A'(t-y)\phi(y)J_n(y)dy.$$

For the sake of its physical interpretation this last equation may be written as:

$$J_{n+1}(t) = -rA(0)\phi(t)J_n(t) + r\frac{d}{dt}\int_0^t A(t-y)\phi(y)J_n(y)dy.$$

Inspection of this equation shows that the term $J_{n+1}(t)$ may be physically interpreted as the difference in the currents flowing in the unvaried network and in a resistance $1/A(0)$ in response to the fictitious E.M.F. $r\phi(t)J_n(t)$. This interpretation is of value in enabling one to write down immediately the corresponding steady-state current in the important case of periodic applied forces and periodic resistance variations.

In the light of the foregoing example the extension of the method of solution to more complicated networks and to the case of simultaneous impedance variations in a plurality of branches of the network should present no difficulties. The appropriate procedure, however, will be briefly illustrated by an example of some practical importance which may be termed:

THE INDUCTION GENERATOR PROBLEM.

In a sufficiently general form, this problem, which includes the fundamental theory of the dynamo, may be stated as follows:

Given an invariable primary and secondary circuit with a variable mutual inductance $Mf(t)$ which is an arbitrary but specified time function, and let the primary be energized by an E.M.F. $E(t)$ impressed in the circuit at the reference time $t = 0$; required the primary and secondary currents.

In operational notation the problem may be formulated by the equations:

$$\begin{aligned} Z_{11}I_1 - pMf(t)I_2 &= E(t) \\ -pMf(t)I_1 + Z_{22}I_2 &= 0 \end{aligned} \tag{11}$$

in which Z_{11} and Z_{22} are the self impedances of the primary and secondary respectively; $Mf(t)$ is the variable mutual inductance; $E(t)$ is the applied E.M.F. in the primary, and p denotes the differential operator d/dt. By aid of the fundamental formula (1) these equations may be written down as the following simultaneous integral equations:

$$\begin{aligned} I_1(t) &= \frac{d}{dt}\int_0^t dy A_{11}(t-y)\left(E(y) + M\frac{d}{dy}[f(y)I_2(y)]\right), \\ I_2(t) &= M\frac{d}{dt}\int_0^t dy A_{22}(t-y)\frac{d}{dy}[f(y)I_1(y)]. \end{aligned} \tag{12}$$

In these equations $A_{11}(t)$ and $A_{22}(t)$ denote the indicial admittances of the primary and secondary circuits respectively (when $M = 0$); that is the currents in these circuits in response to a unit E.M.F. (zero before, unity after time $t = 0$). We of course assume that they are known or can be determined by usual methods.

It follows at once that the formal solution of these equations is the infinite series:

$$I_1(t) = X_0(t) + X_2(t) + X_4(t) + \cdots + X_{2n}(t) + \cdots \tag{13}$$

$$I_2(t) = Y_1(t) + Y_3(t) + Y_5(t) + \cdots \tag{14}$$

in which the successive terms of the series are defined as follows:

$$\begin{aligned} X_0(t) &= \frac{d}{dt}\int_0^t dy A_{11}(t-y)E(y) = I_0(t), \\ Y_1(t) &= M\frac{d}{dt}\int_0^t dy A_{22}(t-y)\frac{d}{dy}[f(y)X_0(y)], \\ X_2(t) &= M\frac{d}{dt}\int_0^t dy A_{11}(t-y)\frac{d}{dy}[f(y)Y_1(y)], \\ Y_3(t) &= M\frac{d}{dt}\int_0^t dy A_{22}(t-y)\frac{d}{dy}[f(y)X_2(y)], \qquad \text{etc.} \end{aligned} \tag{15}$$

In the light of formula (1) the physical interpretation of the series solutions (13) and (14) follows at once. Thus $X_0(t)$ is equal to the current $I_0(t)$ flowing in the *isolated* primary in response to the applied E.M.F. $E(t)$; the first component current $Y_1(t)$ in the secondary is equal to the current which would flow in the isolated secondary in response to the applied E.M.F. $M(d/dt)f(t)X_0(t)$; $X_2(t)$, the second component current in the primary is equal to the current in the isolated primary in response to the applied E.M.F. $M(d/dt)f(t)Y_1(t)$; etc. The resultant currents are thus represented as built up by a to-and-fro interchange of energy between primary and secondary or by a series of successive reactions. In the important case where the applied E.M.F. and the variation of mutual inductance are both sinusoidal time functions of frequency F and f respectively, it is easy to show that each component current becomes ultimately equal to a set of periodic steady-state currents. Thus the component X_0 is ultimately singly periodic of frequency F; Y_1 is ultimately doubly periodic of frequencies $F + f$ and $F - f$; X_2 triply periodic of frequencies $F + 2f$, F and $F - 2f$; Y_3 quadruply periodic of frequencies $F = 3f$, $F + f$, $F - f$, $F - 3f$; etc.

In connection with this solution attention should be called to a discussion of the same problem in a paper by Liebowitz (Proc. Inst. Radio Engineers, Dec., 1915). The method of solution there employed and credited by the author to M. I. Pupin is based entirely on steady-state concepts and is limited to sinusoidal impressed forces and inductance variations. With these restrictions the solution arrived at corresponds term by term to the steady-state part of the complete solution given above. While the two solutions thus become ultimately identical in the region where Pupin's solution is convergent and valid[1] the present treatment is of broader scope in that transient as well as steady-states and arbitrary forces and inductance variations are included in the complete solution.

It is beyond the scope of the present paper to go into a consideration of the energy relations of the induction generator, but a few deductions may be noted. In the case of resistance variations the ignored force which controls the variable resistance element neither supplies nor abstracts energy; consequently in this case all the energy consumed in the system is furnished by the electrical source. In the case of the induction generator, on the other hand, the ignored force which controls the variation of mutual inductance may either supply or abstract energy from the electrical system; in other words the system may act either as an induction generator or an induction motor, and the ignored force correspondingly as a source or sink of energy. In the simple case where the series is so rapidly convergent as to make the component currents of frequencies F and $F + f$ alone of importance the system acts like an electrical generator or motor according as the frequency f of inductance variation is greater or less than the frequency F of the electrical supply. This agrees with the well-known fact that the induction motor acts like a generator when driven above synchronism. When the circuits are appropriately tuned to emphasize the higher harmonics the device is essentially a Goldschmidt generator which has been proposed for the production of radio frequency currents.

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[1] In the paper referred to, incorrect physical conclusions are deduced from the fact that the steady-state series diverges under certain conditions. As explained in the present paper no physical significance can in general be attached to the divergence of this series.

The formal series solutions are absolutely convergent in the majority of actual circuits. In special circuit arrangements, however, such as the Goldschmidt generator, where the primary and secondary are tuned to emphasize the higher harmonic currents, the series may be divergent. In this case some transformation, such as that discussed in connection with the preceding example must be introduced. For example, if the equations (12) are cleared of the differential operator, and if we introduce the functions

$$J_1(t) = I_1(t)/(1 - \mu_1\mu_2 f(t)), \qquad \mu_1 = MA_{11}'(0),$$
$$J_2(t) = I_2(t)/(1 - \mu_1\mu_2 f(t)), \qquad \mu_2 = MA_{22}'(0),$$

it is easily shown that the series solutions in J_1 and J_2 are absolutely convergent.

THE SOLUTION FOR THE STEADY-STATE OSCILLATIONS.

For the very important case of periodic applied forces and periodic variations of circuit elements we are often concerned exclusively with the ultimate steady-state of the system, and not at all with the mode in which the steady-state is approached; that is, attention is restricted to the periodic oscillations which the system executes after transient disturbances have died away. In this case, if the periodic variations of circuit elements are sufficiently small the required steady-state is obtained in the form of a series by replacing each term of the complete series solution by its ultimate steady-state value; a process which is very simple in view of the physical significance of each term of the latter series. The procedure will be briefly illustrated in connection with the *microphone transmitter* problem, which is formulated and solved in equations (1)–(6). The variable resistance element will be taken as $r \cos pt$, and the current $I_0(t)$ as the *real part* of $I_0 e^{iqt}$ where I_0 is in general complex, and the symbol i denotes the imaginary operator $\sqrt{-1}$. The frequency F of the impressed electric force is therefore $q/2\pi$ and the frequency f of resistance variation is $p/2\pi$. The symbolic notation commonly used in the theory of alternating current will be employed and the symbolic impedances of the unvaried network at frequency $(q + np)/2\pi$ will be denoted by Z_n. This is obtained by methods long employed in alternating current calculations by replacing the operator d/dt by $i(q + np)$ in the differential equations which describe the network or system. Similarly the impedance of the unvaried network at frequency $(q - np)/2\pi$ will be denoted by Z_n'.

If reference is now made to equations (6) and their physical significance kept in mind it is evident at once that the component current $I_1(t)$ is equal to the current in the unvaried network in response to the applied E.M.F.

$$rf(t)I_0(t) = (r/2)I_0(e^{i(q+p)t} + e^{i(q-p)t}).$$

Consequently, after transient effects have died away, the component current $I_1(t)$ is replaceable by

$$I_1 = (r/2)I_0\left(\frac{e^{i(q+p)t}}{Z_1} + \frac{e^{i(q-p)t}}{Z_1'}\right).$$

It will be understood, of course, that the real part of this complex expression is the actual solution, and that the imaginary part is to be discarded. Proceeding in precisely the same way with the second component current $I_2(t)$, it is ultimately replaceable by

$$I_2 = (r/2)^2I_0\left\{\frac{e^{i(q+2p)t}}{Z_1Z_2} + \frac{e^{i(q-2p)t}}{Z_1'Z_2'} + \frac{e^{iqt}}{Z_0}\left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right)\right\}.$$

Similarly,

$$I_3 = (r/2)^3I_0\left\{\begin{aligned} &\frac{e^{i(q+3p)t}}{Z_1Z_2Z_3} + \frac{e^{i(q+p)t}}{Z_1}\left(\frac{1}{Z_1Z_2} + \frac{1}{Z_0Z_1} + \frac{1}{Z_0Z_1'}\right) \\ &\quad + \frac{e^{i(q-3p)t}}{Z_1'Z_2'Z_3'} + \frac{e^{i(q-p)t}}{Z_1'}\left(\frac{1}{Z_1'Z_2'} + \frac{1}{Z_0Z_1'} + \frac{1}{Z_0Z_1}\right).\end{aligned}\right.$$

In this way the steady-state series solution is built up term by term, the component currents being poly-periodic as indicated in (10).

For sufficiently small impedance variations this method of solution works very well, and leads to a rapidly convergent solution. In other cases, however, the solution so obtained may be divergent, even when the complete series solution from which it is derived is absolutely convergent. The explanation of this lies in the fact that the steady-state series so obtained is the *sum of the limits* (as t approaches infinity) of the terms of the complete series solution, whereas the actual steady-state is the *limit of the sum*. These are not in general equal; in particular the former may be and often is divergent when the latter is convergent.

In view of the foregoing considerations it is of great importance to develop another method of investigating the steady-state oscillations which avoids the difficulties in the formal series solution. The following method has suggested itself to the writer and works very well in cases where the previous form of solution fails. It should be stated at the outset, however, that the absolute convergence of the solution to be discussed, while reasonably certain in all physically possible systems, has not been established by a rigorous mathematical investigation which appears to present very considerable difficulties.

The method of solution will be elucidated in connection with the example discussed above under the title of the *microphone transmitter problem* and formulated in equations (1)–(6); the extension of the method of solution to more involved problems will be obvious. For the case of an applied E.M.F. of frequency $F = q/2\pi$ and a resistance variation of frequency $f = p/2\pi$ the formal series solution shows that the ultimate steady-state oscillations are of frequency F, $F \pm f$, $F \pm 2f$, $F \pm nf$. If the variable resistance is taken as $r \cos pt$ and the current $I_0(t)$ as the *real part* of $I_0 \exp(iqt)$ (where I_0 is in general complex), the following tentative solution suggests itself:

$$I(t) = A_0 e^{iqt} + \sum_{j=1}^{n} A_j e^{i(q+jp)t} + A_j' e^{i(q-jp)t} + R_n(t).$$

In this expression the coefficients A_0, A_j, A_j', which are to be determined, are in general complex and the real part of the expression is alone to be retained in the final solution. The foregoing is of course equivalent to a trigonometric series but the exponential form is much more convenient to handle. In the summation the upper limiting index n is a finite positive integer which may be assigned any desired value. The "remainder" $R_n(t)$ and the coefficients A_0, A_j, A_j' are to be determined. If the steady-state solution is convergent the remainder $R_n(t)$ must approach zero as the index n is indefinitely increased. The practical value of the solution will therefore depend on the rate of convergence of the series.

If the coefficients A_0, A_j, A_j' are determined in accordance with the process developed below, it may be shown that the remainder $R_n(t)$ for large values of t satisfies the integral equation

$$R_n(t) = -(r/2)\left(\frac{A_n}{Z_{n+1}} e^{i(q+(n+1)p)t} + \frac{A_n'}{Z'_{n+1}} e^{i(q-(n+1)p)t}\right)$$
$$- r\frac{d}{dt}\int_0^t A(t-y)\cdot f(y)\cdot R_n(y)dy.$$

In this expression Z_{n+1} and Z'_{n+1} are the complex expressions for the impedance of the unvaried network at frequencies $(q + (n+1)p)/2\pi$ and $(q - (n+1)p)/2\pi$ respectively. They are obtained in accordance with well-known rules from the differential equations of the unvaried network by replacing the differential operator d/dt by $i(q + (n + 1)p)$ and $i(q - (n + 1)p)$ respectively. The imaginary part of the foregoing expression is of course to be discarded.

The coefficients A_0, A_j, A_j' are now determined by the following set of equations which are obtained by substitution of the assumed solution in the integral equation of the problem and then letting the time t become indefinitely large.

$$\begin{aligned} A_n &= - h_n A_{n-1}, \\ A_n' &= - h_n' A'_{n-1}, \\ A_j &= - h_j(A_{j-1} + A_{j+1}), \\ A_j' &= - h_j'(A'_{j-1} + A'_{j+1}), \qquad j = (n - 1), (n - 2), \cdots, 2, \\ A_1 &= - h_1(A_2 + A_0), \\ A_1' &= - h_1'(A_2' + A_0), \\ A_0 &= I_0 - h_0(A_1 + A_1'). \end{aligned}$$

In these equations the symbol h_j denotes $r/2Z_j$; similarly h_j' denotes $r/2Z_j'$.

It will be observed that starting with A_n, A_n' each coefficient is determinable in terms of the coefficient of next lower order. Thus from the first and second set of equations:

$$A_{n-1} = - h_{n-1}A_{n-2}\frac{1}{1 - h_{n-1}h_n},$$

$$A'_{n-1} = - h'_{n-1}A'_{n-2}\frac{1}{1 - h'_{n-1}h_n'}.$$

Continuing this process it is easy to show that

$$\begin{aligned} A_j &= - h_j C_{jn} A_{j-1}, \\ A_j' &= - h_j' C'_{jn} A'_{j-1}, \qquad j = n, (n - 1), \cdots, 2, \end{aligned}$$

where C_{jn} denotes the terminating continued fraction

$$C_{jn} = \frac{1}{1 - h_j h_{j+1}}\,\frac{1}{1 - h_{j+1}h_{j+2}}\,\frac{1}{1 - h_{j+2}h_{j+3}}\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \frac{1}{1 - h_{n-1}h_n}$$

and C_{jn}' the corresponding expression in h_j', h_n'.

Finally, therefore

$$A_1 = -h_1 C_{1n} A_0,$$

$$A_1' = -h_1' C_{1n}' A_0$$

and

$$A_0 = I_0 - h_0(A_1 + A_1')$$

$$= I_0/(1 - h_0 h_1 C_{1n} - h_0 h_1' C_{1n}').$$

The coefficients are thus all determined in terms of I_0 and the remainder $R_n(t)$ is given by an integral equation. It follows therefore that, provided the series converges, the coefficients A_0, A_j, A_j' are the limits of the foregoing expressions as the index n is made indefinitely large and the terminating continued fractions become infinite continued fractions. The complete solution therefore involves the evaluation of infinite continued fractions.

The practical value of this method of solution will depend, of course, on the rate of convergence of the continued fractions. While no rigorous proof has been obtained, it is believed that they are absolutely convergent for all physically possible systems, but this question certainly requires fuller investigation. Nevertheless any doubt regarding the convergence of the solution need not prevent the use of the method in a great many problems where physical considerations furnish a safe guide. For example this method of solution, when applied to the problem of the induction generator, discussed above leads to the usual simplified engineering theory of the induction generator and motor, besides exhibiting effects which the usual treatment either ignores or fails to recognize.

It seems worth while pointing out that the method of solution just discussed does not exclude the investigation of the transient disturbances which exist when the electrical forces are impressed on the system or when the impedance variations are initiated. To show this in connection with the variable resistance problem, let the steady-state current, as derived above, be denoted by $S(t)$ and the total current $I(t)$ by $S(t) + T(t)$. Substitution in equation (4) gives:

$$T(t) = I_0(t) - S(t) - r\frac{d}{dt}\int_0^t A(t-y)f(y)S(y)dy$$
$$- r\frac{d}{dt}\int_0^t A(t-y)f(y)T(y)dy,$$

which determines the transient disturbance $T(t)$.

NON-LINEAR CIRCUITS.

In the previous examples discussed the variations of the variable circuit elements are assumed to be specified time functions, which is the same thing as postulating that these variations are controlled by ignored forces which do not explicitly appear in the statement and equations of the problem. We distinguish another type of variable circuit element where the variation is not an explicit time function, but rather a function of the current (and its derivatives) which is flowing through the circuit. For example the inductance of an iron-core coil varies with the current strength as a consequence of magnetic saturation. The equation of a circuit which contains such a variable element may be written down in operational notation

$$ZI + \phi(I) = E(t),$$

or

$$ZI = E(t) - \phi[I(t)]. \tag{16}$$

In this equation Z is, of course, to be taken as the impedance of the invariable part of the circuit, the indicial admittance of which is denoted by the usual symbol $A(t)$.

Equation (16) may be interpreted as the equation of the current $I(t)$ in a circuit of invariable impedance Z when subjected to an applied E.M.F. $E(t) - \phi[I(t)]$; consequently by aid of formula (1), $I(t)$ is given by

$$I(t) = \frac{d}{dt}\int_0^t A(t-y)E(y)dy - \frac{d}{dt}\int_0^t A(t-y)\phi[I(y)]dy. \tag{17}$$

The first integral is simply the current in the invariable circuit of impedance Z in response to the applied E.M.F. $E(t)$; denoting this by $I_0(t)$ we have

$$I(t) = I_0(t) - \frac{d}{dt}\int_0^t A(t-y)\phi[I(y)]dy. \tag{18}$$

This is a *functional integral equation*, the solution of which is gotten by some process of successive approximations. For example, provided the sequence converges, $I(t)$ is the limit as n approaches infinity of the *sequence*

$$I_0(t),\ I_1(t),\ I_2(t),\ \cdots,\ I_n(t), \tag{19}$$

where the successive terms of the sequence are defined by the relations:

$$\begin{aligned} I_1(t) &= I_0(t) - \frac{d}{dt}\int_0^t A(t-y)\phi[I_0(y)]dy \\ &\cdots\cdots\cdots\cdots\cdots \\ I_{n+1}(t) &= I_0(t) - \frac{d}{dt}\int_0^t A(t-y)\phi[I_n(y)]dy. \end{aligned} \tag{20}$$

With this brief sketch, the method will now be applied to *The Problem of the Three-Element Vacuum Tube.*[1]

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[1] The reader is assumed in the following to be acquainted with the theory of the vacuum tube in its broad outlines. For a very able account of the theory and physics of the tube see Van der Bijl's papers, PHYSICAL REVIEW, Sept., 1918, and Proc. Inst. Radio Engineers, April, 1919. For some mathematical methods of dealing with the device see my paper, Proc. Inst. Radio Engineers, April, 1919.

In this device the output circuit or plate current is assumed to be a known function of the grid-filament and plate-filament potential differences. It is further assumed that the output or plate circuit is closed through an impedance Z (whose circuit elements are invariable) and that a specified potential difference $E(t)$ is applied between the grid and filament. We denote by $V(t)$ the potential difference between plate and filament and by $I(t)$ the unknown plate current[1] which we are to determine. We assume that $I(t)$ is a known function of $E(t)$ and $V(t)$ and therefore write

$$I(t) = F[E(t),\ V(t)], \tag{21}$$

But since this same current flows into the output circuit impedance Z (of indicial admittance $A(t)$) across whose terminals the potential difference is $V(t)$, we have also

$$I(t) = \frac{d}{dt}\int_0^t A(t-y)V(y)dy. \tag{22}$$

Equating (20) and (21) we get the functional equation:

$$F[E(t),\ V(t)] = \frac{d}{dt}\int_0^t A(t-y)V(y)dy. \tag{23}$$

Now in the actual tube the work of Van der Bijl and others has shown that over a considerable part of the characteristic the current is given by the approximate relation

$$I(t) = \frac{\mu E(t) - V(t)}{R}, \tag{24}$$

where μ is a physical parameter of the tube commonly termed the amplification factor and R is the "internal resistance" of the tube. This suggests that the characteristic function be written as:

$$F(E,\ V) = \frac{\mu E - V}{R} + \phi(E,\ V). \tag{25}$$

Whence by substitution in (23) and rearrangement we get

$$V(t) = \mu E(t) + R\phi[E(t),\ V(t)] - R\frac{d}{dt}\int_0^t A(t-y)V(y)dy. \tag{26}$$

Which is a functional integral equation in the unknown potential difference $V(t)$. With this function determined the current $I(t)$ is given by (20) or (21).

The solution of this functional integral equation is obtained by a process of successive approximations; in the present problem the most convenient and physically significant mode of approximation is to take $V(t)$ as the limit (as the index n increases indefinitely) of the sequence

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[1] The potential differences $E(t)$, $V(t)$ and the current $I(t)$ are to be understood as denoting not the total values but rather their variations from the normal or steady condition. Thus $I(t)$ does not include the steady d.c. current.

$$V_0(t),\ V_1(t),\ V_2(t),\ \cdots,\ V_n(t), \tag{27}$$

where the successive approximations are defined by the equations:

$$\begin{aligned} V_0(t) &= \mu E(t) - R\frac{d}{dt}\int_0^t A(t-y)V_0(y)dy \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ V_{n+1}(t) &= \mu E(t) + R\phi[E(t), V_n(t)] - R\frac{d}{dt}\int_0^t A(t-y)V_{n+1}(y)dy. \end{aligned} \tag{28}$$

The physical significance of these approximations, which is of importance in the following discussion, may be seen as follows: Consider a circuit of resistance R in series with an impedance Z (of indicial admittance $A(t)$) with a potential difference $E(t)$ impressed on the terminals of the circuit, and let $V(t)$ denote the unknown voltage developed across the impedance Z. The following expressions for the current $I(t)$ in the circuit can be written down at once:

$$\begin{aligned} I(t) &= \frac{1}{R}(E(t) - V(t)) \\ &= \frac{d}{dt}\int_0^t A(t-y)V(y)dy. \end{aligned}$$

Equation of these two expressions gives the following integral equation for $V(t)$:

$$V(t) = E(t) - R\int_0^t A(t-y)V(y)dy. \tag{29}$$

Comparison of this equation with (28) shows at once that the first term $V_0(t)$ of the sequence (27) is simply equal to the voltage developed across the output circuit impedance Z on which an E.M.F. μE is impressed through a resistance R. Similarly $V_{n+1}(t)$ is equal to the potential developed across Z when an E.M.F. $\mu E + R\phi(E, V_n)$ is impressed thereon through a resistance R. It follows, therefore, that the resistance R is to be regarded as the "internal resistance" of the tube. In the engineering theory of the amplifier this is usually defined by the relation $1/R = \partial F(E, V)/\partial V$, but in the present discussion its value is not so limited. A second consequence of the above is that, so long as the characteristic is approximately a straight line the equivalent circuit of the tube is simply a resistance R in series with an impedance Z on which an E.M.F. μE is impressed. This agrees with the engineering theory of the vacuum tube amplifier[1] and shows that the simple theory is equivalent to the first approximation of the complete solution.

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[1] See papers by Van der Bijl and the writer in Proc. I. R. E., April, 1919, and also a paper by H. W. Nichols, PHYSICAL REVIEW, June, 1919.

This physical interpretation is of substantial advantage in the following discussion of the convergence of the sequence. From the physics of the vacuum tube and the way in which the ϕ function is formed from the characteristic function $F(E, V)$ of the tube, it may be shown that

$$\phi(E, V) \leqq (\mu E - V)/R$$

and

$$\phi(E, V_n) - \phi(E, V_{n+1}) \leqq (V_{n+1} - V_n)/R.$$

If $V_{n+1}(t) - V_n(t)$ is denoted by $d_{n+1}(t)$, subtraction of the nth from the $(n + 1)$st equation of (28) gives

$$d_{n+1}(t) + R\frac{d}{dt}\int_0^t A(t - y)d_{n+1}(y)dy \leqq d_n(t).$$

In the light of the physical interpretation of the integral equation of this type, it is immediately evident that $d_{n+1}(t)$ is related to $d_n(t)$ as the voltage across an impedance Z is to the potential impressed thereon through a resistance R. It follows therefore from physical considerations that $d_{n+1} \angle d_n$ in all circuits of practical importance and that the sequence is convergent.

Referring to equations (28) it will be observed that each stage of the approximation formally requires the solution of an integral equation. As a matter of fact this operation can be dispensed with and the solution written as

$$V_0(t) = \frac{d}{dt}\int_0^t K(t - y)\mu E(y)dy$$

. . . .

$$V_{n+1}(t) = V_0(t) - R\frac{d}{dt}\int_0^t dyK(t - y)\phi[E(y), V_n(y)].$$

The function $K(t)$ may be itself evaluated from the integral equation

$$K(t) = 1 - R\frac{d}{dt}\int_0^t A(t - y)K(y)dy$$

Since this equation, physically interpreted, states that $K(t)$ is the voltage developed across the impedance Z when a "unit E.M.F." (zero before, unity after, time $t = 0$) is impressed thereon through a resistance R, it is legitimate to regard $K(t)$ as a datum of the problem, like the indicial admittance $A(t)$, since it can be evaluated by processes applicable to the theory of invariable circuits.

In view of the physical significance of the successive terms of the sequence, the actual evaluation of the integral equations and the definite integrals may be dispensed with if we are concerned only with steady-state phenomena, and the integral equations of (28) may be replaced by equivalent operational formulæ which are calculable by usual methods. Thus if $E(t)$ is a periodic time function and we ignore transient states, we have

$$V_0 = \mu E \frac{Z}{R + Z}, \tag{30}$$

which follows at once from the physical interpretation of the sequence. V_0 is thus a periodic function of the same frequency as that of the impressed E.M.F. E. Having calculated V_0, we have in operational notation:

$$V_1 = (\mu E + R\phi(E, V_0)) \frac{Z}{R + Z}. \tag{31}$$

This can be solved for V_1 by usual steady-state methods provided that $\phi(E, V_0)$ is expanded either as a Fourier series, which is always possible, or preferably as a power series in E and V_0 which is usually possible over the operating range of the characteristic. Higher approximations V_2, V_3, $\cdots$ follow by straightforward operations.

CONCLUSION.

The purpose of the present paper has been to illustrate in a few representative problems the application of integral equations to the solution of those problems in electric circuit theory in which variable circuit elements are involved. Integral equations have been employed for some time by the writer in the solution, both formal and numerical, of practical problems in circuit theory and have proved to be a serviceable instrument. In the present paper the emphasis has been placed on general methods and the physical interpretation of the solutions, and no attempt has been made to discuss the appropriate methods of numerical solution. A considerable experience, however, has convinced the writer that the application of integral equations to the problems of circuit theory is attended by marked advantages in actual engineering calculations.

DEPARTMENT OF DEVELOPMENT AND RESEARCH,
AMERICAN TELEPHONE AND TELEGRAPH COMPANY,
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THE BLACKENING OF A PHOTOGRAPHIC PLATE AS A FUNCTION OF INTENSITY OF LIGHT AND TIME OF EXPOSURE.

By P. S. Helmick.

Synopsis.

Blackening of Photographic Plates as a Function of Light Intensity, Wave-length and Exposure.—*Three emulsions*, a slow one (Seed 23), a rapid one (Seed 27X) and a very rapid one (Seed Graflex) were coated on plate glass and illuminated for various lengths of time with various intensities of monochromatic light of wave-length 450, 550 and 650 $\mu\mu$. The results are given in curves and are found to be well represented, for a given emulsion and wave-length, by the *empirical equation for blackening*, $D = (1/a) \log [b - (b - 1)e^{-ct}]$, where $\log c = A + B \log I + c \log^2 I$, and a, b, A, B and C are constants. A graphical method of determining these constants employing a monographic chart is described. Schwarzschild's exponent p is shown to be equal to $(B + C \log I)^{-1}$, and is found to vary from 0.7 to 1.95 according to the emulsion, wave-length and intensity. In using the *photographic method of measuring the intensity of a source of light*, it is obviously important to determine the constants of the equation for the blackening of the emulsion for the proper wave-length. The method of doing this is explained in detail.

THE object of this research has been the expression of the blackening of a photographic plate as a function of the intensity of monochromatic light and time of exposure to light, other factors being constant.

Such an expression finds practical application in photometry of monochromatic sources. By exposing a plate to a light source of unknown intensity for any known length of time, measurement of the resulting blackening will enable the unknown intensity of light to be calculated.

Apparatus and Materials.

The plates used in the work were coated on special plate glass, and the variations of density due to unevenness of coating were of the order of 1 per cent. Three emulsions were used: "A" a slow emulsion (Seed 23), "B" a rapid emulsion (Seed 27X), and "C" a very rapid emulsion (Seed Graflex).

The source of light was a Hilger Monochromatic Illuminator. Light from a constant-voltage tungsten lamp was focussed upon the collimating slit of the illuminator, and was again focused upon the second slit of the instrument, before emerging into the exposing box.

Variations in intensity of light were obtained by varying the distance between the plate and the second slit of the illuminator, assuming the inverse square law, and applying the necessary correction for finite width of source.[1]

Intensities were measured in watts/sq. meter by a thermopile calibrated against a standard lamp whose radiation was certified to by the Bureau of Standards.

Exposures were made by cutting one of the 5 x 7 plates into 30 strips, and exposing 12 of these, the remaining strips being used to obtain the amount of fog, as explained later. Three wave-lengths of light were used: 450 $\mu\mu$, 550 $\mu\mu$, and 650 $\mu\mu$. A hand-operated shutter was mounted in front of the slit of the illuminator, and by listening to a telephone receiver clicking seconds, exposures consisting of any integral number of seconds could easily be made.

Plates were developed at a practically constant temperature in a developer compounded after Brush's formula.[2] The maximum variation in temperature during development of the plates which make up one of the sets of curves shown later on was never more than 1°.8 C., generally it was only a few tenths of a degree. A time of development of 6 minutes at 23° C. was selected as standard. This variation in temperature was corrected for by the use of Watkins' formula[3]

$$r = (s_1/s_2)^{\frac{10}{t_2-t_1}},$$

where r = temperature coefficient, t_1 = low temperature, t_2 = high temperature, s_1 = time of development at t_1, and s_2 = time of development at t_2.[4]

The density of a photographic plate is defined as $\text{Log}_{10}\,o$, where o, the opacity, is the ratio of the incident to the transmitted light. Densities were measured in a modification of Lemon's spectro-photometer,[5] in which the prism is replaced by two mirrors inclined to one another so as to reflect two beams of light into the observing telescope. The density corresponding to an angle θ between transmitting planes of the nicols when rotated so as to cut down one light beam to the same intensity which the interposed plate of unknown density cuts down the other light beam is given by the expression $\text{Log}_{10}\,\text{Sec}^2\,\theta$.

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[1] See Bull. Bur. Standards, 3, 81; 1907.

[2] PHYS. REV., 31, 243; 1910.

[3] The A. Photographer and Photo. News, London, May 17, 1910.

[4] According to Watkins, the value of "r" for any hydroquinone developer is 2.25, so this value was accordingly taken for the "r" of the hydroquinone developer used in this work. For example, if the temperature were 22° C., the time of development allowed was given by the solution of the equation

$$2.25 = \left[\frac{S_1}{6}\right]^{\frac{10}{33-22}}.$$

[5] Astrophys. Journ., 39, 204; 1914.

Densities were measured with light of 535 μμ, obtained by placing two "monochromatic" color filters in front of the observing telescope. Density readings on a single plate were repeated until the probable error of the mean was reduced to a very small value.

Brand A.

Wave-length 550 μμ. Distance 226.2 cm. 1.53 × 10^{-4} watts/sq. m. Temperature 22°.0. Time Devel. 6^m 30^s.

Plate.	Time.	*D*	Plate.	Time.	*D*
1500		0.35	1514		0.29
1501	15^m	6.31	1515	15^s	0.32
1502		0.39	1516		0.26
1503	8^m	4.03	1517	8^s	0.25
1504		0.37	1520		0.25
1505	4^m	2.88	1521	4^s	0.27
1506		0.42	1522		0.27
1507	2^m	1.91	1523	2^s	0.28
1510		0.34	1524		0.27
1511	1^m	1.35	1525	1^s	0.26
1512		0.31	1526		0.24
1513	30^s	0.58	1527		0.28

The preceding table is a sample of one set of exposures. All of the separate small plates were cut from the same 5 x 7 plate. The small plates were numbered to show how they were cut with respect to the original plate. For example 1513 lay adjacent to 1512, 1514, 1503 and 1523 in the original plate. The even numbers were not exposed, and were used to obtain the fog. These unexposed fog strips were developed along with the exposed plates in order to find the blackening of the exposed plates which was not due to exposure to light. This fog density was then subtracted from the densities of the exposed plates in the calculations and curves.

The plates were of brand "*A*," exposed to light of wave-length 550 μμ at a distance of 226.2 cm. from the slit of the monochromatic illuminator, the flow of energy on the plate being 1.53 × 10^{-4} watts/sq. m. The plates were developed at a temperature of 22°.0 for a time of 6^m 30^s.

The table gives the plate number, the time of exposure, and the mean of the density determinations of the plate.

THEORETICAL.

In 1890, Hurter and Driffield[1] developed an equation of the following form for the density *D* of a photographic plate exposed for a time *t* to light of constant intensity *I*:

$$D = \alpha \operatorname{Log}_e [\beta - (\beta - 1)e^{-\gamma^{n}}],$$

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[1] Journ. Soc. Chem. Ind., 9, 455; 1890.

where α, β, and γ are parameters dependent upon the character of the plate and the developer, $-\beta$ is equal to the opacity of the unexposed plate. They assumed that the amount of silver made developable by the light was proportional to $I \cdot t$, and that only the light absorbed by the unaltered silver particles contributed to the building up of the latent image.

This expression would lead to the so-called "Reciprocity Law" of Bunsen and Roscoe[1] ($I \cdot t$ equals constant for equal blackening) which Abney,[2] Miethe,[3] Eder,[4] Michalke and Schiener, Schwarzschild,[5] Kron,[6] Lemon,[7] and others have shown to be only a very rough approximation to the facts.

However, for a constant intensity of light, the expression gives a very good relationship between the density of the plate and the time of exposure, as both Hurter and Driffield, and Sheppard and Mees,[8] have shown.

In this work the equation of Hurter and Driffield is made use of in the following way:

Given a set of curves showing the relationship between density and time of exposure for intensities of light $I_1 = k_1$, $I_2 = k_2$, $\cdots$, $I_n = k_n$.

We will rewrite the equation of Hurter and Driffield in the form:

$$D = 1/a \operatorname{Log}_e [b - (b - 1)e^{-ct}].$$

Then for the curves $I_1, I_2, \cdots, I_n$, we can find values of the parameters a, b, c_1, c_2, $\cdots$, c_n, respectively which will give us this set of n curves. But c is a function of I, and if we can find this function, we can write our equation in the form:

$$D = 1/a \operatorname{Log}_e [b - (b - 1)e^{-f(I) \times t}]$$

and thus the density is completely determined in terms of the intensity of light and time of exposure.

Hurter and Driffield gave no hint as to the method of determination of the parameters a, b, and c, and Sheppard and Mees give a method applicable only when $a = 1$. Two methods are here developed which may be used to find the values of these parameters from a given experimental curve.

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[1] Ann. der Phys., 117, 538; 1862.

[2] Phot. Journ., Oct., 1893.
Journ. Camera Club, 8, 46.

[3] Inaug. Diss. Gottingen, 1899.

[4] Handbuch, Bd. 2.
Jahrbuch, 1899, 457.

[5] Phot. Corr., 1899. Beitrage zur Phot. Photem. d. Gestirne. Astrophys. Journ., 11, 89; 1900.

[6] Ann. der. Phys., 41, 755; 1913.

[7] Loc. cit.

[8] Investigations on the Theory of the Photographic Process. Longmans, p. 210.

Assuming that the experimental curve is of the form given above, select four points, (t_1, D_1), (t_2, D_2), (t_3, D_3), and (t_4, D_4), so that $t_2 = t_1 + \Delta t$, $t_3 = t_1 + 2\Delta t$, $t_4 = t_1 + 3\Delta t$.

Then

$$e^{a(D_1-D_2)} = \frac{b-(b-1)e^{-ct_1}}{b-(b-1)e^{-ct_1}e^{-c\Delta t}},$$

$$e^{a(D_1-D_3)} = \frac{b-(b-1)e^{-ct_1}}{b-(b-1)e^{-ct_1}e^{-2c\Delta t}},$$

$$e^{a(D_1-D_4)} = \frac{b-(b-1)e^{-ct_1}}{b-(b-1)e^{-ct_1}e^{-3c\Delta t}},$$

$$\frac{b-(b-1)e^{-ct_1}e^{-c\Delta t}}{e^{a(D_2-D_1)}} = \frac{b-(b-1)e^{-ct_1}e^{-2c\Delta t}}{e^{a(D_3-D_1)}} = \frac{b-(b-1)e^{-ct_1}e^{-3c\Delta t}}{e^{a(D_4-D_1)}},$$

$$\frac{e^{a(D_3-D_1)} - e^{a(D_4-D_1)}}{e^{a(D_2-D_1)} - e^{a(D_3-D_1)}} = e^{-c\Delta t},$$

(1) $$c = (1/\Delta t)\,\mathrm{Log}_e \frac{e^{aD_2} - e^{aD_3}}{e^{aD_3} - e^{aD_4}},$$

(2) $$(e^{aD_1} - e^{aD_2})(e^{aD_3} - e^{aD_4}) = (e^{aD_2} - e^{aD_3})^2,$$

(3) $$b = \frac{e^{aD} - e^{-ct}}{1 - e^{-ct}}.$$

These last three equations are sufficient to determine a, b, and c.

The practical difficulties connected with this method are quite material, so a simple graphical method was accordingly developed.

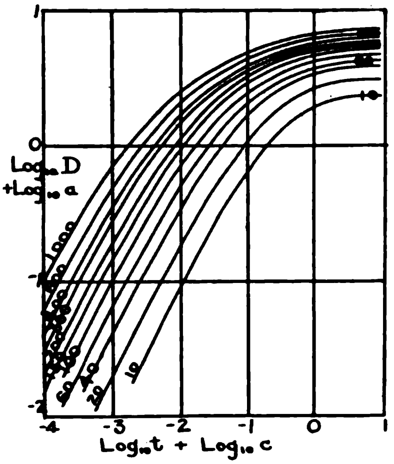

Fig. 1.

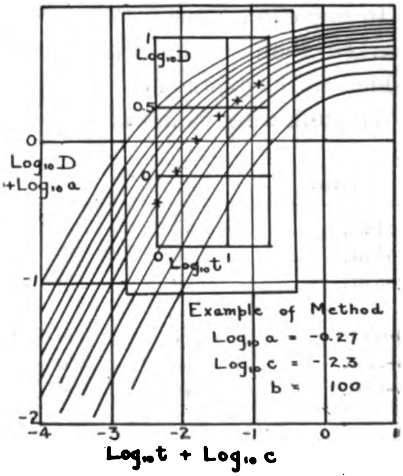

Fig. 2.

A family of 11 curves

$$D = 1/a\ \mathrm{Log}_e\,[b - (b-1)e^{-ct}]$$

was plotted as shown in the figure, with b a parameter ranging in value from 10 to 1,000, as in Fig. 1.

Log aD was plotted along the y-axis, and Log ct along the x-axis. The experimental curve was plotted upon tracing paper with Log D and Log t as ordinate and abscissa, respectively. The tracing paper was shifted over the family of curves preserving parallelism of axes, until it coincided with one of the family. b was determined by the particular curve of the family which gave coincidence, and the a and the c were given by the intercepts of the axes of the experimental curve upon the axes of the family of curves. (Fig. 2.)

In actual practice, especially for large ranges of times of exposures, it was not difficult to find the position of the tracing paper which gave the optimum fit of the experimental curve to one of the theoretical curves,—*i.e.*, that the values of a, b, and c are unique.

Log c was next expressed as a function of Log I by Campbell's "Method of the Zero Sum,"[1] assuming that

$$\text{Log}_{10}\, c = A + B\, \text{Log}_{10}\, I + C\, \text{Log}_{10}^2\, I.$$

This relationship was selected because the curvature of the Log c — Log I curve was very small, and because the equation readily lends itself to methods of adjustment of constants, and to the determination of the exponent "p" in Schwarzschild's "Law."[1] ($I \cdot t^p$ equals constant for equal blackening.)

In one case, namely the set of plates A–650, the curvature was more marked than in the other cases, and a term involving $\text{Log}_{10}^3\, I$ was added to the expression.

The following values of the parameters were obtained by this method:

Plate.	a	b	A	B	C	D
A–450	0.63	200	3.99	1.95	0.10	
A–550	0.63	200	0.10	1.34	0.06	
A–650	0.40	200	−1.37	2.56	0.91	0.16
B–450	1.41	200	5.92	2.57	0.16	
B–550	1.12	200	−0.94	0.37	−0.12	
B–650	1.26	200	−1.00	1.38	0.11	
C–450	1.58	400	4.46	1.94	0.10	
C–550	1.12	100	0.51	1.00	−0.03	
C–650	1.58	200	−0.26	1.87	0.15	

$$\text{Log}_{10}\, c = A + B\, \text{Log}_{10}\, I + C\, \text{Log}_{10}^2\, I + D\, \text{Log}_{10}^3\, I.$$

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[1] Phil. Mag., 39, 177; 1920.

[2] Loc. cit.

In Fig. 3 the $\text{Log}_{10}\, c - \text{Log}_{10}\, I$ curves are plotted for the 9 different sets of plates. Points show values of "c" determined graphically.

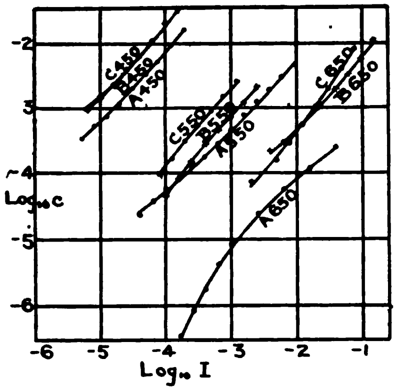

Fig. 3.

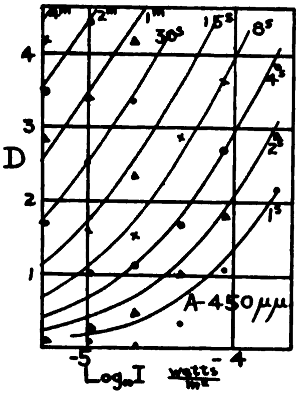

Fig. 4.

Curves show values of "c" computed from

$$\text{Log}_{10}\, C = A + B\, \text{Log}_{10}\, I + C\, \text{Log}_{10}^2\, I.$$

Figs. 4 to 12 show values of blackenings computed from the above

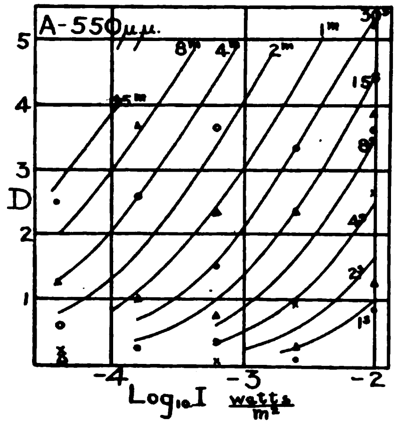

Fig. 5.

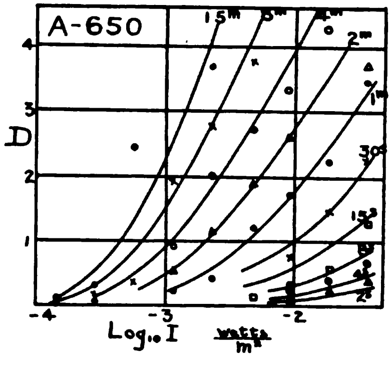

Fig. 6.

parameters, and the points represent the experimentally determined values.

The "p" of Schwarzschild's so-called "Law" can be very easily obtained from the equation which the writer has used to express the relation between the density of a plate and the intensity of light and time of exposure.

Schwarzschild states that the product $I \cdot t^p$ must be constant for equal blackening, but Kron[1] and Lemon[2] have shown that p is a function of the intensity of light.

Expressing the blackening of a plate in the form

$$D = 1/a[\text{Log}_e b - (b - 1)e^{-ct}],$$

where

$$\text{Log}_{10} c = A + B \text{ Log}_{10} I + C \text{ Log}_{10}^2 I,$$

then $c \cdot t$ must be constant for equal blackening, or

$$t \cdot 10^{A + B \text{ Log}_{10} I + C \text{ Log}_{10}^2 I} = \text{constant}.$$

or

$$t \cdot I^{B + C \text{ Log}_{10} I} = \text{constant}.$$

So $(B + C \text{ Log}_{10} I)^{-1}$ is equivalent to the "p" of Schwarzschild's expression.

The following extreme values of p were computed in this manner, and show the variation in the value of p with the intensity of light.

Plate.	Range of Log_{10} Intensity.			Range of p.		
A–450........	−3.7	−5.3		1.09	1.01	
A–550........	−2.0	−4.4		1.22	1.06	
A–650........	−1.4	−3.8	(−2.92 Max.)	1.59	1.36	(1.23 Max.)
B–450........	−3.8	−5.2		1.95	1.72	
B–550........	−2.6	−4.0		0.68	0.85	
B–650........	−0.8	−2.4		1.30	1.13	
C–450........	−3.8	−5.2		1.58	1.44	
C–550........	−2.9	−4.1		1.05	1.08	
C–650........	−1.1	−2.7		1.71	1.48	

PROCEDURE IN USING METHOD FOR PHOTOMETRIC WORK.

"Calibration" of Plates.

1. Plates are exposed to a known intensity of monochromatic light for various times of exposure and a Log D − Log t curve plotted. Values of the constants a and b are determined from this curve with the aid of the monographic chart.

2. Plates are exposed for constant times of exposure to various known intensities of the monochromatic light and a Log D − Log t curve plotted as in 1. Values of c as a function of I are now determined with the monographic chart.

3. Log c is plotted against Log I.

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[1] Loc. cit.

[2] Loc. cit.

Determination of Intensity.

1. A plate is exposed for any length of time to the monochromatic light of unknown intensity.

2. Knowing the density of the plate, the time of exposure, and the values of the constants *a* and *b*, the value of *c* for the particular intensity of light is determined from the monographic chart.

3. From the determined value of *c*, the value of *I* is found from the Log *c* — Log *I* curve.

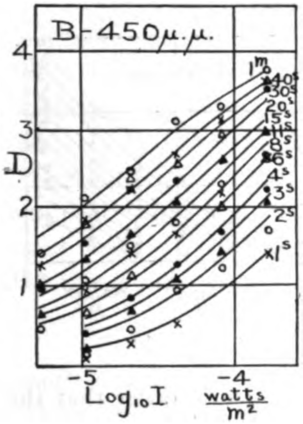

Fig. 7.

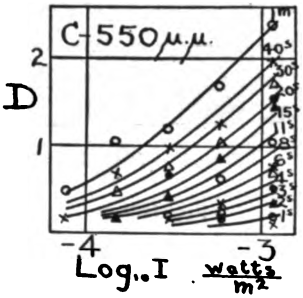

Fig. 8.

SOME GENERAL CONSIDERATIONS.

In plotting photographic blackening curves, the standard practice has always been to plot density of plate against logarithm of time of exposure, intensity being a parameter. In the light of the foregoing developments,

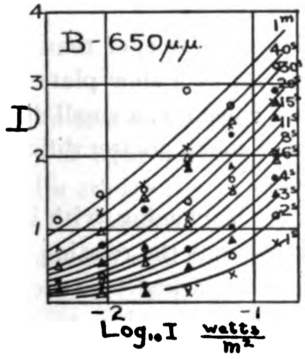

Fig. 9.

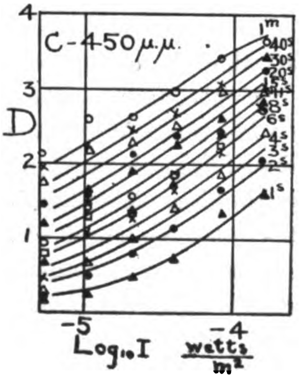

Fig. 10.

as a and b are constant for a given set of curves, it would be more logical to plot logarithm of density against logarithm of time,—for in that case the result would consist of a family of parallel curves. The actual drawing of such a family would be greatly facilitated, because the curves would be identical in shape. In this work however, the curves have been plotted according to the standard practice in order that they may be comparable to previously published curves.

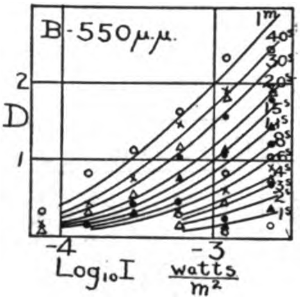

Fig. 11.

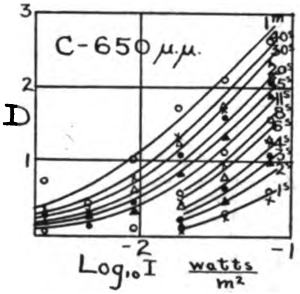

Fig. 12.

A set of exposures such as has been made throws much light upon the characteristics of fast and slow plates.

Consider first the relationship between brand of plate and blackening, with equal times of exposures and intensities of light, other factors being constant. In general, the curves of the slower plates are steeper than those of the faster ones.

The result is, that for a small intensity of light, the fast plate attains a greater density for a given time of exposure than does a slower plate. But for a longer time of exposure, the slower plate may attain a greater density than does the fast one. It is common knowledge among photographers that the longer the exposures, the less the difference that can be noticed between fast and slow plates.

The fact that the curves of the slow plates are steeper than those of the fast ones accounts for the greater contrast of a slow plate as compared to a fast one. With time of exposure constant, a small difference of intensity of light on the slow plate means a much greater difference in density in the slow plate than in the fast one.

The effect of wave-length of light upon the blackening, with intensity and time constant, is of course, very apparent, but the relation between slope of curves with wave-length of light is not marked. In fact, Leimbach[1] claims, though others have questioned his conclusions, that the slope, or "gradation" is independent of the wave-length. Upon closer inspection, the writer's curves show a tendency to have the least gradation for that wave-length where the plate has its greatest sensibility, as Abney,[1] Stark[2] and others have found.

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[1] Zeit. wiss. Phot., 7, 157 u. 181; 1909.

Certain generalizations can also be made with regard to the Log c – Log I curves. First, the curves depart more or less from a straight line. If Schwarzschild's expression were valid,—*i.e.*, $I \cdot t^p =$ constant for equal blackening, the Log c – Log I curves would be straight lines, as may be proved from the expression $p = (B - C\,\mathrm{Log}_{10}\,I)^{-1}$. The curves may therefore be regarded as giving some indication of the amount of error which is involved in assuming Schwarzschild's expression.

If Bunsen and Roscoe's "Law" held,—*i.e.*, $I \cdot t =$ constant for equal blackening, the curves would be straight lines with slope equal to unity, or c would equal I (Hurter and Driffield's assumption). The curves show that this is only a very rough approximation.

CONCLUSIONS.

Two methods of obtaining the mathematical equation of the blackening of a photographic plate as a function of intensity of monochromatic light and time of exposure have been pointed out.

Using the method of the monographic chart, the blackening equations of three different brands of plates have been calculated for three different wave-lengths of light. In these nine different cases, there is fairly good agreement between the experimental and the calculated values.

It is a pleasure to acknowledge the encouragement received from the department of physics of the State University of Iowa, and particularly from Professors L. P. Sieg, and H. L. Dodge, the latter now of the University of Oklahoma.

Ross[3] has pointed out an apparent inconsistency in the assumptions underlying the Hurter and Driffield equation made use of in the formulae above, and as a substitute develops the equation

$$D = d_m\left[1 - \frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s kIt}\right].$$

D is the density of the plate, d_m the density for infinite exposure, I the intensity of light (assumed monochromatic, t the time of exposure, and k, r, and n are certain parameters. A method is given of determining n if d_m can be found by experiment, if arbitrary values assigned to r and k. The expression will hold only for one intensity of light unless the Reciprocity "Law" ($I \cdot t =$ constant for equal blackening) be assumed.

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[1] Treatise on Photography, 10th Ed. Longmans, p. 413.

[2] Ann. der Phys., 35, 474; 1911.

[3] Journ. Optical Soc. of Am., 4, 261; September, 1920.

Ross' formula can be made the basis of expressing the blackening of a plate exposed to monochromatic light as a function of intensity of light and time of exposure according to the graphical method already outlined above if the formula is written in the form

$$D = \frac{1}{a}\left[1 - \frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^{s}ct}\right],$$

and families of Log *aD* − Log *ct* curves are plotted with *n* and *r* as parameters.

A set of experimental Log *D* − Log *t* curves is obtained (*I* having a different value for each curve), and a particular Log *aD* − Log *ct* curve is chosen so that the *n*, *r*, and *a* will give a good fit to each of the Log *D* − Log *t* curves by allowing *c* to vary. The *c* and the *a* are found from the *x* and the *y*-intercepts of the axes of the Log *D* − Log *t* curves on the axes of the Log *aD* − Log *ct* axes, and the *n* and the *r* are determined by the particular Log *aD* − Log *ct* curve chosed to give the best fit. If *c* be next expressed as a function of *I*, we have a mathematical expression for the blackening of a photographic plate as a function of the intensity of monochromatic light and time of exposure.

The possible advantages of a graphical means of determining the parameters of Ross' equation seems to lie in the fact that by a graphical method the reader can easily judge the accuracy of fit of the experimental to the theoretical curve, and can make the curves fit particularly well at any desired point, that he can find at once the values of the parameters *r*, *n*, and *k*, and that he is not limited to using one particular value of *r* or *k*.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA,
June, 1920.

SOME APPLICATIONS OF THE METHOD OF IMAGES—I.

By Jakob Kunz and P. L. Bayley.

Synopsis.

Solution of Some Electrostatic Problems by the Method of Images. (*a*) *Charged Wire between Two Parallel Plates.*—In Part I. of this paper the authors obtain expressions for the potentials at any point between an infinitely long charged wire and two conducting infinite planes parallel to it, for the surface density of the induced charge at any point on the plates, and for the capacity per unit length of such a condenser. These expressions contain only circular and hyperbolic functions. (*b*) *Charged Wire Inside an Infinite Rectangular Tube.*—In Part II., the potential at any point inside of the tube is obtained by a single infinite summation of the potentials due to each of a singly infinite set of images as given by the expression in Part I. Thus, although the problem is essentially one of doubly periodic functions, the solution appears in circular and hyperbolic functions. Comparing a square tube with a circular tube of the same capacity, each with a small wire of a given size through its center, the square tube has the larger perimeter. Tables are given showing the variation of the capacity of a certain rectangular tube with the size of the wire at its center and with the position of a certain sized wire. (*c*) *Two or Four Charged Wires Inside a Rectangular Tube.*—In the case of certain symmetrical positions this problem can be solved immediately from the preceding results.

Introduction.

Green's theorem in the potential theory states that if the potential $V(x, y, z)$ in every point of a closed surface is given and also the value of ΔV in every point of the enclosed volume, then the potential V is uniquely determined in every point of that volume. This is the principle of the method of images, which we shall apply in the following cases:

I. The Capacity of an Infinite Wire between Two Infinite Parallel Planes.

Let a linear charge of e units per unit length be placed at $(a, 0)$ between the two earthed plates of infinite extent A and B of Fig. 1. Let the origin of the complex plane be taken at 0 and P be the point $z = x + iy$ at which the potential is required.

The images are shown at points z_1, z_{-1}, z_2, z_{-2}, etc., whose distances from P are r_1, r_{-1}, r_2, r_{-2}, etc. The potential at P due to the charge at a and the induced charges on the plates A and B will be calculated from the charge itself and its infinite set of images. The potential at P due to the charge at a is $V = c - 2e \log r_0$. The resultant potential at P is then the sum of such expressions, one for each charge.

$$V_P = C + 2e[-\log r_0 + \log r_1 + \log r_{-1} - \log r_2 - \log r_{-2} + \cdots],$$

$$V_P = C + 2e \log \frac{r_1 r_{-1} r_3 r_{-3} r_5 r_{-5} \cdots}{r_0 r_2 r_{-2} r_4 r_{-4} \cdots},$$

$$V_P = C + 2e \log \frac{\prod\limits_{n=-\infty}^{+\infty} r_{2n-1}}{\prod\limits_{n=-\infty}^{+\infty} r_{2n}}.$$

But $r_0 = |z - z_0|$, $r_1 = |z - z_1|$, $r_{-1} = |z - z_{-1}|$, $\cdots$, $r_n = |z - z_n|$, $r_{-n} = |z - z_{-n}|$; moreover $\log r = \log |z| = R \log z$, where R denotes the real part of the logarithm. For all images of positive charge

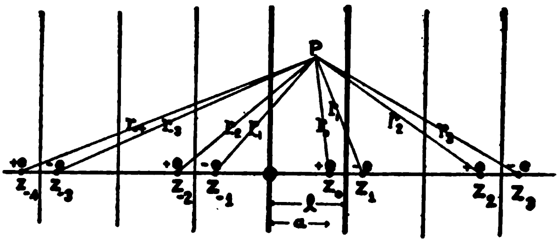

Fig. 1.

$z_{\pm 2n} = \pm 2nl + a$. For all images of negative charge $z_{\pm 2n-1} = \pm 2nl - a$. For a point taken on the plate A the potential is zero and the distances $r_0 = r_{-1}$, $r_1 = r_{-2}$, etc. Then $C = 0$; hence

$$V_P = 2e \log \frac{\Pi |z - z_{2n-1}|}{\Pi |z - z_{2n}|},$$

$$V_P = 2eR \log \frac{(z + a)(z + a - 2l)(z + a + 2l)(z + a - 4l)\cdots}{(z - a)(z - a + 2l)(z - a - 2l)(z - a + 4l)\cdots},$$

$$V_P = 2eR \log \frac{\frac{\pi}{2l}(z + a)\left[1 - \left(\frac{z + a}{2l}\right)^2 \frac{\pi^2}{\pi^2}\right] \times \left[1 - \left(\frac{z + a}{2l}\right)^2 \frac{\pi^2}{4\pi^2}\right]\left[1 - \left(\frac{z + a}{2l}\right)^2 \frac{\pi^2}{(3\pi)^2}\right]\cdots}{\frac{\pi}{2l}(z - a)\left[1 - \left(\frac{z - a}{2l}\right)^2 \frac{\pi^2}{\pi^2}\right] \times \left[1 - \left(\frac{z - a}{2l}\right)^2 \frac{\pi^2}{(2\pi)^2}\right]\left[1 - \left(\frac{z - a}{2l}\right)^2 \frac{\pi^2}{(3\pi)^2}\right]\cdots},$$

$$V_P = 2eR \log \frac{\sin \pi \left(\dfrac{z+a}{2l}\right)}{\sin \pi \left(\dfrac{z-a}{2l}\right)},$$

$$V_P = 2eR \log \frac{\sin \dfrac{\pi}{2l}(x+a) \cosh \dfrac{\pi}{2l} y + i \cos \dfrac{\pi}{2l}(x+a) \sinh \dfrac{\pi}{2l} y}{\sin \dfrac{\pi}{2l}(x-a) \cosh \dfrac{\pi}{2l} y + i \cos \dfrac{\pi}{2l}(x-a) \sinh \dfrac{\pi}{2l} y},$$

$$(1) \quad V_P = e \log \frac{\cosh \dfrac{\pi}{l} y - \cos \dfrac{\pi}{l}(x+a)}{\cosh \dfrac{\pi}{l} y - \cos \dfrac{\pi}{l}(x-a)}.$$

Now let us consider a wire of finite radius r having the same charge e per unit length. The potential of the surface of the wire is assumed to be the same as the potential at a point r units distant from z_0. This is equivalent to assuming that the equipotential surface at a distance r from z_0 is a circle and is quite accurately true for small values of r when the wire is not too near one of the plates. For the potential of the wire we choose

$$y = r; \qquad x = a;$$

$$V_w = e \log \frac{\cosh \dfrac{\pi}{l} r - \cos \dfrac{\pi}{l} 2a}{\cosh \dfrac{\pi}{l} r - \cos \dfrac{\pi}{l}(a-a)}.$$

The capacity per unit length of the system is therefore

$$C = \frac{e}{V_w} = \frac{1}{\log \dfrac{\cosh \dfrac{\pi}{l} r - \cos \dfrac{\pi}{l} 2a}{\cosh \dfrac{\pi}{l} r - 1}},$$

If $2a = l$, the capacity will be equal to:

$$C = \frac{1}{\log \dfrac{\cosh \dfrac{\pi}{l} r + 1}{\cosh \dfrac{\pi}{l} r - 1}}.$$

Expanding the cosh in infinite series we obtain approximately:

$$C = \frac{1}{2 \log \dfrac{4a}{\pi r}}.$$

From equation (1) we find the electric force in the x direction E_x as follows:

$$\frac{\partial V}{\partial x} = -E_x = e\frac{\pi}{l}\left\{\frac{\sin\frac{\pi}{l}(x+a)}{\cosh\frac{\pi}{l}y - \cos\frac{\pi}{l}(x+a)} - \frac{\sin\frac{\pi}{l}(x-a)}{\cosh\frac{\pi}{l}y - \cos\frac{\pi}{l}(x-a)}\right\}.$$

The surface density at any point on the plate A is found by

$$\sigma = -\frac{1}{4\pi}\left(\frac{\partial V}{\partial x}\right)_{x=0} = \frac{e}{2l}\,\frac{\sin\frac{\pi}{l}a}{\cos\frac{\pi a}{l} - \cosh\frac{\pi}{l}y}.$$

σ is a maximum for $y = 0$

$$\sigma_{y=0} = \frac{e}{2l}\,\frac{\sin\frac{\pi}{l}a}{\cos\frac{\pi a}{l} - 1}$$

$$= \frac{e}{2l}\operatorname{cotg}\frac{\pi a}{2l}.$$

II. THE CAPACITY OF A CIRCULAR WIRE IN A RECTANGULAR CYLINDER.

We shall begin with the potential due to a linear charge. Let the cylinder have a height of l cm. and breadth of l' cm. (Fig. 2). The charge is at z_{00} or simply $z_0 = x_0 + iy_0$ and has e units per unit length. We shall remove the walls with their induced charges and consider in their stead all the infinite sets of images, which are distributed so that one of

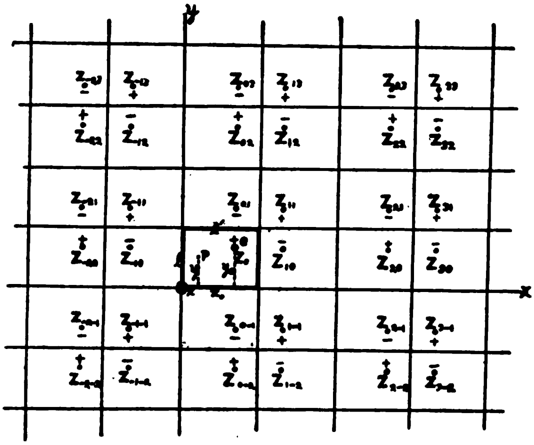

Fig. 2.

them is in each rectangle throughout the whole complex plane. The potential at a point $P(z = x + iy)$ will first be calculated by obtaining formula (1) in generalized form applying to any row of images and then taking a single summation for all rows.

The expression for the potential due to any row of images is given by formula (1) when y is replaced by the perpendicular distance from the point P to the row of images. In Fig. 2 such a distance is represented for any row by one of the expressions:

$$(a)\quad y - y_0 + 2nl,$$

$$(b)\quad y + y_0 + 2nl,$$

where n is any positive or negative integer. The distances $x + a$, $x - a$, and l in Fig. 1 become $x + x_0$, $x - x_0$, and l' when applied to Fig. 2. The potential at P is the algebraic sum of the potentials due to all such rows of images. Noting that the potentials due to the rows whose distances are given by (b) are negative, we have

$$(2)\quad V_P = e \sum_{n=-\infty}^{+\infty} \log \frac{\left[\cosh\frac{\pi}{l'}(2nl + y - y_0) - \cos\frac{\pi}{l'}(x + x_0)\right] \times \left[\cosh\frac{\pi}{l'}(2nl + y + y_0) - \cos\frac{\pi}{l'}(x - x_0)\right]}{\left[\cosh\frac{\pi}{l'}(2nl + y - y_0) - \cos\frac{\pi}{l'}(x - x_0)\right] \times \left[\cosh\frac{\pi}{l'}(2nl + y + y_0) - \cos\frac{\pi}{l'}(x + x_0)\right]}.$$

This problem is essentially that of the conformal transformation of the rectangle with a singular point within of character log $1/r$ and is treated in many texts on elliptic functions. See Greenhill, Elliptic Functions, §§ 273–275; and Kneser, Die Integral Gleichungen und ihre Anwendung in der Math. Physik, p. 137. The usual solution of this problem in elliptic functions treats the whole set of images at once and is expressible in sigma or theta functions. The same expressions may be derived as follows by associating with each image a factor of a sigma function.

The coördinates of the images (Fig. 2) are given as follows: for positive images

$$z_{\pm 2m,\, \pm 2n} = z_0 \pm 2ml' \pm i2nl,$$

$$z_{\pm 2m-1,\, \pm 2n-1} = - \bar{z}_0 \pm 2ml' \pm i2nl,$$

where $z_0 = x_0 + iy_0$ and $\bar{z}_0 = x_0 - iy_0$; for negative images

$$z_{\pm 2m,\, \pm 2n-1} = \bar{z}_0 \pm 2ml' \pm i2nl,$$

$$z_{\pm 2m-1,\, \pm 2n} = - z_0 \pm 2ml' \pm i2nl,$$

then

$$(2a)\qquad V_P = 2eR\log\prod_{\substack{m=-\infty\\n=-\infty}}^{+\infty}\frac{(z+\bar{z}_0+\Omega)(z-\bar{z}_0+\Omega)}{(z+z_0+\Omega)(z-z_0+\Omega)},$$

where $\Omega = 2ml' + i2nl$. Dividing each term by Ω and multiplying and dividing by a proper exponential factor, we may associate each term with a sigma function

$$\sigma(z) = z\Pi'\left(1+\frac{z}{\Omega}\right)\epsilon^{z/\Omega+\frac{1}{2}(z/\Omega)^2}.$$

Reduction leaves

$$V_P = 2e\log\frac{\sigma(z+\bar{z}_0)\sigma(z-\bar{z}_0)}{\sigma(z+z_0)\sigma(z-z_0)},$$

a well-known formula which with proper interpretation has been applied to vertex motion and the flow of heat.

The sigma functions may be expanded into a single convergent product as follows:

$$\sigma(z) = \epsilon^{\eta_1 z^2/2\omega_1}\frac{2\omega_1}{\pi}\sin\frac{\pi z}{\omega_1}\prod_{n=1}^{\infty}\frac{\sin\pi\left(\frac{2n\omega_2-z}{2\omega_1}\right)\sin\pi\left(\frac{2n\omega_2+z}{2\omega_1}\right)}{\sin^2\pi n\frac{\omega_2}{\omega_1}},$$

where

$$\eta_1 = \frac{\pi^2}{2\omega_1}\left[\frac{1}{6}+\sum_{n=1}^{\infty}\frac{1}{\sin^2\left(i\pi n\frac{\omega_2}{\omega_1}\right)}\right],$$

$\omega_1 = l'$, and $\omega_2 = il$. After considerable reduction we get identically equation (2).

It was suggested to the authors that for purposes of computation it is sometimes easier to use theta functions instead of the sigma function. The functions are connected in the following way:

$$\sigma(z) = A\epsilon^{2\eta_1\omega\left(\frac{z}{2\omega_1}\right)^2}\vartheta_1\left(\frac{z}{2\omega_1}\right).$$

Hence,

$$\frac{\sigma(z+\bar{z}_0)\sigma(z-\bar{z}_0)}{\sigma(z+z_0)\sigma(z-z_0)}$$

$$= \epsilon^{2\eta_1\omega_1\left[\left(\frac{z+\bar{z}_0}{2\omega_1}\right)^2+\left(\frac{z-\bar{z}_0}{2\omega_1}\right)^2-\left(\frac{z+z_0}{2\omega_1}\right)^2-\left(\frac{z-z_0}{2\omega_1}\right)^2\right]}\frac{\vartheta_1\left(\frac{z+\bar{z}_0}{2\omega_1}\right)\vartheta_1\left(\frac{z-\bar{z}_0}{2\omega_1}\right)}{\vartheta_1\left(\frac{z+z_0}{2\omega_1}\right)\vartheta_1\left(\frac{z-z_0}{2\omega_1}\right)}$$

$$= \epsilon^{\frac{-4\eta_1}{\omega_1}ix_0y_0}\frac{\vartheta_1\left(\frac{z+\bar{z}_0}{2\omega_1}\right)\vartheta_1\left(\frac{z-\bar{z}_0}{2\omega_1}\right)}{\vartheta_1\left(\frac{z+z_0}{2\omega_1}\right)\vartheta_1\left(\frac{z-z_0}{2\omega_1}\right)}.$$

Substituting this expression in equation (2) and using the classical expansion of the theta functions we find essentially the same result as before.

Now let us replace the linear charge by a wire of finite radius r. If the surface of the wire coincided with an equipotential surface, then the potential could be calculated exactly from formula (2). This is very nearly the case when the dimensions l and l' are large compared with r and when the wire is not too near the walls of the cylinder. At the point P of Fig. 3 we have:

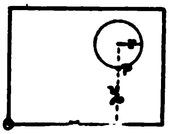

Fig. 3.

$$\begin{aligned} x + x_0 &= 2x_0, \\ x - x_0 &= 0, \\ y - y_0 &= -r, \\ y + y_0 &= 2y_0 - r. \end{aligned}$$

Then

$$(3)\quad V_w = e\sum_{-\infty}^{+\infty}\log\frac{\left[\cosh\frac{\pi}{l'}(2nl - r) - \cos\frac{\pi}{l'}2x_0\right]\times\left[\cosh\frac{\pi}{l'}(2nl + 2y_0 - r) - 1\right]}{\left[\cosh\frac{\pi}{l'}(2nl - r) - 1\right]\times\left[\cosh\frac{\pi}{l'}(2nl + 2y_0 - r) - \cos\frac{\pi}{l'}2x_0\right]}.$$

By definition the capacity C of the system per unit length is equal to

$$C = \frac{e}{V_w}.$$

Instead of (3) we may write:

$$(4)\quad V_w = e\left\{\begin{aligned} &\sum_{n=0}^{\infty}\log\frac{\cosh\frac{\pi}{l'}(2nl + r) - \cos\frac{\pi}{l'}2x_0}{\cosh\frac{\pi}{l'}(2nl + r) - 1} \\ &+\sum_{n=1}^{\infty}\log\frac{\cosh\frac{\pi}{l'}(2nl - r) - \cos\frac{\pi}{l'}2x_0}{\cosh\frac{\pi}{l'}(2nl - r) - 1} \\ &+\sum_{n=0}^{\infty}\log\frac{\cosh\frac{\pi}{l'}(2nl - 2y_0 + r) - 1}{\cosh\frac{\pi}{l'}(2nl - 2y_0 + r) - \cos\frac{\pi}{l'}2x_0} \\ &+\sum_{n=1}^{\infty}\log\frac{\cosh\frac{\pi}{l'}(2nl + 2y_0 - r) - 1}{\cosh\frac{\pi}{l'}(2nl + 2y_0 - r) - \cos\frac{\pi}{l'}2x_0}. \end{aligned}\right.$$

Calculations from formulæ (1) to (5) are made simple by use of the Smithsonian Institute Tables of Hyperbolic Functions. Where more than three or four significant figures are desired, the hyperbolic functions can be built up by means of the tables of exponentials in the same book.

We shall next compare the relative dimensions of circular and square cylindrical condensers of the same capacity. The capacity per unit length of a condenser formed of two concentric cylinders is given by

$$C = \frac{1}{2 \log \frac{R}{r}},$$

where R is the radius of the outer cylinder and r that of the inner cylinder. Now let the outer cylinder have a square cross-section and the inner cylinder be a wire of the same radius r as before. In the square cylinder $l = l' = 2x_0 = 2y_0$. If r/l is small in comparison to unity, it may be dropped from each term of equation (4) except in the denominator of the first term when $n = 0$. We obtain:

$$(5)\quad V_0 = e\left\{\begin{aligned} &\log 2 - \log\left(\cosh\frac{\pi r}{l} - 1\right) + 2\log\frac{\cosh\pi - 1}{\cosh\pi + 1} \\ &+ 2\sum_{n=1}^{\infty}\log\frac{\cosh 2\pi n + 1}{\cosh 2\pi n - 1} + 2\sum_{n=1}^{\infty}\log\frac{\cosh(2n-1)\pi - 1}{\cosh(2n-1)\pi + 1}\end{aligned}\right\}.$$

All the terms except the second are purely numerical quantities, which may be represented by $\log N$, hence

$$V_0 = e\log\frac{N}{\cosh\frac{\pi r}{l} - 1}.$$

The capacities of the square and circular condensers are equal if

$$2\log\frac{R}{r} = \log\frac{N}{\cosh\frac{\pi r}{l} - 1}$$

$$= \log\frac{N}{\frac{1}{2!}\left(\frac{\pi r}{l}\right)^2 + \frac{1}{4!}\left(\frac{\pi r}{l}\right)^4 + \cdots}.$$

Neglecting powers above the second:

$$\frac{R^2}{r^2} = \frac{2N}{\left(\frac{\pi r}{l}\right)^2} \qquad \text{or} \qquad R = \frac{\sqrt{2N}}{\pi}l,$$

$$N = 1.43555 \qquad \text{and} \qquad R = 0.539364l.$$

The circular cylinder lies partly outside the square cylinder. The perimeter of the circular cylinder is $2\pi R = 3.36l$, which is smaller than that of the square cylinder $4l$. This might have been expected from the fact that the average distance from the center to all points on the square is less than the radius of a circle of the same perimeter.

If we take a square of diagonal 2 cm. and a wire in the center of radius $r = 0.001$ cm. the formulæ (4) and (5) give the same result to within 0.2 per cent.

By means of formula (3) or (4) the capacity of a square outer cylinder of side $l = l' = 2\pi$ was calculated per unit length for different sizes of wire placed at the center. These values are given in Table I.

Table I.

r in cm.	0.0005	.001	.002	.01	.02	.06	.1
C in E.S.U.	.300	.326	.3565	.455	.518	.657	.755

Table II. contains the values of the capacity of a rectangular cylinder π cm. high ($l' = \pi$) 2π cm. wide ($l = 2\pi$) with a wire of radius $r = 0.01$ cm. half way between the upper and lower walls. Values of C are given for different positions of the wire as it is moved toward the side wall. The capacity changes but little for positions of the wire near the center.

Table II.

$y_0 = \pi/2$; $l = 2\pi$, $l' = \pi$, $r = .01$ cm.

x_0 in cm.	π	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{19}{12}\pi$	$\frac{5}{3}\pi$	$\frac{11}{6}\pi$
C in E.S.U.	.501	.503	.508	.514	.523	.580

The electric force in the x direction $E_x = -(\partial V_P/\partial x)$ in any point P, and the surface density of the induced charge in any point of the side wall ($x = 0$) is equal to

$$\sigma = -\frac{1}{4\pi}\left(\frac{\partial V}{\partial x}\right)_{x=0}.$$

From (2) we deduce:

$$-E_x = \frac{\partial V_P}{\partial x} = e\frac{\pi}{l'}\sum_{n=-\infty}^{+\infty}\left\{\frac{\sin\frac{\pi}{l'}(x+x_0)}{\cosh\frac{\pi}{l'}(2nl+y-y_0)-\cos\frac{\pi}{l'}(x+x_0)}\right.$$

$$\left.-\frac{\sin\frac{\pi}{l'}(x-x_0)}{\cosh\frac{\pi}{l'}(2nl+y-y_0)-\cos\frac{\pi}{l'}(x-x_0)}\right.$$

$$
\left. + \frac{\sin \frac{\pi}{l'}(x - x_0)}{\cosh \frac{\pi}{l'}(2nl + y + y_0) - \cos \frac{\pi}{l'}(x - x_0)} - \frac{\sin \frac{\pi}{l'}(x + x_0)}{\cosh \frac{\pi}{l'}(2nl + y + y_0) - \cos \frac{\pi}{l'}(x + x_0)} \right\}.
$$

We shall finally apply the previous results to the two and four wire cable in rectangular conduits. From the distribution of images in Fig. 2 it is evident at once, that the capacity of a system represented by Fig. 4 can be found as follows: Let the charges per unit length be $+e$ and $-e$ and placed at equal distances from the walls AF and CD and on a line parallel to AC. The distribution of the field is not changed by inserting a conducting partition BE. This plate would become charged positively on the right-hand side and negatively on the left-hand side and the system appears as two equal condensers connected in series. Then if equation (3) is written in the form $V_w = ef(x_0, y_0, l, l', r)$, the capacity of $ABEF$ is equal to $(e/V_w) = (1/f)$ and the capacity of $ACDF$ containing both wires is equal to $1/2f$.

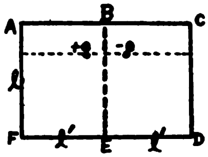

Fig. 4.

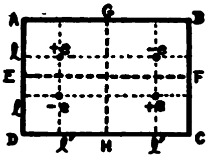

Fig. 5.

If four wires are placed symmetrically as shown in Fig. 5, two walls, EF and GH, may be inserted without disturbing the original field of force. We have two sets of condensers joined in series, each set consisting of two condensers in parallel. This combination has the capacity of any single condenser.

URBANA, ILLINOIS,
June 3, 1920.

SOME APPLICATIONS OF THE METHOD OF IMAGES—II.

By C. M. Hebbert.

Synopsis.

Distribution of Potential around a Charged Wire within a Rectangular Tube as Determined by the Method of Images.—The author derives the general equation for potential, given by Kunz and Bayley in the preceding paper, directly from Euler's infinite product expansion for the sine, thus avoiding the use of elliptic functions. Equipotential lines and lines of force are shown for two cases, a square and a rectangular tube; also curves giving the fall of potential along certain important lines.

At Dr. Kunz's suggestion I have carried out the calculations of potentials given by equation[1] (2) for the special cases in which the wire is at the center of a square cylinder and at the point $(l/2, l/10)$ in a rectangular cylinder of dimensions $2l \times l$. Tables and curves are given for these cases.

I have also applied Euler's infinite product expansion for the sine, as used in deriving equation (1), to the derivation of the general equation (2). This method avoids the use of the elliptic functions and arrives at the result in a direct way.

Equation (2*a*) may be written (taking out value for $m = 0$):

$$V_P = 2eR \log \prod_{\substack{m=-\infty \\ n=-\infty}}^{\infty} \frac{\begin{matrix}(z + \bar{z}_0 + 2inl)(z - \bar{z}_0 + 2inl) \\ \times (z + 2ml' + 2inl + \bar{z}_0)(z + 2ml' + 2inl - \bar{z}_0)\end{matrix}}{\begin{matrix}(z + z_0 + 2inl)(z - z_0 + 2inl) \\ \times (z + 2ml' + 2inl + z_0)(z + 2ml' + 2inl - z_0)\end{matrix}},$$

where $m \neq 0$

$$= 2eR \sum_{n=-\infty}^{\infty} \log \prod_{m=1}^{\infty} \frac{\begin{matrix}(z + \bar{z}_0 + 2inl)(z - \bar{z}_0 + 2inl) \\ \times (z + 2ml' + 2inl + \bar{z}_0)(z - 2ml' + 2inl + \bar{z}_0) \\ \times (z + 2ml' + 2inl - \bar{z}_0)(z - 2ml' + 2inl - \bar{z}_0)\end{matrix}}{\begin{matrix}(z + z_0 + 2inl)(z - z_0 + 2inl) \\ \times (z + 2ml' + 2inl + z_0)(z - 2ml' + 2inl + z_0) \\ \times (z + 2ml' + 2inl - z_0)(z - 2ml' + 2inl - z_0)\end{matrix}}$$

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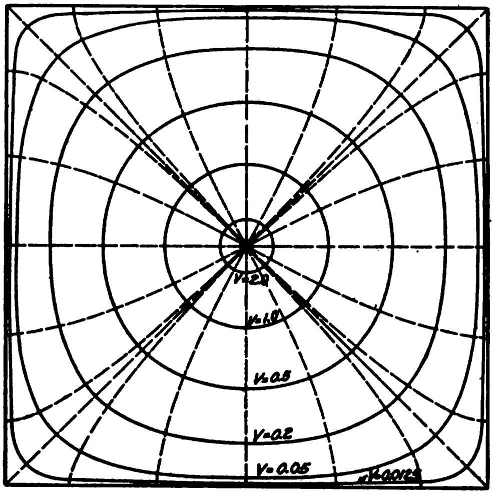
[1] The numbers refer to equations in the preceding paper.

$$= 2eR\sum_{n=-\infty}^{\infty}\log\prod_{m=1}^{\infty}\frac{\dfrac{\pi(z+\bar{z}_0+2inl)}{2l'}\left[1-\left(\dfrac{z+\bar{z}_0+2inl}{2ml'}\right)^2\right]\times\dfrac{\pi(z-\bar{z}_0+2inl)}{2l'}\left[1-\left(\dfrac{z-\bar{z}_0+2inl}{2ml'}\right)^2\right]}{\dfrac{\pi(z+z_0+2inl)}{2l'}\left[1-\left(\dfrac{z+z_0+2inl}{2ml'}\right)^2\right]\times\dfrac{\pi(z-z_0+2inl)}{2l'}\left[1-\left(\dfrac{z-z_0+2inl}{2ml'}\right)^2\right]}$$



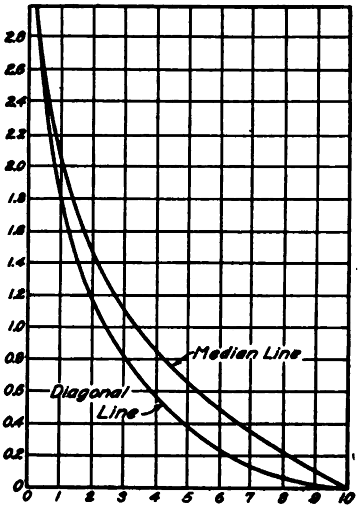


Fig. 1. Fig. 2.

FIG. 1. Shows the equipotential curves and lines of force for a wire at the center of a square cylinder. The coördinates were read directly from the curves in Fig. 2.

FIG. 2. Shows the fall of potential along the median and diagonal lines of the square in Fig. 1. The horizontal unit represents tenths of median and diagonal respectively, measured from the center.

$$= 2eR\sum_{n=-\infty}^{\infty}\log\frac{\sin\dfrac{\pi(z+\bar{z}_0+2inl)}{2l'}\cdot\sin\dfrac{\pi(z-\bar{z}_0+2inl)}{2l'}}{\sin\dfrac{\pi(z+z_0+2inl)}{2l'}\cdot\sin\dfrac{\pi(z-z_0+2inl)}{2l'}}$$

$$= e\sum_{n=-\infty}^{\infty}\log\frac{\left[\cosh\dfrac{\pi}{l'}(y+2nl-y_0)-\cos\dfrac{\pi}{l'}(x+x_0)\right]\times\left[\cosh\dfrac{\pi}{l'}(y+2nl+y_0)-\cos\dfrac{\pi}{l'}(x-x_0)\right]}{\left[\cosh\dfrac{\pi}{l'}(y+2nl+y_0)-\cos\dfrac{\pi}{l'}(x+x_0)\right]\times\left[\cosh\dfrac{\pi}{l'}(y+2nl-y_0)-\cos\dfrac{\pi}{l'}(x-x_0)\right]}$$

which is equation (2).

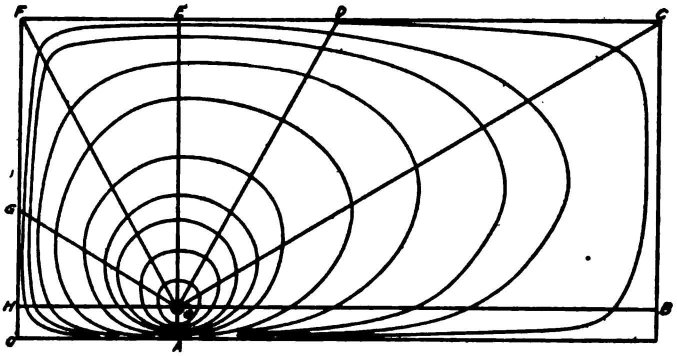
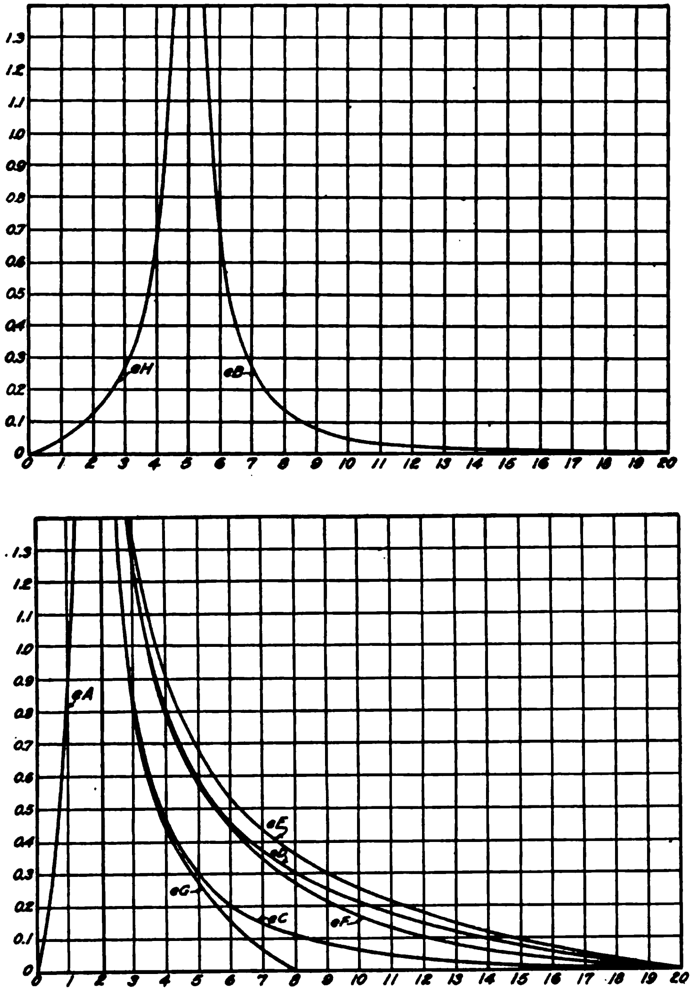


Fig. 3. Same as Fig. 1 for wire at $l/2$, $l/10$ in rectangular cylinder $2l \times l$.



Figs. 4 and 5 Show potential drop along lines eA, eB, eC, etc. of Fig. 3. Horizontal unit in Fig. 4 is $l/10$ along x-axis and in Fig. 5 is $l/20$ along y-axis of Fig. 3.

Below are tables of calculated values for a few points. There may be errors in these values but the smoothness of the curves obtained indicates that the errors are not serious. The first six terms of the series gave sufficiently accurate results for the square, and ten terms for the rectangle.

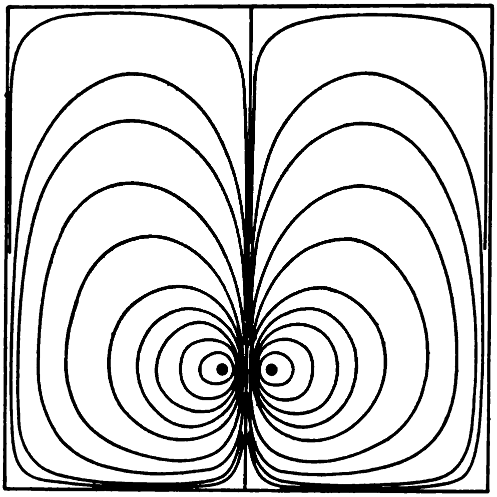


Fig. 6.

Shows the curves of Fig. 3 and their images with respect to the wall *OA*. The coördinates of the curves in Figs. 3 and 6 were read from the curves of Figs. 4 and 5.

The curves given in Figs. 1 to 6 are based on calculations for a large number of points not given in the tables.

TABLE I.

Calculated Potentials for Square Cylinder with e = 1 at Center. (See Figs. 1 and 2.)

Distance from center in tenths of line given	1	3	5	7	9
Median $V =$	2.066	1.111	.664	.360	.115
Diagonal $V =$	1.765	.813	.383	.135	.015

TABLE II.

Calculated Potentials for Rectangular Cylinder 2l × l with e = 1 at (l/2, l/10). (See Figs. 3–6.)

$k =$	1	2	3	4	5	6	8	10	14	15	17	18
Point $(l/2, kl/20)$, $V =$	.943	∞	1.366	.911	.683	.539	.362	.254	.123			.037
Point $(kl/10, l/10)$, $V =$	.051	.123		.674	∞	.680	.143	.050		.007	.003	

URBANA, ILLINOIS,
June 5, 1920.

THE ELECTRICAL RESISTANCE OF METALS.

By P. W. Bridgman.

Synopsis.

Electrical Resistance of Eighteen Elements.—The paper contains a brief summary of an extensive series of measurements which are to be published in detail elsewhere made to determine the *effect of pressures up to* 12000 *kg. per sq. cm. and of temperatures from* 0° *to* 275° *C.* on the resistance of *lithium, sodium, potassium, gallium, bismuth, mercury, calcium, strontium, magnesium, titanium, zirconium, arsenic, tungsten, lanthanum, neodymium, carbon (amorphous and graphitic), silicon, and black phosphorus.* The data for tungsten and magnesium are improvements on data previously published; the data for the other substances are new. The first six of these elements were studied in both the liquid and the solid states. The pressure coefficients of solid calcium, solid strontium, and both solid and liquid lithium are positive; the coefficient of bismuth is positive in the solid state, but negative in the liquid.

Modified Electron Theory of Metallic Conduction.—A previous theoretical discussion of measurements of the effect of pressure on resistance suggested most strongly that in metallic conduction the electrons pass through the substance of the atoms, and that the mechanism by which resistance is produced is *intimately connected with the amplitude of atomic vibration.* This view is here given quantitative form. The classical expression for conductivity, $\sigma = (e^2/2m)(nl/v)$, is retained; the number of free electrons is supposed to remain constant, their velocity is taken to be that of a gas particle of the same mass and temperature, and their mean free path is supposed to be many times the distance between atomic centers. The variations of path are then computed in terms of the variations of amplitude, and thus the variations of resistance are obtained and *checked with experimental results.* It is shown that the theory in this form explains Ohm's law, gives the correct temperature coefficient and the most important part of the pressure coefficient, avoids the difficulty of the classical theory with reference to specific heats, indicates a vanishing resistance at low temperatures, leaving open the possibility of super-conductivity, and retains the classical expression for the Wiedemann-Franz ratio. Besides these quantitative checks, the theory is shown to be entirely consistent qualitatively with all the new data; in fact, many of these new results, particularly the effect of pressure and temperature on the relative resistance of solid and liquid, seem to demand uniquely this conception of metallic conduction.

In a forthcoming number of the Proceedings of the American Academy I shall give the results of measurements of the effect of pressure and temperature on the resistance of 18 elements and several alloys. This is additional to my previous results for 22 elements. The numerical data will be briefly summarized in the Proceedings of the National Academy. In a previous paper in this journal[1] I drew certain inferences with regard to the mechanism of electrical conduction from the previous data for 22 substances. In this paper I desire to extend this work in the light of the new data now available.

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[1] P. W. Bridgman, Phys. Rev., 9, 269–289, 1917.

It will pay to briefly indicate the extent of the new data; the numerical results will be referred to in the following as the need arises. Previous to these measurements there were data for the effect of pressure on the resistance of no metal in both the liquid and the solid state, and the data were known for only one liquid metal, mercury. I have now obtained results for the variation of resistance under both pressure and temperature of the following six metals in both the liquid and the solid states: mercury, lithium, sodium, potassium, gallium, and bismuth. Previously none of the alkali or alkali earth metals had been measured; it is just these for which the largest pressure effects would be expected. I now have the data for lithium, sodium, potassium, calcium, and strontium. The effect of pressure was not known on any of the rare earth metals. I now have the data for La and Nd. The complete list of elements covered by the new work is: Li, Na, K, Mg, Ca, Sr, Hg, Ga, Ti, Zr, Bi, As, W, La, Nd, C, I, and P (black). Perhaps the most striking of the new results are as follows. Three more metals have been added to the list of those whose resistance increases with increasing pressure; these are Li, Ca, and Sr. Bi and Sb were the only ones previously known. Of these Li was a particular surprise, because its compressibility is so high. The resistance of liquid as well as solid Li increases under pressure. The resistance of liquid Bi, on the other hand, decreases under pressure, although that of the solid increases. I expected that the resistance of gallium would also increase under pressure, because this substance is abnormal in expanding on freezing, but its resistance decreases normally with pressure in both the solid and the liquid states. The effects of pressure on the resistance of Na and K are much larger than for any other metals, Na decreasing 40 per cent. in resistance under 12,000 kg., and K decreasing by 70 per cent. The decreases of these metals are insignificant compared with that of black phosphorus, however, which decreases to 3 per cent. of its initial resistance under the same pressure. The change is entirely reversible. The pressure coefficient of carbon has opposite signs in the amorphous and graphitic phases.

In my previous theoretical paper I directed especial emphasis to one fact brought out by the previous measurements, namely that the most important single factor in affecting the resistance of a metal is without doubt the amplitude of vibrations of the atoms. It turned out that the relative change of resistance under a change of either temperature or pressure was equal approximately to twice the corresponding change of amplitude of atomic vibration. The relation was fairly accurate for all metals with regard to the temperature coefficient (which is nearly the same for all metals and equal to the reciprocal of the absolute temperature), and was not so exact for changes of pressure, but nevertheless was somewhat more accurate than any other relation previously proposed for the pressure change. This fact seemed to me to indicate that the mechanism of conduction was by a passage of electrons from atom to atom through the substance of the atom itself. The atom is to be thought of as normally offering no resistance to the passage of the electron (super-conductivity at absolute zero), but resistance may be encountered in passing from atom to atom. The assumption of paths within the atom which are resistanceless need occasion no alarm in these days of non-radiating quantum orbits. In fact there may be an intimate connection between the two. I had called this theory the "gap" theory of resistance. Not only the quantitative fact that the resistance varies as the square of the atomic amplitude, but a large number of qualitative facts also, were in accord with this point of view. These qualitative facts were many of them brought into line by the conception that the "gap" may function in two ways. At large mean distances of separation of the atoms it may happen that passage of electrons from atom to atom is made easier by temperature agitation of unusual violence, which brings the surfaces of the atoms closer together than normal during part of their vibration, whereas at small mean distances of separation, the passage of electrons is on the average hindered by temperature agitation. Except for the deduction of the expressions for the variation of atomic amplitude with temperature and pressure, the theory as hitherto expounded was qualitative rather than quantitative. In particular, I did not attempt to give any detailed picture of the way in which the gaps between atoms might offer resistance to the passage of electrons, or what the character of the resistance might be. The theory as previously given also made no attempt to explain the Wiedemann-Franz ratio, although I pointed out that an explanation was not inconsistent with the elements of the theory.

It is now possible to cast this point of view into quantitative form, at least as far as temperature variations of resistance go; the pressure changes cannot be so easily dealt with for a reason that will appear later.

In the first place it is interesting to observe that the proportionality of resistance to the square of the amplitude of atomic vibration holds also at low temperatures; the previous considerations were entirely confined to ordinary temperatures. It has been noticed by Grüneisen,[1] after an examination of the best data, that down to very low temperatures the quotient of resistance by absolute temperature is proportional to specific heat, the factor of proportionality being different for different metals. Let us compare this experimental variation of resistance with a variation as the square of the amplitude of atomic vibration at low temperatures. The specific heat formula which best fits the facts at low temperatures is that of Debye. This gives an energy content proportional to the fourth power of the absolute temperature, and of course a specific heat proportional to the third power, since the specific heat is obtained by differentiating the energy. We have, therefore, the expressions,

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[1] E. Grüneisen, Verh. D. Phys. Ges., 15, 186–200, 1913.

$$E = At^4 \quad \text{and} \quad C = 4At^3,$$

where E and C are the energy and the specific heat of the atom respectively. We may also express the energy of the atom in terms of its frequency and amplitude of vibration. This gives the equation

$$2\pi^2 m\nu^2\alpha^2 = At^4,$$

where α is the amplitude, and ν the frequency of atomic vibration. Now differentiate this expression logarithmically with respect to t, giving

$$\frac{2}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_p = \frac{4}{\tau} - \frac{2}{\nu}\left(\frac{\partial\nu}{\partial\tau}\right)_p.$$

The value of $1/\nu(\partial\nu/\partial\tau)_p$ was found in the previous paper. Substituting this value gives

$$\frac{2}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_p = \frac{4}{\tau} - 2\left(\frac{\partial v}{\partial\tau}\right)_p^2 \Big/ C_v\left(\frac{\partial v}{\partial p}\right)_\tau.$$

An examination of the previous deduction of $1/\nu(\partial\nu/\partial\tau)_p$ shows that the connection between temperature and energy did not enter, so that the same expression is valid at low as well as high temperatures. But now it is an empirical fact that at low temperatures the compressibility approaches a constant value, and that the ratio of the thermal expansion to the specific heat also approaches a constant value. Hence in the limit, $(\partial v/\partial\tau)_p^2/C_v(\partial v/\partial p)_\tau$ becomes proportional to $(\partial v/\partial\tau)_p$, and hence vanishes. We have, therefore, at low temperatures,

$$\frac{2}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_p = \frac{4}{\tau}.$$

At high temperatures $2/\alpha(\partial\alpha/\partial\tau)_p$ was also proportional to $1/t$, but the factor of proportionality was 1 instead of 4.

Now our previous empirical observation was that $2/\alpha(\partial\alpha/\partial\tau)_p$ gives the variation of resistance with temperature. Compare this with

Grüneisen's empirical observation. He has

$$R/t = BC,$$

$$R = BCt$$

$$= 4ABt^4,$$

where R is the resistance and B a constant. Hence

$$\frac{1}{R}\frac{dR}{dt} = 4/t,$$

which checks precisely with the value of twice the variation of atomic amplitude.

Although we have found a variation at high and low temperatures proportional to the variation of the square of the amplitude, we cannot set the resistance at all temperatures equal to a constant times the square of the amplitude (that is if Grüneisen's observation is correct) for the factor of proportionality changes on passing from low to high temperature. The relation of proportionality must therefore fail at some intermediate temperature.

Returning now to the task of making the "gap" theory more definite, we in the first place make specific the action of the gap in imparting resistance by picturing precisely the same sort of mechanism as that operating during a collision of the classical theory. There cannot be any doubt that the electron encounters difficulty in getting free from the atom when the atoms are separated from each other as in a gas, for there is a definite ionizing potential, which involves an amount of energy large in comparison with that ordinarily available in the form of temperature energy of agitation. It is also evident that in some way the electrons do get free from the atoms in the solid state, because the solid is conducting for any E.M.F., no matter how small, and at low temperatures its Ohmic resistance vanishes. This means that under certain conditions forces may act on the electron when in the act of passing from one atom to another, whereas while the electron is passing through the interior of the atom no forces act. During the action of a force between electron and atom, there is a chance for the transfer of energy from one to the other, so that we have a tendency to equipartition. In other words, the "gap" produces resistance by interfering with or terminating the free path of the electron, precisely as did the "collision" in the classical theory. The amount of interference with the free path will depend on the amplitude of vibration, and so the resistance will depend on the amplitude. Such a picture as this enables us to carry over immediately much of the analysis of the classical theory. For instance, the classical expression for the resistance at once holds, namely,

$$R = \frac{2m}{e^2} \frac{v}{nl}, \tag{1}$$

where e is the charge on the electron, m the mass of the electron, n the number of electrons describing paths per cm.^{3}, l the mean free path of the electron, and v the undisturbed velocity of translation of the electron.

Ohm's law at once follows on the same basis as in the classical theory. Previously I had to leave Ohm's law unexplained, with only the remark that there was no inconsistency.

This picture of the mechanism of interference with the free path is so different physically from that of the classical theory, and yet gives such similar results when substituted into the mathematical expression, that it will pay to stop for a moment to inquire what is our justification for setting up so definite a picture. In particular, we may compare this picture with that of Wien,[1] who also has a free path mechanism of conduction and Ohm's law, but imagines the electrons interfered with in their flight by collisions with the centers of the atoms. Many facts are equally understandable from either point of view. There are a number, however, which are more naturally explained by supposing that the interference encountered is in passing through the surface from one atom to the next. Many of these facts will be given later; I mention only the two most striking. In the first place it is a universal fact that if a metal changes form, as by melting, or by a polymorphic transition, the phase with the smaller volume has the smaller resistance. This holds for all known normal meltings, in which the liquid has the larger volume, and also holds for the abnormal meltings of Bi, Sb, and Ga, in which the liquid has the smaller volume, and also the smaller resistance. It holds also for the normal polymorphic transitions of Zn and Ni, and for the abnormal transition of Tl, in which the high temperature phase has the smaller volume. Now this is difficult to understand from Wien's point of view, for it would seem that in the phase with the smaller volume there must be more chance of collision with the atomic centers, and so a higher resistance. The difficulty cannot be turned by supposing that the energy of temperature agitation is different in the two phases, for the specific heat of liquid bismuth is greater than that of the solid, and therefore in the liquid the atoms have more chance of terminating the free path both because they are more numerous, and because their amplitude of temperature agitation is greater, so that they effectively cover more territory. On the other hand, the fact that the resistance of the phase with the smaller volume is the smaller is most naturally explained from the "gap" point of view simply by the observation that the atoms are closer together, and the electrons find it easier to pass from one to another. Another fact not readily understandable from Wien's point of view is that the temperature coefficient of resistance of the liquid is nearly always less than that of the solid. In some cases it may be very much less, or may even be negative, instead of positive. Now the increased violence of temperature agitation in the liquid, as shown by the higher specific heat, would seem to demand a higher temperature coefficient, according to Wien's view. There is a most natural explanation in terms of the gap theory, as has been explained in the preceding paper, or as will be elaborated further in the following.

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[1] W. Wien, Columbia Lectures, 1913, 29–48.

So much for the mechanism by which resistance is produced, and the explanation of Ohm's law. To get further, we have to know the precise manner of variation of n, l, and v with temperature and pressure. The classical theory supposed that v was the value given by the equipartition of energy, treating the electron as a gas particle, that l was at least of the order of magnitude of the distance between atomic centers, and n could look out for itself, being determined by the necessities of the case. The weaknesses of the old theory are well known. One of the most serious is that the n needed to give the observed values of specific resistance is of the order of magnitude of the number of atoms itself, or even may be considerably in excess, which leads to the insuperable difficulty of the specific heat. An n of the same order of magnitude was indicated by the application of the theory to the optical theory of metals. Many attempts have been made to avoid this difficulty, but as yet without success. Nearly all attempts at replacing the classical theory have failed in the endeavor to give even an approximate explanation of the Wiedemann-Franz ratio. In order that the thermal conductivity of a metal shall have its high value, and in order that the Wiedemann-Franz ratio may have approximately its experimental value, it seems to be necessary to suppose not only that temperature energy is carried by the electrons, but that the amount so carried is precisely the amount which would be carried if the energy of the electrons were the equipartition energy on the old classical basis.

Apparently the most promising attempt at another explanation of the Wiedemann-Franz ratio is the recent theory of Borelius,[1] which does give a result of approximately the right magnitude. But although his explanation may be mathematically satisfying, I do not think that it can be considered satisfactory physically. His explanation is on the basis of the Debye theory of heat conduction.[1] Debye's theory is that heat is conducted by elastic waves, which are dissipated by atomic irregularities. A quantity analogous to the mean free path may be defined in terms of the rate of dissipation, so that the higher the rate of dissipation the shorter the equivalent free path, and the less the conductivity. Debye has applied this theory to insulators, like rock salt, and has been able to account for the experimental fact that the conductivity varies inversely as the absolute temperature. Borelius has taken over this theory for metallic conduction, by means of the assumption that the amount of energy dissipated at each atom as the elastic wave passes over it is equal to the ratio of the thermal energy of the atom to that of the electron. This gives a thermal conductivity of the right order of magnitude. In combination with his theory of electric conduction he gets a Wiedemann-Franz ratio of approximately the right magnitude, and with the right dependence on temperature. But he has neglected the dissipation of the elastic waves due to the atoms. If Debye's analysis is correct, this should be present in the metal as well as in an insulator, and is very much larger than the dissipation supposed by Borelius. Hence, taking account of the neglected atomic dissipation, the thermal conductivity of a metal would turn out to be actually less than that of an insulator, for there is dissipation not only by the atoms but also by the electrons.

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[1] G. Borelius, Ann. Phys., 57, 278–286, 1918.

Compared with the classical conception of conduction as performed by a swarm of electrons playing in the free spaces between the atoms, the view at which we have arrived of conduction as performed by electrons passing freely through the substance of the atoms places us in a much more advantageous position, for it allows the possibility of very long free paths (in fact at absolute zero there is no resistance to the motion and the paths may be indefinitely long), and hence enables us to get along with many fewer electrons. In this way the specific heat difficulty may be avoided. But in order to account for the facts of thermal conduction, it seems necessary to take over the classical idea that the electrons are moving with the energy of gas particles at the same temperature.

The theory developed here proceeds on the following assumptions. We take over the classical expression (*I*) for resistance in terms of n, l, and v. We suppose that v is the same as that given by the classical equipartition theory, and we suppose l large enough to avoid the specific heat difficulty by allowing a small value of n. We try to deduce from our fundamental expression the variation of resistance with temperature and pressure in terms of the variations of v, which are given by the classical expression, and the variations of l, which we get from our picture of the electrons jumping from atom to atom across a region capable of exercising interference with the path. The quantity n we suppose to stay constant. One reason for this supposition is that we do not need to assume any variation in order to account for the facts. However, it does not seem unplausible that n should stay approximately constant. As temperature is increased, the tendency to an increased n due to the increased chance of getting an electron out of an atom by increasing violence of collision is counterbalanced by the increasing difficulty of getting the electron out of the atom because of the increased distance of separation of atomic centers. Or again, it may be that the number of migrating electrons is determined by a sort of spontaneous atomic disintegration in the outer part of the atomic structure, over which changes of temperature or of pressure can have no control.

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[1] P. Debye, Wolfskehlstiftung Vorträge, B. G. Teubner, 1914, 17–60.

There is evidence as to the magnitude of n given by optical theories of metals. We shall for the present merely disregard this. We are much more justified in doing this now than we would have been several years ago, for quantum theory has made it exceedingly uncertain whether we are justified in keeping our old model of a vibrating electron as a source of light. Quantum theory has shown,[1] for example, that the success of the electron in giving the classical expression for the Zeeman separation was due to a quite accidental cancelling out of the factor h from the result, and that the factor h will not so conveniently cancel out in treating certain other phenomena, as the Stark effect. Furthermore, the number of electrons demanded by optical theory must now seem impossibly high in the light of our knowledge of the structure of the atom, so that we are probably justified in disregarding the line of attack from the optical side until quantum theory has become more developed.

Concerning our assumption of the equipartition value for v, putting it equal to $\sqrt{2(\kappa\tau/m)}$, there cannot be much question at moderate temperatures, so long as n is small enough to avoid the specific heat difficulty, but there may be question as to what the limit of temperature is to be. It is possible that the classical expression for the energy may hold to lower temperatures for the electron than for the atom. Hydrogen or helium at low temperatures in the gaseous condition in contact with the solid walls of the container continues to have the classical energy, although this is no longer the equipartition energy as compared with the atoms of the solid walls. However, the classical expression cannot continue to hold indefinitely to extremely low temperatures, for even a few electrons with the classical energy would ultimately make trouble with the specific heat. We will not concern ourselves further with the situation at extremely low temperatures except to remark that the assumptions of this theory will have to be modified, and to indicate that such a modification is not at all impossible. For instance, the electron in coming from the atom, leaves behind it a positively charged ion. As the electron wanders through the metal it must at some time come again in contact with an ion, and may recombine with it. Throughout the metal, therefore, there is going on a continuous process of emission of electrons by the atoms and reabsorption. Now the mechanism of the ejection of the electron by the atom within the solid is something which we do not understand, but it is not unlikely that it is determined from within the atom, and is not greatly affected by outside conditions, at least as far as the velocity of emission is concerned. The electron may, therefore, be expelled with a definite velocity independent of temperature. It then travels from atom to atom, and is jostled about, until it ultimately acquires the equipartition energy. The time required to accomplish equipartition may well be less the greater the violence of agitation. Presently, the electron recombines with an ion. At low temperatures, because of the feebleness of temperature agitation, it may be that the electron is absorbed before it has acquired the equipartition energy, or that it has possessed this energy for only a relatively short time. The specific heat difficulty will not then appear, and the conductivity will be even higher than that corresponding to Grüneisen's empirical observation, because the mean velocity will not decrease so rapidly with decreasing temperature. And it is of course a fact that some metals show superconductivity at extremely low temperatures, and that superconductivity is not covered by Grüneisen's formula.

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[1] A. Sommerfeld, Atombau und Spektrallinien, Vieweg, 1919, 422–440.

So much for the assumptions with regard to v and n. Now for a deduction of the variation of l, the mean free path.

There is a point implied in our assumption of a long free path which may detain us for a moment. We have pictured the electron as passing through the substance of many atoms before its path is arrested. Now in a cubic crystal this means that the electronic paths may be confined to three directions mutually at right angles, along the crystalline axes. The classical deduction for the expression for conductivity assumes of course that the directions of the paths are distributed at random throughout space. Is a specialized distribution in only three directions consistent with the known fact that the resistance of a cubic crystal is independent of the direction? A simple analysis immediately shows that there is no inconsistency here. Imagine the applied electric force making the angles α, β, and γ with the crystal axes, and find the contribution made to the conductivity by the electrons moving along the X axis. The component of force along this axis is $E \cos \alpha$, so that the velocity imparted to the electron is smaller by the factor $\cos \alpha$ than it would be if the force were·along the axis. And the component of this velocity in the direction of the force is further diminished by the factor $\cos \alpha$, so that the total contribution of the electrons moving along the X axis is diminished by the factor $\cos^2 \alpha$. Similarly the electrons moving along the other two axes contribute to the conductivity terms equal to $\cos^2 \beta$ and $\cos^2 \gamma$. The sum of the three terms is 1, which is thus independent of direction.

On the other hand, there is no necessity in supposing that a single free path is straight; the electrons may perform Virginia reels about the nuclei of the atoms in quantum orbits. Something of this sort probably has of necessity to take place in non-cubic crystals.

We return to the question of the variation of l. For the moment we consider that the solid is maintained at constant volume. As temperature is increased, the amplitude of vibration is increased, and it may be that during part of the vibration the atoms are so far separated that the electron cannot pass. That is, when the atoms are in vibration, there is a certain chance that the free path of the electron may be terminated in the passage from one atom to the next. This chance is a function both of the amplitude of atomic vibration, and of the mean distance of separation of atomic centers. Call β the chance that the path will be terminated, r the mean distance of separation of atomic centers and α the amplitude of atomic vibration. Then we may expand the unknown function in powers of α and write

$$\beta = f(r, 0) + \alpha \frac{\partial f(r, 0)}{\partial \alpha},$$

provided that α is small. α may be calculated, and it turns out that for ordinary temperatures α is a small fraction of r. Now it is our fundamental assumption that when the atoms are at rest no resistance is encountered in passing from atom to atom, provided that the distance of separation of atomic centers is not too great, as we suppose it is not for a solid. This means that $f(r, 0) = 0$, and we may write

$$\beta = \alpha \frac{\partial f(r, 0)}{\partial \alpha}.$$

That is, so long as the volume is kept constant, the chance that the path will be terminated in passing from atom to atom is proportional to α. In unit distance the chance that the path will be terminated is equal to β

multiplied by the number of atoms in unit length, again giving a constant times α. Now the probability definition of the mean free path is merely the reciprocal of the chance that the path will be terminated in unit distance, so that we have at once

$$l = \text{Const}/\alpha.$$

To compute the variation of l at constant volume it is sufficient to calculate the variation of α at constant volume.

Now the variation of α was already computed in the previous paper. The expressions there given were for the variation of α with pressure and temperature, but of course from these derivatives the derivative at constant volume may at once be found if we know the compressibility and thermal expansion of the substance.

The computation of the change of amplitude is the only place in the theory into which the quantum hypothesis explicitly enters. The assumptions at the basis of the computation were these. The energy of the atom is the classical amount, $\kappa\tau$ (so that the deduction does not hold for low temperatures), the frequency is a function of volume only (the forces on the atoms are on the average a function of volume only), and the entropy of the atom is the same as the entropy of an ideal linear oscillator at the same temperature, which by quantum considerations is shown to be a function of τ/ν only.

For convenience of reference the values previously found are reproduced here.

$$\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial p}\right)_\tau = -\left(\frac{\partial v}{\partial\tau}\right)_p\Big/C_v,$$

$$\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_p = \frac{1}{2\tau} - \left(\frac{\partial v}{\partial\tau}\right)_p^2\Big/C_v\left(\frac{\partial v}{\partial p}\right)_\tau.$$

The change of amplitude at constant volume may at once be found

$$\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_v = \frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_p + \frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial p}\right)_\tau\left(\frac{\partial p}{\partial\tau}\right)_v.$$

But

$$\left(\frac{\partial p}{\partial\tau}\right)_v = -\left(\frac{\partial v}{\partial\tau}\right)_p\Big/\left(\frac{\partial v}{\partial p}\right)_\tau.$$

Substituting this value above gives at once the simple result

$$\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial\tau}\right)_v = \frac{1}{2\tau}.$$

Now let us find the change of resistance at constant volume with temperature. Our formula for the specific resistance is

$$R = \frac{2m^{1/2}(2\kappa)^{1/2}}{e^2n}\frac{\tau^{1/2}}{l}$$

$$= \text{Const}\frac{\tau^{1/2}}{l}.$$

Differentiating this logarithmically gives

$$\frac{1}{R}\left(\frac{\partial R}{\partial \tau}\right)_v = \frac{1}{2\tau} - \frac{1}{l}\left(\frac{\partial l}{\partial \tau}\right)_v.$$

Using the value found for l in terms of α gives

$$\frac{1}{l}\left(\frac{\partial l}{\partial \tau}\right) = -\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \tau}\right)_v,$$

and substituting this gives

$$\frac{1}{R}\left(\frac{\partial R}{\partial \tau}\right)_v = \frac{1}{\tau}.$$

Thât is, the temperature coefficient of resistance at constant volume is the reciprocal of the absolute temperature. This of course, is in accord with the experimental facts. In my preceeding theoretical paper I gave the values of $1/R(\partial R/\partial \tau)_v$. The difference between $1/R(\partial R/\partial \tau)_v$ and $1/R(\partial R/\partial \tau)_p$ is not large, but is in the direction to make $1/R(\partial R/\partial \tau)_v$ even more nearly equal to $1/\tau$ than $1/R(\partial R/\partial \tau)_p$.

The change from the point of view of the last paper is to be noticed. We previously thought of the change of α as the only significant feature, and noticed that $1/R(\partial R/\partial \tau)_v$ was equal to $2/\alpha(\partial \alpha/\partial \tau)_v$. We did not see any particular reason why the change of resistance should be proportional to twice the change of α, that is, why the resistance should be proportional to the square of the amplitude, although we advanced reasons which made it seem not improbable. Our present expression for R demands that we analyze the change of resistance into two effects; one is a temperature effect, due to the term $\tau^{1/2}$, and the other is an effect due to the change of l, which involves the change of α. Now it turns out that the change of l with α, which again changes with temperature, is of such a nature that the sum of the two effects is as before exactly equal to twice the change of α. Hence the previous analytical result stands, but our physical analysis is different.

We now let fall the condition that the volume be kept constant during the change, and consider the change of resistance with temperature at constant pressure, which is the coefficient usually directly determined by experiment. The formal work of differentiation of our fundamental expression for resistance may be carried through as before. We encounter difficulty in determining the variation of the mean free path with temperature, for the condition no longer holds that the volume is constant, and β, the probability of the termination of the path, can no longer be put equal to a constant times the amplitude, but the factor multiplying the amplitude is an unknown function of the volume. I have not found any universal assumption as to the dependence of β on the distance of separation of atomic centers which seems to be plausible, but it would seem that β might vary in any way with the structure of the particular atom. For instance, it is conceivable that as the distance of separation of atomic centers is increased the atoms acquire rotational movement with respect to each other, so that parts of the atomic surfaces are brought into opposition which are not so favorable for the passage of electrons. The magnitude of this effect will depend entirely on the structure of the particular atom. Or again, as the atoms are brought closer to each other, the ease of passage will be affected by the deformation of the atom, and this again varies in an unknown way from element to element. The only fact about this unknown effect of changing distance between atomic centers which it seems fairly safe to assume as common to most elements is the sign; it is likely that as the atoms are brought closer together at constant amplitude the ease of passage is increased, and so the mean free path is increased.

The unknown effect of changing volume will evidently be much greater for those changes of volume relatively large compared with the change of amplitude. This means that the unknown effect will be much larger for changes of pressure than for changes of temperature. The magnitude of these changes was considered in the previous paper. It was there shown that the pressure coefficient of amplitude is from six to nine times the pressure coefficient of distance of atomic separation (linear compressibility), whereas the temperature coefficient of amplitude was from 50 to 650 times as great as the temperature coefficient of atomic separation (linear dilatation). In view of the largeness of this latter ratio, it would seem that we are justified for most substances in neglecting the effect as far as variations of temperature at constant pressure are concerned. We will, as a matter of fact, make this assumption. We then have

$$\frac{1}{l}\left(\frac{\partial l}{\partial \tau}\right)_p = -\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \tau}\right)_p,$$

and analysis exactly like the preceding would give

$$\frac{1}{R}\left(\frac{\partial R}{\partial \tau}\right)_p = \frac{1}{\tau} - \left(\frac{\partial v}{\partial \tau}\right)_p^2 \Big/ C_v \left(\frac{\partial v}{\partial p}\right)_\tau .$$

Now for all solid metals the second term is small compared with the first, and we may neglect it. We may now replace R, the specific resistance by ρ, the observed resistance (that is, the resistance of a piece of wire with fixed terminals), since the difference of the two coefficients is equal to the linear expansion, which may be neglected in comparison with $1/\tau$, and obtain as our final approximate result

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial \tau}\right)_p = \frac{1}{\tau}.$$

This equation, of course, agrees with experimental fact. The usual temperature coefficients are always somewhat larger than $1/\tau$, which is the direction of variation which the above considerations would lead us to expect. I know of no theory which gives the departure of the temperature coefficient from $1/\tau$. Wien's theory is the only one which makes the attempt, and this cannot be regarded as successful. Of course the classical theory does not account for the temperature coefficient at all, but is driven to unplausible assumptions as to the variation of the free path with temperature in order to be consistent.

Now to find the pressure coefficient, we differentiate our expression for resistance with respect to pressure at constant temperature, getting

$$\frac{1}{R}\left(\frac{\partial R}{\partial p}\right)_\tau = -\frac{1}{l}\left(\frac{\partial l}{\partial p}\right)_\tau.$$

The only statement which we can make about $1/l(\partial l/\partial p)_\tau$ is that it is at least as large as $1/\alpha(\partial \alpha/\partial p)_\tau$, so that we expect the pressure coefficient of resistance to be at least as large numerically as $(\partial v/\partial \tau)_p/C_v$ (substituting the value found above for $1/\alpha(\partial \alpha/\partial p)_\tau$) and to be negative. In the previous paper it was shown that the pressure coefficient is given on an average for a large number of metals by $2/\alpha(\partial \alpha/\partial p)_\tau$. Our present theory leads us to replace this by the statement that there is a lower limit one half the value previously found. Now it is true that twice the amplitude represented a somewhat better approximation to the average of all the results than did the expression of Grüneisen, for example, which was the best of other theoretical expressions for the pressure coefficient of resistance, but there were deviations from it in both directions. It is on the other hand true that there are no cases known, either among the substances of the previous work or among the new elements of my more recent work, except those abnormal metals for which the coefficient is positive, and which will be dealt with separately later, in which the coefficient falls as low as $1/\alpha(\partial \alpha/\partial p)_\tau$. So that although our present theory is not complete because of the unknown element of atomic structure, it is at least more satisfactory than the former attempt in that it is true without exception.

So much for the formal expression of the theory with regard to temperature and pressure changes of resistance. The classical expression for thermal conductivity is to be taken over without change, and need not be written down explicitly. It gives the right order of magnitude for the Wiedemann-Franz ratio, but is not a complete expression of the facts because it neglects the part of conduction done by the atoms, and hence will be expected to fail particularly at low temperatures, where the atomic conduction becomes important.

The theory as outlined is not inconsistent with a retention of the classical expression for the Peltier heat. In particular the theory is in accord with the classical theory in at least two aspects of thermoelectric phenomena. The magnitude of the pressure coefficient of the thermoelectric force would mean, according to the classical theory, that the number of free electrons does not change much with increasing pressure. This is in exact accord with the assumptions of our theory. Furthermore, the small Peltier heat between a solid and a liquid metal makes it likely that the number of free electrons is not greatly different in solid and liquid. As will be seen in the following, our theory gives an account of the difference of resistance between solid and liquid in terms of a difference of free path only, without supposing a difference in the number of electrons. There was here a weak point of the classical theory, because to account for the great difference of conductivity between solid and liquid it had to suppose a number of electrons much greater in the solid than in the liquid, which was not consistent with the thermo-electric phenomena. At the same time I do not believe that it is desirable to take over entirely the whole classical picture of the mechanism of thermo-electric action. There are, of course ions as well as wandering electrons present in the metal, and these ions may play a part in thermo-electric action, although because of their relatively small translational velocity their part in conduction may be altogether insignificant. That some such modification is necessary is indicated by the fact that the classical expression for the Thomson heat demands that the number of free electrons increase as the square root of the absolute temperature, whereas we have assumed that the number is constant. However, the Thomson heat is relatively a small matter, and I have not attempted to bring it within the range of the present theory. If the above considerations are sound it ought to be possible to get it in without essentially modifying the mechanism which we have set up to account for conductivity.

This is as far as I have got at present with the quantitative development of the theory. Its advantages are obvious. It gives an unforced and inevitable description of the variation of resistance with temperature, predicts the right sign and an important part of the numerical magnitude of the pressure coefficient, avoids the difficulty of the classical theory with the specific heats while retaining the classical explanation of the Wiedemann-Franz ratio, and leaves open the possibility of much greater conductivity or even superconductivity at low temperatures, which was not possible to the classical theory. Further development of this theory seems to demand more intimate knowledge of atomic structure than we have at present.

But in addition to these quantitative facts, the theory is able to bring a very large number of facts qualitatively into line, as was emphasized in the preceding paper. In the following I give a survey of the new facts brought out by the new experimental work, and the relation of the theory to these facts.

1. The view of conduction as due to the passage of electrons from atom to atom through the substance of the atom receives confirmation from a group of phenomena not considered in detail in the previous paper, namely the phenomena of the resistance of alloys. Alloys fall into two main groups, according as they do or do not form mixed crystals. Those alloys which do not form mixed crystals solidify by the separation of the two components each in a pure condition, so that the solid alloy consists of a mechanical mixture of microscopic crystals of the two components. It would be expected that the resistance of a mixture of this sort would be the mean of the resistance of its components, and it is indeed the experimental fact that the resistance of such alloys can be computed by the rule of mixtures. The other class of alloys is one in which mixed crystals are formed, the atoms of the two metals entering side by side into the same crystal edifice. This is possible because of a certain degree of resemblance of the two kinds of atoms. The resemblance is not complete, however, and the indiscriminate use of either kind of atom in the crystal edifice is possible only with a certain amount of distortion in the final result. It is a fact that most mixed crystals will not accept an unlimited amount of the foreign ingredient, but the two atoms will crystallize side by side only up to certain limiting proportions. In the mixed crystal structure we would expect, therefore, a certain amount of imperfect fitting between adjacent atoms, with the result that the electrons encounter difficulty in passing from atom to atom, so that, because of the extra resistance of the "gaps" between the atoms, the resistance of the alloy is greater than that computed from the components. This is in exact accord with the experimental facts.

Furthermore, the temperature coefficient of alloys is strikingly in accord with this view. The temperature coefficient of those alloys which do not form mixed crystals is the same as that of the pure metals, which is what we would expect, but the temperature coefficient of those alloys in which there are mixed crystals is very much less than that normal to a pure metal. A large number of binary alloys satisfy Matthiesen's rule in this regard. If we write the resistance of the alloy in the form

$$R_A = R_M + \Delta R,$$

where R_A is the resistance of the alloy, R_M the resistance which would be computed according to the rule of mixtures, and ΔR the resistance which must be added to the computed resistance to give the actual resistance, then Matthiesen's rule states that the temperature coefficient of the alloy is given by the expression

$$\frac{dR_A}{dt} = \frac{dR_M}{dt}.$$

This means that $d\Delta R/dt = 0$, or the additional resistance is not affected by changes of temperature. This is in precise accord with our point of view, for the additional resistance (ΔR) is due to the lack of perfect fit between adjacent atoms of different kinds, and this would be expected to be relatively little affected by changes of temperature.

It is also an experimental fact that the resistance of alloys tends to a finite value at 0° Abs., instead of vanishing like that of a pure metal. This merely means that the gaps between the different kinds of atoms persist to low temperatures, as we would certainly expect.

2. The fact that the resistance of a metal increases on hard drawing is consistent with this point of view. During hard drawing the crystal grains are broken up, and the fitting of the atoms is rendered less exact on the average. This is proved by the universal fact that during hard drawing the density of the metal decreases. But such a disarrangement of the fitting of the atoms means an increased chance of interference when the electrons pass from atom to atom, and consequently an increase of resistance.

Tammann[1] has given an explanation of this fact on the basis of a difference of resistance in different directions within the crystal grains. It would seem, however, that this explanation must be rejected, because the resistance of a cubic crystal is independent of direction (see Voigt, Krystal Physik, pp. 311, 313), whereas the phenomena of increase of resistance on hard drawing is shown by all metals, regardless of their crystal system.

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[1] G. Tammann, Lehrbuch der Metallographie, Leopold Voss, 1914, 117–125.

3. It has been observed that as the proportions of the components change through a series of alloys forming mixed crystals the electrical resistance increases in that direction in which the mechanical hardness also increases. Now an increased mechanical hardness means an increased staggering in the positions of the atoms in the crystalline grains, so that it is more difficult to produce sliding of one part of the crystal on another, and by the same token an increase in the difficulty of the electrons in making the leap from atom to atom.

4. In the previous paper a suggestion was made as to the possible explanation of the positive pressure coefficient of bismuth and antomony· The idea was that the amplitude might increase with increasing pressure instead of decreasing as normal. It was shown in that paper that the fact that both metals expand on freezing indicates that there is a certain relative position of the atoms in which the repulsive forces are unusually large at an unusual distance of separation, and that the atoms crystallize in this relative position. From a grossly material point of view this may be expressed by saying that the atoms have knobs, and that the metal crystallizes with the knobs in contact. On the average, except for these knobs, the bismuth atom may be much like that of other metals. The fact that the repulsive forces are unusually large at an unusual distance of separation of the atomic centers is compensated for by an unusually slow decrease of the repulsive force as the distance between atomic centers is decreased. This slow change of the repulsive force was shown in the preceding paper to be consistent with a decreasing frequency of atomic vibration as the centers are brought closer together, and hence consistent with an increasing amplitude with increasing pressure.

This view receives interesting numerical confirmation from recent work of Grüneisen[1] on the equation of state of solids. He has shown that it is possible to a good degree of approximation to explain the behavior of solids by supposing that at least over a small range the forces between atoms are represented to a sufficient approximation by the expression

$$f = \frac{A}{r^2} + \frac{B}{r^m},$$

where the first represents a force of attraction, and the second a force of repulsion. In general m will be much larger than 2. Its magnitude will give an idea of how rapidly the repulsive force increases as the atoms approach. Grüneisen gives an equation for m in terms of atomic volume, atomic heat, thermal expansion, and compressibility. It is not necessary to reproduce the expression here. Grüneisen finds that *m* varies for normal metals from 7.5 for iron to 15.6 for gold. If we apply Grüneisen's formula to bismuth and antimony we find that *m* for the latter is 3.8, and for the former 4.5, both very much less than for normal metals. Our point of view is confirmed, therefore, that the repulsive forces in bismuth and antimony increase more slowly than normal as the distance between atomic centers is decreased, and therefore we have the possibility of an increasing amplitude with increasing pressure and so an increasing resistance.

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[1] E. Grüneisen, Ann. Phys., 39, 257–306, 1912.

5. The fact that the temperature coefficients of solid bismuth and antimony are normal is quite in accord with our view, for with rising temperature we have seen that the effect of temperature on increasing amplitude quite overshadows any pure volume effect, so that we would expect the temperature coefficients of all metals to be nearly the same, irrespective of the behavior of the pressure coefficient.

6. The fact that the pressure coefficient of liquid bismuth is normal in being negative is significant. It indicates that the positive coefficient of the solid is in some way connected with the crystalline structure. The picture which we have given of the mechanism of the positive coefficient of the solid has this property, for we have ascribed the increasing amplitude with increasing pressure in the solid to the fact that the atoms are held in fixed orientations with respect to each other, and that in this particular orientation the forces are abnormal in character. Such a fixity of orientation is possible only in the crystal. In the liquid there is no definite relation of orientation, the localities of abnormal force play a relatively unimportant part, and the liquid behaves normally. Not only is the pressure coefficient of liquid bismuth normal, but the temperature coefficient is also normal for a liquid, and is less than that of the solid, and less than the reciprocal of the absolute temperature.

7. It was shown in the previous paper that the abnormality of force between the atoms of bismuth made possible a crystal with abnormally large volume, and hence a crystal which expands on freezing. By analogy I was prepared for a positive pressure coefficient in gallium, which is also abnormal in expanding on freezing. The facts are the opposite, however, and the pressure coefficient of gallium is negative, as is normal. Our previous argument was entirely qualitative, however, and merely indicated the tendency toward an increasing amplitude with increasing pressure without setting up a criterion as to whether the tendency might be strong enough to counteract the normal tendency in the opposite direction or not. In the absence of a definite criterion therefore, we have only the right to expect the same tendency in gallium without

actually being sure whether the tendency will be strong enough to make the coefficient positive. Now as a matter of fact, if the expected magnitude of the pressure coefficient is calculated by the method of the previous paper, it will be found that the discrepancy for gallium is greater than for any other substance with negative coefficient, the calculated value being about twice the observed value. This means that the pressure coefficient is much less than we would expect from the behavior of normal substances, so that here we evidently have the tendency which we are looking for toward a positive coefficient.

In other respects the behavior of gallium is as we would expect. The temperature coefficient of the solid is normal, and the temperature coefficient of the liquid is also normal in being less than that of the solid. Furthermore, the pressure coefficient of the liquid is normal, and is greater numerically than that of the solid, as we would expect, because the tendency to abnormality in the solid is due to a particular orientation of the atoms, and this disappears in the liquid.

8. The behavior of lithium is of a type not shown by any other substance yet known. Its pressure coefficient is positive in both solid and liquid, and the coefficient of the liquid is greater than that of the solid. The temperature coefficient behaves normally. Furthermore, the melting of lithium is normal, in that the liquid has a larger volume than the solid. The data seem not to have been previously determined. In the paper on new resistance data will be found the melting data for lithium which I determined for this particular purpose. I have followed the melting curve up to 8,000 kg., and it seems normal in every respect. Our picture of the mechanism of conduction in lithium must probably, therefore, be different from that of bismuth or antimony or gallium. It is of course possible that the explanation of the abnormal coefficient follows on the same lines as for other metals, namely an increasing amplitude of atomic vibration with increasing pressure. If this explanation is adopted, the abnormality of the atom must not be thought of as confined to certain localities which function only in the crystalline phase, but the abnormality must be one of the atom as a whole, for the liquid as well as the solid is abnormal. This is not an impossible view, for if m be calculated for lithium by Grüneisen's formula, the value 5 will be found, which is low compared with most other metals, but is not low compared with bismuth and antimony. But now the question arises as to the interpretation to be put on the value of m. m itself merely is a measure of the rapidity with which the repulsive force increases as the atom is approached. A low value may be due either to an abnormality of a particular part of the atom, as we have supposed the case with bismuth

and antimony, or it may indicate that the atoms are separated by more than the usual distance, and the repulsive force is not important. We would expect the latter view to be more nearly correct for substances which are very compressible. This seems indicated by the fact that m is as small for the very compressible elements sodium and potassium as it is for lithium, and sodium and potassium have very large negative pressure coefficients of resistance. Now lithium is also one of the most compressible metals. It seems likely to me that the low value of m is more probably connected in some way with the high compressibility than with an abnormality which might result in an increasing amplitude of atomic vibration with increasing pressure.

Now there is a picture of conduction offered by the theories of Wien[8] and Lindemann[1] which gives an alternative explanation, which may quite probably be correct for this particular substance. Their picture is of electrons as well as atoms located on a space framework, and of the space framework of electrons moving bodily through the framework of the atoms when an external force is applied. The mean free path of the electrons in this motion is interrupted by collisions with the nuclei of the atoms. This picture means a positive pressure coefficient of resistance for most substances, for as pressure is increased the channels of passage of the electrons become more restricted. The reason for this is that the closing in of the channels because of the decreasing distance between atomic centers more than neutralizes the opening of the channels because of decreasing amplitude of atomic vibration. This is in spite of the fact that the relative change in atomic amplitude is much greater than the relative change in the distance between atomic centers, because the distance between atomic centers is absolutely much greater than the amplitude. It will be found as a matter of fact that if the relative magnitudes of atomic distance and amplitude and of the changes of atomic amplitude and distance with pressure be calculated for lithium, Wien and Lindemann's picture would lead us to expect a positive pressure coefficient. Evidently the positive pressure coefficient suggested by this picture remains true for the liquid state so long as the electrons continue to move between the atoms.

Apart from the argument from the pressure coefficient, this picture of Wien and Lindemann seems to have a particular probability of being correct for lithium because of its atomic structure. The structure is a simple nucleus consisting of a central positive charge and two electrons rotating about it, and a single other electron connected in some way with the nucleus at a relatively large distance. If the superficial electrons of the atom are ever to arrange themselves into a space lattice, it would seem that here is their chance.

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[1] F. A. Lindemann, Phil. Mag., 29, 127–140, 1915.

The fact that the pressure coefficient of liquid lithium is larger than that of the solid is simply explained as due to the greater compressibility of the liquid. The compressibility of the liquid has never been measured, but there is no case known in which the liquid does have a smaller compressibility than the solid, and furthermore the behavior of the melting curve, its direction of curvature, and the direction of the difference of volume between solid and liquid, makes it almost inevitable that the compressibility of the liquid be greater than that of the solid.

The fact that the pressure coefficient of resistance becomes greater at higher pressures is also intelligible from this point of view. The channels through which the electrons slip are the spaces which are left between atomic centers after the inpenetrable nucleus has been subtracted. Hence a given proportional decrease in the distance between atomic centers will mean a greater proportional decrease in the channel when the atoms are close together (high pressures) than when they are further apart.

9. On the other hand, the increase of the pressure coefficient of bismuth and antimony with increasing pressure is at least perfectly consistent with the picture presented of their conduction as performed by electrons passing through the atoms instead of between them. If reference is made to Fig. 1 of the previous theoretical paper it will be seen that it is quite possible that as the atomic centers approach the restoring force per unit displacement becomes weaker. In fact such a behavior is inevitable at some value of the volume, but we have no criterion for deciding whether this takes place at a volume greater or less than the volume at which the crystal is stable. Now such a weakening of the restoring force when the volume decreases means an amplitude of vibration becoming progressively larger at the higher pressures (smaller volumes), and hence a pressure coefficient increasing with increasing pressure.

10. The pressure coefficients of calcium and strontium were also found to be positive. It is difficult to decide from the data at hand whether this is more probably due to the lithium or the bismuth type of mechanism. The melting data are not known for either of these metals, so that we do not know whether the liquid or the solid has the greater volume. Neither are the data known for the effect of pressure on the resistance of the liquid. It is true that the value of the "*m*" of the atomic force calculated for calcium is somewhat lower than for the ordinary run of metals, being 6, but the compressibility of calcium is also high, and the *m*

for strontium is distinctly high, being 9.5. It is, of course, probable that the mechanisms of calcium and strontium are the same, because of their similar positions in the periodic table. I believe that at present the probability is in favor of the lithium rather than the bismuth type of conduction. The melting data would be a great help toward this decision.

11. There is an interesting relation between the temperature coefficients of solid and liquid lithium. At the melting point of lithium, 180°, the temperature coefficient of the solid is 0.00251 (obtained by a linear extrapolation of the resistances at 0° and 100° and therefore somewhat uncertain, but the best that we can do), and the coefficient of the liquid is 0.00150. The ratio of these two coefficients is 1.69, which is almost exactly the ratio of the specific resistance of the liquid to that of the solid at the melting temperature, for which I found the value 1.68.

This can be easily understood in the light of Wien and Lindemann's picture. The resistance of the liquid is higher than that of the solid because the regular channels between the atoms are broken up by the haphazard arrangement of the atoms in the liquid. Now this factor of random arrangement of the atoms in the liquid is one that will persist at all temperatures, producing a permanent difference of resistance between solid and liquid. This can be given mathematical expression. Write the equation

$$R_L = R_S + \Delta R,$$

where R_L is the resistance of the liquid, R_S the resistance of the solid, and ΔR the increment of resistance on passing from solid to liquid. Now differentiate with respect to the temperature

$$\frac{1}{R_L}\frac{dR_L}{dt} = \frac{R_S}{R_L}\cdot\frac{1}{R_S}\frac{dR_S}{dt} + \frac{\Delta R}{R_L}\cdot\frac{1}{\Delta R}\frac{d\Delta R}{dt}.$$

If the experimental relation

$$\frac{1}{R_L}\frac{dR_L}{dt} = \frac{R_S}{R_L}\cdot\frac{1}{R_S}\frac{dR_S}{dt}$$

is to hold, then we must have $d\Delta R/dt = 0$. That is, the increment of resistance when the solid passes to the liquid is independent of temperature, which is what our picture suggested.

12. The same relation between the temperature coefficient of the solid and the liquid holds for some other metals. Thus for sodium at the melting point at atmospheric pressure I found for the coefficient of the solid 0.00414, and for the liquid 0.00310. The ratio of these is 1.34, and I found experimentally the ratio of the resistance of the liquid to that of the solid to be 1.45. Mercury has the largest ratio observed for the

ratio of specific resistance of liquid to solid, and we would therefore expect an unusually small temperature coefficient for the liquid. It is in fact unusually small, being only 0.00090 at 0° C. If we assume that the coefficent of the solid is $1/\tau$, we find for its coefficient at the melting point (− 38.85°) 0.00409, and for that of the liquid at the same temperature 0.000934. The ratio of these is 4.38 and the ratio of the resistance of the liquid to the solid found by Onnes[1] is 4.22. On the other hand, the relation breaks down for solid and liquid potassium. I find that the temperature coefficient of the liquid is actually slightly greater than that of the solid at the melting point, which is abnormal, whereas the resistance of the liquid is 1.56 times greater than that of the solid. It might be mentioned that my figures for the temperature coefficients of solid and liquid do not agree with those of other observers. The values of Northrup[1] are most favorable to the hypothesis being urged here. He makes the coefficient of liquid potassium less than that of the solid, but the ratio of the two coefficients as found by him is only 1.27, whereas the ratio of specific resistances would demand 1.56.

The only other metals for which I have determined the coefficients, namely bismuth and gallium, cannot possibly satisfy the relation because the temperature coefficients of the solid are greater than those of the liquid, as is normal, but the resistance of the liquid is less than that of the solid. This is because of the abnormal volume relation, so that we are not surprised that the relation breaks down.

There are also some data of Northrup[3] for other metals. For gold his data give a ratio of the temperature coefficients at the melting point of 2.15, and he found for the ratio of the resistances 2.28.

Now with regard to the explanation of this relation between the temperature coefficients, our point of view would lead us to expect an invariable element, unaffected by changes of temperature, in the relative resistances of solid and liquid, and hence, according to the analysis, a ratio of the coefficients equal to the inverse ratio of the resistances. When the solid melts to the liquid the atoms lose their regular mutual arrangement, which becomes haphazard. In passing from atom to atom in the liquid the electrons cannot avail themselves of particularly favorable localities, but must use the average of the entire atom. This element will be a permanent difference between solid and liquid, unaffected by changes of temperature. However, the relations are so complicated that this can be only part of the picture, and in fact the relation does not always hold. There is a specific effect due to the "τ" term in our formula for resistance. This may to some extent be neutralized by the tendency of the "gaps" in the liquid condition to function in the second way, decreasing instead of increasing the resistance with increasing amplitude. Furthermore, it is not certain to what extent our deduction of the variation of amplitude with temperature and pressure is valid for the liquid. We assumed that the energy of atomic vibration was either potential or else kinetic energy of to and fro motion. Now in the liquid there is probably some rotational energy as well. The exact relation between the translational and the rotational energies in the liquid is not yet clear, so that we do not know how large an effect to expect on this account. It seems evident, however, that the direction of the effect will be to decrease somewhat the temperature coefficient of amplitude, because part at least of the energy will be rotational, and therefore the translational energy, and so the amplitude, cannot increase as rapidly with increasing temperature as if we had supposed all the energy to be translational.

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[1] K. Onnes, Kon. Akad. Wet. Proc., 4, 113–115, 1911.

[2] E. F. Northrup, Trans. Amer. Elec. Chem. Soc., 20, 185–204, 1911.

[3] E. F. Northrup, Jour. Fran. Inst., 177, 287–292, 1914.

The entire theoretical significance of this observation as to the ratio of the temperature coefficients of solid and liquid may therefore not yet be completely clear, but at any rate the observation itself is to be kept in mind. I am not aware that this relation has been previously noticed.

13. The considerations of the last section have an application to the temperature coefficient of resistance of the liquid at high temperatures. Write the formula for the resistance in the form

$$R_L = \text{const}\ \tau^{1/2}/l_L.$$

Now the free path, l_L, is inversely proportional to the chance, β, that the free flight of the electron will be terminated in passing from atom to atom. Let us analyze this chance of termination into two parts. One will depend on the haphazard orientation of the atoms with respect to each other, and will be independent of temperature, and the other will depend on the amplitude, and will, as before, be proportional to the amplitude, if we can neglect the pure volume effect, and the tendency of the gaps to function in the second way. Hence we may write

$$\beta = A_1 + A_2\alpha,$$

which gives

$$R_L = \text{const}\ \tau^{1/2}(A + \alpha).$$

Differentiate this with respect to the temperature, giving,

$$\frac{1}{R_L}\frac{dR_L}{d\tau} = \frac{1}{2t} + \frac{\alpha}{A + \alpha} \cdot \frac{1}{2\tau}.$$

Hence we see that at high temperatures, where α is large, the temperature coefficient again approaches $1/\tau$, although at lower temperatures it is less than $1/\tau$. These considerations apply only to those liquids for which the functioning of the gaps by decreasing resistance at large distances of separation may be neglected. Now there are liquids for which this last factor is certainly important, as zinc and cadmium, whose temperature coefficients immediately after melting are negative, but there are other liquids, as tin and lead, whose resistance in the liquid shows no peculiarity. The resistance of these liquids increases linearly with the temperature. In fact the linear increase of the resistance of tin is so striking that Northrup and Suydam[1] have proposed this as the basis of a resistance thermometer. Now it is evident that the temperature coefficient of any metal which increases linearly in resistance with temperature must eventually become equal to $1/\tau$, according to the above formula.

14. The very large difference of resistance between solid and liquid mercury is striking, particularly in view of the fact that the change of volume on melting is not larger than usual. This would suggest that the structure of the mercury atom is such that the element of haphazard orientation is particularly important. This element we saw has a tendency to constancy. Consider now the variation of resistance with pressure. I found experimentally that the pressure coefficient of the liquid is less numerically than that of the solid, a somewhat unexpected result. Write

$$R_L = R_S + \Delta R.$$

Differentiate with respect to the pressure

$$\frac{1}{R_L}\left(\frac{\partial R_L}{\partial p}\right)_\tau = \frac{R_S}{R_L}\cdot\frac{1}{R_S}\left(\frac{\partial R_S}{\partial p}\right)_\tau + \frac{\Delta R}{R_L}\cdot\frac{1}{\Delta R}\left(\frac{\partial \Delta R}{\partial p}\right)_\tau.$$

This equation shows at once that the experimental relation

$$\frac{1}{R_L}\left(\frac{\partial R_L}{\partial p}\right)_\tau < \frac{1}{R_S}\left(\frac{\partial R_S}{\partial p}\right)_\tau$$

demands that

$$\frac{1}{\Delta R}\left(\frac{\partial \Delta R}{\partial p}\right)_\tau < \frac{1}{R_S}\left(\frac{\partial R_S}{\partial p}\right)_\tau,$$

which is what our picture led us to expect. Notice that we have disregarded the pure volume effect, so that these considerations would not be expected to apply to a highly compressible metal. Mercury, however, is relatively incompressible, and the considerations have a certain force.

15. A similar consideration applies to the pressure coefficient of alloys. The pressure coefficient of resistance of the alloys of the relatively in-

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[1] E. F. Northrup and V. A. Suydam, Jour. Fran. Inst., 175, 153–161, 1913.

compressible metals is usually less than that of the components. This is to be ascribed to a relative insensitiveness to pressure of the feature which is responsible for the increased resistance of the alloy. This feature is the failure of the exact register of the different kinds of atoms, and this feature will be insensitive to pressure unless the atoms themselves are highly deformable. The data for alloys are very restricted, and it is not known how the alloys of such very compressible metals as sodium and potassium would behave under pressure.

16. The same considerations also show why the ratio of resistance of solid to liquid is relatively constant along the melting curve. For the six substances which I measured this ratio suffered relatively little change, although the accuracy of the measurement was not as great as desirable in some cases. This constancy is to be ascribed to the fact that the effect of haphazard orientation in the liquid as opposed to the regular arrangement in the crystal is an intrinsic difference between solid and liquid, and is not affected by temperature and pressure changes.

17. In my previous paper some significance was attached to the fact that the temperature coefficient of liquid mercury at constant volume is negative. This was ascribed to the second manner of functioning of the gaps at the increased volume of the liquid. It was suggested that such might be found to be the case for all liquid metals. At the same time there was no necessity in the suggestion, and there was no criterion which could show whether the second manner of functioning of the gaps would be more important than the normal method for other metals or not. It appears that this is not the case for the new metals. The computation could be made only for sodium, potassium, and bismuth. The values of the thermal expansion are not known for liquid gallium. For lithium, because of the abnormal pressure coefficient of the liquid, the temperature coefficient of the liquid at constant volume is even larger than at constant pressure, and is of course positive. The computation for the three metals above is somewhat uncertain because the compressibility and thermal expansion of the liquids are not known with any great accuracy. A discussion will be found in the new American Academy paper. I find for sodium that the coefficient at constant pressure is 0.00325 against 0.00170 at constant volume; for potassium the respective coefficients are 0.0044 and 0.0025, and for bismuth they are 0.000475 and 0.000015. The uncertainty in the fundamental data is not so great but that there can be no doubt that the coefficients at constant volume of both liquid sodium and potassium are positive, but there may be considerable question in the case of bismuth.

Sodium and potassium are among the most compressible and expansible of the elements, and it is perhaps not surprising that for them the amplitude continues to function in the normal manner. The example of bismuth makes it not unlikely, however, that the coefficient at constant volume of the ordinary liquid metals may be negative, as it is for mercury, instead of positive. It is in any event significant that the difference between the coefficients at constant volume and constant pressure of all the liquid metals is greater than the difference for the solid, so that the tendency of the gap to function in the second way is manifest.

The second manner of functioning of the gap in liquid mercury may at first seem inconsistent with the relative constancy of the gap under pressure, which we invoked to explain the pressure coefficient of the liquid being less than that of the solid. But it is to be remembered that a change of temperature is always more effective than a change of pressure (compare the magnitudes of the compressibility and the dilatation) so that the two facts are not inconsistent. It is also to be kept in mind that the difference of compressibility between solid and liquid mercury is abnormally small.

18. The behavior of the alkali metals sodium and potassium is of interest because of the unusual magnitude of the effect, and demands special discussion.

In the first place I may mention in the interest of candidness that I had anticipated a possible positive pressure coefficient for potassium on the basis of the similarity of the structure of its atom to that of lithium. According to Langmuir's picture,[1] lithium consists of a positive kernel surrounded by one shell, with a single electron outside that shell, whereas potassium consists of a positive kernel, two surrounding shells, and a single electron outside the two shells. This similarity would seem to give some justification for the expectation of a positive pressure effect. However, the facts are the exact reverse. Not only is the pressure coefficient of potassium negative, as is normal, but it is larger than for any other metal as yet measured. It would seem, therefore, that the greater size of the central portion of the atomic structure of the atom of potassium prevents the arrangement of the superficial electrons in a space lattice, or at least is not favorable to the ready mobility of such a lattice, if it exists. The mechanism of conduction in the case of potassium is probably normal, in that the electrons pass through the substance of the atoms. It must be said, however, that potassium is abnormal in several respects. My measurements make the temperature coefficient of the liquid at the freezing point greater than that of the solid, and the pressure effects are abnormal in that the coefficient of the liquid decreases with increasing temperature. It is therefore not impossible that part of the mechanism of conduction of potassium is abnormal.

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[1] I. Langmuir, Jour. Amer. Chem. Soc., 41, 868–934, 1919.

If the expected magnitude of the pressure coefficients of sodium and potassium is computed on the basis of the preceding paper from twice the pressure coefficient of amplitude, it will be found in both cases that the computed values are very much too small. The observed coefficient of sodium is 1.75 times the computed, and the observed value for potassium is 2.4 times that computed. According to our view, this means an unusually large volume effect in addition to the amplitude effect. Now not only are the compressibilities of sodium and potassium among the highest of the elements, but the change of atomic distance with pressure is an unusually large fraction of the change of amplitude, so that for this additional reason we would expect a large volume effect. If the ratio of the pressure coefficient of amplitude to the coefficient of linear compressibility be computed, by the use of formulas already given, the values 3.8 and 3.6 will be found for sodium and potassium respectively. This is unusually small, we have already seen that the run of values for the ordinary metals is from 6 to 9. Lithium is the only metal which is markedly lower, and for this the ratio is 0.95, but since its conduction mechanism is abnormal, we need not consider it further. It is, then, just for such metals as sodium and potassium that our picture would lead us to expect the largest discrepancy between the amplitude and the total effects, and it is just here that we find them.

Sodium and potassium are also unusual in the large decrease of the pressure coefficient with increasing pressure. Between atmospheric pressure and 12,000 kg. the instantaneous coefficient of sodium has decreased by a factor of 2.5, and that of potassium by 2.6. In the previous work the largest decrease was for lead, with a factor 1.38. The large decreases for sodium and potassium are evidently connected with the unusually high compressibilities. It is a universal result of experiment that those substances with a high compressibility show not only an unusually large absolute decrease of compressibility with rising pressure, but the relative decrease is also unusually high. The actual data have not been determined for sodium and potassium, but we have every reason to expect that the same will be found to hold for them also. This means that the volume effect becomes relatively less at high pressures, and that the pressure coefficient decreases by an unusually large amount.

In addition to the unusually large decrease of pressure coefficient with increasing pressure, there is an unusually large change of the coefficients with temperature. The pressure coefficient increases with rising temperature by an unusually large amount, whereas the metals of the previous paper showed relatively little change with temperature. The coefficients of sodium at atmospheric pressure increase by a factor of 1.18 between 0° and 80°. The corresponding comparison cannot be made for potassium, because it is not solid over a wide enough range, but a comparison of the results at higher pressures shows that the change for potassium is even more rapid than for sodium. Of course a mathematical consquence of a pressure coefficient changing rapidly with temperature is a temperature coefficient changing rapidly with pressure. This effect again can be explained on the basis of the volume effect. Sodium is unusually expansible at atmospheric pressure, and we would expect a temperature coefficient considerably greater than that computed on the basis of the amplitude effect alone, that is, a coefficient greater than $1/\tau$. And as a matter of fact, the average coefficient of sodium between 0° and 100° is 0.00475, larger than $1/\tau$ by an unusual amount. Now at higher pressures, although the actual measurements have not yet been made, it is exceedingly probable that the volume expansion will show an unusually large decrease, and that sodium will approach more nearly the behavior of the more staid metals. The behavior of the temperature coefficient is in entire accord with this expectation, at 12,000 kg. it has dropped from 0.00475 to 0.00408, a value still somewhat larger than $1/\tau$, but not any larger than for many of the other metals at atmospheric pressure.

These considerations would lead us to expect the temperature coefficient to become not less than $1/\tau$ for any pressure, no matter how high, as long as the atomic vibration continues to function in the normal way in decreasing the probability of an undisturbed flight of the electron. It is conceivable that at exceedingly high pressures the atom itself may become so much compressed that a moderate amount of temperature disturbance is not sufficient to interfere at all with the chance of passage of an electron from atom to atom. The kinetic energy of temperature agitation in such a solid would consist in a quivering of the nucleus within the outer structure of the atom, which is prevented from much superficial motion by the close packing of the adjacent atoms. This state of affairs demands an unusually small thermal expansion. If such a state of affairs should occur, we might expect the temperature coefficient of resistance to become less than $1/\tau$. Now this is actually what does occur with potassium. Its mean temperature coefficient between 25° and 60° decreases from 0.00454 at atmospheric pressure to 0.00184 at 12,000 kg., the final value thus being much less than $1/\tau$. This speculation is most attractive, but more weight cannot be attached to it until the dilation of potassium has been measured at high pressures. It may be said that if this effect is to exist anywhere, it has the best chance with potassium of the metals measured, since it is by far the most compressible. Potassium is a trifle more than twice as compressible as the nearest metal sodium. And in further support of this view it may be noticed that the temperature coefficient of potassium at 6,000 kg. has dropped only to 0.0034, and it is therefore still a little higher than $1/\tau$ ($\tau = 298°$ Abs.), so that the decrease of the temperature coefficient becomes increasingly rapid at higher pressures, a most unusual behavior for a pressure effect. In caution it may be said that the experimental accuracy in determining the variation of the coefficients of potassium was not so high as for some other substances. It will be most interesting to attempt the verification of this observation on cæsium, which is almost twice as compressible as potassium.

If this observation should be justified by the measurements of thermal expansion, it would be a further most important verification of the view that the free flight of the electrons is interferred with during the passage from one atom to another, and that the interference is not due to the vibration of the nucleus, for the temperature agitation of the nucleus must continue to exceedingly high pressures.

19. The new data cover the non-metallic elements carbon and phosphorus (black). It is probably too early to extend this theory to all non-metallic substances, but it is interesting that in two important particulars the behavior of carbon and black phosphorus is what we would expect. We have thought of the gaps between atoms as functioning in two ways. Normally in metals the electrons leap easily across the gaps, and their passage is made more difficult by temperature agitation. The gaps, when in this condition, have a low specific resistance. But when the atoms are separated by more than the usual amount the electrons usually find difficulty in leaping the gaps, which are thus of high specific resistance, but the passage may be made more easy by increasing violence of temperature agitation, which brings the atoms closer together during part of their vibration. If the gaps are on the average in this second condition, we expect a high specific resistance for the substance as a whole, and a negative temperature coefficient of resistance (at least at constant volume, and presumably at constant pressure). Now these are the facts for both carbon and black phosphorus. Their specific resistance is much higher than that of any metal, but their temperature coefficient is negative.

It is also natural to expect that in such a substance the effect of pressure will be to drive the gaps from the second condition to the first as the atoms are brought closer together. This means an unusually large decrease of resistance with pressure. This is the fact for black phosphorus. The pressure effect is larger than for any other substance, and is so large that at 12,000 kg. the resistance is only 3 per cent. of its value at atmospheric pressure. Even after this enormous decrease the resistance is still much higher than for metals. The specific resistance of black phosphorus normally is 1 ohm per cm. cube, and therefore at 12,000 kg. it is still 0.03. The temperature coefficient at 12,000 kg. is still negative, but has dropped numerically from 0.0058 to 0.0030, which is in the direction to be expected as the resistance approaches that of a metal in character. The pressure effect on black phosphorus is different in another respect from that of metals, in that the relative coefficient $1/R(\partial R/\partial p)_\tau$ is very nearly independent of pressure. We saw that for metals this coefficient decreases with increasing pressure, and we gave as the explanation the decreasing compressibility with increasing pressure. It is evident that the mechanism is different in the case of black phosphorus. A constant coefficient would mean that a constant increment of external pressure always drives the same fraction of the total number of gaps from the second into the first group. We would expect this condition to hold as long as the number of electrons in the second group is still large. That this number still is large, even under 12,000 kg., is shown by the fact that under 12,000 kg. the resistance is still very much higher than for metals.

Conclusion.

We have now considered a great many of the phenomena of the electrical resistance of metals, and found them all consistent with the view of the nature of electrical conduction previously advanced. The facts considered in this paper were nearly all not known when the view was first suggested and are different in character from any previously available, having largely to do with the pressure and temperature changes of resistance of both the liquid and the solid metal, particularly of the alkali metals, in which the effects are very large.

The view of conduction is that the free paths of the electrons are interfered with in jumping from atom to atom, but that throughout the interior of the atom there is no resistance to their motion. We have put this conception into quantitative form, partly on the basis of the classical theory. At ordinary temperatures we have assumed the equipartition velocity of the electrons, that their number is independent of the temperature, and that the free path varies with the amplitude in a way which may be computed. This gives Ohm's law, the correct temperature coefficient, the Wiedemann-Franz ratio, an important part of the pressure effect, vanishing resistance at 0° Abs., and avoids the specific heat difficulty. In addition there are certain features which can be discussed only qualitatively; there is a specific effect of changing volume of which we may be pretty sure of the sign, but which depends in an unknown way on atomic structure, and there is a second manner of functioning of the "gap" at large volumes. Within the restricted range open to quantitative discussion the facts are in accord with the theory, and throughout the much wider domain open to qualitative discussion I have not found a single fact which is inconsistent, and many which I believe demand this view uniquely.

The probability seems great that the view contains the most important elements operative in the phenomena of resistance and thermal conduction in metals. This theory does not exclude the presence of other factors which may be important for the thermo-electric and the Hall effects. In fact our theory has made it inevitable that there should be in the metal charged ions as well as the electrons, and there is no reason why these should not be important in certain phenomena, although because of their low velocity of translation they cannot be expected to play a large part in conduction.

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THE LUMINOSITY OF MERCURY VAPOR DISTILLED FROM THE ARC IN VACUO.

By Norman H. Ricker.

Synopsis.

Afterglow of Mercury Vapor from an Arc in Vacuum.—(1) The *effect of an electric field on the spectrum* was studied. It was found that charged grids decreased the luminosity without changing the color; but when, under certain conditions, the luminosity was increased, there was a change in the relative intensity of the lines. (2) The *velocity of the vapor was measured by a stroboscopic method.* The luminosity was decreased intermittently at a known frequency and the resulting "puffs" as they moved along the tube, were observed stroboscopically. From this velocity and the rate of distillation, the density of the vapor was computed. (3) *Decay of luminosity along the tube* was observed. (4) *Theoretical discussion* of these results leads to the conclusion that recombination of the positive and negative ions produced by the discharge, is the most probable cause of the afterglow.

I. Introduction.

It has been observed by a number of experimenters that, when an electrical discharge has been passed through a gas, the luminosity frequently persists for some time after the discharge has ceased. In many cases this luminosity has been shown to be due to a readjustment of chemical equilibrium in the gas, which has been upset by the discharge. This is very noticeably the case in the electrical discharges in oxygen, in hydrogen, and in nitrogen. Strutt[1] has extensively investigated the discharge in these gases and shown that the persistence of luminosity in oxygen is due to the chemical action of ozone, formed in the discharge, with traces of impurities present in the gas. The afterglow in nitrogen was shown to be due to the formation of atomic nitrogen which gives off light upon recombining to form molecular nitrogen, or in reaction with impurities in the gas. Also an active form of hydrogen seems to be formed by the discharge in hydrogen, which active modification, upon interacting with impurities in the gas, produces an afterglow.

However there are many other cases in which a chemical action could hardly be expected to explain the phenomenon. Of this type of afterglow, one of the most interesting examples is the case of mercury vapor.

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[1] Proc. Roy. Soc., Vol. 85 (1911), p. 219; Vol. 85 (1911), p. 377; Vol. 85 (1911), p. 537; Vol. 86 (1912), p. 56; Vol. 86 (1912), p. 105; Vol. 86 (1912), p. 262; Vol. 86 (1912), p. 529; Vol. 87 (1912), p. 179; Vol. 86 (1912), p. 531.

It was observed by Stark and Reich[1] in 1903, that, if a side tube were connected to a mercury arc in vacuo, so that the mercury vapor in the arc could, by virtue of its vapor pressure, be blown out into a condensation chamber (Fig. 1), this vapor which distills away remains luminous during its journey away from the arc. If this side tube is not of too small a diameter, the vapor may, under proper conditions, be made to remain noticeably luminous for a considerable distance along its journey. Stark[2] showed that this vapor was intensely ionized, and that the intensity of its luminosity could be diminished by having the vapor move through an independent electric field. This vapor was later studied by Mathies[3] and by Child.[4] The fact that the luminosity may be quenched by means of an electric field indicates that the luminosity is intimately connected with the ions present in the vapor. Just how these ions are responsible for the luminosity is a more difficult question to answer.

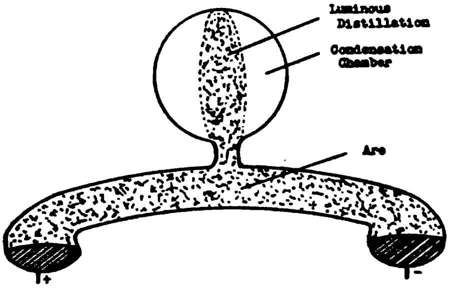

Fig. 1.

A series of very careful, important, and interesting experiments have been done by Strutt[5] on this vapor, as well as the vapors from other substances in the electric discharge. Strutt[6] has shown that, if the vapor is allowed to flow past the electrodes of an independent electric circuit (as in Figs. 2 and 3), the luminosity of the vapor stops abruptly at the cathode, regardless of whether the anode or the cathode is upstream. That is to say, the luminosity is unaffected upon passing the anode, but is quenched upon passing the cathode. Moreover the current in this independent circuit depends only upon the position of the cathode—regardless of the disposition of the anode. Strutt reasons in the following manner. Since the luminosity stops abruptly at the cathode and does not reappear after the vapor has passed out of the electric field, the quenching of the luminosity must be due to the removal of the positive ions at the cathode. An equal number of negative ions must be removed at the anode or else the independent electrical circuit would continually charge up. The removal of these negative ions at the anode (Fig. 3) does not apparently affect the luminosity. Now, if the light were produced by the recombination of positive and negative ions, the rate of production of light would be proportional to the rate of recombination of ions, which, in turn, is proportional to the product of the numbers of positive and negative ions per unit volume of the vapor. Thus the removal of either positive or negative ions should stop the luminosity. Since this is not observed, Strutt comes to the conclusion that the positive ions themselves emit the light as positive ions—the radiation being started while the ions are in the arc. This necessitates that the emission last during the time the vapor takes in moving from the arc to the place of observation, a time which Strutt[1] calculates to be of the order of one thousandth part of a second, which is not to be expected if the electrons in the atom radiate according to Lorentz's calculations. Lorentz[2] calculates that an electron radiating light of wave-length $6. \times 10^{-5}$ cm. should lose amplitude in the ratio $1/e$ in $4. \times 10^{-8}$ secs., and could not therefore carry its luminosity with it for any appreciable distance. However as Strutt[3] remarks, if the radiation is due to a uniformly revolving ring of equally spaced electrons, the system will radiate its energy very much more slowly than a single electron, and might therefore travel a considerable distance before extinction.[4] Strutt also suggests that perhaps the positive ion, while in the arc, is set into a sort of unstable state, such that it may later emit radiation by some "hang-fire" mechanism—the

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[1] Phys. Zeit., Vol. 4 (1903), p. 321.

[2] Phys. Zeit., Vol. 4 (1903), p. 440.
Ann. d. Phys., Vol. 14 (1904), p. 520.
Ann. d. Phys., Vol. 18 (1905), p. 231.

[3] Ann. d. Phys., Vol. 30 (1909), p. 633.
Verh. d. Deut. Phys. Ges., Vol. 12 (1910), p. 754.

[4] Phys. Rev., Vol. 22 (1906), p. 221.
Phil. Mag., Vol. 36 (1913), p. 906.
Phys. Rev., Vol. 4 (1914), p. 387.

[5] Proc. Roy. Soc., Vol. 88 (1912), p. 110; Vol. 90 (1914), p. 364; Vol. 91 (1914), p. 92.

[6] Proc. Roy. Soc., Vol. 91 (1914), p. 92.

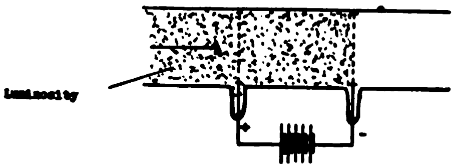

Fig. 2.

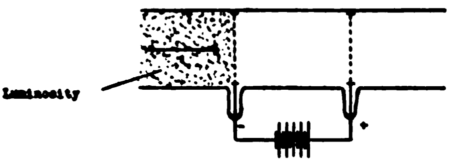

Fig. 3.

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[1] Loc. cit.

[2] Theory of Electrons (1909), p. 259.

[3] Loc. cit.

[4] Loc. cit.

actual radiation taking place over a very short interval of time, during which the ion does not travel an appreciable distance. That is to say, a positive ion leaves the arc with a certain amount of stored up energy, but does not radiate. Due to its critical state it may radiate later. After it has moved some distance, it emits its radiation in a sort of flash which indicates its position at the time. (The length of time before the light is emitted might be expected to be controlled by the laws of probability.) Thus the numerous ions emitting light after different intervals of time have elapsed, give an apparently continuous decay in luminous intensity along the column of distilled vapor. This might be expected on the Bohr theory of the atom. In any case, because of the electrical properties of the vapor, as cited above, Strutt holds firmly to the view that the light is emitted by the positive ions as positive ions and rejects the recombination theory.

Child[1] criticises these conclusions of Strutt by suggesting that, while the total number of negative ions (see Fig. 3) may be decreased by the removal of a fraction of them at the anode, the density of the ions in the space between the electrodes will be unchanged if the velocity of the ions in this space is decreased by the same fraction. The number of ions passing through unit area perpendicular to the stream per second is the product of the density of the ions by their velocity. If this number of ions is altered by a given factor, the density of the ions will be unchanged if their velocity is altered by this same factor. Moreover the direction of the electric field is such as to oppose the motion of the negative ions, and so decrease their velocity.

Child[2] describes an experiment in which the luminosity is diminished in the neighborhood of an electrode charged positive with respect to the arc circuit, but offers no explanation of the phenomenon. He also notices that, if the potential of this electrode is sufficient, the luminosity is increased, being now of a greenish color. Later, by means of an alternating current arc and a stroboscopic wheel, Child[3] finds that the luminosity persists for about a thousandth of a second after the current has ceased. Child[4] again criticises Strutt's work in a later paper. In another paper Strutt[5] presents evidence to the effect that the emitting centers are negatively charged particles, in contradiction to his previous conclusions.

In view of this difference in opinion as to the manner in which this luminosity is produced, it was thought advisable to investigate this luminosity further, with the view of deciding between several possible sources of the radiation.

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[1] Phil. Mag., Vol. 31 (1916), p. 139.

[2] PHYS. REV., Vol. 4 (1914), p. 387.

[3] PHYS. REV., Vol. 9 (1917), p. 1.

[4] Phil. Mag., Vol. 37 (1919), p. 61.

[5] Proc. Roy. Soc., Vol. 94 (1918), p. 88.

II. Experimental Arrangements.

In the investigation of this vapor, the first apparatus used is shown in Fig. 4. In this apparatus (made of glass) the arc was maintained between the mercury pool, *K*, and the cylindrical anode, *An*, made of nickel, and sealed into the glass by means of a platinum wire. The vapor from the arc passed through this anode and into the larger tube in which the four auxiliary electrodes *A*, *B*, *C*, *D*, made of a nickel wire grid of rather coarse mesh, were sealed. These electrodes were made by stretching fine nickel wires across a nickel ring 3 cms. in diameter. The wires were all parallel, and spaced about 2 mms. apart. The vapor condensed at the end of the tube at *K′*, and, by means of the connecting tube, flowed back to the cathode, *K*. A mercury reservoir was connected to this system, so that by varying the pressure of the air over the mercury, the level of the mercury could be altered. The wide tube, containing the auxiliary electrodes, was surrounded by a cylindrical electric heater to prevent the mercury condensing on the walls of the tube. A slit about one centimeter wide was cut along the side of this heater parallel to its axis, through which the luminosity of the vapor could be observed. The tube in which the arc was maintained was about 2 cms. internal diameter, and the arc was about 10 cms. long. The large tube was 3.7 cms. internal diameter and 23 cms. long. The heated portion of this

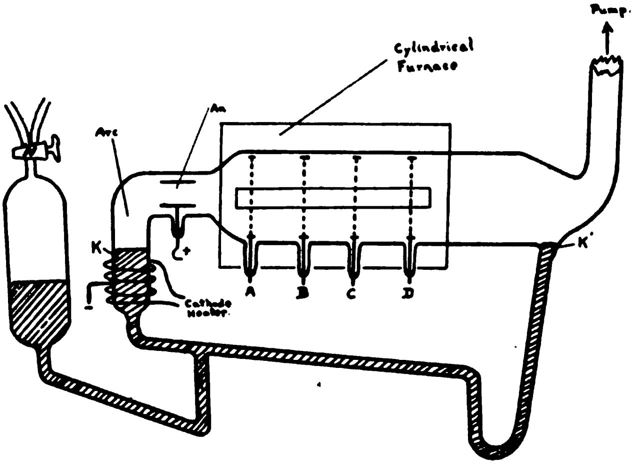

Fig. 4.

tube was 16 cms. long, and the auxiliary electrodes were spaced 3.5 cms. apart. A coil of nichrome wire was wound about the mercury cathode so that it could be heated, and the vaporization of the mercury thereby controlled. The apparatus was evacuated by means of a combination motor-driven oil pump and Langmuir condensation pump made by the General Electric Company. With this pump an extremely high vacuum could be obtained. The arc was started by heating the cathode until the mercury was vaporizing rapidly, and a discharge from an induction coil was then passed between the anode, *An*, and the electrode, *A*. The arc was maintained by a battery of Edison storage cells, giving about 80 volts, with a suitable resistance in series. An ammeter was inserted to measure the current through the arc, and a voltmeter was placed across the arc to measure the potential drop across the arc. The arc had a great tendency to form between the anode, *An*, and the mercury pool, K', but, by repeated trials, the arc could be made to "strike" in the proper manner. When running steadily with a current of three amperes, the drop in potential across the arc was about 12.9 volts.

When the arc is in operation, the vapor pressure in the arc is quite appreciable, and causes some of the vapor to be blown through the cylindrical anode, *An*, into the wider tube in which the auxiliary electrodes are placed. As the vapor flows down this tube, it can be clearly seen by its reddish luminosity, for the entire length of the tube. The luminosity is, under proper conditions, quite bright, and its reddish color stands out in sharp contrast to the light from the arc itself, which is of a bluish color. However, when examined with a small pocket spectroscope, it is seen that the spectra are the same, except for the relative intensities of the lines. If the cathode heater is stopped, so that the cathode may cool down, the luminosity of the distilled vapor becomes very faint. Thus it seems that it is necessary to supply more mercury vapor than the arc normally forms, if the luminosity is to be at all intense. Keeping the current through the arc constant, as the cathode is heated, so as to supply more vapor to the arc, the potential drop across the arc increases, and, if the cathode is very hot, so that the mercury vapor in the arc is very dense, the potential drop mounts to about 25 volts or slightly more, when the arc suddenly goes out. If the cathode heater is stopped, so that the vapor is distilling very slowly, the arc frequently goes out. Thus it seems that it is best to have the heater running for the steady maintenance of the arc as well as for producing a bright stream of distilled vapor. As well as the author is aware, this is the first time that, in such experiments, the cathode has been independently heated.

An inductance was placed in series with the arc to further steady it. It was observed that the auxiliary electrodes, *A*, *B*, *C*, *D*, tend to charge up to a negative potential of about a volt with respect to the anode, *An*, which was connected to earth.

The effect was tried of applying various potentials to the auxiliary electrodes.

III. The Positive Effect.

If any one of the electrodes is made positive, and this positive potential slowly increased, while all other electrodes are left insulated so as to take up what potential they may, the following effects are observed. First there is a quenching of the luminosity near the electrode on the upstream side, which quenched region broadens, reaching further and further upstream until the luminosity in almost the entire region between the electrode and the arc's anode, *An*, has been quenched. While this has been going on, a quenching of the luminosity near the electrode on the downstream side is noticed, and, as the potential is increased, this region of quenching reaches further and further downstream. Thus there is a quenching of the luminosity in the neighborhood of the electrode and on both sides of it. All of these effects have occurred at potentials up to two volts. At two volts the luminosity in the region between the electrode and the anode, *An*, has become too faint to observe. It may be mentioned here that the boundary of the quenched region is not at all sharp, but is a gradual shading off. If the potential is further increased a bluish glow starts on the upstream side of the electrode, and reaches further and further upstream as the potential is increased, becoming brighter all the while. Also, during this process, the bluish luminosity passes through the electrode, and, as the potential is increased, reaches further and further downstream. The important thing to notice is that the reddish luminosity is diminished on both sides of the electrode, but is not permanently quenched, for it reappears further down the tube. Moreover the bluish luminosity which in all probability is due to ionization by collision, occurs on both sides of the electrode, in its immediate neighborhood. Both of these effects may be easily explained in the following manner. As the potential of the electrode is increased, the electric field on the upstream side is increased, and the electrons in the vapor are accelerated. As the velocity of the electrons is increased the density is accordingly diminished and (assuming that the light is produced by the recombination of positive and negative ions) the luminosity is therefore decreased. As this increased velocity probably persists for some time after the electrons have left the field, the quenching on the downstream side is explained. As the electrons gradually lose this increased velocity the density increases, and the luminosity is restored. If, during this procedure, the velocity of the electrons relative to the vapor has reached a sufficient value to ionize, the production of the bluish glow is explained—the glow marking the region in which the velocity of the electrons relative to the vapor is equal to or greater than the ionizing velocity. This explanation of the quenching may be objected to on the grounds that the number of electrons going to the electrode is greatly increased by the electric field drawing a large number of ions out of the arc, which, as we shall seel ater, is most certainly the case. However the increase in velocity under the electric field is of higher order than the increase in the number of electrons, so that the explanation still holds. This will be discussed again later. This positive effect is, in all probability, the effect observed by Child[1] when the auxiliary electrode is made positive with some part of the arc circuit. The current to this electrode is rather large, as may be seen from the curve in Fig. 5, which shows the variation of the current to the electrode with the potential of the electrode. The current for five volts is seen to be over one ampere. As

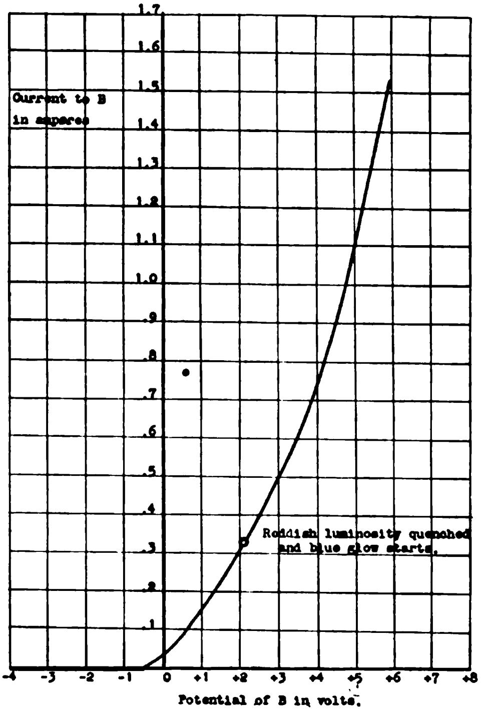

Fig. 5.

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[1] Phys. Rev., Vol. 4 (1914), p. 387.

will be seen later, this is very much larger than when the electrode is negative.

IV. The Negative Effect.

If any one of the auxiliary electrodes is given a negative potential, and this potential is slowly increased (negatively), while all other electrodes are left insulated so as to take up what potential they may, it is found that the luminosity on the upstream side is unaffected even with potentials as high as 300 volts, except within about half a millimeter of the electrode. However the luminosity on the downstream side is diminished by this procedure. If the density of the stream of distilled vapor is not too great, the luminosity on the downstream side may be, as well as can be judged, completely quenched. The potentials required for this effect vary greatly with the density of the vapor stream. If the density of the vapor stream is very small, so that the luminosity is feeble, small potentials of the order of one to three volts are sufficient to quench the luminosity. On the other hand, if the density of the vapor stream is large, potentials as high as 300 volts will not completely quench the luminosity. This seems to run parallel with the current to the electrode. In Fig. 6 is shown the way in which the current to the electrode varies with the potential of the electrode for negative potentials. From these curves it may be seen that, if the vapor density is small, a few volts seem to be sufficient to saturate the current. However, as the density of the vapor increases, it becomes more and more difficult

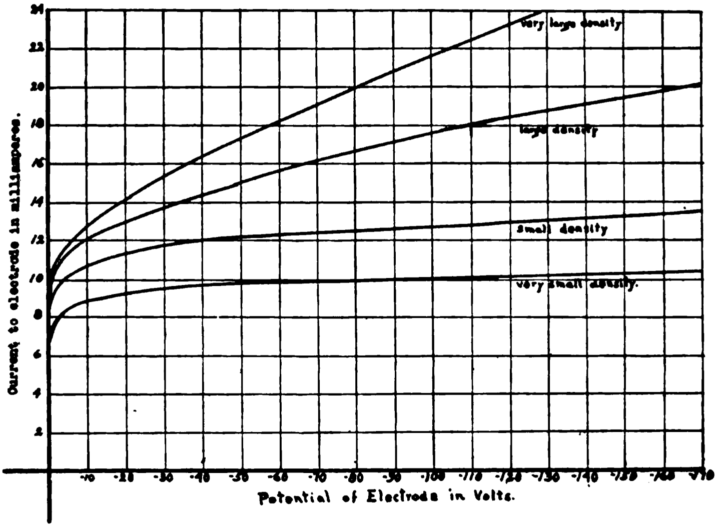

Fig. 6.

to obtain saturation, and the curve then resembles the curves obtained in the conductivity of flames. As the potential difference is increased the current continues to increase and the luminosity continues to decrease. However the current never becomes saturated and the luminosity is never completely quenched. This behavior of the current with respect to the potential is, in all probability, to be explained in the same manner as in the case of flames, by the large drop in potential in the neighborhood of the cathode. This is supported by the observations of Strutt[1] to the effect that almost the entire drop in potential occurs within a region very close to the cathode. It is to be expected that, as the density of the vapor increases, there are more ions swept out of the arc by the vapor stream, and as the density of the ions increases the effect should become more prominent. From Fig. 6 it will be noticed that the currents obtained in the negative effect range from ten milliamperes or lower to about twenty-five milliamperes. This is about one hundredth of the current obtained in the positive effect.

If the electrode, *B*, in Fig. 4, is maintained at a certain negative potential, with a milliammeter in series to measure the current, and the electrode, *A*, is then given a negative potential, the current to *B* is seen to decrease. This effect is much more marked when the density of the vapor is small. If the vapor density is large, the application of a negative potential to *A* does not appreciably affect the current to *B*. This is what we should expect from the above consideration of the currents to the electrodes, for with a dense vapor flowing past the electrode, *A*, only a very few positive ions are removed, and so the number of ions reaching the electrode, *B*, is practically the same as when *A* is uncharged. The important thing to notice in the case of the negative quenching is that, once the luminosity has been quenched, its quenching is permanent. That is to say, the luminosity does not reappear after the vapor has passed out of the electric field. This, together with the parallel relations of the quenching of the luminosity and the current to the electrode, seems to indicate that the quenching of the luminosity is dependent, in some manner, upon the removal of the positive ions at the negative electrode.

V. THE STRUTT ARRANGEMENT.

In Strutt's[2] experiments the distilled vapor was allowed to flow past the electrodes of an independent circuit, insulated from the arc circuit, as indicated in Figs. 2 and 3. These experiments of Strutt's have been discussed to some extent in the introduction. Strutt[1] found that, as the vapor flowed past the cathode of an auxiliary circuit, it had part of its luminosity removed; while, as it flowed past the anode, no change in luminosity was observed. Also the current in the auxiliary circuit is determined by the potential difference between the electrodes, and the position of the cathode. That is to say, regardless of the position of the anode, the current for a given cathode is the same. Also, if the vapor is made to flow past two such independent circuits in succession, increasing the current in the first circuit by increasing the potential difference between the electrodes of that circuit, decreases the current in the second circuit as is to be expected, as then more ions are removed by the first circuit, and so fewer reach the second. All of these experiments of Strutt's have been repeated by the present writer and his observations verified. Moreover the current in the auxiliary circuit seems to be exactly the same, for a given potential difference, as the current in the circuit formed by the same electrode as cathode, and the anode of the arc itself as anode. That is to say, the current is the same in the Strutt arrangement as in the negative effect, if the electrode used in the negative effect is the same electrode as the one used as cathode in the Strutt arrangement. Also, as well as can be judged, the quenching of the luminosity in the Strutt arrangement is the same as in the negative effect. It may be well to remark here that the presence of the mercury pool, K', at the end of the tube and in metallic connection with the cathode of the arc, does not seem to interfere with any of the electrical effects, either positive effect, negative effect, or the Strutt arrangement; for the connecting tube was later removed and all previously described work repeated. No differences in the results were noticed. From the shape of the curves in Fig. 6 it does not seem that an appreciable amount of ionization by collision goes on in connection with the negative effect. This is to be expected, as the large drop in potential occurs only in the immediate neighborhood of the negative electrode, and only the positive ions, which have small mobilities, are accelerated here. This also is the case with the Strutt arrangement. Occasionally, however, in working with the negative effect, ionization by collision does become appreciable, and then an arc forms. Whenever this happened the circuit was quickly broken to extinguish the arc and the circuit reformed. This does not happen often, however. Whenever the arc forms the color of the light is bluish, quite different from the reddish luminosity of the distilled vapor. An arc was never formed in working with the Strutt arrangement.

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[1] Proc. Roy. Soc., Vol. 91 (1914), p. 92.

[2] Loc. cit.

VI. DISCUSSION OF THE ELECTRICAL EFFECTS.

The positive effect has already been discussed in some detail and an explanation of the effect given. There are, however, a few points of difficulty. In the first place we assumed that the negative ions are accelerated as they approach the positive electrode. By this we mean merely that they have their velocities relative to the vapor increased. Of course the electrons are suffering collisions during their motion, but their velocities relative to the vapor will be the product of their mobilities into the strength of the electric field. In order that this average velocity of the electrons should increase it is necessary that the electric field near the positive electrode be stronger, which is not to be expected if a large drop in potential occurs at the negative electrode. Now it is found in the conductivity of flames that placing an easily ionized salt on the cathode increases the current enormously, and makes the potential gradient more nearly uniform along the flame. In the positive effect the cathode of the auxiliary circuit is the anode of the arc itself. This anode, being in the arc itself, is surrounded by intensely ionized vapor. This should produce the same effect as a salt on the cathode of a circuit through a flame. Thus it is reasonable to suppose that, in this case, the largest drop in potential should occur at the auxiliary electrode used in the positive effect. Now it may be seen by comparing currents in the positive and negative effects, that the currents involved in the positive effect are from twenty to one hundred times the currents involved in the negative effect. This increased conductivity must certainly be due to the cathode of the auxiliary circuit (anode of the arc) being surrounded by intensely ionized vapor. Now from later considerations it follows that there are 1.51×10^{16} molecules of mercury vapor per cubic centimeter in the distilled vapor. This corresponds to the number of molecules per cubic centimeter of any gas at 0° C. and at a pressure of .418 mm. of mercury. Now if we assume that, when the positive quenching occurs, we have an electric field intensity of one volt per centimeter, the value of X/p is 2.4. Townsend[1] gives a series of values for the velocities of electrons in air at low pressures as a function of X/p. For $X/p = 2$, $U = 1.75 \times 10^6$ centimeters per second. For $X/p = 5$, $U = 3 \times 10^6$ centimeters per second. Now the velocity of electrons through mercury vapor might be expected to be smaller than through air, because the mercury atom is larger than the atoms making up air. However the mercury molecule consists of a single atom, while the molecules in air consist of two atoms, and so there is probably not a great amount of difference between the sizes of the mercury and air molecules. It is then probably safe to assume that the velocities of electrons in mercury vapor are the same as in air: at least they are of the same order. If we interpolate between Townsend's values of the velocity we have $U = 2 \times 10^6$ centimeters per second, approximately. Initially the velocity of the electrons is the same as the velocity of the distilled vapor, which, from later considerations, is 3,740 centimeters per second. When the electric field has been established the velocity of the electrons is 2×10^6 centimeters per second. This means that the velocity has been increased 535 times. Now by a comparison of the currents obtained in the positive and negative effects it is seen that the current in the positive effect is about thirty times the current in the negative effect, when the luminosity has been quenched. Since the current in the negative effect is a measure of the total number of positive ions flowing past per second (when the vapor density is not too large), and therefore a measure of the number of electrons when no electric field is acting (since there must then be equal numbers of positive and negative ions), it follows that, by virtue of the electric field, the total number of electrons flowing has been increased thirty times, while the velocity has been increased 535 times. The density of the electrons has then been decreased about 18 times. As the number of positive ions is probably not altered, due to their small mobilities, the luminosity should, as a first approximation, be proportional to the density of the electrons. Therefore the intensity of the luminosity should be decreased about 18 times. This explains the decrease in luminosity on the upstream side of the electrode. In regard to the diminution of the luminosity on the downstream side there are several possibilities.

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[1] Electricity in Gases (1915). p. 124

In the first place, if the mean free paths of the electrons in the vapor are sufficiently large, the electrons which do not strike the electrode (which are equal in number to the positive ions which pass through the electrode) will continue with their high velocities for a certain distance before their increased velocities are lost by collisions with the vapor Jeans[1] gives a series of numerical values of the mean free paths of molecules in various gases. His table does not include mercury vapor, but, for the heavy gases, it may be seen that the values are not widely different. The value given for xenon is 3.5×10^{-6} centimeters at 0° C. and 760 mms. pressure. Now both xenon and mercury vapor are monatomic gases, and their atomic weights do not differ so very widely. We can therefore take the mean free path of mercury atoms in mercury vapor to be 3.5×10^{-6} centimeters at 0° C. and 760 mms. pressure (if such were possible). The density of the vapor in the experimental tube was 1/1,816 of the value under standard conditions. We must therefore increase the mean free path, λ, 1,816 times (since λ is inversely proportional to the density) so that we have $\lambda = .0545$ millimeters. This is for mercury atoms. For electrons this value must be increased by the factor $4\sqrt{2}$, which gives $\lambda = .309$ millimeters. This means that, in going one centimeter, the electron must make about 32 collisions. So, unless the electron could suffer a number of collisions without having its velocity and direction sensibly altered, we should not expect the quenching to extend this far. There are, however, several ways in which this region of quenching could be increased.

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[1] Dynamical Theory of Gases (1916), p. 341.

1. A considerable fraction of the electrons, of a given mean free path, will have actual free paths several times the mean free path. This should lengthen the region and produce a gradual shading off of the luminosity, as is actually observed.

2. When the velocity of an electron relative to the vapor is large it is difficult for recombination to occur, and, if the velocity is not too high, the collisions between the electrons and molecules of mercury vapor will be almost perfectly elastic. Therefore, while collisions will deflect the path of the electron in all possible directions, so that its velocity relative to the vapor is zero, still the numerical value (regardless of direction) of its velocity will decrease very slowly. By the time its velocity has dropped to a value at which recombination can take place, it has been carried along by the vapor as it flows along the tube. (This effect might also occur to some extent on the upstream side.)

3. If the electric field on the upstream side of the electrode increases near the electrode there must be an excess of negative charge in the vapor. The vapor is therefore acted upon by a force in the direction of the electrode—resulting in the vapor as a whole being accelerated, causing accordingly a decrease in density of both positive and negative ions and thereby a decrease in intensity of luminosity.

4. The absolute temperature in the experimental tube is probably twice the absolute temperature at 0° C., and this should cause a slight increase in the mean free path.

5. It is perhaps not unreasonable to suppose that, at high velocities, the electrons may collide with the atoms without being greatly deflected, for electrons of much higher velocities (β rays) most certainly do. This would have the same effect as lengthening the free paths of the electrons.

These five explanations are all in the right direction to explain the large region of downstream quenching. They probably all do help to increase this region—the extent to which they actually contribute is however debatable. At any rate, the downstream quenching is decidedly real, and the quenched region extends downstream from a few millimeters to two or three centimeters. It is difficult to explain the positive effect on the theory that the positive ions are the light emitters, as positive ions.

The negative effect is easily explained on either theory. If the light emitters are the positive ions themselves, the removal of the positive ions at the electrode should quench the luminosity. If the light is due to the recombination of positive and negative ions, the removal of the positive ions at the electrode will prevent them later recombining with negative ions, and so stop the luminosity.

The only effect that seems at all to stand in the way of the recombination theory is the quenching of the luminosity with the Strutt arrangement. Referring again to Figs. 2 and 3, it is easy enough to explain the case (Fig. 2) where the negative electrode is upstream on either theory, for the removal of the positive ions should quench the luminosity. In the case shown in Fig. 3, however, the two theories, on first consideration, predict different effects. On the theory that the light is given out by the positive ions we should expect that the luminosity should be unaffected until the vapor has reached the negative electrode, where the positive ions are removed and the luminosity accordingly quenched, as is observed. However, on the recombination theory, the removal of either positive or negative ions should diminish the luminosity. Since the number of electrons removed at the positive electrode must be equal to the number of positive ions removed at the negative electrode, we should expect that the luminosity should be diminished at the positive electrode when the electric field is established. This, however, is not observed. The conclusions drawn from this by Strutt have already been discussed, as well as Child's criticism of them. It appears to the present writer that Child's criticisms are just. Due to the high mobilities of the electrons at low pressures it is possible for their velocities to be very greatly altered by the electric field, even when it is weak. This will tend to prevent a decrease on the density of the electrons. We can show that the densities of the positive and negative ions must be very nearly equal in the following manner.

Suppose that all of the electrons were removed at the upstream anode. The postive ions will then continue on their path until they reach the cathode. The density of the positive ions in the space between the electrodes will probably then not vary greatly from point since recombination cannot occur. Let there be n ions per cubic centimeter in this region, and let e be the charge on each. Then if X be the electric field, and x the distance from the anode, measured toward the cathode, we will have

$$\frac{dX}{dx} = 4\pi ne,$$

and so

$$X = 4\pi nex + X_0$$

and

$$V = \int 4\pi nex \cdot dx + \int X_0 \cdot dx$$

or

$$V = 2\pi nex^2 + X_0 \cdot x + V_0,$$

where V is the potential difference between the anode and the point x. Let us take $V_0 = 0$, and let $x = a$ at the cathode. Also suppose that X_0 is negligible. Then

$$V = 2\pi nea^2.$$

Now neu is of the order of the maximum current involved in the experimental work, and is about 10 milliamperes across a cross section of about 10 square centimeters, or one milliampere across unit area. Therefore

$$ne = \frac{i}{u} = \frac{3 \times 10^6}{3{,}740} \text{ E.S.U.} = 8 \times 10^2 \text{ E.S.U.},$$

since the velocity, u, of the positive ions is the same as the velocity of the vapor, which, from later considerations, is shown to be about 3,740 centimeters per second. Since $a^2 = 10$, this gives

$$V = 5 \times 10^4 \text{ E.S.U.}$$

or 1.5×10^7 volts.

The actual potential difference between the electrodes is of the order of 50 volts, however. To give this value ne should be 2.7×10^{-3} E.S.U. or 3.3×10^{-6} of the value obtained on the assumption that no electrons entered the space between the electrodes. That is to say, the value 2.7×10^{-3} E.S.U. represents $(n_1 - n_2)e$, where n_1 is the number of positive ions per cubic centimeter, and n_2 is the number of electrons per cubic centimeter.

We then have

$$\frac{(n_1 - n_2)}{n_1} = 3.3 \times 10^{-6},$$

which shows that the number of positive ions per cubic centimeter and the number of electrons per cubic centimeter are very nearly equal. If n_1 and n_2 are nearly equal it follows that the velocities of the electrons must be decreased by the electric field. Now a very weak field is sufficient to make a great change in the velocities of the electrons, as stated before. This can be shown in the following manner. The initial velocity of the electrons is the velocity of the vapor, 3,740 centimeters per second. The velocity of the electrons relative to the vapor is the product of the mobility of the electrons into the electric field. As the field tends to retard the electrons, the actual velocity of the electrons will be $(u - kX)$ where k is the mobility. This velocity can thus be reduced to zero by a field of .0018 volts per centimeter, if we assume that the mobility of the electrons is 2×10^6 centimeters per second for a field of one volt per centimeter. The electrons are not brought completely to rest, and so the actual field must be less than this. This is in agreement with Strutt's observations that almost the entire drop in potential occurs within a region very close to the cathode. Thus, in spite of the large drop in potential at the cathode, the weak field between the electrodes would be sufficient to slow down the electrons so that their density is unchanged. With these considerations, then, the quenching with the Strutt arrangement can be explained on either theory.

In the case in which the cathode is upstream, the electrons will be accelerated by the electric field, and their density will accordingly decrease about one thousand times. Thus if only one thousandth of the positive ions reaching the cathode enter the region between the electrodes the densities of positive and negative ions in this region will be equal. The luminosity will then be reduced to one millionth of its previous value.

The conclusions to be drawn from the electrical effects are these. The negative effect and the quenching of the luminosity with the Strutt arrangement can be explained on either theory. The positive effect can be explained very easily on the recombination theory, but an explanation on the other theory does not immediately present itself.

VII. Relative Quenching of the Lines in the Visible Part of the Spectrum.

The following lines may easily be observed in the spectrum of the distilled vapor:

* yellow $\lambda = 5790.$ A.U.
* yellow $\lambda = 5769.$ A.U.
* green $\lambda = 5461.$ A.U.
* violet $\lambda = 4358.$ A.U.
violet $\lambda = 4078.$ A.U.
violet $\lambda = 4047.$ A.U.

The lines marked with an asterisk are the most prominent, and are accordingly the ones upon which observations were made. In addition to these are two rather faint orange lines and a somewhat faint greenish-blue line.

These lines are the same lines as occur in the spectrum of the mercury arc itself, the only difference in the spectra being in the relative intensities of the lines.

The question that immediately arises is whether all lines are quenched in the same ratio, or whether some lines are more easily quenched than others. To be sure the color of the luminosity, as observed by the eye, does not seem to change when the luminosity is diminished by the electric field. Strutt also is of the opinion that the lines are quenched in the same ratio, but no quantitative measurements to determine this point were made by him. The present writer has measured the relative intensities of the four most prominent lines before quenching to their intensities after the application of the electric field, by means of a Nutting polarization photometer in connection with a Hilger wave-length spectroscope. The conclusions reached in these experiments were that all lines are quenched in the same ratio; or, if not, the differences are very small.

VIII. Intensities of the Visible Lines in the Spectrum of the Bluish Glow as Compared with Those in the Luminosity of the Distilled Vapor.

When observed with a spectroscope the light from the bluish glow shows the same lines as the reddish luminosity of the distilled vapor. However, since there is a marked difference in the color of the two types of luminosity as viewed by the eye, one would expect that this difference is due to a difference in the relative intensities of the lines in the two cases. These relative intensities have been measured by means of the Nutting polarization photometer in connection with a Hilger wave-length spectroscope. The results of these experiments showed that, when the yellow line $\lambda = 5{,}790$ was of the same intensity in the two cases, the yellow line $\lambda = 5{,}769$ was about six per cent. more intense in the blue glow than in the reddish luminosity, the green line $\lambda = 5{,}461$ was about 76 per cent. more intense in the blue glow, and the violet line $\lambda = 4{,}358$ was about three times as intense in the blue glow as in the reddish luminosity of the distilled vapor. It seems then that the process producing the blue glow is different from the process concerned in the production of the persistent reddish luminosity.

IX. The Addition of Un-ionized Mercury Vapor to the Luminous Vapor.

The previously described experiments on the electrical quenching of the luminosity show that the luminosity is intimately connected with the positive ions present in the vapor, so that the removal of the positive ions stops the luminosity. This would indicate that the light is not produced by the influence of the intense light from the arc, for if this were the case the luminosity should occur beyond the electrodes used in the negative quenching effects. It was thought, however, that this point might well be investigated in a different manner. Accordingly the apparatus shown in Fig. 7 was used. In this apparatus, also made of glass, the arc was maintained between the mercury cathode and the cylindrical anode made of nickel. The cathode was provided with an independent electrical heater so that the volatilization of the mercury could be controlled. The vapor from the arc moves up the vertical tube leading to the pump. Connected with this tube on one side was a tube containing mercury which could be heated by an independent electric heater. The mercury vapor from this tube passes through a glass jet and is blown across the main tube where it is exposed to the direct action of the light from the arc itself. It then passes into the side tube at the left where it finally condenses. Grids of fine nickel wire forming the electrodes *A* and *B* were placed in this tube so that the vapor passing through them could be studied.

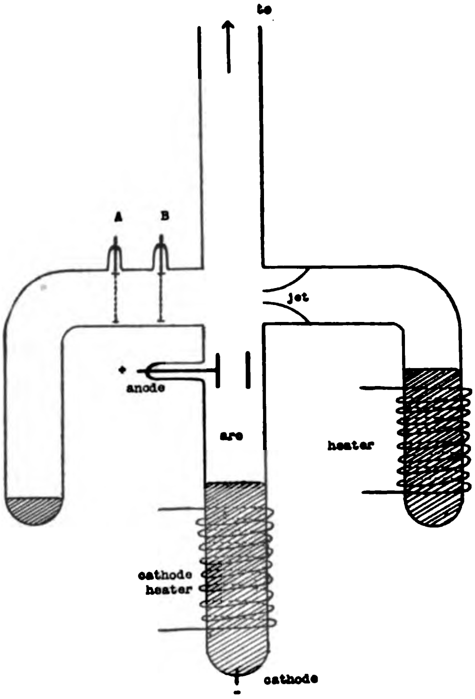

Fig. 7.

The arc was run with about 2.4 amperes with the heater only hot enough to prevent the arc going out. In this condition the luminosity in the vertical tube above the anode was too faint to observe. The part of the tube in which the arc was formed was wrapped with asbestos, as was the first experimental tube (Fig. 4), to prevent the brilliant light from the arc interfering with observations. When the mercury in the side tube was heated, the mercury vapor was forced through the jet and into the side tube to the left where it could be seen condensing. A very faint luminosity appeared in this tube in the region near the electrodes *A* and *B*. This luminosity was too faint for its color to be determined, but it was found that it could be quenched by establishing a difference in potential between the electrodes *A* and *B*. It is highly probable that this luminosity is due merely to the horizontal stream of mercury vapor carrying with it some of the luminous vapor which has distilled away from the arc. This vapor was too faint to observe when it was in the vertical tube, for there was considerable light from the arc itself which had been reflected from the walls of the tube. This light is probably much brighter than the faint luminosity of the distilled vapor, and so the latter is obscured. However as soon as this vapor has been carried into the side tube where it is fairly dark, its luminosity is more easily observed.

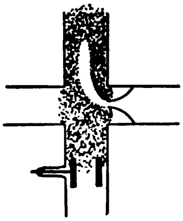
Fig. 8.

When the cathode of the arc was strongly heated, a very bright column of distilled vapor could be seen in the vertical tube. In this case, when the un-ionized mercury vapor was forced through the glass jet, the resulting stream of un-ionized vapor was deflected upwards and carried along by the current of luminous vapor. The stream of un-ionized vapor could be traced through the luminous vapor by its lack of luminosity. The effect is illustrated in the adjoining figure (Fig. 8). This seems to show that the intense light from the arc itself does not produce any great amount of luminosity in un-ionized mercury vapor.

In this connection the dependence of the intensity of the luminosity in the distilled vapor upon the rate of volatilization of the mercury at the cathode of the arc is worthy of consideration. Due to the more rapid volatilization of the mercury the potential drop across the arc increases and, in consequence, the energy expended in the arc increases. If this energy is used up in ionization we should expect the number of ions to be increased. However the potential drop across the arc is increased by only a few per cent. by the increased vaporization of the mercury, and as the intensity of the luminosity is increased many times, we should not expect that the corresponding increase in ionization should account for the increase in luminosity. The part played by this increased supply of mercury vapor is probably to place a larger number of neutral molecules between the positive and negative ions, and so diminish their rate of recombination. This permits them to travel a greater distance away from the arc before recombining, and thereby losing their ability to produce light. When very little vapor is distilling away from the arc there are not many neutral molecules between the ions, and so the ions recombine quickly. Accordingly they have become very few in number before they have left the arc for any appreciable distance. The change in the velocity by the different rate of volatilization and also the corresponding alteration in the densities of the ions contribute to the effect.

X. The Velocity of the Vapor.

In all of these experiments with the luminosity of the distilled vapor it is highly desirable to know the velocity with which the vapor flows along the tube. The velocity of this vapor was calculated by Stark[1] from the potential gradient set up in the vapor due to a magnetic field perpendicular to the direction of flow. The value given by Stark is 28,000 centimeters per second. Strutt[2] calls attention to the fact that the velocity of the vapor cannot exceed the molecular velocity of agitation at the temperature of the lamp. Taking this temperature to be 273° C. Strutt gives 50,000 centimeters per second for the velocity of molecular agitation and therefore sets this as an upper limit for the velocity of the vapor.

It was thought highly desirable to attempt a direct measurement of the velocity of the vapor. Accordingly the first apparatus (Fig. 4) was arranged as illustrated in Fig. 9. The anode of the arc was earthed and an audion oscillator arranged so as to cause the potential of one of the electrodes (B) to vary harmonically from zero to about 100 volts negative potential. In this way the electrode would alternately stop the luminosity and let it through. Accordingly "puffs" of luminosity were sent down the tube with the velocity of the vapor. The tube was then observed through the holes of a stroboscopic wheel. When the stroboscopic wheel was run at such a speed that the frequency of intermittance of vision was the same as the frequency of the audion oscillator the "puffs" of luminosity appeared to remain stationary so that the distances between them could be measured. If n is the frequency of the oscillator and λ the distance between similar parts of two successive puffs, then the velocity, U, of the vapor in the center of the tube is given by

$$U = n \cdot \lambda.$$

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[1] Phys. Zeit., Vol. 4 (1903), p. 440.

[2] Proc. Roy. Soc., Vol. 91 (1914), p. 92.

The puffs as experimentally observed appeared to be equally spaced, showing that the velocity of the vapor did not change rapidly as the vapor moved along the tube. However only three or four puffs could be seen at one time, and so this point deserves further consideration. The frequency of the oscillator was adjusted until λ was 2.0 centimeters.

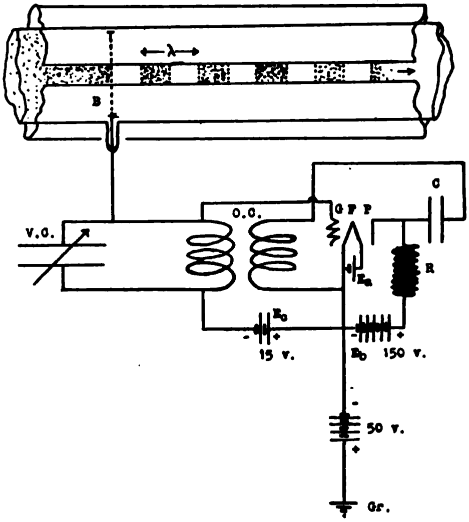

Fig. 9.

Potential of electrode *B* made to fluctuate between 0 and −100 volts by means of the audion oscillator. Luminosity observed through a stroboscopic wheel with frequency of intermittance same as frequency of oscillator. Luminous puffs then appear to stand still and can be measured.

The frequency of the oscillator was then found to be 1,870 vibrations per second. This gives U to be 3,740 centimeters per second.

This method of measuring the velocity could well be used in studying the way in which the velocity of the vapor varies as the vapor flows along the tube. A study might also be made by this method of the transition from viscous to turbulent flow. The advantage of this method is that the motion of the vapor can be traced visually.

In the determination of the velocity the current through the arc was 3 amperes and the cathode was just sufficiently hot to produce a good distillation of luminous vapor. This was the condition under which most of the conductivity and quenching effects were studied.

XI. The Density of the Vapor.

If we know the velocity of the vapor as it flows along the tube, and the cross-sectional area of the tube, we can calculate the density of the vapor by measuring the rate of distillation of the mercury. To measure the rate at which the mercury distills, the original apparatus (Fig. 4) was altered as shown in Fig. 10. The tube connecting the mercury pool K' with the cathode K, was cut and separate tubes forming barometric columns were run down from each to the vessels R and S filled with mercury. In the side of the vessel S was an overflow tube which led to the beaker, T. The arc was run at a current of three amperes with the cathode heater at the same temperature as when the previous experiments were done. With the apparatus in normal operation the mercury in the vessel R was kept at a fixed level by pouring mercury in as it distilled away from the cathode. As the mercury condensed at the end of the tube at K' it flowed into the vessel S, and the overflow was caught in the beaker, T. The mercury which overflowed in a known time was caught and weighed. The rate of distillation of the mercury could then be determined. Let r be the rate of distillation of the mercury in grams per second. Then if α is the cross-sectional area of the tube and V the average velocity of the vapor over the cross-section, the density, d, of the vapor is given by

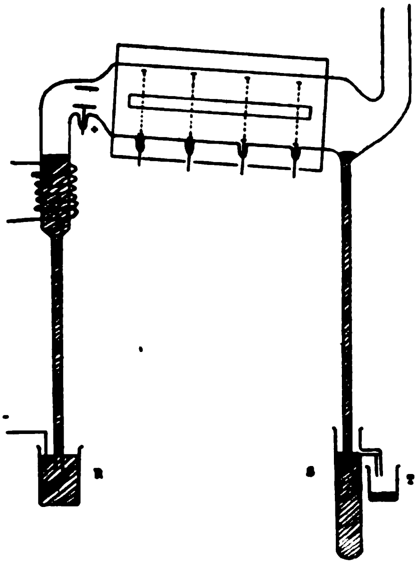

Fig. 10.

$$r = \alpha \cdot V \cdot d.$$

Now V will be $\frac{1}{2}U$ if the flow is not so fast as to be turbulent, or 1,870 centimeters per second. The diameter of the tube was 3.7 centimeters, which gives $\alpha = 10.76$ square centimeters. In one hour 356.91 grams of mercury distilled over—giving the rate, $r = .0991$ grams per second. Thus $d = 4.95 \times 10^{-6}$ grams per cubic centimeter. The mass of the mercury atom is 3.25×10^{-22} gram. There are then 1.51×10^{16} molecules of mercury per cubic centimeter of the vapor.

XII. The Nature of the Flow of the Vapor along the Tube.

It has been shown by Reynolds[1] that, when a fluid flows along a uniform circular tube, the flow will be governed by the laws of steady viscous flow provided the average velocity of the fluid across a cross-section does not exceed a certain critical velocity. If the average velocity exceeds this critical velocity, the flow will be turbulent. If W is this critical velocity, W is given by

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[1] Phil. Trans., Vol. 174: 3 (1883), p. 935.

$$W = \frac{1000\mu}{a\rho},$$

where μ is the viscosity; a, the radius of the tube; and ρ, the density of the fluid. Taking the viscosity of the mercury vapor at 273° centigrade to be 5×10^{-4}, the radius of the tube 1.85 centimeters, and the density of the vapor 4.95×10^{-6} grams per cubic centimeter, we have $W = 54{,}000$ centimeters per second, which is many times our experimentally determined velocity. Thus we are safe in assuming that the flow of the vapor is governed by the ordinary viscous equations. If a gas of viscosity μ flows along a uniform circular tube of radius a and length L, then, if p_1 is the pressure at one end and p_2 the pressure at the other end, the relation connecting these quantities is

$$\mu p v = \frac{(p_1^2 - p_2^2)}{8L}(a^2 - r^2),$$

where v is the velocity of the vapor at a point where the pressure is p—remembering that pv must be constant along a stream line to prevent the accumulation of gas at a point. r is the radial distance of the point from the axis of the tube. Since we know v at some point near the beginning of the tube, and from the density and temperature can calculate p, we know pv and can take p here to be p_1. We then have

$$p_2^2 = p_1^2 - \frac{8\mu p_1 v_1}{a^2} L$$

at the center of the tube. Now a density of 4.95×10^{-6} grams per cubic centimeter at a temperature of 273° centigrade corresponds to a pressure p of .836 millimeters of mercury, or 1.116×10^3 dynes per square centimeter. So

$$p_1^2 = 1.245 \times 10^6,$$

$$v_1 = 3740,$$

$$\mu = 5 \times 10^{-4},$$

$$a^2 = 3.42.$$

So

$$p_2^2 = 1.245 \times 10^6 - 4.88 \times 10^3 L.$$

When

$$L = 0 \text{ centimeters}, \qquad p_2 = 1.116 \times 10^3.$$

When

$$L = 40 \text{ centimeters}, \qquad p_2 = 1.026 \times 10^3,$$

$$\frac{p_1 - p_2}{p_1} = \frac{.09}{1.116} \qquad \text{or 8 per cent.}$$

Thus there is not a very great decrease in density in a distance of 40 centimeters along the tube. It must also be borne in mind that a decrease in density necessitates an increase in velocity. However the velocities with which we are dealing are of the order of 1/7th of the velocity of molecular agitation, and therefore of the maximum possible velocity. This approach toward the limiting velocity should tend to prevent the velocity increasing as much as out theory indicates. Attention might well be called to the fact that the relation between the pressure and the distance along the tube is parabolic, and so if the pressure falls by 8 per cent. in forty centimeters, the fall in pressure for the first part of the tube will be less than the proportional amount given by a uniform pressure gradient.

These calculations, together with the experimental observations that the luminous "puffs," in the velocity determinations, are equally spaced should permit us to take the velocity of the vapor along a streamline as constant, without any great error. We may then use the simpler equations for the case of an incompressible liquid.

XIII. The Theory of the Decay in the Intensity of the Luminosity as the Vapor Flows along the Tube.

Let us consider a number of possible ways in which the positive ions could be responsible for the luminosity of the distilled vapor.

1. In the first case let us assume the hypothesis that the positive ions do not emit the light during their entire journey from the arc, but that they are set into some sort of unstable state by the arc, such that when they emit light it is by some "hang-fire" method. That is to say, the ions travel away from the arc without emitting light. Suddenly, while they are still positive ions, they emit light—the time during which the light is emitted being so small that the ion does not travel an appreciable distance during emission. The ions are, however, continually disappearing by recombination. The light intensity is thus proportional to the number of positive ions per cubic centimeter at any point.

2. In the second case we will take as our hypothesis that the light is produced during recombination of the positive ions with electrons to form neutral molecules. Thus the light intensity is proportional to the rate of recombination, which in turn is proportional to the square of the number of positive ions per cubic centimeter (since there are equal numbers of positive and negative ions).

3. In the third case we will take as our hypothesis that the positive ions themselves emit the light—the emission occurring continually during their journey from the arc. The number of ions, however, is continually decreasing because of recombination. Thus the intensity of the light should be proportional to $ne^{-\lambda t}$, where n is the number of positive ions per cubic centimeter, and t is the time.

These various ways in which the positive ions could produce the luminosity should give rise to different laws of decay as the vapor flows along the tube. Accordingly it was hoped that, by comparing the rate at which the luminosity decayed experimentally with the rates predicted by these several hypotheses, the correct hypothesis could be determined. For the present we shall consider only cases (1) and (2). Case (3) will be discussed in the next chapter.

Let us consider a tube of uniform circular cross-section along which the vapor is flowing. Let us suppose also that the velocity, v, of the vapor is constant at all points in the tube. Let x represent the distance of any point in the vapor from some fixed plane placed perpendicular to the axis of the tube, the distance being measured in the direction of flow of the vapor. Let there be N_0 positive ions per cubic centimeter at $x = 0$. At any point at a distance x along the tube we will have

$$\frac{dn}{dt} = -\alpha n^2, \tag{1}$$

where n is the number of positive ions per cubic centimeter. But

$$\frac{dx}{dt} = v,$$

so

$$dn = -\alpha n^2 dt$$

becomes

$$dn = -\frac{\alpha}{v} n^2 dx.$$

Integrating

$$\int_{N_0}^{n} \frac{dn}{n^2} = -\frac{\alpha}{v}\int_0^x dx,$$

$$\frac{1}{N_0} - \frac{1}{n} = -\frac{\alpha}{v}x,$$

from which

$$n = \frac{1}{\frac{1}{N_0} + \frac{\alpha}{v}x}. \tag{2}$$

Case (1) then gives

$$I \propto \frac{1}{(A + Bx)}. \tag{3}$$

Case (2) then gives

$$I \propto \frac{1}{(A + Bx)^2}. \tag{4}$$

In the development of these equations we have assumed that the velocity of the vapor is uniform across a cross-section, while we have good reason to believe that the flow of the vapor is governed by the equations of steady viscous flow. Nevertheless the above equations will hold regarding the luminosity along a fine elementary tube of infinitesimal cross-sectional area with its axis along a streamline of the actual tube.

Let us now consider the vapor as flowing along the tube as governed by the equations of steady viscous flow. Let us also suppose that the velocity is independent of x, and that at $x = 0$ the number of positive ions per cubic centimeter is uniform across the cross-section of the tube and is equal to N_0. Then, if U is the velocity of the vapor along the axis of the tube, the velocity v at a radius r from the axis will be given by

$$v = U\frac{a^2 - r^2}{a^2}, \tag{5}$$

where a is the radius of the tube. The number of ions per cubic centimeter now becomes a function of both x and r.

$$n = \frac{1}{\dfrac{1}{N_0} + \dfrac{\alpha a^2}{U(a^2 - r^2)}x}, \tag{6}$$

which may be written as

$$n = \frac{A}{C - Dr^2} - \frac{Br^2}{C - Dr^2}, \tag{7}$$

where

$$A = UN_0a^2, \tag{8}$$

$$B = UN_0, \tag{9}$$

$$C = Ua^2 + N_0\alpha a^2x, \tag{10}$$

$$D = U. \tag{11}$$

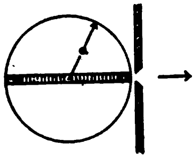

Fig. 11.

Now in making observations on the intensity of the luminosity in the tube, the light which is observed is the light from all points on a diameter of the tube. The total light observed may then be written in the form of an integral.

$$I = 2\int_0^a I_r dr, \tag{12}$$

where I_r is the intensity of the light from a point at a radial distance r from the axis of the tube. In the above, x is held constant. Let us now take case 1, where I_r is proportional to n. Except for a constant we may then write

$$I_1 = \int_0^a n\cdot dr$$

the subscript 1, referring to case 1.

$$I_1 = A\int_0^a \frac{dr}{C - Dr^2} - B\int_0^a \frac{r^2dr}{C - Dr^2}. \tag{13}$$

We may, at any time, drop any constant factor, since the unit in which I is measured is arbitrary.

$$I_1 = \frac{A}{D}\int \frac{dr}{\frac{C}{D} - r^2} - \frac{B}{D}\int \frac{r^2dr}{\frac{C}{D} - r^2}.$$

Let $C/D = b^2$ and for r substitute $b\cos\beta$ then

$$dr = -\,b\sin\beta\cdot d\beta.$$

We then have

$$I_1 = \frac{-\,A}{Db}\int_{r=0}^{r=a} \frac{d\beta}{\sin\beta} + \frac{Bb}{D}\int_{r=0}^{r=a} \frac{\cos^2\beta}{\sin\beta}\cdot d\beta.$$

Upon integrating and dividing out by constant multipliers we have

$$I_1 = 1 + \frac{b^2 - a^2}{2ab}\log\frac{b - a}{b + a}.$$

Substituting the value of b and calling $N_0\alpha/U = k$ this becomes

$$I_1 = 1 - \frac{kx}{2(1 + kx)^{1/2}}\log\frac{\sqrt{1 + kx} + 1}{\sqrt{1 + kx} - 1}. \tag{14}$$

Let us now take case 2, where I_r in (12) is proportional to n^2. Except for a constant factor we may then write

$$I_2 = \int_0^a n^2dr,$$

which by (7) becomes

$$I_2 = A^2\int_0^a \frac{dr}{(C - Dr^2)^2} - 2AB\int_0^a \frac{r^2dr}{(C - Dr^2)^2} + B^2\frac{r^4dr}{(C - Dr^2)^2}. \tag{15}$$

If we let $C/D = b^2$ and $r = b\cos\beta$ and integrate we obtain

$$I_2 = 1 + \frac{kx}{1 + kx} - \frac{1 + \frac{3}{4}kx}{(1 + kx)^{3/2}}kx\log\frac{\sqrt{1 + kx} + 1}{\sqrt{1 + kx} - 1}, \tag{16}$$

where $k = N_0\alpha/U$.

XIV. Experimental Verification of the Theory.

In order to measure experimentally the rate of decay in luminous intensity with the distance along the tube, the apparatus shown in Fig. 12 was made. This apparatus consisted of a glass tube 2.5 centimeters in diameter in which the arc was formed. The vapor from this arc distills upward through the anode and into a tube 3.7 centimeters inside diameter and 65 centimeters long. In the lower part of this tube is an auxiliary grid, *A*, for use in measuring the velocity of the stream. A long cylindrical electric furnace was placed about the greater part of this tube to prevent the mercury vapor condensing on the walls. There was a long slit along the side of the furnace, parallel with its axis, and about one centimeter wide, through which the luminosity was observed. The arc was "struck" by vaporizing the mercury at the cathode by means of the cathode heating coil, and starting the arc by means of an induction coil between the electrode *A* and the anode of the arc. The arc operated well with a current of three amperes with the cathode heater going. The potential drop across the arc varied from 13 volts to 20 volts, depending upon the rate of volatilization of the mercury at the cathode. The potential drop was generally about 15 volts.

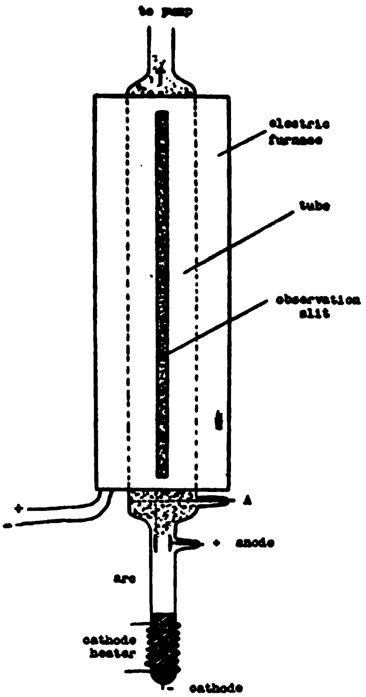

Fig. 12.

The luminous vapor could be traced for the entire length of the tube. The inside walls of the furnace were carefully blackened, and in this way reflected light from the walls was almost completely done away with. In fact when very little vapor was distilling, the upper part of the tube appeared quite dark. The mercury vapor condensed on the cold parts of the tube well above the furnace. The condensed drops of mercury rain down and fall back to the cathode. However the drops have to be fairly large before they can do this. If the extreme upper part of the tube is carefully observed, the small droplets may be seen to fall into the tube. However, before they have descended more than about a centimeter, the uprushing current of mercury vapor strikes them and sends them up again. Thus the mercury must descend in a few large drops instead of a large number of small ones. This is a good thing, for if there were a large number of small droplets continually raining down into the tube, the light from the arc, which they would reflect, would interfere with the observed luminosity of the distilled vapor. An occasional large drop does not matter. For a given mass of mercury, the area of the reflecting surface is much smaller when the mercury is in one large drop than when it is divided up into a large number of small ones.

The luminosity as a function of the distance along the tube was measured in the following manner. A Nutting polarization photometer made by Adam Hilger was turned on its side and mounted on the carriage of a cathetometer. Aluminium tubes 10 centimeters long were affixed to the apertures of the photometer as shown in Fig. 13. These tubes were carefully blackened on the inside with soot in order to minimize the effect of scattered light entering the photometer. The apertures on the photometer were 3.8 centimeters apart. The device was so arranged that the light from the lower aperture could be decreased by the crossed nicols and so matched with the light from the upper aperture. In order to measure the intensity of the luminosity along the vapor stream the ratio of intensities between these two points was observed. The photometer was then moved up so that the lower tube came into the position previously occupied by the upper tube. The ratio was again determined, and so on for about two thirds of the length of the tube. Ten such ratios were observed along the tube at intervals of 3.8 centimeters. The ends of the tube were avoided as it was feared that conditions might not be steady and uniform near the ends. The distance between the photometer tubes, 3.8 centimeters, was taken as unit distance, and at $x = 0$ the intensity of the light was taken as unity. So the intensity at any point could be calculated as the product of all previous ratios. Several series of readings were made of the luminosity as a whole. These readings agreed very well with one another. Other readings were made with the mercury green line alone by holding a mercury green line filter in front of the eyepiece. The decay of the mercury green line gave a curve almost identical with the decay of the luminosity as a whole. In Fig. 14, I_x shows the manner in which the luminosity decays as the vapor flows along the tube.

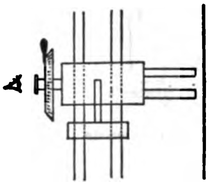

Fig. 13.

Now to compare these experimental curves with the theory of the previous chapter. It will be seen in the expressions for I_1 and I_2 that their graphs will both pass through the point $x = 0$, $I = 1$, and that the luminosity vanishes as x becomes infinite. There is, however one arbitrary constant in each of the expressions (14) and (16). This means that, by properly choosing the constants, we can make the curves pass through one arbitrary point on the experimental curve. If in (14) we take $k = 1.6$ and in (16) we take $k = .36$ the graphs of these equations will both pass through the point $x = 5$, $I = .076$ approximately. These curves are also shown in Fig. 14. It will be seen that neither of these two theoretical curves fit the experimental curve very well. However it will be noticed that the curve I_2 is not so far away from the experimental curve as is I_1. That is to say, if we are to decide between recombination producing the light, and the first case considered in the previous chapter, the evidence is decidedly in favor of the recombination theory.

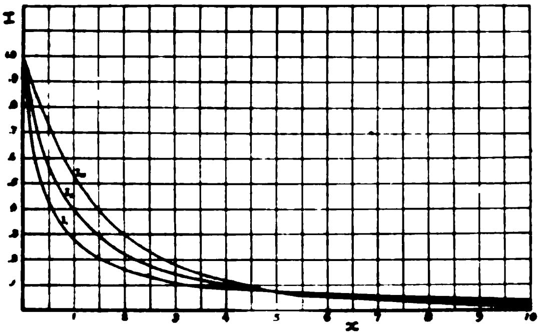

Fig. 14.

In the previous chapter it was stated that the third case would be considered later. This case assumes that the positive ions emit the light continually during their journey from the arc. If the light from each ion decays exponentially with respect to the time we should expect the luminosity at a point to be proportional to $ne^{-\lambda t}$. So

$$I \alpha \frac{e^{-\lambda x}}{1 + Ax}.$$

Equations of this form will involve two arbitrary constants, and, after an integration procedure as in cases 1 and 2 has been carried out, the two arbitrary constants will enable us to pass the curve through two arbitrary points of the experimental curve. Thus this theory might be made to agree with the experimental curve. However it must be borne in mind that as soon as a number of arbitrary constants enter we can make an equation fit almost any sort of curve. Thus the equation becomes little more than an empirical one. We cannot then give as much weight to such a curve fitting the experimental curve as when but a single arbitrary constant is involved.

In matching intensities with the Nutting photometer it was found that there was a color difference in the lights entering the two apertures. The light entering the lower aperture appeared to be yellower than that entering the upper aperture. Thereupon the following experiment was tried. The photometer was kept in a fixed position and the individual lines matched separately. This was done by holding filters before the eyepiece so that only the light under observation was transmitted. In this way the decay in the intensities of the separate lines over the distance between the photometer tubes could be measured. These ratios were measured for the yellow lines, the green line, and the violet line—$\lambda = 4{,}358$. The average of a large number of observations showed that, for equal intensities starting at the first aperture, for each unit of the violet luminosity reaching the second aperture, .873 units of the green luminosity, and .786 units of the yellow luminosity reach the second aperture. This shows that the violet line persists longer than the green line and that the green line persists longer than the yellow lines.

XV. CONCLUSION.

In the preceding chapters it has been shown that the observed phenomena with the luminosity and electrical conductivity of the distilled vapor can be explained on the assumption that the luminosity is produced during the recombination of positive and negative ions to form neutral molecules, and that the intensity of the luminosity is accordingly proportional to the rate of recombination of the ions. Some of the phenomena can be explained by the positive ions themselves emitting the light; and with one phenomenon (positive effect) it does not seem possible to explain the effects except on the recombination theory. On the whole the evidence is very much in favor of the recombination theory.

In conclusion the author wishes to express his most sincere thanks to Professor H. A. Wilson for his kindly interest in these experiments, and for his helpful advice during the course of the investigation.

PHYSICS LABORATORIES,
THE RICE INSTITUTE,
HOUSTON, TEXAS.

HENRY ANDREWS BUMSTEAD

Henry Andrews Bumstead, Professor of Physics and Director of the Sloane Physical Laboratory at Yale University, Chairman of the National Research Council, Past-president of the American Physical Society and a former member of the editorial board of the Physical Review, died on the train on the thirty first day of December, 1920, while returning from the Chicago meeting of the Physical Society and the American Association for the Advancement of Science. Professor Bumstead was in his fifty first year.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE CLEVELAND MEETING, NOVEMBER 26 AND 27, 1920.

THE 105th regular meeting of the American Physical Society was held in Cleveland, Ohio, on November 26 and 27, 1920. The morning session of November 26 was held in the Physical Laboratory of Western Reserve University and the sessions in the afternoon of November 26 and in the morning of November 27 were held in the Physical Laboratory of Case School of Applied Science. The presiding officers were President Ames and Vice-President Lyman. There were about 125 members and friends in attendance. There was a dinner at the University Club on the evening of November 26, attended by about sixty-five persons; an enjoyable feature of the dinner being the attendance of Dr. T. C. Mendenhall. Complimentary luncheons, given by Western Reserve University and Case School of Applied Science, were served on the days of meetings. The college laboratories were inspected by many members during the intermissions, and after adjournment on November 27 many members accepted the invitation to inspect the Nela Research Laboratories. The meeting was considered one of the most enjoyable of the smaller meetings.

Two Committees of the National Research Council took advantage of the occasion to hold meetings of their respective committees, the Committee on Acoustics and the Committee on Methods and Technique of Physical Research.

At the meeting of the Council on November 26, attended by six members, the following business was transacted:

The proposal of the Board of Editors to issue a general index to the PHYSICAL REVIEW covering the issues from 1893 to 1920 inclusive was approved.

Nomination of officers and preparation for the program of the Annual Meeting were made according to other announcements of the Secretary.

One hundred and sixteen persons were elected to *Associate Membership*, as follows: Paul Anderson, Jeanette Armstrong, Edward J. Baldes, A. F. Beal, Arthur L. Becker, George Becker, A. E. Bellis, Donald M. Bennett, S. L. Boothroyd, Frederick S. Brackett, Ben E. Brown, W. Byron Brown, Charles Brush, Jr., A. B. Bryan, Jay W. Buchta, Johanna Busse, Lois Carr, R. A. Castleman, Jr., Elliott W. Cheney, Paul P. Cioffi, J. R. Collins, Jean P. Cooley, Howard L. Coyne, J. W. Doolittle, Philip H. Dowling, Katherine M. Downey, James A. Duncan, John B. Esmaker, Louisa L. Eyre, Joseph L. Finkelstein, Peter E. Fossum, Willena Foster, James B. Friauf, Hugo Fricke, Paul H. Geiger, L. H. Germer, Victor Guillemin, Rowland V. Hagen, Lloyd B. Ham, A. Baird Hastings, Lucy J. Haynes, George C. Henny, Erik Hetle, W. J. Hitchcock, Jean Huddleston, C. Judson Humphreys, Charles W. Jarvis, Edward T. Johnson, A. L. Johnsrud, E. W. Kellogg, K. H. Kingdon, Elias Klein, E. J. Knapp, Chitting Kwei, C. E. Lane, Earl E. Libman, Ernest F. Ling, Laurice L. Lockrow, R. E. Lofton, William V. Lovell, Erwin F. Lowry, William A. Lynch, Hector J. MacLeod, Leopold D. Mannes, John Richard Martin, R. Earl Martin, Frederick A. May, Wm. Walter Merrymon, Melvin Mooney, Charles R. Moore, Clement Moran, Harold M. Mott-Smith, Waldemar Noll, Clarence A. Nordquist, Russell M. Otis, Samuel R. Parsons, George M. Pearsall, James O. Perrine, C. J. Pietenpol, H. H. Plaskett, George A. Pomeroy, James R. Randolph, Leo G. Raub, F. W. Reynolds, Martin E. Rice, Harold F. Richards, Donald E. Richmond, Paul Rood, P. M. Roope, Wilfred N. St. Peter, Otto Sandirk, Ralph A. Sawyer, Merit Scott, Marcus L. Sherin, Roger H. Sinden, John C. Slater, T. S. Sligh, Russell P. Smith, Sinclair Smith, Theodore A. Smits, B. L. Steele, I. Melville Stein, Homer G. Tasker, A. Hadley Taylor, Lucien B. Taylor, Harry C. Thompson, Jesse L. Van Horn, Robert O. Von Nardroff, John H. Van Vleck, Albert S. Walton, John S. Ward, Yu Ching Wen, R. B. Wilsey, Evelyn Wise, E. W. Woolard, Franklin B. Wright.

The following program of thirty-one papers was presented, five being read by title:

The Crystalline Structures of Sputtered Metallic Films. H. KAHLER.

The Effect of Adsorbed Air on the Aging of Thin Metal Films. LEWIS R. KOLLER.

Constancy of Hall Coefficient in Thin Silver Films. G. W. STEWART.

The Hall Effect and the Specific Resistance in Silver Films. G. R. WAIT.

The Hall Effect of the Nernst Effect in Magnetic Alloys. ALPHEUS W. SMITH.

Conductivity of Insulating Materials Near the Breakdown Voltage. J. E. SHRADER.

Magneto-Resistance Effects in Films of Bismuth. L. F. CURTISS.

The Magnetic Mechanical Analysis of Manganese Steel. SIR ROBERT HADFIELD, F.R.S. (London), and S. R. WILLIAMS and I. S. BOWEN.

Pulsating and Thermionic Discharges in Evacuated Tungsten Lamps. A. G. WORTHING.

Effect of Strong Electrostatic Fields on the Vaporization of Tungsten. A. G. WORTHING, and W. C. BAKER.

A Compensation Scheme for Electrostatic Measurements. W. F. G. SWANN. (Read by title.)

Note on Electromagnetic Induction and Relative Motion—A Rejoinder. W. F. G. SWANN. (Read by title.)

The Electrostatic Charges of the Earth and Sun and Their Relation to Terrestrial Magnetism. FERNANDO SANFORD.

The Spectro-Photoelectric Sensitivity of Proustite. W. W. COBLENTZ. (Read by title.)

Color Filters for Photographic Uses. CHARLES D. HODGMAN.

Comparison Between the Fundamental Equations for Ponderomotive Force for Point Charges Due to Larmor-Lorentz and to Mega Nad Saha. ALBERT C. CREHORE.

The Newtonian Laws of Gravitation Deduced from the Saha Electromagnetic Theory Applied to the Copernican Atom. ALBERT C. CREHORE.

On the Propagation of the Sound Wave from the Muzzle of a Large Gun. DAYTON C. MILLER.

A New Tone Generator. C. W. HEWLETT.

A Method of Studying Sound Waves by Means of a Synchronous Commutator. L. O. GRONDAHL.

On the Relative Positions of Lines in X-Ray Spectra. WILLIAM DUANE and R. A. PATTERSON.

X-Ray Analysis of Three Series of Alloys. MARY R. ANDREWS.

The Relation Between the Emissive Power of a Metal and Its Electrical Resistivity. C. DAVISSON and J. R. WEEKS.

Mass-Absorption Coefficients as a Function of Wave Length Above and Below the K X-Ray Limit of the Absorber. F. K. RICHTMYER.

Graphical Determination of Hexagonal and Tetragonal Crystal Structure from X-Ray Data. ALBERT W. HULL and WHEELER P. DAVEY.

The Absorption and Scattering Coefficients for Homogeneous X-Rays in Several Elements of Low Atomic Weight. C. W. HEWLETT.

The Current-Temperature Relation for Different Pyrometer Filaments. W. E. FORSYTHE.

On the Absorption Spectrum of HCl. WALTER F. COLBY and CHALES F. MEYER.

Flame Excitation of Luminescence. E. L. NICHOLS and D. T. WILBER.

A Continuous Spectrum from Mercury Vapor. C. D. CHILD. (Read by title.)

Note on Sheet Striae as Thin Films in Optical Glass. L. E. DODD. (Read by title.)

The abstracts of all the papers on the above program are given in the following pages.

DAYTON C. MILLER,
Secretary.

THE CRYSTALLINE STRUCTURE OF SPUTTERED METALLIC FILMS.

BY H. KAHLER.

USING the X-ray method of crystal analysis cathodically sputtered films[1] of bismuth, silver, gold, cobalt and platinum, have been examined. *All have been found to be crystalline with a random orientation of crystal axes.*

---

[1] Prepared by L. F. Curtiss.

The lattice structures and dimensions of these films have been determined and in every case have been found to be identical with those of the unsputtered metals. The cases of cobalt, and selenium present complications due to their allotropic forms. The nature of these complications is now being studied.

A comparison is also being made between the crystalline structures of sputtered and evaporated films of silver.

Cornell University,
November, 1920.

The Effect of Adsorbed Air on the Ageing of Thin Metal Films.

By Lewis R. Koller.

THIN metal films deposited by cathode sputtering in vacuo undergo a decrease in resistance with time. This process has been called ageing. In the course of some senior research on ageing in this laboratory it was noted that the films apparently increased in resistance when air was admitted to the sputtering jar and again decreased when the jar was reëxhausted.

In this investigation thin platinum films were prepared by sputtering in air, and the ageing was studied first in the best obtainable vacuum and then in air at atmospheric pressure. In the vacuum the resistance of the film at first decreases very rapidly and then more slowly, finally reaching a nearly constant value. The curve of resistance plotted against time is regular in form and values can be closely duplicated for different films. The effect of admitting air at any time during the process is practically to stop the ageing. When the air is admitted the resistance of the film shows a slight initial rise (along a definite curve) after which it remains practically constant.

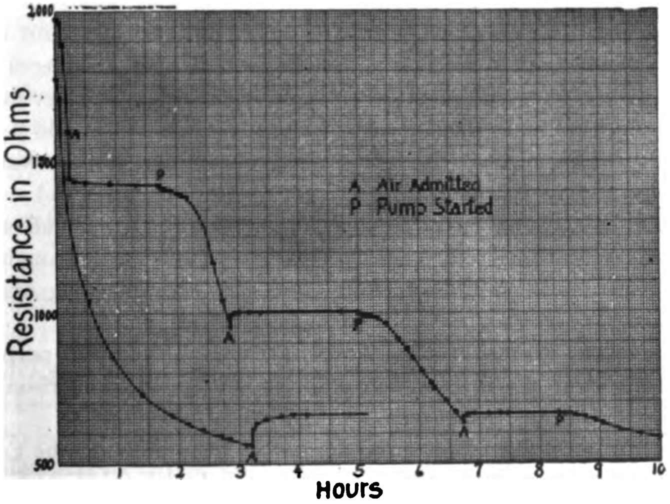

Fig. 1.

If a sputtered film is considered as composed of many small particles the initial aging in vacuo is probably due to the agglomeration of these particles. The effect of admitting air is to surround each of these particles with a film of adsorbed gas, thus preventing further agglomeration. If a film which has been allowed to adsorb air is again placed in a vacuum, the adsorbed air will be given off and the film will continue to age as before. The ageing can again be halted at any point by admitting air and can be continued again by pumping out the jar. The accompanying curves show this process of ageing in stages and normal ageing in vacuo. This effect in particular is a strong indication that adsorbed gases play a large part in determining the properties of thin metal films.

When the cover of the sputtering jar is removed and the film is allowed to come into contact with the air of the room its resistance undergoes a sudden decrease. It is not likely that this is due to the condensation of a water film as the decrease is very much greater than would be caused by a water film. This effect is at present under investigation.

The writer is pleased to acknowledge assistance from a grant from the Rumford Fund to Prof. F. K. Richtmyer.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
November, 1920.

CONSTANCY OF HALL COEFFICIENT IN THIN SILVER FILMS.

BY G. W. STEWART.

IN the preceding abstract Dr. Wait announces that the Hall coefficient retains the same value as in bulk silver even though the film used may have a specific resistance many hundred times that of bulk silver. The constancy of the coefficient suggests a theoretical examination in order to ascertain the simplest set of assumptions that will account for the result and yet without conclusions not in accord with any results obtained upon the variation of specific resistance of thin films.

If the film be supposed to consist of granules, each treated as bulk silver, and the current to be conducted from one granule to the next through areas of "contact" across which the electrons pass, then the following assumptions seem reasonable:

1. That, in any normal cross section of the film, the current perpendicular to the cross section flowing through granules is large in comparison with the current flowing in the contact gaps.

2. That the total fall of potential across the gaps between the granules in the direction of the Hall E.M.F. is small compared with this E.M.F.

With these two assumptions it is readily found that the Hall coefficient is independent of the thickness of the films. There is no qualitative disagreement of the theory with experiments upon specific resistance; it is therefore important to pursue experiments suggested by the theory. Films should be obtained having much smaller granules, so small that the contact gaps are comparable in size, and the constancy of the coefficient tested. Tests for constancy should be made in films of metal having a coefficient of opposite sign.

Physical Laboratory,
State University of Iowa.

The Hall Effect and the Specific Resistance in Silver Films.

By G. R. Wait.

The purpose of the investigation was to establish with greater certainty the variation of the Hall effect in thin silver films and simultaneously to compare the variation of specific resistance. The work was limited to chemically deposited films that were hard and, in contrast to previous experiments, mechanically permanent. The thickness as determined by weighing the silver and also by weighing the silver iodide resulting therefrom, varied from about 20 $\mu\mu$ to 200 $\mu\mu$. The chief results were as follows:

1. The specific resistance rapidly increased with decreasing thickness at 24 $\mu\mu$ to 30 $\mu\mu$. This result is similar to those of previous investigations.

2. There was found a constant Hall coefficient; *i.e.*, the Hall effect was proportional to the primary current and to the magnetic field and inversely proportional to the thickness of the films. Moreover, this coefficient proved to be, within errors of experiment, the same as that found in bulk silver. This result is new.

3. Incidentally but importantly a modified process of deposition was found which produced films that were hard, that were reproducible and that remained unchanged with time, chemical action excluded.

It can be shown that the assumption of conduction between adjoining granules makes possible the explanation of the constancy of the Hall coefficient. Moreover with this assumption we can explain qualitatively the change of specific resistance with thickness and with temperature.

Physical Laboratory,
State University of Iowa.

The Hall Effect and the Nernst Effect in Magnetic Alloys.

By Alpheus W. Smith.

The Hall Effect.—Observations have been made on the Hall effect in iron-copper, nickel-copper and iron-nickel alloys. The addition of small quantities of copper to iron or nickel increases the Hall effect. In the iron-copper series the Hall effect has its largest value in an alloy containing 1.5 per cent. copper. A further increase in the amount of copper causes a decrease in the Hall effect. The variation of the Hall constant in this case is very similar to the variation of the electrical resistance under corresponding conditions. In the nickel-copper series the addition of copper continues to increase the Hall effect until the alloy contains a little more than 26 per cent. of copper. This alloy corresponds to the compound $CuNi_3$. When the concentration of copper is increased beyond that necessary for this compound, the Hall effect drops suddenly to a small fraction of its largest value, and then decreases gradually with a further increase in the concentration of copper. The Hall effect is positive in iron and negative in nickel. Nevertheless the addition of nickel to iron causes a rapid increase in the Hall effect so that an alloy containing 13.11 per cent. of nickel shows a Hall effect which is six times its value in pure iron. There is an approximate proportionality between the Hall effect and the concentration of nickel in the alloy for alloys containing not more than 13.11 per cent. of nickel.

The Nernst Effect.—The curve showing the relation between the Nernst effect and the concentration of copper in a nickel-copper series of alloys has nearly the same form as the corresponding curve for the Hall effect. Each curve shows a break where the concentration is that necessary for the compound $CuNi_3$. The Nernst effect is negative in iron and positive in nickel. When nickel is added to iron the Nernst effect decreases in magnitude, becomes zero for a concentration of about 2.2 per cent. nickel and then reverses its direction for higher concentrations of nickel. When the alloy contains 13.11 per cent. of nickel the effect has the direction which it has in pure nickel and its magnitude is about five times as large as in pure iron.

Reversal of the Hall Effect in Alloys.—An attempt is then made to find in the theory of Borelius an explanation of the reversal of the Hall effect in certain alloys in which the effect is negative for small values of the magnetic field and positive for large values. The observed effect can be broken up into two parts, a positive part which is proportional to the magnetic field and a negative part which at first increases with the magnetic field and then reaches a limiting value. The theory requires that this part of the Hall electromotive force be proportional to the magnetic field and this is in agreement with the observations. By assuming that the negative part required by the theory reaches a limiting value when the magnetic field is sufficiently increased, satisfactory agreement between theory and observation is again obtained.

Conclusion.—The direction and magnitude of the Hall effect and the Nernst effect must be thought of as determined by the crystal lattice and the fields of force in the intermolecular spaces rather than by the characteristics of the metal atoms or the deflection of free electrons in the interstices between the atoms.

OHIO STATE UNIVERSITY.

CONDUCTIVITY OF INSULATING MATERIALS NEAR THE BREAKDOWN VOLTAGE.

BY JAMES E. SHRADER.

A METHOD of testing insulation whereby the values of conducting are obtained just before breakdown has been devised which provides against danger to the recording instruments in case of puncture.

This method consists in placing an insulated hot cathode rectifier in series with the sample which is being tested by voltage from a D.C. source or rectified A.C. By controlling the temperature of the cathode of the rectifier used as a current limiting device the value of the current flowing is always under the control of the operator. When under a given voltage the current does not reach a constant value upon raising the temperature of the cathode, this is an indication that the voltage has reached the breakdown point. Actual breakdown can occur by this device without damage to the recording instruments.

With this device samples of insulating material have been tested up to the point of breakdown. From inspection of current-voltage curves it is observed that these materials fall into three general classes.

1. Materials whose conductivity varies directly with voltage according to Ohm's Law.
2. Materials whose conductivity varies according to Ohm's Law up to the neighborhood of breakdown after which the conductivity increases at an accelerated rate to breakdown.
3. Materials whose conductivity increases over the whole range at an accelerated rate to breakdown.

Westinghouse Research Laboratory,
November 10, 1920.

Magneto-Resistance Effects in Films of Bismuth.

By L. F. Curtiss.

THE work on films of bismuth obtained by cathodic sputtering, of which a report[1] has previously been made, has been continued with some interesting results.

Films prepared in essentially the same way as those used previously were mounted in flat glass tubes with leads sealed through the glass. These tubes were evacuated for several hours at a temperature of 200°–230° with a Langmuir mercury vapor pump. They were then sealed off and the films thus mounted were heated in an oil bath for three or four hours at about the same temperature and at intervals several days apart. The purpose in this was to complete the ageing and to bring about the steady conditions which the previous work had indicated as possible. After this result had been attained the films were placed in a temperature bath between the poles of the magnet and measurements made of the change of resistance at various field strengths and at several constant temperatures from 20° to 230°.

This procedure yielded several new results. First, and perhaps most striking, was the fact that the films throughout this process of heating as described above steadily *decreased* in resistance down to a final value after the heating had been continued long enough. After this state had been reached accurately reproducible results in the measurement of resistance at various temperatures could be obtained provided the film was not raised to a higher temperature than the previous maximum. Both these facts are at variance with previous experience with films of this metal for, as will be recalled, all former films (with one or two exceptions) increased in resistance on standing after preparation and this rate of increase was hastened by heating. Furthermore there seemed to be no limit short of infinity for the value of the resistance of a film after successive repetitions of the heating, so that reproducible measurements were never attained. The explanation for this difference seems obviously to lie in the difference in the manner of protecting the films in the two cases. Formerly the films were heated in oil which undoubtedly contained some air so that the films were oxidized whereas in the present case the air was pretty well removed from the tube in which the film was placed and thus oxidation was prevented to a greater extent. In addition the oil taken up by the film in the former case and the removal of a certain amount of adsorbed air from the film in the second case contributed also to this difference without doubt. Thus the present results indicate that bismuth films behave during ageing in the same way as films of platinum, gold, and silver have been found to behave by other investigators, at least as far as the decrease of the resistance of the film is concerned.

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[1] F. K. Richtmyer and L. F. Curtiss, Phys. Rev., XV., p. 465, 1920.

It was found further, in the measurement of the resistance of the films at various temperatures, that the negative temperature coefficient became smaller in value as the temperature increased, finally becoming zero and reversing its sign, and attaining a positive value which gradually increased in magnitude as the temperature was further increased. Thus the temperature-resistance curve has a minimum, the location of which depends on the initial resistance of the film, being in general at a lower temperature the less the resistance of the film, *i.e.*, the thicker the film, since all the films had the same length and breadth. This curve is, moreover, perfectly symmetrical with respect to an axis parallel to the y-axis (plotting resistance on the y-axis and temperature on the x-axis). The greater the resistance of a film the larger the value initially of the negative temperature coefficient.

Films treated in this manner also showed a much greater increase of resistance in the magnetic field than did any of the films in the former work. The maximum obtained was about 17½ per cent. at a field of about 16,000 gauss and at a temperature of about 20°. For the previous work the corresponding maximum was approximately 4 per cent. These measurements were also reproducible provided the heating had been carried far enough so that the temperature-resistance curve had attained its final form. The increase of resistance is not proportional to the square of the field strength, as it seemed to be before. When the percentage increase of resistance (y-axis) is plotted against the square of the field (x-axis) the observed points outline a curve concave towards the x-axis, resembling very closely the corresponding curve for bismuth in bulk, except for the fact that the effect has only about one quarter the magnitude. When the percentage increase of resistance is plotted directly against the field the curve bends upward quite perceptibly at first but finally takes up a definite direction so that the greater part is a straight line, again agreeing with the behavior of bismuth in bulk. This similarity is further shown in the results obtained at various temperatures. The effect becomes less as the temperature is increased above room temperature, at first dropping rapidly, so that if the percentage increase of resistance (y-axis) is plotted against the temperature (x-axis) curves of gradually decreasing curvature are obtained, convex towards the x-axis, hence very similar in form to those for bismuth in bulk under the same conditions.

Cornell University,
November 1, 1920.

The Magnetic Mechanical Analysis of Manganese Steel.

By Sir Robert Hadfield, F.R.S., S. R. Williams and I. S. Bowen.

By proper heat treatment, so-called non-magnetic manganese steel may be made magnetic. From a group of six manganese steel rods, drawn from the same source, three were heat treated and three untreated. Rods, numbered 5 and 2 were selected as representatives of these two classes respectively.

These rods were used in a series of comparative tests between the reciprocal magnetic and mechanical properties, known as the Joule effect (change in length due to a magnetic field) and the Villari effect (change in magnetic intensity due to a longitudinal stress).

The heat treated rod showed an increase in length for all field strengths employed, consequently there was no Villari reversal but an increase in magnetic intensity for these same field strengths when the rod was stretched.

The so-called non-magnetic sample showed some magnetic intensity when subjected to a magentic field. If we multiply the values of I for the non-magnettc rod by the factor 36 we practically duplicate the values of I for the magnetic rod. It would indicate that the heat treatment produced an added quantity of something which was very meagerly supplied to the non-magnetic rod.

The remainder of the paper deals with the importance of magnetic mechanical analysis, (1) from the standpoint of applied science, (2) for its bearing on a comprehensive magnetic and atomic theory.

London, England,
Oberlin, Ohio.

Pulsating Thermionic Discharges in Evacuated Tungsten Lamps.

By A. G. Worthing.

The starting point of this investigation has been the observation of peculiar, periodic, automatic changes in brightness of a certain tungsten ribbon filament in an evacuated lamp bulb. Several other lamps were investigated, only a few of which showed similar discharges, and none to the extent shown by the one in which the discovery was made.

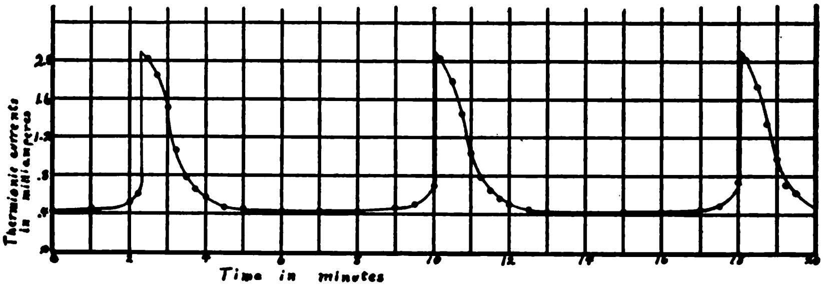
All of the lamps tested contained a third insulated terminal by means of which thermionic discharges in the lamp could be measured. Simultaneous observations of thermionic currents and of filament brightnesses by means of an optical pyrometer showed conclusively that the original variations noted in the filament brightness were directly connected with changes in the thermionic discharges in the lamp. These discharges may take place either from the filament to the insulated terminal or from one leg of the filament to the other.

The tungsten ribbon filament in the lamp tested was V-shaped with sides about 2.5 cm. long and with an apex slightly truncated so as to give an approximately square surface 1 mm. on edge which in the mounting faced the third terminal also of tungsten. The lamp was evacuated in the ordinary manner.

In one test, a direct current of 8.2 amperes was used in heating the filament to a temperature of about 2280° K. The negative terminal of a 44-volt battery was connected to one of the leads to the ribbon filament through a voltmeter with 5,000 ohms resistance; the positive terminal of the battery was connected to the insulated terminal in the bulb.

The thermionic currents represented by the voltmeter readings showed discharges of a pulsating character (see accompanying figure) in which, without any alteration of conditions by the experimenter, the currents rose from a moderately steady minimum at first slowly and then with great suddenness to a maximum value at least 4.8 times the minimum value, and then fell more or less gradually to the original minimum value. The period of the discharge, eight minutes, was constant for nine successive intervals to within 4 per cent. Small characteristic variations in the form of the curves were repeated also.

Different temperatures of operation, different applied voltages and resistances in the voltmeter circuit yielded discharges differing in period, minimum and maximum current values, and ratios of maximum to minimum current values. Temperatures beyond a certain value and resistances below a certain minimum in the thermioinc current circuit, other conditions remaining unchanged, tended toward pulsations which gradually diminished in intensity and disappeared leaving a steady thermionic discharge.



No definite explanation is offered for this action. However, the blue glow discharges which were obtainable in the lamps used would seem to indicate that it might be connected with the residual gases in the lamps.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
November, 1920.

Effect of Strong Electrostatic Fields on the Vaporization of Tungsten.

By A. G. Worthing and W. C. Baker.

THE general procedure has consisted in noting the rate of change of resistance of a tungsten filament operated in vacuo at some fixed current first when exposed to a weak electrostatic field opposing thermionic emission and second when exposed to a similarly directed strong electrostatic field. For well-seasoned filaments, the changes in resistance with continued burning, at least for moderate electrostatic fields, are due largely to changes in filament cross section resulting from vaporization. Therefore, any difference in the rates of change of resistance for the two conditions of operation is naturally to be ascribed to the effect of the strong electrostatic field on the rate of vaporization of the material.

The most satisfactory mounting for the filaments studied contained two insulated circuits leading through the stem of a well-evacuated, hard-glass lamp-bulb. One of these circuits included the filament to be investigated held straight and taut by springs of larger tungsten wire, in the axis of a helical coil of still larger tungsten wire which formed a part of the other insulated circuit. All wires leading into the lamp bulb, as well as all wires within, were of tungsten. All junctions were fused. The strong electrostatic field was obtained by connecting the insulated coil to the negative terminal of a small motor-driven Winshurst machine while the hot filament and the frame of the machine were earthed.

In the most satisfactory test conducted, a 0.060 mm. filament 3.5 cm. long, mounted in a 6 mm. helix 3.2 cm. long was operated at 2780° K. alternately for several time intervals of equal length, first with a potential difference between filament and coil of 30 volts and second with an average potential difference of 11,000 volts. This latter value corresponded to an electrostatic field strength of 800,000 volts/cm. at the surface of the filament.

The measured resistances platted as functions of time of burning showed a straight line relationship for each interval of burning. The averages of the slopes, four against three, showed no change to within ½ per cent. due to electrostatic field. Hence, we conclude that, for the conditions specified, the effect of the electrostatic field on the vaporization of tungsten is negligible.

According to generally accepted views a filament connected to earth becomes deficient in negative electrons when exposed to a steady negative electrostatic field. This means that there exists in the filament an excess of positive ions during the continuation of the electrostatic field. These are probably located on the surface and may here be called "surface ions" to distinguish them from the very temporary ions throughout the filament which are the counterparts of the temporary free electrons. A field strength of 800,000 volts/cm., mentioned above, means a "surface-ion" density of 44×10^{10} ions/cm.^{2}, one atom of every 70,000 surface atoms being ionized.

Due to the pull of the electrostatic field these positive "surface ions" might be expected to be more easily detached, that is to be more readily vaporized than neutral surface atoms. Just what forces are called into play and how they are directed is difficult to say since we have no way as yet of determining whether these "surface ions" sustain the whole or only a part of the pull of the electrostatic field on the filament. Thus (1) the tubes of induction from the coil may terminate only on surface ions; (2) the tubes may terminate indifferently on neutral atoms and surface ions, with a readjustment of the tubes within and between the atoms of the filament; or (3) the tubes may congregate in bundles with relatively a large portion of them terminating on "surface ions," and with the remainder terminating on neutral atoms, but with a selective tendency for atoms in close proximity to "surface ions." On supposition (1) the "surface ions" sustain the total pull due to the electrostatic field; on supposition (2) the pulls sustained by a surface ion and a neutral atom are equal; on supposition (3) the pull sustained by the "surface ions" is intermediate between those sustained in cases (1) and (2).

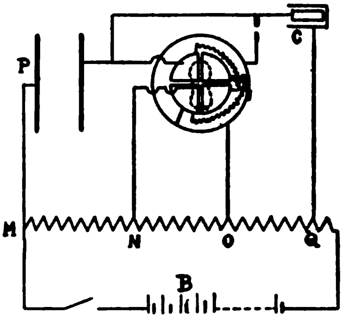
Supposition (3) seems most probable. Some speculation on the basis of (1), however, is perhaps permissible, since in that case the computations, in addition to entailing only a moderate amount of labor, will likely give some idea of the magnitudes of the actual electrostatic forces to which the "surface ions" are subjected. For a field strength of 800,000 volts/cm., the pull on an ion in a moderately smooth filament surface is computed to vary from about 63×10^{-8} dynes to roughly twice that amount, depending on how far out the individual ion may be from the surface described by the remaining "surface ions." This force is of the magnitude of the force computed from the estimated yield stress for a tungsten wire at incandescence reduced to a single atom (yield stress ÷ cross-sectional surface density of atoms). This approximate coincidence together with the failure to detect an effect of an electrostatic field on vaporization does not necessarily indicate any error in supposition (1) or in the computations made, since, on the "amorphous phase theory" of the binding material between grains the yield stress may depend on the weak amorphous phase while the resistance to vaporization depends largely on the cohesive forces within the crystal. Moreover, vaporization takes place by atoms, whereas the yielding of a stressed wire takes place by layers or "en bloc." On the other hand, the approximate coincidence leads us to hope that, by increasing the field strength, or by a modification of conditions, a positive influence of electrostatic fields on vaporization may be found.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
November, 1920.

A COMPENSATION SCHEME FOR ELECTROSTATIC MEASUREMENTS.

BY W. F. G. SWANN.

IN the measurement of a saturation current in a gas, it is customary to connect one plate of the ionization chamber to one pole of a high-potential battery, the other plate being connected to one quadrant of an electrometer whose other quadrant, together with the remaining pole of the battery is connected to the electrometer case. If, under these conditions, the electrometer is to be free from fluctuations resulting from inductive action of the high-potential plate, a very high battery-constancy is essential. Again, while a quadrant electrometer is usually adjusted so as to show no deflection on application of the needle potential, with the quadrants connected together, this independence of the position of the needle on the needle potential does not persist when one quadrant is insulated as in actual use, so that here again we are confronted with the necessity of high battery constancy. All difficulties arising from fluctuations in battery potentials may be eliminated by the very simple device indicated in the diagram.



The high-potential battery B is connected to the ends of a high resistance, which may be conveniently made of coils of I.A.I.A. wire immersed in oil. The high potential plate P of the ionization chamber, the electrometer needle, electrometer case, and outer member of a subsidiary condenser C are connected to different points M, N, O, Q, along the resistance. Thus, both the ionization-chamber potential and the needle potential are provided from the same battery. By adjusting the capacity of C, or the position of the point Q, it is readily possible to arrange so that on throwing on the whole battery B there is no inductive deflection of the needle; the various capacities then compensate in their inductive actions, and the whole set-up is absolutely independent of fluctuations in the battery, provided that the resistance ratios remain constant. The relative order of the points M, N, O, Q, may vary with the capacities involved; and, it is even possible, in certain cases, to dispose of C and let the capacity between needle and quadrant function in its stead.

Another useful feature of the above arrangement is its adaptibility in a case where one has a charge between the plates, which it is desired to measure. Ordinarily, it would be necessary to apply the potential to P with the electrometer quadrants connected to the case so as to avoid inductive action. One would then endeavour to release one of the quadrants from earth *as quickly as possible* after applying the potential so as to catch all the ions as they came across. There must always be a certain loss in such an experiment. With the present arrangement, however, the inductive action is compensated, so that one may release the quadrant from earth before throwing the potential on P, and thus avoid losing any of the charge which it is desired to measure.

By inserting an insulated potentiometer-system, between Q and C for example, it is easy to reduce the measurements to a null method.

The principle of compensation described may of course be extended to such appliances as the tilted-leaf electroscope, etc.

UNIVERSITY OF MINNESOTA.

NOTE ON ELECTROMAGNETIC INDUCTION AND RELATIVE MOTION.
A REJOINDER.

BY W. F. G. SWANN.

AT the Washington, D. C., meeting of the American Physical Society, April 23–24, 1920, Dr. S. J. Barnett presented a paper[1] in which he takes exception to a paper by myself presented before the Physical Society on December 30, 1919[2], and dealing with an experiment performed by him on Electromagnetic Induction and Relative Motion.[3] As Dr. Barnett appears to consider that my conclusion is irrelevant, and that I have overlooked certain former papers of his own which, it is claimed, cover the point in question, I may perhaps be permitted to restate what the conclusion really was.

Dr. Barnett sought to detect the effect of the electric intensity arising from the uniform rectilinear motion of a magnet. My contention was that, according to recognized electromagnetic theory, the case surrounding his detecting condenser shielded the latter completely from the effects of the electric intensity. In other words, the electric intensity in question resulted in an induced charge-distribution on the surface of the case; and, the portion of the resultant electric intensity inside the case, due to this induced charge, completely cancelled the other portion, whose effect was the primary object of the investigation. This conclusion rested upon the fact that, in Dr. Barnett's experiment, the electric intensity in question was of a type derivable from a potential. I had read Dr. Barnett's papers to which he refers, and have reëxamined them since, but have failed to find any reference to this matter, which constitutes the whole point of my paper, and which indeed appears to me to render obvious, from the start, the null result to be expected from the experiment.

I may add that my desire was to show that Dr. Barnett's result was entirely consistent with electromagnetic theory as expressed by the Maxwell-Lorentz equations, a point which seemed to me worthy of mention because Dr. Barnett considers his experiment inconsistent with the theory of relativity with which, at any rate in its correctly stated form, all conclusions properly derived from electromagnetic theory must be consistent.

UNIVERSITY OF MINNESOTA.

[1] PHYS. REV., June, 1920, pp.527–528.

[2] PHYS. REV., March, 1920, p. 227. I have given a further account of the point concerned, in a paper covering the whole subject of Unipolar Induction, and published in PHYS. REV., V. 15, pp. 365–398, 1920.

[3] PHYS. REV., August, 1918, pp. 95–114.

The Electrostatic Charges of the Earth and Sun and their Relation to Terrestrial Magnetism.

By Fernando Sanford.

Up to the present time no one has proposed any explanation of the earth's magnetic field except the possible rotation of a negative charge with the earth.

If the magnetic fields of the sun and earth are due to the rotation of their negative charges, these charges should be great enough to exert an inductive influence upon each other, and the earth should be less electronegative upon its day side than upon its night side. The existence of this phenomenon has been shown by a series of photographic records of the diurnal changes in the earth's potential at Palo Alto, California, for the months of August, September and October and up to this writing on November 16.

These records have been made by means of a quadrant electrometer, one pair of whose quadrants is connected to the water system of Palo Alto while the other pair is joined to an insulated capacity enclosed in an earthed metal cage.

The average diurnal change in potential for the period mentioned has been about 70 millivolts, as registered by the electrometer. What relation this bears to the actual change in the potential of the earth is not known. The maximum negative potential has occurred at about 8 P.M., while the maximum positive potential which occurred between 9 and 10 A.M., in August has changed progressively to a later period until it occurred at noon in October. If the potential at 6 A.M., be taken as the zero potential for the day then the earth's potential is negative throughout the night and positive throughout the day. This is shown graphically by Fig. 1, which represents the diurnal potential curve for 29 days from August 6 to September 4, 1920.

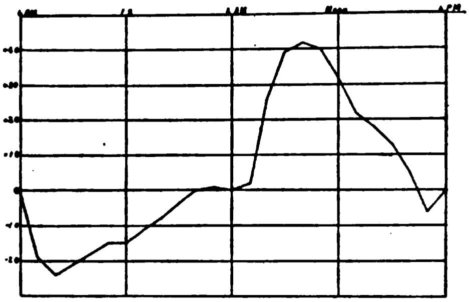


Fig. 1.

Since the magnetic field induced by the rotation of the earth's charge should be parallel to its axis of rotation, the magnetic variation produced by a change in the earth's potential should appear most plainly in the variation of the north-south component of the magnetic field for the same period. Since the electric charge which is assumed to produce the earth's magnetic field must be negative, the N magnetic component should be greater where the earth is more negative, and *vice versa*. The curve for the N magnetic component should then resemble the electric potential curve with its signs inverted.

The only data which I have been able to find for the diurnal variation of the N magnetic component for the month of August are published by Chree in Volume II. of the Collected Researches of the National Physical Laboratory, of Teddington, England. These data are taken from the records of the Falmouth Observatory for the five most quiet days of each August for the twelve years, 1891–1902. They are represented graphically in the dotted curve in Fig. 2, the continuous curve being the electric potential curve at Palo Alto, as shown by Fig. 1, except that the mean potential of the day is taken as zero, instead of the potential at 6 A.M. The relation between the two curves, representing, as they do, conditions at places one third the circumference of the earth distant in longitude, 14 degrees distant in latitude and 20 years distant in time, is so close as to preclude any possibility of their not being related to the same physical cause.

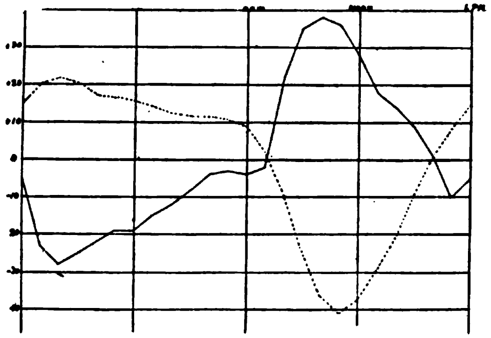


Fig. 2.

The same relation is shown equally well by the corresponding curves for September and October, an interesting fact being that the time of greatest deviation in both sets of curves has moved progressively from about ten o'clock in August to Noon in October.

A further semblance to terrestrial magnetism is shown in the disturbances which resemble "magnetic storms." Fig. 3 shows photographic copies of

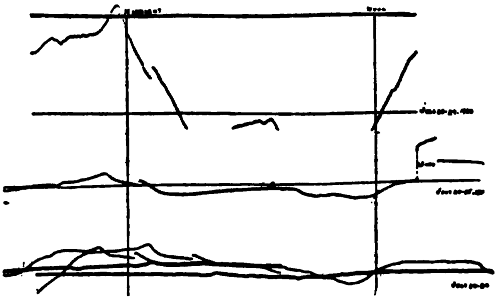


Fig. 3.

three daily records during the course of one of these disturbances. The break in the middle record marked 1.5 volt shows the effect of introducing a dry cell of this voltage between the electrometer and the earth. The lower record shows parts of two days. The heavy line in this record is a magnetometer record showing change in declination.

Fig. 4 shows copies of three successive daily records from August 3 to 6.

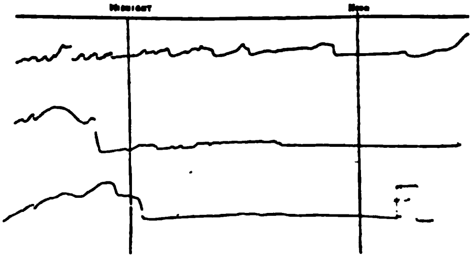


Fig. 4.

In these records the disturbances give the suggestion of interfering wave-trains. The repetition of the "bay" in the two later records shows a common characteristic of magnetic storms. The effect of introducing 1.5 volts between the electrometer and the earth is shown in the lower record. The original records are 20 inches long.

Palo Alto, California,
November 16, 1920.

THE SPECTROPHOTOELECTRIC SENSITIVITY OF PROUSTITE.

By W. W. Coblentz.

Continuing the investigation of the spectrophotoelectrical properties of substances, an examination was made of the mineral proustite, Ag_3AsS_3, using the apparatus described in previous papers.

The samples examined were beautiful semitransparent crystals of a light vermilion color, indicating a high absorptivity for wave-lengths less than 0.55 μ.

At + 20° to − 50° the spectrophotoelectric sensitivity curve of proustite has a slight maxim at about 0.61 μ and a marked sensitivity with a maximum in the extreme violet. As the temperature is lowered (to − 100° C.) the maximim in the ultra-violet is more and more obliterated by a new maximum (the 0.61 μ band) which occurs at about 0.58 μ. The position of this new maximum remains quite constant as the temperature is decreased to − 170° C. No photoelectric sensitivity was observed for radiation stimuli of wave-lengths extending from 1 μ to 2 μ in the infra-red.

The general behavior of the spectrophotoelectric reaction in proustite is quite similar to that of cuprous oxide;[1] and it is entirely different from silver sulphide,[1] which has the maximum of its photoelectrical sensitivity at 1.2 μ in the infra-red.

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[1] Pfund, Phys. Rev., (2) 7, p. 289; 1916.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
November 10, 1920.

COLOR FILTERS FOR PHOTOGRAPHIC USES.

BY CHARLES D. HODGMAN.

IN the course of a somewhat extended study of the processes of three-color photography a considerable number of filters of various types have been worked out. They include a series of three-color filters of varied extent and abruptness of absorption, various yellow or orange contrast or compensation filters and others for various special uses such as photomicrography and contrast control in the photography of colored objects.

The filters constructed were of the ordinary type, each consisting of two glass plates flowed with colored gelatine and, when dry, cemented together with balsam. The factors determining the selective absorption are the strength and dye content of the gelatine solution and the volume flowed per unit area. The flowing solution consisted in each case of six per cent. by weight of clarified gelatine, a definite quantity of one or more aqueous dye solutions and distilled water to make the whole up to 100 per cent. Any filter is thus fully determined by giving for each of its two components (1) the dye used and the strength of aqueous stock solution, (2) the quantity of this dye solution used in 100 parts of the final mixture, and (3) the quantity flowed on unit area. The gelatine solution is filtered and poured while still warm on the previously cleaned and leveled glass plates.

The action of the filters is shown by photographs of their absorption spectra. It is possible however to indicate the characteristics approximately by giving the limits of action in the spectrum. Such an attempt is made in the accompanying table in connection with sufficient data for exact reproduction.

In choosing filters for any specific purpose it should be noted that the action obtained is limited by the sensitiveness of the photographic plate used and the character of the light source. Due allowance must be made for these factors. The following data refers to absorption spectra for which the light source was an incandescent tungsten filament in a gas filled bulb of light blue color. This lamp is supposed to produce an approximation to daylight and is commercially known as "Mazda C 2." The spectrum of this source on the brand of panchromatic plate used is continuous from about 0.35 μ to 0.72 μ. A region of less action exists at about 0.52 μ. The ordinary plates, not color sensitized, showed action without filter from 0.35 μ to 0.55 μ. In the case of the orthochromatic plates the action was extended to about 0.63 μ.

The first group listed is a series of red filters differing principally in the position of the limit of absorption toward the blue. While originally intended

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[1] Coblentz & Kahler, B. S. Bulletin, 15, p. 213; 1919.

Filter.	Formula.			Plate.	Limits of Action, Microns.
	Dye Solution.	Quantity.	Coating.		
		c.c.	c.c.		
110	*a* Rhodamine B 1%	85	0.094	Pan.	.60–.71
	b Orange G 1%*	50	.125		
103	*a* Rose Bengal 0.5%	13.6	.094	Pan.	.58–.71
	b Orange G 1%*	50	.125		
12	*a* Erythrosine 0.5%	20	.094	Pan.	.57–.71
	b Orange G 1%*	50	.125		
5	*a* Ponceau 2R 1%	15	.094	Pan.	.56–.71
	b Auramine 0.5%	20	.094		
4	*a* Ponceau 2R 1%	12	.094	Pan.	.55–.71
	b Orange G 1%*	20	.094		
115	*a* Fluoresceïne 0.5%*	94	.156	Pan.	.54–.71
	b Picric acid 0.5%	33	.094		
113	*a* Ponceau 2R 1%	5	.094	Pan.	.54–.71
	b Metanil yellow 0.5%	16	.125		
13	*a* Naphthol green 0.5% Brilliant green 0.5%	12 2	.094	Pan.	.48–.62
	b Picric acid 0.5% Naphthol yellow sat. sol.	30 50	.125		
8	*a* Naphthol green 0.5% Brilliant green 0.5%	6 3	.094	Pan.	.48–.62
	b Picric acid 0.5%	94	.125		
7	*a* Methylene green 1%	15	.125	Pan.	.47–.62, slight action
	b Auramine 0.5%	20	.094		.66–.71
121	*a* Picric acid 0.5% Naphthol yellow sat. sol.	30 50	.125	Ortho.	.46–.63
	b Uncoated glass				
17	*a* Naphthol yellow sat. sol.	50	.125	Ortho.	.47–.63
	b Naphthol yellow sat. sol.	50	.125		
14	*a* Nacht blau 1%	15	.094	Pan.	.40–.49
	b Gentian violet 0.5%	10	.094		
10	*a* Methylene green 0.5%	15	.125	Pan.	.37–.49
	b Gentian violet 0.5%	20	.094		
11	*a* Iodine green 1%	10	.094	Pan.	.35–.51
	b Gentian violet 0.5%	10	.078		
119	*a* Picric acid 0.5%	5	.078	Ord.	.39–.50
	b Erythrosine 0.5%	20	.094		
123	*a* Aesculin 1%	5	.094	Ord.	.39–.50
	b Erythrosine 0.5%	15	.094		
118	*a* Aesculin 1%	5	.094	Ord.	.39–.52
	b Uncoated glass				
20	*a* Orange G 1%*	23	.094	Pan.	.53–.70
	b Naphthol yellow sat. sol.	50	.094		
19	*a* Orange G 1%*	8	.094	Pan.	.52–.70
	b Naphthol yellow sat. sol.	50	.094		
17	*a* Naphthol yellow sat. sol.	50	.125	Pan.	.49–.70
	b Naphthol yellow sat. sol.	50	.125		

Filter.	Formula.			Plate.	Limits of Action, Microns.
	Dye Solution.	Quantity.	Coating.		
		c.c.	c.c.		
121	*a* Picric acid 0.5%	30	.125	Pan.	.47–.70
	Naphthol yellow sat. sol.	50			
	b Uncoated glass				
122	*a* Aesculin 1%	15	.125	Pan.	.42–.70
	b Naphthol yellow sat. sol.	50	.078		
118	*a* Aesculin 1%	5	.094	Pan.	.39–.70
	b Uncoated glass				
16	*a* Naphthol yellow sat. sol.	50	.062	Pan.	.35–.70,
	b Uncoated glass				max. .56
18	*a* Orange G 1%*	1.5	.094	Pan.	.40–.70,
	b Naphthol yellow sat. sol.	50	.062		max. .56
23	*a* Gentian violet 0.5%	10	.094	Pan.	.64–.71
	b Orange G 1%*	50	.125		
101	*a* Naphthol green 0.5%	20	.125	Pan.	.59–.65
	Orange G 1%*	50			
	b Erythrosine 0.5%	94	.125		
104	*a* Fluoresceïne 0.5%*	94	.125	Ortho.	.54–.61
	b Picric acid 0.5%	33	.094		
20	*a* Orange G 1%*	23	.094	Ortho.	.52–.61
	b Naphthol yellow sat. sol.	50	.094		
17	*a* Naphthol yellow sat. sol.	50	.125	Ord.	.47–.53
	b Naphthol yellow sat. sol.	50	.125		
107	*a* Nacht blau 1%	20	.094	Pan.	.46–.51
	b Picric acid 0.5%	33	.078		
106	*a* Rose Bengal 0.5%	16	.125	Ord.	.46–.50
	b Picric acid 0.5%	33	.078		
111	*a* Picric acid 0.5%	33	.094	Pan.	.46–.48
	Erythrosine 0.5%	20			
	b Nacht blau 1%	20	.094		
108	*a* Eosine yellowish 1%	14	.094	Ord.	.42–.47
	b Aesculin 1%	15	.125		
22	*a* Eosine yellowish 1%	14	.094	Ord.	.37–.45
	b Eosine yellowish 1%	14	.094		
106	*a* Rose Bengal 0.5%	16	.125	Pan.	.46–.50, .59–.71
	b Picric acid 0.5%	33	.078		
123	*a* Erythrosine 0.5%	15	.094	Pan.	.39–.51, .55–.71
	b Aesculin 1%	5	.094		
119	*a* Picric acid 0.5%	5	.078	Pan.	.39–.48, .56–.71
	b Erythrosine 0.5%	20	.094		
108	*a* Eosine yellowish 1%	14	.094	Pan.	.42–.47, .56–.71
	b Aesculin 1%	15	.125		
22	*a* Eosine yellowish 1%	14	.094	Pan.	.38–.45, .56–.71
	b Eosine yellowish 1%	14	.094		

* Orange G and fluoresceïne will not dissolve in water in the proportion stated. Ammonium hydroxide is added drop by drop until solution occurs.

for three-color work there are many other uses for which such filters are suitable."

The next two groups comprise green and blue three-color filters of various characteristics. Among these are certain yellow and negative green filters giving with ordinary or orthochromatic plates a range of action desirable in the blue and green sensation negatives in three-color work.

The series of orange and yellow contrast filters is in effect a continuation of the red series, absorbing decreasing amounts of the blue and violet. Two, Nos. 16 and 18 show an approach to the distribution of visual intensity.

The fifth group comprise filters which of themselves or when combined with the limited sensitiveness of ordinary or orthochromatic plates restrict the action to relatively narrow limits.

The last five filters, previously listed for use with ordinary or orthochromatic plates, are here used as negative green filters with panchromatic plates.

Characteristics and Formulæ of Filters.

For each component part of each filter, indicated as "*a*" and "*b*," there are given (1) the name of the dye used and the concentration of the aqueous stock solution, (2) the quantity of stock dye solution used for each 100 c.c. of the final mixture and (3) the volume of coating solution for each sq. cm. area. In addition to the dye solutions each coating mixture contains six per cent. by weight of gelatine and distilled water to make 100 per cent.

CASE SCHOOL OF APPLIED SCIENCE.

COMPARISON BETWEEN THE FUNDAMENTAL EQUATIONS FOR THE PONDEROMOTIVE FORCE FOR POINT CHARGES DUE TO LARMOR-LORENTZ AND TO MEGH NAD SAHA.

BY ALBERT C. CREHORE.

THE following five points are deduced from the form of the expressions for the ponderomotive forces due to Larmor-Lorentz and to Megh Nad Saha for point charges. They may each be taken as in favor of the probable truth of the Saha form in preference to the Larmor-Lorentz form. The equations under discussion are

$$\mathbf{F} = e_1\left(\mathbf{E} + \frac{1}{c}\mathbf{q}_1 \times \mathbf{H}\right), \tag{1}$$

where

$$\mathbf{E} = \frac{e_2(1-\beta_2^2)}{(RA_2)^3}\left\{\mathbf{R} - \frac{R}{c}\mathbf{q}_2 + \frac{\left\{\mathbf{f}_2 \times \left(R - \frac{R}{c}\mathbf{q}_2\right)\right\} \times \mathbf{R}}{c^2(1-\beta_2^2)}\right\}, \tag{2}$$

$$\mathbf{H} = \frac{e_2(1-\beta_2^2)}{(RA_2)^3}\left\{-\frac{\mathbf{R} \times \mathbf{q}_2}{c} + \frac{\left\{\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2\right)\right\} \times \mathbf{R}}{Rc^2(1-\beta_2^2)}\right\}, \tag{3}$$

$$A_2 = 1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \quad \text{and} \quad A_1 = 1 - \frac{\mathbf{q}_1 \cdot \mathbf{R}}{cR}, \tag{4}$$

The Saha equation is

$$\mathbf{F} = \frac{e_1e_2(1 - \beta_2^2)}{(RA_2)^3(1 - \beta_1^2)^{1/2}}\left\{\left(1 - \frac{\mathbf{q}_1\cdot\mathbf{q}_2}{c^2}\right)\mathbf{R} - \frac{RA_1}{c}\mathbf{q}_2\right\}. \qquad (5)$$

1. The Larmor-Lorentz form (1) makes the force upon a moving point charge due to a stationary point charge equal to the simple electrostatic force. That is to say, the force does not depend upon the state of motion of the first charge.

Under the same conditions the Saha equation (5) makes the force depend upon the velocity of the first point. These statements are shown to be true by putting $\beta_2 = 0$ in the two sets of equations, when the Larmor-Lorentz form gives

$$\mathbf{F} = \frac{e_1e_2}{R^3}\mathbf{R}, \qquad (6)$$

and the Saha form gives

$$\mathbf{F} = \frac{e_1e_2}{R^3(1 - \beta_1^2)^{1/2}}\mathbf{R}. \qquad (7)$$

A criticism of the Larmor-Lorentz equation on these grounds was given on pages 465–6, PHYS. REV., June, 1917. The Saha equation has supplied this deficiency.

2. The force upon the first point charge due to the second in the Larmor-Lorentz form obeys the inverse first power of the distance, when the distance is great. In the Saha form it obeys the inverse square of the distance law. This may be shown from the equations. In the expressions (2) and (3), the last term or the vector containing the acceleration, $\left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2\right)\right] \times \mathbf{R}$, contributes a term varying as R^2, and this divided by the R^3 in the coefficient gives a result R^{-1}. Now, any term varying as R^{-1} may be made larger than one varying as R^{-2} or higher powers by taking the distance large enough.

On the other hand the Saha equation contains no acceleration term, and no term varying as R^{-1}, the lowest order being R^{-2}. Again, these inverse first power terms in the Larmor-Lorentz form may be picked out from the equation when the electron has a circular motion. The first term of the force (1), namely $e_1\mathbf{E}$, is given in full by equations (48), (49) and (50), page 453 PHYS. REV., June, 1917. If we suppose that the first point is stationary $\mathbf{q}_1 = 0$, and the magnetic term in (1) vanishes. The whole force at great distances exerted upon a stationary charge by a revolving charge is, therefore, according to the Larmor-Lorentz form, neglecting the inverse square of the distance and higher powers as inappreciably small in comparison,

$$\mathbf{F}_i = \frac{e_1e_2\beta_2^2}{R^2A_2^3a_2}\left\{-\beta_2 y\cos\alpha + \frac{1}{R}[(z^2\cos\alpha - xz\sin\alpha)S_2 \right.$$
$$\left. + y^2S_2\cos\alpha - xyC_2]\right\}i, \qquad (8)$$

$$\mathbf{F}_j = \frac{e_1e_2\beta_2^2}{R^2A_2^3a_2}\left\{\beta_2(x\cos\alpha + z\sin\alpha) + \frac{1}{R}[-xyS_2\cos\alpha + x^2C_2 + z^2C_2 - yzS_2\sin\alpha]\right\}j, \quad (9)$$

$$\mathbf{F}_k = \frac{e_1e_2\beta_2^2}{R^2A_2^3a_2}\left\{-\beta_2y\sin\alpha + \frac{1}{R}[-yzC_2 + y^2S_2\sin\alpha - (xz\cos\alpha - x^2\sin\alpha)S_2]\right\}k. \quad (10)$$

For the definition of terms reference may be made to the paper cited.

3. As to the forces exerted by one body upon another, let it be assumed that each body is made up of neutral atoms, each atom containing as much charge that is stationary as it does charges that revolve. When the Larmor-Lorentz equation is applied to such bodies, and it is considered that the action of any pair of charges is unaffected by the presence of all the others, and a summation taken, the result is that the total force is zero, that is in the line joining centers. It does not matter whether the negative or the positive charges are considered as the rotators.

On the other hand the Saha equation applied to the two bodies under the same conditions gives a residual attraction between the neutral bodies. Moreover, this attraction obeys the Newtonian law of equal action and reaction, the force being inversely as the square of the distance. It is shown under another title that this result is directly traceable to the extra factor $(1 - \beta_1^2)^{-1/2}$ in the Saha equation (5).

4. The tangential force upon one selected electron in a ring of equally spaced electrons is in the direction of rotation of the ring in the Larmor-Lorentz form. The force referred to is that due to the other electrons in the ring. This fact has probably been responsible for the use of rings in an atomic model more than any other circumstance. But, suspicions might well have been more vigorously aroused as to the correctness of this result, because the equation has never led to an exact solution of an equilibrium speed, where the velocity is uniform. There is always a radiation of energy demanded, and its source of supply must have been the internal energy of the electrons, which must, therefore have been changing.

The Saha equation leads to a diametrically opposite result. The force along the tangent line has the opposite direction to that deduced from the Larmor-Lorentz equation, and tends to stop the ring.

Being willing to follow the lead of the new ideas suggested by the Saha equation, the author has come to the conception of an atom without revolving rings of electrons, but with a central positive nucleus revolving, a Copernican system instead of a Ptolemaic system. This topic is discussed under another title.

5. The Saha ponderomotive force equation possesses much greater simplicity than that of Larmor-Lorentz. There are but two instead of four directions in the component vectors composing the force as expressed by (5), namely that

of the line joining centers and of a direction opposite to the velocity of the second point. From a mathematical viewpoint the Saha equation possesses the advantage of perfect symmetry in each of the four components of the force resolved along the four axes of the four-dimensional space. With a mathematician this point possibly outweighs all of the others mentioned.

NELA RESEARCH LABORATORIES.

THE NEWTONIAN LAWS OF GRAVITATION DEDUCED FROM THE SAHA ELECTROMAGNETIC THEORY APPLIED TO THE COPERNICAN ATOM.

BY ALBERT C. CREHORE.

IN a previous paper[1] the Larmor-Lorentz form of ponderomotive force was applied to rings of electrons at a great distance apart, and the author was led to the conclusion that the fundamental ponderomotive force equation requires modification. The evidence for this seemed so strong that he deliberately modified the resulting expression but not the fundamental equation, obtaining for the force exerted upon a first ring by a second ring

$$F = \frac{E_1E_2}{2k}\beta_1^2\beta_1^2\{1 - (-X\sin\alpha + Z\cos\alpha)^2\}r^{-2}, \qquad (1)$$

X and Z being the direction cosines of position of the center of the second ring with respect to the first, α the angle between their axes of revolution, and k the specific inductive capacity of the medium. E_1 and E_2 represent the total charges in the rings, and β_1 and β_2 their respective speeds relative to the velocity of light.

When the directions of the axes of the rings are oriented on the supposition that all directions are equally probable, the brace in (1) becomes 2/3, thus reducing the expression to

$$F = \frac{E_1E_2}{3k}\beta_1^2\beta_2^2r^{-2}. \qquad (2)$$

The new ponderomotive force equation due to Saha leads to precisely this equation (1) as the force exerted upon a first neutral atom due to a second neutral atom, assuming that (1) is summed for each pair of rings in each of two atoms. Using the Saha equation the average force for all orientations exerted by a second ring upon a first ring is

$$F = \frac{E_1E_2}{k}[-1 + (\tfrac{2}{3}\beta_2^2 - \tfrac{1}{2}\beta_1^2) + (\tfrac{3}{20}\beta_2^4 + \tfrac{1}{3}\beta_1^2\beta_2^2 - \tfrac{3}{8}\beta_1^4)\cdots]r^{-2}, \qquad (3)$$

where E_1 and E_2 represent the total charge per ring. Using the Larmor-Lorentz equation this force is

$$F = \frac{E_1E_2}{k}[-1 - \tfrac{2}{3}\beta_2^4 - \tfrac{4}{5}\beta_2^6 - \cdots]r^{-2}. \qquad (4)$$

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[1] PHYS. REV., June, 1917, p. 445. "The Atom," D. Van Nostrand Co., New York, p. 118, equation (201).

The important difference to be noticed is that the Saha equation contains a term with the product of the two speeds, β_1^2, and β_2^2, whereas the Larmor-Lorentz equation does not. The result is that, when all the forces between two bodies composed each of neutral atoms is summed up, the whole force vanishes at great distances for an equation like (4), while for a form like (3) the product terms will remain and represent the whole force. To show this let us write by menas of (3) the total force exerted by all revolving charges in body B upon all revolving charges in A. We have

$$F_1 = \frac{1}{k}[-\Sigma E_1 \Sigma E_2 + \tfrac{2}{3}\Sigma E_1 \Sigma(E_2\beta_2^2) - \tfrac{1}{2}\Sigma(E_1\beta_1^2)\Sigma E_2 + \tfrac{3}{20}\Sigma E_1\Sigma(E_2\beta_2^2)$$
$$+ \tfrac{1}{3}\Sigma(E_1\beta_1^2)\Sigma(E_2\beta_2^2) - \tfrac{3}{8}\Sigma(E_1\beta_1^4)\Sigma E_2]r^{-2}. \quad (5)$$

The force due to the revolving charges in B upon all the stationary charges in A is obtained by putting $\beta_1 = 0$ and changing the sign of (5), giving

$$F_2 = \frac{1}{k}[\Sigma E_1 \Sigma E_2 - \tfrac{2}{3}\Sigma E_1 \Sigma(E_2\beta_2^2) - \tfrac{3}{20}\Sigma E_1 \Sigma(E_2\beta_2^4)]r^{-2}. \quad (6)$$

The force due to the stationary charges in B upon the revolving charges in A is obtained from (5) by making $\beta_2 = 0$, and changing the sign, giving

$$F_3 = \frac{1}{k}[\Sigma E_1 \Sigma E_2 + \tfrac{1}{2}\Sigma(E_1\beta_1^2)\Sigma E_2 + \tfrac{3}{8}\Sigma(E_1\beta_1^4)\Sigma E_2]r^{-2}. \quad (7)$$

The force due to the stationary charges in B upon the stationary charges in A is obtained from (5) by making both β_1 and β_2 equal to zero, and not changing the sign, giving

$$F_4 = -\frac{1}{k}\Sigma E_1 \Sigma E_2 r^{-2}. \quad (8)$$

The total force of B on A is the sum of these four parts. (5) to (8), giving

$$F = \frac{1}{3k}\Sigma(E_1\beta_1^2)\Sigma(E_2\beta_2^2)r^{-2}. \quad (9)$$

This is precisely the same as (2) above when E_1 and E_2 stand for integral numbers of electrons, and assuming that the force expressed by (2) is summed over the two bodies A and B. Moreover, the form would have come out the same as (1), had we started with the equation from which (3) has been derived that is, before the average for orientation was taken.

It has been shown[1] that the force expressed by (2) or by (9) or by (1) is an exact statement of Newton's laws of gravitation provided we concede that all rotators in a crystal may be divided into four equal groups with axes of rotation parallel respectively to the four medial lines of a regular tetrahedron. With this arrangement of the axes of rotation the force becomes independent of the orientation of the crystal. The force is always an attraction, and obeys the inverse square law. When, however, the values of the speeds which electrons were supposed to have in rings are substituted in this equation, the force comes out of the order 10^{27} times too large to be the gravitational force.

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[1] "The Atom," Chapter XIII. and following.

But, having obtained such a result from the Saha equation, it is well to give heed to some other matters that this equation reveals. Chief among these is the fact that the force upon a single electron in a ring due to all the other electrons in the ring is in the direction opposite to its motion instead of in the same direction as the Larmor-Lorentz equation requires. In following the Saha theory I have, therefore, abandoned the employment of rings of electrons as a feature of an atomic model. Previous to this investigation the author had already concluded that the nucleus of an atom must revolve. The Lorentz mass formula applied to a positive charge of $2e$, uniformly distributed as in the solid Lorentz electron, gives for its radius

$$ak = \frac{4}{5m}\left(\frac{2e}{c}\right)^2 = \frac{16}{5m}\left(\frac{e}{c}\right)^2. \tag{10}$$

The reciprocal of $e^2/m_H c^2$ is twice the Rydberg constant, $2K$, namely

$$2K = m_H\left(\frac{c}{e}\right)^2. \tag{11}$$

If this is a true physical equation, and $2e$ is the charge on the nucleus of the hydrogen atom, then by (10) and (11)

$$a_H k = 8/5K. \tag{12}$$

These relations are satisfied dimensionally by giving the specific inductive capacity, k, the dimensions $L^{-1}T$, the reciprocal of a velocity. But, (12) connects the radius of the nucleus, a, immediately with the Rydberg constant, and shows that there ought to exist within the nucleus the characteristic property of a frequency, K. The value $2K$ is the frequency that the author had formerly used as the frequency of revolution of the electrons in hydrogen, and it also agrees with the Bohr value of the frequency of revolution of his single electron.

By changing over from the Ptolemaic or ring system to the Copernican, or rotating nucleus system for an atomic model, and retaining the same frequency of rotation, $2K$, for the nucleus as before used for the negative electron in an orbit, the magnitude of the gravitational force given by (9) is found exact without any modification. Moreover, when the positive nucleus is set in rotation it becomes equivalent to a closed ring of point charges, so far as great distances are concerned, and a ring even according to the Larmor-Lorentz theory is known to lose no energy through radiation. The two greatest difficulties with an atomic model are thus at once overcome.

The comparison between the magnitudes of the actual and theoretical attractions, therefore, reduces to a comparison between the actual radius of the nucleus of an atom and the theoretical radius of a ring which gives the same attraction. Let us make the comparison using two hydrogen atoms assuming that the nucleus has a positive charge $2e$. Its radius by the Lorentz formula (10) is then $a = 4.86 \times 10^{-16}$ cm. The Newtonian attraction on the average between two hydrogen atoms is

$$F = k' m_H^2 r^{-2}. \tag{13}$$

The theoretical attraction is according to (9)

$$F = \frac{e^2}{3K}\beta^4 r^{-2}. \tag{14}$$

Equating gives

$$\beta^4 = 3kk'\left(\frac{m_H}{e}\right)^2; \qquad \beta^2 = \sqrt{3kk'}\,\frac{m_H}{e}. \tag{15}$$

With the numerical values, $k = 1$; $k' = 666. \times 10^{-10}$; $m_H = 1.658 \times 10^{-24}$ and $e = 4.763 \times 10^{-10}$, we find

$$\beta^2 = 0.778 \times 10^{-18} \qquad \text{and} \qquad \beta = 0.882 \times 10^{-9}. \tag{16}$$

Hence the linear velocity is $v = 26.5$ cm. per sec. approximately. Using the frequency of rotation, $2K$, as above stated, we obtain

$$v = 4\pi a K, \qquad \text{and} \qquad a = \frac{v}{4\pi K} = 6.4 \times 10^{-16} \text{ cm.} \tag{17}$$

The best that can be done at present is to compare this radius with 4.86×10^{-16} cm., that obtained from the mass formula on the assumption of no rotation. The value obtained for the equivalent ring is about one third larger. This, however, is considered to be in remarkable agreement in view of the fact that a charge set in rotation has not been investigated theoretically. There is little doubt in the author's opinion that such a charge set in rotation changes its shape, possibly assuming the form of an oblate spheroid, or some other more complicated form. At any rate, up to a certain limit, any change in shape would seem likely to bring the two figures just given into closer accord.

The two radii would have been $4\sqrt{2} = 5.66$ times further from agreement had we assumed that the nucleus of the hydrogen atom has a single charge of e, and used the same frequency.

It is scarcely necessary to point out that this Copernican atom possesses features that are likely to prove of considerable interest to chemists, as well as to those who have seen advantages in the so-called magneton, for the nucleus becomes in effect a magneton.

Nela Research Laboratories.

On the Propagation of the Sound Wave from the Muzzle of a Large Gun.

By Dayton C. Miller.

In 1918 and 1919 an extended series of experiments was carried out at Sandy Hook Proving Ground, on the various characteristics—pressure, velocity, and wave form—of the air waves generated by large guns in action. One group of observations, consisting of seventy sets of measures, relates to the velocity of propagation of the wave. This problem has three phases: (1) The velocity of an explosive sound of great intensity; (2) the normal velocity of sound in free air, and (3) the form of the wave front and its propagation from the muzzle of the gun. The calculations relating to the last group of observations have now been completed, including corrections for the meteorological conditions.

The observations were made, by means of a microphone and string-galvanometer method, along the axis at intervals of 100 feet up to 1,000 feet from the gun, and also along lines making angles of about 45°, 90°, and 180° with the axis. In other experiments observations were made to distances as great as 2,200 feet from the gun. The observations have been reduced by means of a least squares solution, and are completely satisfied by the following construction for the wave front.

The trace of the expanding wave front on a plane containing the line of fire is always a circle. The center of the circle moves forward from the muzzle of the gun along the axis with a velocity, the initial value of which depends upon the kind and size of gun and the charge of powder. The velocity of displacement of the center diminishes rapidly, being an exponential function of the time. The radius of the circle at any given time after the origin of the wave is the distance sound would travel in air in this time with the uniform normal velocity of sound for the given meteorological conditions. In other words, the wave front develops in such a manner that if the distance which the sound is considered to have traveled is measured from the virtual instantaneous center, instead of from the muzzle of the gun, the velocity of the wave front is always the normal velocity of sound. The equation of the wave front when the muzzle of the gun is taken as the origin, is then, that of a circle with a moving center, as follows:

$$[x - a(1 - e^{-bt})]^2 + y^2 = v^2t^2,$$

a being the maximum displacement of the center along the axis, b the damping coefficient of the air for this displacement, t the time elapsed since the origin of the sound at the muzzle of the gun, and v the normal velocity of sound in air under the given meteorological conditions. For the complete determination of the wave front it is only necessary to determine experimentally the propagation along the axis for the given gun and charge of powder. The position of the wave front on the line of fire at a given time t is, then

$$x_t = vt + a(1 - e^{-bt}),$$

and the velocity is,

$$\frac{dx_t}{dv} = v + abe^{-bt},$$

while the initial velocity when $t = 0$, is $v + ab$.

For a 10-inch rifle, firing the service charge, the total displacement of the center, a, has been found to be about 72 feet, which value is attained in about 0.2 second after the discharge of the gun, and the damping coefficient, b, has the value 27.

Corrections for temperature and humidity are applied in the usual manner. If there is a wind, since the medium moves as a whole, the wave-front remains a circle, but the center is displaced in accordance with the velocity of the wind.

These equations have been found to satisfy all the observations very closely, the largest single residual corresponding to 0.4 foot in the position of a station, which is equivalent to 0.0004 second in time, up to distances of about a thousand feet.

Case School of Applied Science,
November 20, 1920.

A New Tone Generator.

By C. W. Hewlett.

THE instrument consists of two equal pancake coils of insulated wire placed close together, with a thin aluminum diaphragm in between. Each coil is built up of annular coils with annular air spaces between to allow sound waves to pass out from either side of the diaphragm. The coils and diaphragm are held in place by insulating materials so that magnetic hysteresis is eliminated, and eddy currents cut down to a minimum. This is one of the chief features of the instrument as it enables it to be used at frequencies where the ordinary telephone receiver would fail, on account of the tremendous losses owing to hysteresis and eddy currents.

To operate the instrument a direct current battery is connected to the two coils in series in such a way that the north face of one coil faces the north face of the other. The resulting magnetic field in the region occupied by the diaphragm lies in the direction of the radii of the diaphragm and may be referred to as the "polarizing" field of the instrument. Two fairly large condensers are connected in series and this is connected to the two terminals of the battery. If now an alternating E.M.F. is impressed on this circuit at the common junction of the condensers, and the common junction of the two coils, the diaphragm will vibrate with the frequency of the alternating current. The reason is as follows: The alternating current passes through the coils so that at every instant the adjacent faces of the two coils so far as the alternating fields in the two coils are concerned, are of opposite sign. The resulting alternating magnetic field in the region of the diaphragm is normal to the diaphragm, and consequently this alternating magnetic field induces alternating currents in the diaphragm which are circular, and concentric with the center of the diaphragm. These alternating currents in the diaphragm react with the polarizing radial field and cause the diaphragm to vibrate with the same frequency and character as the alternating current. This instrument is most conveniently used in connection with a vacuum tube oscillator. By shunting the circuit as described above with a choke coil of very high impedance for the frequencies to be used, it may be included as part of the tuned portion of a vacuum tube oscillator circuit. By simply changing the tuning of this circuit the frequency of the sound produced may be varied over a wide range. The whole audible limit can be covered with this instrument.

The operation of the instrument has been investigated mathematically and it appears that it should be possible to use the instrument as a precision source of sound. On account of the elimination of magnetic hysteresis, and the absence of eddy currents except in the diaphragm where the loss due to this cause can readily be calculated in any given case; it is possible to calculate the periodic force acting on the diaphragm at all places, for any given frequency; and from this to calculate the energy of virbation delivered to the air. Experiments are in progress along this line to determine the minimum intensity of sound for audition as a function of the frequency.

The instrument has also been used successfully as a transmitter and receiver of voice currents. The instrument should be particularly useful in the transmission and reception of speech on account of the elimination of magnetic hysteresis and eddy current losses which distort the voice currents in other types of apparatus.

STATE UNIVERSITY OF IOWA,
November 6, 1920.

A METHOD OF STUDYING SOUND WAVES BY MEANS OF A SYNCHRONOUS COMMUTATOR.

BY L. O. GRONDAHL.

IN a study of transmission of sound through the human body that was undertaken at the suggestion of and with the coöperation of Dr. William Charles White of the Tuberculosis League of Pittsburgh, there was developed a method of studying sound waves which seems worth while making known as it may have a number of applications in other sound problems. As it was necessary for the writer to leave the work in the initial stage, no applications have been carried out.

The method was designed to determine the relative transmission of sound from the open mouth through the lungs to different portions of the surface of the chest. The sound producer was in this case a telephone receiver excited by a current from a 900-cycle airplane generator. This diaphragm was held over the open mouth and the sound transmitted was received in a stethoscope bell held against the chest. The stethoscope bell was attached by means of a rubber tube to a specially designed microphone, kindly loaned for the purpose by the Western Electric Company. The current variations produced in the microphone circuit were received by the primary of a transformer, the secondary of which was connected to a two-stage amplifier. From the amplifier the alternating current was then brought back to a crown commutator mounted on the shaft of the alternator. The pulsating direct current thus produced was read on a D'Arsonval galvanometer.

It is evident that the deflection obtained depends on the setting of the brushes on the commutator. If commutation takes place at the current minimum, the galvanometer deflection is a maximum, and if it takes place at the current maximum, galvanometer deflection is zero. At successive current minima, the galvanometer deflection is alternately positive and negative. An illustration of this was immediately encountered in application to our problem when it was found that the reading with the stethoscope bell over the trachea was the negative of that with the bell over the lobe of the lung.

It is always possible to set the brushes on the commutator so as to get zero deflection on the galvanometer. That enables one therefore to determine the exact phase relation between the sound received at one position and that received at any other position in the medium. It makes it possible to map a sound wave front and to measure wave-lengths. If the source is stationary and the stethoscope bell is movable, one can choose a position of zero deflection in the galvanometer and follow it around this source. This might be interesting in such a problem as that of Abbott & Clarke on the velocity of sound from a moving source, or in a study of the effect of attaching a horn to a source of sound.

It can be shown that the method is independent of overtones and other sounds of a frequency different from the one that is being investigated.

If a number of commutators are mounted on the same shaft it makes it possible to analyze the sound that is used for the experiment. Commutators on which the number of segments are in the ratio of 1 : 2 : 3 : 4 etc. can be used to give readings corresponding to the fundamental, and to any overtone for which there is a corresponding commutator. It will also give the phase relation between the different harmonics. The fact that it is possible to separate out the components in this way makes it possible to study each one entirely independently of the rest. In a study of the relative intensities of components of different frequencies, it would, of course, be necessary to make a careful study of the characteristics of all the vibrating parts.

The work then suggests the following:

1. A D'Arsonval galvanometer method of comparing sound intensities that is independent of all sounds of frequencies that are different from the one that is being studied, and useful in the study of sound transmission.
2. A method of mapping sound waves in air.
3. A method of studying wave-lengths and sound velocities in a medium without depending on standing waves.
4. A method of analyzing the sound that is used in an experiment where the sound is produced directly or indirectly by a rotating mechanism.

Carnegie Institute of Technology,
September, 1920.

On the Relative Positions of Lines in X-Ray Spectra.

By William Duane and R. A. Patterson.

Many phenomena indicate that the lines in the L series of X-rays divide themselves into at least three groups. The lines in any one group are associated together in some way. For instance they may be produced by the same subatomic mechanism.

Three critical absorption wave-lengths have been found in the L series of each chemical element (of high atomic number), and these belong respectively to the three corresponding groups of emission lines.

It has been shown by experiments[1] that in the K series the critical absorption has a wave-length a fraction of one per cent. shorter than that of the γ emission line, and no emission lines shorter than the γ line have been observed in the K series of any chemical element.

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[1] Phys. Rev., Oct. 1919, p. 369 and Proceedings of the Nat. Ac., Sept., 1920.

Our experiments on the spectrum of tungsten indicate that the third critical absorption in the L series, La_3, has a wave-length slightly shorter than that of the shortest L emission line, $L\gamma_4$. With regard to the position of La_3, therefore, the L series has the same structure as the K series.

When we examine the positions of the other two L critical absorptions, however, relative to the emission lines in their groups, we find a somewhat different situation. The first L group, L_1, contains the lines l, α_2, α_1, β_2 and β_5, β_5 having the shortest wave-length. In order to determine definitely the exact position of the first L critical absorption, La_1, with regard to these emission lines we performed the experiment in such a way as to measure the wave-lengths of La_1 and of the emission lines in a single series of readings. The graph shown on the slide[1] represents the electric current in the X-ray spectrometer's ionization chamber as a function of the grazing angle of incidence (and, therefore, approximately of the wave-length, λ). The sharp drop in this graph which corresponds to the critical absorption La_1 lies between the peaks that represent the two emission lines β_2 and β_5. La_1, therefore, must have a longer wave-length than β_5.

The second group of lines in the L series, to which the second critical absorption La_2 belongs, contains among others the emission line γ_1 and probably the emission line γ_2. The graph shown in the second slide[2] indicates the relative position of these lines. The sharp drop corresponding to La_2 lies between the peaks representing the lines γ_1 and γ_2. The line γ_2 has a shorter wave-length than the line γ_1, and, therefore, a shorter wave-length than the critical absorption La_2.

We may arrive at a theoretical explanation of the fact that a line may have a shorter wave-length than that of the critical absorption belonging to its group by assuming that the critical absorption corresponds to the transfer of an electron from an L orbit to an orbit at the periphery of the atom, where it can find a vacant place. This does not mean a transfer to an indefinitely great distance from the atom. According to this idea a line of shorter wave-length than the critical absorption would be due to an electron falling from an orbit outside of the periphery.

This conception does not contradict the quantum theory, provided that the critical potential required to produce such a line of short wave-length slightly exceeds that required to produce the other lines in the group, and that the frequency required to produce such a line in X-ray fluorescence slightly exceeds that required to produce the other lines.

There are other more or less plausible explanations.

The positions of the critical ionization wave-lengths in the L series are now being investigated.

HARVARD UNIVERSITY.

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[1] See Proceedings of the Nat. Acad., Sept., 1920, p. 521.

[2] See Proceedings of the Nat. Acad., Sept., 1920, p. 522.

X-ray Analysis of Three Series of Alloys.

By Mary R. Andrews.

USING the Hull method of X-ray analysis the three series of alloys, nickel-iron, cobalt iron and copper-zinc were examined.

The nickel-iron alloys show centered cubic (iron) structure from 0 to 20 per cent. Ni, a mixture of centered cubic and face-centered cubic (Ni) to about 35 per cent. Ni, and face-centered cubic alone from 35 per cent. to 100 per cent. Ni.

A non-magnetic nickel-iron showed almost pure face-centered cubic structure in the non-magnetic condition, but strongly centered-cubic form after magnetization by chilling.

The cobalt-iron alloys show the centered cubic (iron) form from 0 to 75 per cent. Co, the face-centered cubic from 85 to 98 per cent. Pure cobalt crystallizes in both face-centered cubic and in hexagonal forms.

The zinc copper alloys show three structures: 0 to 40 per cent Zn, face-centered cubic (copper form); 40 to 50 per cent. Zn, face-centered and centered cubic mixture; 50 to 70 per cent. Zn, nearly pure centered cubic; 70 to 80 per cent. Zn—mixture of centered cubic and another form (probably rhombohedral); 80 to 90 per cent. Zn—this form nearly pure; 90 to 95 per cent. Zn mixture of this form and hexagonal (Zn form).

Research Laboratories,
General Electric Company.

The Relation between the Emissive Power of a Metal and Its Electrical Resistivity.

By C. Davisson and J. R. Weeks.

MAXWELL'S electromagnetic theory of light, Kirchoff's radiation law and Planck's law of power distribution in the black body spectrum together constitute sufficient basis for calculating in detail the characteristics of the thermal radiation from a metal. The only special data required are the temperature of the metal and its electrical resistivity.

On this basis Foote has derived an expression for the total emissive power of a metal defined in terms of normal intensity of radiation, and has shown that emissive powers calculated from this expression agree very exactly with observed values in the case of platinum.

It does not seem to be generally recognized, however, that Foote's formula is not appropriate for the calculation of total emissive powers defined in terms of power radiated per unit area, or, which amounts to the same thing, that the departures from Lambert's cosine law in the infra-red are by no means negligible.

The difference to be expected between these two differently defined emissive powers may be arrived at from the following considerations. The coefficient of reflection of a metal of resistivity r (ohm-cm.) for unpolarized radiation of wave-length λ (cm.) meeting its surface at angle of incidence Φ turns out, on

Maxwell's theory, to be

$$R = \frac{\sqrt{F(\Phi)} - \sqrt{2}\cos\Phi}{\sqrt{F(\Phi)} + \sqrt{2}\cos\Phi} \cdot \frac{G(\Phi)}{G(\Phi) + \sqrt{2}\sin\Phi\tan\Phi\, F(\Phi)},$$

where

$$F(\Phi) = (3600\lambda^2/r^2 + \cos^4\Phi)^{1/2} + \cos^2\Phi$$

and

$$G(\Phi) = F(\Phi) - \cos^2\Phi + \sin^2\Phi\tan^2\Phi.$$

The emissive power of the metal for this wave-length and angle is $E = (1 - R)$ and the total power radiated per unit area at temperature T is

$$W_T = 2\int_0^\infty J_\lambda \int_0^{\pi/2} (1 - R)\sin\Phi\cos\Phi\, d\Phi\, d\lambda,$$

where J_λ is Planck's distribution function.

The total emissive power of the metal defined in terms of total power radiated is

$$E = \frac{2\int_0^\infty J_\lambda \int_0^{\pi/2} (1 - R)\sin\Phi\cos\Phi\, d\Phi\, d\lambda}{\int_0^\infty J_\lambda d\lambda}.$$

On the other hand the total emissive power defined in terms of normal intensities is

$$E' = \frac{\int_0^\infty J_\lambda(1 - R_0)d\lambda}{\int_0^\infty J_\lambda d\lambda},$$

where R_0 is the value of R for $\Phi = 0$.

One has, therefore, for the ratio of the two emissive powers

$$\frac{E}{E'} = \frac{\int_0^\infty J_\lambda 2\int_0^{\pi/2} (1 - R)\sin\Phi\cos\Phi\, d\Phi}{\int_0^\infty J_\lambda(1 - R_0)d\lambda}.$$

Figure 1 shows the ratio

$$\frac{2\int_0^{\pi/2} (1 - R)\sin\Phi\cos\Phi\, d\Phi\, d\lambda}{(1 - R_0)}$$

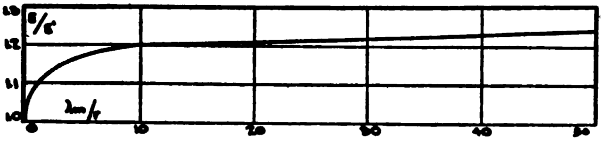

Fig. 1.

as a function of λ/r. It will be noticed that this ratio changes only slowly with λ except for λ/r small. For this reason the ratio E/E' is approximately equal to the above ratio calculated for $\lambda/r = \lambda m/r$ where λ_m has its usual meaning, and is given by $\lambda_m = (0.2940/T)$.

The total emissive power of a metal defined in terms of total power radiated may, therefore, be calculated by first calculating its emissive power defined in terms of normal intensities by Foote's formula,

$$E' = 0.5736(rT)^{1/2} - 0.1769(rT)$$

and then multiplying the value of E' by the ratio E/E' taken from Fig. 1 and corresponding to $\lambda m/r = (0.2940/(rT))$.

Theoretical and observed results for platinum are shown in Fig. 2. The curve r gives the experimentally determined resistivity-temperature relation

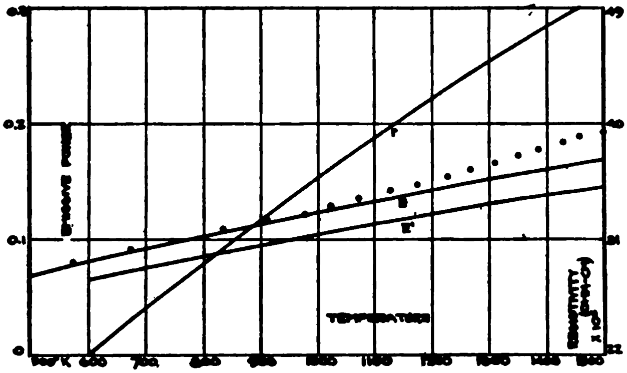

Fig. 2.

for a sample of pure platinum wire. E' is the emissive power curve for this sample calculated from Foote's formula. E is the curve of emissive power defined in terms of total power radiated and obtained from the curve E' as described above. The circles indicate values of emissive power determined by dividing the power dissipated in vacuo by a uniformly heated length of the wire by the power radiated by an equal area of black body at the same temperatures. The points are, therefore, experimentally determined values corresponding to curve E. It will be noticed that the agreement between observed and calculated values is good up to about 1000° K. and that above this temperature the observed values are above the calculated curve. This departure is taken to be due to the variation of resistivity with frequency which becomes important as the bulk of the radiation moves toward short wave-lengths.

It appears, therefore, from both theoretical considerations and from observation that very great departures from Lambert's cosine law occur in the infrared, resulting in the case of platinum in differences between the two differently defined total emissive powers of as much as 20 per cent.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE
AND TELEGRAPH COMPANY AND THE WESTERN
ELECTRIC COMPANY, INCORPORATED, NEW YORK CITY.

MASS-ABSORPTION COEFFICIENTS AS A FUNCTION OF WAVE-LENGTH ABOVE AND BELOW THE K X-RAY LIMIT OF THE ABSORBER.

BY F. K. RICHTMYER.[1]

IN a paper[2] read at the April (1920) meeting of the Physical Society data was presented showing the relation between X-ray wave-lengths and the mass-absorption coefficient μ/ρ for wave-lengths from 0.1 Å. up to 0.5 Å. μ/ρ was shown to be a linear function

$$\left(\frac{\mu}{\rho} = A\lambda^3 + \frac{\sigma}{\rho}\right)$$

of the cube of the wave-length for the three substances investigated; aluminum, copper, and molybdenum. The present paper is an extension of this work to longer wave-lengths, and to two substances of higher atomic number, silver and lead. In the case of molybdenum, silver, and lead the law of absorption has been studied on the long wave-length side of the K absorption limit.

Aluminum.—The previous equation: $\mu/\rho = 14.45\,\lambda^3 + .15$ was found to give values of μ/ρ slightly too high for wave-lengths above 0.5 Å., there being a slight systematic variation above this point. The equation

$$\frac{\mu}{\rho} = 14.3\,\lambda^3 + .16$$

gives μ/ρ within about 1 per cent. up to 0.8 Å., but does not fit quite so well as the original equation at the shorter wave-lengths.

Molybdenum.—Values of μ/ρ have been obtained up to $\lambda = .85$ Å., thus passing through the K absorption limit at $\lambda = .622$ Å. Below this limit a graph between μ/ρ and λ^3 is very slightly concave toward the λ^3 axis, the original equation

$$\frac{\mu}{\rho} = 450\,\lambda^3 + .4$$

giving values, very exact up to .3 Å. but deviating increasingly above that value.

As is well known the absorption takes an abrupt drop at the gamma line of the K series, but even on the long wave-length side of the gamma line the linear relation between μ/ρ and λ^3 still holds with different value of the coefficient of λ^3 but, so far as data can show, *with the same values of σ/ρ*. Above the gamma line the equation is

$$\frac{\mu}{\rho} = 51.5\,\lambda^3 + .5.$$

Silver.—Values of μ/ρ have been measured up to $\lambda = .7$ Å. Below the K absorption limit the equation

$$\frac{\mu}{\rho} = 603\,\lambda^3 + .7$$

represents absorption quite accurately, especially at the shorter wave-lengths.

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[1] This work was done at the Research Laboratory of the General Electric Co., Schenectady, to which the writer wishes to make grateful acknowledgment.

[2] PHYSICAL REVIEW, Vol. XV. (1920), p. 547.

Above the K limit the values of μ/ρ are given by

$$\frac{\mu}{\rho} = 86\,\lambda^3 + .6.$$

Lead.—Only the region on the long wave-length side of the K absorption limit has been meausred. The equation

$$\frac{\mu}{\rho} = 510\,\lambda^3 + 1$$

fits the data reasonably well from .17 Å. up to .7 Å. It is to be pointed out that in the region from the K limit of lead ($\lambda = .148$ Å.) up to the K limit of silver ($\lambda = .491$ Å.) silver is a better absorber than lead.

In the previous paper it was shown that A, in the equation

$$\frac{\mu}{\rho} = A\lambda^3 + \frac{\sigma}{\rho}$$

is proportional to the cube of the atomic number of the absorbing material for values of λ less than the critical K-absorption limit, using data on Al ($N = 13$), Cu ($N = 29$), and Mo ($N = 42$). This has been now extended to include Ag ($N = 47$). It is to be pointed out, however, that for each substance there seems to be a slight deviation from this law as the K_γ line is approached, the values of μ/ρ being increasingly less than would be predicted from this equation.

If we represent the absorption above the K-series limit by

$$\frac{\mu}{\rho} = L\lambda^3 + \frac{\sigma}{\rho},$$

σ/ρ, as noted above, seems to have the same numerical value as below that limit, while the ratio of A to L is of the order of 7 : 1. Data herein contained is not sufficiently accurate to determine whether L is proportional to the cube of the atomic number. The observations seem to indicate a power somewhat higher than the cube.

From the above therefore it seems possible to represent the mass absorption coefficient of any substance by an equation of the form

$$\frac{\mu}{\rho} = [K\lambda^3]_0^K + [L\lambda^3]_0^L + \frac{\sigma}{\rho},$$

where $K = A - L$. The term $[K\lambda^3]_0^K$ is to be used from $\lambda = 0$ to $\lambda = K_\gamma$, the absorption limit of the K series. The term $[L\lambda^3]_0^L$ is to be used from $\lambda = 0$ up to the first absorption limit of the L series, beyond which L is probably split up into components. If radiation of wave-length $\lambda < K_\gamma$ falls on an absorber the first of these two terms determines the proportion of total energy reradiated as K radiation. The second determines the proportion reradiated as L, M (etc.) radiation.

In conclusion, the simplicity of absorption laws in the X-ray region as compared with the corresponding laws (whatever they may be!) in the visible and ultra-violet region point to the desirability of approaching the latter by gradually extending our knowledge of the former to longer and longer wave-lengths.

Cornell University,
October, 1920.

GRAPHICAL DETERMINATION OF HEXAGONAL AND TETRAGONAL CRYSTAL STRUCTURE FROM X-RAY DATA.

BY ALBERT W. HULL AND WHEELER P. DAVEY.

X-RAY diffraction patterns from powdered crystals may be interpreted very easily in case the crystals are cubic. In case the crystals belong to any other system, the interpretation has, until now, been easy only in case correct quantitative crystallographic data could be found. The method here reported makes the tretagonal and hexagonal systems as easy of interpretation as the cubic.

The spacings of all possible planes (within a predetermined range) are plotted[1] as a function of the axial ratio. These spacings are plotted on a logarithmic scale (shown on the bottom of each plot) so that they may be compared with the experimental values without regard to the absolute values of the spacings.

To use one of the plots, a straight-edged piece of paper is placed beneath the logarithmic scale of abscissas and the experimental values of planar spacings are laid off on the edge. This paper strip is then moved over the plot with the edge always parallel to the axis of abscissas until a position is found where an exact match is obtained. When this is accomplished the correct crystal system and type of lattice has been found, viz.: that for which the plot was calculated; the axial-ratio is shown by the intercept on the axis of ordinates; and the crystal form corresponding to each experimental spacing may be read off at the top of the plot.

Three plots are given for the hexagonal system, and three for the tetragonal, as follows:

Hexagonal System.

(1) Single triangular prism lattice.
(2) Two triangular prism lattices in close packed arrangement.
(3) Three triangular prism lattices in rhombohedral arrangement.

Tetragonal System.

(1) Single tetragonal lattice.
(2) Body centered tetragonal lattice.
(3) Face centered tetragonal lattice.

The rhombohedral and tetragonal plots may also be used for cubic crystals, and the axial ratios at which these may be read off are indicated on the graphs.

The use of the plots is illustrated by the analysis of the diffraction patterns of zinc, cadmium, and indium. The arrangement of atoms in zinc is shown to be that of hexagonal close packing with axial-ratio 1.86, and side of unit triangular prism 2.670 Å.

The arrangement in cadmium is that of hexagonal close packing with axial ratio 1.89, and side of unit triangle 2.98 Å.

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[1] These plots will be published in full in the PHYSICAL REVIEW.

The lattice of indium is face centered tetragonal with axial-ratio
1.06, and side of
unit square 4.58 Å.

Crystallographic data for the last of these is lacking in the literature. For the first two, the axial ratios in the literature are incorrectly given as 1.356 and 1.335 respectively.

Research Laboratory,
General Electric Co.,
Schenectady, N. Y.,
November 17, 1920.

The Absorption and Scattering Coefficients for Homogeneous X-Rays in Several Elements of Low Atomic Weight.

By C. W. Hewlett.

THIS work was done during the summer of nineteen hundred twenty at the Research Laboratory of the General Electric Company in Schenectady, New York.

Homogeneous X-rays were secured by means of a Coolidge water cooled tube and rock salt spectrometer. X-rays from 0.10 to 1.05 Ångstrom wave-length were used. The total mass absorption coefficients were measured in water, aluminum, carbon, oxygen, nitrogen, lithium and iron. The absorption coefficients were found to be proportional to the cube of the wave-length with certain important exceptions. The mass scattering coefficients were all found to be less than that calculated for electrons 10^{-13} cm. in diameter, although Compton's[1] Theory of an electron 1.85×10^{-10} cm. diameter does not satisfactorily account for the facts. A complete discussion of the investigation is to be published very shortly.

State University of Iowa,
Iowa City, Iowa,
October 5, 1920.

The Current-Temperature Relation for Different Pyrometer Filaments.

By W. E. Forsythe.

IN the disappearing-filament type of optical pyrometer the calibration curve consists of the relation between the temperature of the standard source used and the current through the pyrometer filament. For convenience in reading, everything else being equal, the change in current for a particular change in temperature should be as large as possible. It has often been stated that carbon pryometer filaments were better than tungsten filaments because carbon at low temperatures gave a more open scale, that is, a greater change in current for a given change in temperature.

The relation between the pyrometer current and the corresponding temperature was determined for a range of temperatures from 1828° K. to about 950° K. for a number of tungsten and carbon filaments. For very short tungsten filaments where the effect due to the conduction of heat away from the filament by the ends reached to the central part of the filament the current-temperature relation was found to be much smaller than when the filaments were longer. The same effect also is true for short carbon filaments. However, if the carbon or tungsten filaments were long enough so that the end cooling did not effect the temperature of the central portion of the filament, there was not much difference in the ratio.

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[1] A. H. Compton, Physical Review, 14, 20–43, 247–259, 1919.

The ratio of the current range for a hairpin tungsten filament, length of loop 12 cm. was 4.2 for a temperature range from 1828° to 950° K., the ratio for a carbon hairpin loop 10 cm. long was 4.5 for the same temperature range. Both filaments were long enough so that the end cooling could be neglected.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
November, 1920.

ON THE ABSORPTION SPECTRUM OF HCL.

BY WALTER F. COLBY AND CHARLES F. MEYER.

BECAUSE of the theoretical interest involved, the authors have undertaken to add to existing experimental data regarding the absorption of HCl in the infra-red. The method of investigation is the same as that used by Imes,[1] except that a longer absorption chamber is used, and provision is made for heating the gas.

Observations have thus far been made, chiefly on the short wave-length side of the center of the "fundamental," *i.e.*, in the region $3.47\,\mu$ to $3.16\,\mu$. No evidence has yet been found of a definite head to the absorption band, but it is not said that none exists.

A part of the observations were made with a slit width of .25 mm. (half of that used by Imes), as it was thought that further structure of the lines constituting the band might be revealed. But the results give a first indication that there is no structure beyond that shown by Imes' curves.

The wave-lengths of twenty lines (absorption maxima) on the short wave-length side of the center have been measured. The agreement with Imes' measurements of the first twelve lines is fairly close, but the remaining eight lines show a more rapid convergence than he had prophesied. The twenty lines are represented by the formula:

$$\nu = 28{,}869 + 208.7n - 3.52n^2.$$

The fact that the frequencies are still represented by a parabola with the same degree of exactness, in passing from twelve members to twenty members, is strong evidence that this law is a real law of spacing.

It is hoped to improve the sensibility of the apparatus and to make further studies of the absorption of HCl and other halogen acids.

UNIVERSITY OF MICHIGAN, November, 1920.

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[1] Imes, Astrophys. Journ. 48, p. 125, 1918.

Flame Excitation of Luminescence.

By E. L. Nichols and D. T. Wilber.

THIS new type of excitation, which was discovered in the course of a recent study of luminescence at high temperatures, has the following characteristics:

1. Luminescence is produced by actual contact of certain zones of a hydrogen flame with the material to be excited.

2. The active region of the flame is the boundary between reduction and oxidation.

3. Rapid reduction and oxidation appear to be the essential features of this process of light production. This is inferred from the fact that the best known flame is the hydrogen flame, the flames of alcohol, ether, sulphur, carbon disulphide, etc., having been found inactive while, on the other hand, a full supply of free oxygen in the atmosphere surrounding the flame is likewise essential.

4. The reduction occurring at the surface of the body thus brought to fluorescence by the action of the flame is presumably preliminary to oxidation which is probably to be regarded as the direct cause of luminescence.

This view of the phenomenon is supported by the following facts:

5. The effect cannot be produced by heating in air or in hydrogen outside the flame.

6. Strong electrostatic fields, variously applied, fail to modify the effect, thus apparently excluding ionization as an essential feature.

7. *Photo-excitation, as an explanation, is excluded* since:

(*a*) Many of the most striking instances of flame excitation occur in substances not capable of being excited by light: *e.g.*, CaO, MgO, ZnO, SiO_2, ZrO_2, Al_2O_3, exceptionally pure zinc sulphide and calcium sulphide, etc.

(*b*) Many strongly photo-luminescent compounds like calcium tungstate and willemite are inactive under flame excitation.

(*c*) Where photo-luminescent substances are excited as in the case of Sidot blende and certain of the phosphorescent sulphides of Lenard and Klatt, flame excitation continues to temperatures far above those at which photo-luminescence ceases.

(8) *The effect is not of the nature of temperature radiation*:

(*a*) It is in many cases visible below the red heat.

(*b*) It does not follow Wien's law but exists only between perfectly definite temperature limits.

(*c*) The spectrum is a characteristic luminescence spectrum and not a temperature spectrum. It consists of the usual overlapping bands which in turn are made up of components forming series of equal frequency intervals.

(*d*) The afterglow, which is very brief is of the type of vanishing phosphorescence.

Physical Laboratory of Cornell University,
October, 1920.

A CONTINUOUS SPECTRUM FROM MERCURY VAPOR.

BY CLEMENT D. CHILD.

UNDER certain conditions the discharge through mercury vapor gives a green glow which shows the ordinary line spectrum of mercury together with one which is continuous from the red into the ultra-violet being very prominent in the green. This is the same spectrum as that shown by fluorescent mercury.

There are two conditions necessary for obtaining this by electrical discharge with any considerable brightness; the discharge must pass through hot mercury vapor that is not very far from the point of condensation and the current density must be small. As the current density is increased the line spectrum becomes more prominent and the continuous spectrum less so. With a condenser in the circuit only the line spectrum appears. Either alternating or direct current may be used. The glow at the cathode shows the line spectrum more prominently than does the positive column.

The presence of the continuous spectrum does not depend on the kind of glass, on the electrode used, nor on the purity of the mercury.

When the vapor is in the proper condition for giving the continuous spectrum it differs from the vapor at room temperature in the following respects. It fluoresces under the action of ultra-violet radiation and as a result ionization is not necessary in order to produce visible radiation. It is ionized at a lower potential. The carriers of the continuous spectrum are not charged. The spectrum is radically different. The glow continues for a longer time after the excitation ceases.

The relative brightness of different parts of the spectrum is apparently the same under all conditions.

These facts can be explained by assuming that the carriers of the continuous spectrum are molecules of two or more atoms of mercury and that these molecules are more easily excited when the temperature is somewhat above that of room temperature.

The carrier of the continuous spectrum is probably excited in the case of electric discharge as in the case of fluorescent mercury by ultra-violet radiation of wave-length 2536. This radiation is no doubt excited by the impact of the electron on the mercury atom.

A continuous spectrum is also found in the light produced by the discharge through sodium vapor.

COLGATE UNIVERSITY,
October, 1920.

NOTE ON SHEET STRIÆ AS THIN FILMS IN OPTICAL GLASS.

BY L. E. DODD.

SHEET striæ in optical glass are very thin regions where the index differs slightly from that of the glass outside [them. Extreme thinness and definiteness of bounding surfaces are indicated by observations such as Michelson's.[1] Samples have been obtained in the optical glass factory of the

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[1] Michelson, Bureau of Standards Scientific Paper, No. 333, pp. 33 and 34.

Bureau of Standards, demonstrating the high tenacity, or cohesiveness, of these striæ, which enables them to maintain their sheet structure when subject to agents that deform them. Instead of piercing and passing through such a sheet, a bubble, or a foreign particle of solid matter of some size, will drag the sheet locally along with it, giving at first the appearance of say a rock suspended in a net in air, or a croquet ball in a hammock. As the bubble or solid particle moves farther along it continues to drag the stretching sheet with it, and locally transforms it to a stria of the tubular, or the hair, type.

Because of this definiteness of the bounding surfaces and the high cohesiveness the samples demonstrate, a sheet stria seems to be a true film in the main body of the glass, as a soap bubble is a thin film in air. The glass film is remarkable as its substance is supposedly very near physically and chemically to that of the medium in which it exists immersed. Surface tension forces would be expected under the circumstances to be much weaker. The present samples indicate clearly the striking individuality of the films.

Since in these glass films definite bounding surfaces and surface tension are indicated, the question arises as to the physical relation between the film surfaces and the surrounding mass of glass. Does the glass mass "wet" the film, like water in contact with dry wood, or does it act like a drop of mercury on the same surface? The magnitudes of the forces coming into play in the dragging of the film through the glass mass by either bubble or solid particle, depend upon which case holds. If there is no adhesion there must be more nearly pure slippage, with viscosity forces playing a somewhat different rôle than in the other event.

These striæ offer an interesting study in molecular mechanics. Rayleigh[1] describes some experiments having to do with slippage and non-slippage between solid surfaces in which it is shown that there can be a sticking between two such surfaces when perfectly cleaned, which a slight greasing destroys. It would be interesting to know the extent to which there is such a sticking between the glass film and the glass mass in the molten state, or whether there is present a constituent in such a state that it has a greasing effect, or whether there is present a thin layer giving the "incipient seizing" or whether there is entire absence of adhesion.

Finally, a proposed experiment is the production of a film, if not the blowing of a bubble, of suitable liquid material in a suitable liquid, to approximate the sheet striæ in optical glass.

Bureau of Standards,
Washington, D. C.,
November 6, 1920.

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[1] Lord Rayleigh, "On the Lubricating and Other Properties of Thin Oily Films," Philosophical Magazine. 35. 157, 1918.

NEW BOOKS.

Advanced Lecture Notes on Light. By J. R. ECCLES. Cambridge: The University Press, 1919. Pp. 1 + 141.

The topics treated by the author are given in the following order: Rainbows, Magnifying Power, Chromatic and Spherical Aberration, Wave Theory, Interference, Diffraction, Polarization.

Some of these items are worked out in considerable detail with numerical examples. The volume is printed in quarto and on one side of the paper so that it can be used as an actual notebook.

Études de Photochemie. By VICTOR HENRI. Paris: Gauthier-Villars et Cie, 1919. Pp. vii + 218.

This is the first installment of an extended treatise covering ten years of work by the author and his colaborers.

The six chapters included in the present volume are devoted to absorption spectra of organic compounds; with special reference to the relations between absorption bands in the ultra-violet and the infra-red.

La Matérialisation de L'Énergie. By LOUIS ROUGIER. Paris: Gauthier-Villars et Cie, 1919. Pp. xii + 148.

A philosophical essay on Relativity and the Quantum Theory with special reference to what the author calls the *dematerialization* of matter and the *materialization* of energy.

Theorie der Strahlung und der Quanten. By DR. ARTHUR MARCH. Leipzig: Johann Ambrosius Barth, 1919. Pp. vii + 182. Price, M. 14.

This treatise on Radiation and Quanta, is in two nearly equal parts. The first deals with thermo-dynamic and electro-dynamic theories of radiation, including the introduction of the quantum concept. On this basis specific heats; spectral series, including the ultimate structure of line spectra; the Stark effect and entropy from the standpoint of the quantum theory are discussed.

Second Series *March, 1921* *Vol. XVII, No. 3*

THE

PHYSICAL REVIEW.

THE DYNAMICS OF CAPILLARY FLOW.

BY EDWARD W. WASHBURN.

SYNOPSIS.

Penetration of Liquids into Cylindrical Capillaries.—The rate of penetration into a small capillary of radius r is shown to be: $dl/dt = P(r^2 + 4\epsilon r)/8\eta l$, where P is the driving pressure, ϵ the coefficient of slip and η the viscosity. By integrating this expression, the *distance penetrated by a liquid flowing under capillary pressure alone* into a horizontal capillary or one with small internal surface is found to be the square root of $(\gamma rt \cdot \cos\theta/2\eta)$, where γ is the surface tension and θ the angle of contact. The quantity $(\gamma\cos\theta/2\eta)$ is called the coefficient of penetrance or the penetrativity of the liquid.

Penetration of Liquids into a Porous Body.—(1) *Theory.* If a porous body behaves as an assemblage of very small cylindrical capillaries, the volume which penetrates in a time t would be proportional to the square root of $(\gamma t/\eta)$. (2) *Experiments with mercury, water and other liquids* completely verify the theoretical deductions.

Dynamic capillary method of measuring surface tension is described. It possesses certain advantages on the static method of capillary rise.

1. INTRODUCTION.

THE statical problems connected with the rise of liquids in capillary tubes have been investigated on both the theoretical and the experimental side, but the dynamical aspects of the subject do not appear to have received much attention. Aside from the theoretical interest attaching to the subject, the dynamics of capillary flow have certain practical aspects in connection with the movement of water or oil through soils, the impregnation of wood and other porous materials with liquids, and the determination of the porosity and true density of porous bodies, as well as offering a new method for measuring the surface tension or viscosity of a liquid. The present paper is a contribution to the theory of the subject.

2. THE VELOCITY OF CAPILLARY FLOW.

In the following development of the laws governing capillary flow we shall simplify our problem initially to the extent of assuming that we are dealing with a single capillary tube of uniform internal circular cross-section throughout, the radius being r. The tube may be of any length or shape otherwise. Such a tube may be represented by AB in Fig. 1.

The tube is so arranged that the end A may, at any desired moment, be placed in contact with a liquid having the depth h above the center of the opening at this end. The end B may either be open to the atmosphere or the tube may be closed at this end and completely evacuated previous to establishing connection with the liquid at A. In the former case the pressure of the atmosphere will be neutralized but the tube will offer a small though calculable resistance to the efflux of the air which is displaced by the entering liquid. The equation covering this case will therefore contain a small correction term for the resistance of the air.

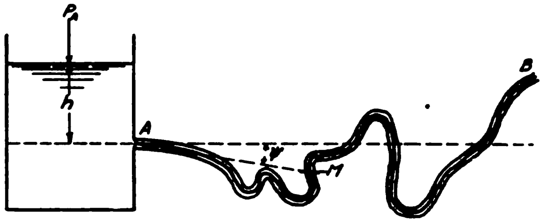

Fig. 1.

At the time 0 let connection be established with the liquid at the point A. At some time t_0 thereafter, the meniscus will have penetrated a distance l_0 at which point its velocity, which was initially very high owing to the small resistance encountered, will have dropped to such a value that the conditions of flow postulated in Poiseuille's law will have been established and these conditions will thereafter persist. Previous to the establishment of these conditions the velocity of flow will be governed by the laws of hydraulics, and an experimental investigation of this portion of the flow would perhaps disclose the two regions of turbulent flow and of slip flow, respectively, whose laws have been investigated by Reynolds, Sorkau[1] and others, and which are separated from each other and from the Poiseuille region by rather pronounced breaks. A theoretical and experimental study of the dynamics of capillary flow in these two regions offers much of interest but will not be entered into in this paper which will be confined to capillaries so small that the Poiseuille region covers practically the whole of the flow.

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[1] Sorkau, Phys. Z., 14, 739 (1913).

It has been found, by Osbourne Reynolds[1] that for a liquid of density D, the distance l_0 is determined by the relation

$$\frac{2rD}{\eta}\frac{dl}{dt} = k, \tag{1}$$

where the constant k, a pure number, was found by experiment to have the value 2000. If we combine this relation with the expression for (dl/dt) to be developed below (see equation 9) we obtain the relation

$$l_0 = \frac{\left(P_A + g\cdot D\cdot h + \frac{2\gamma}{r}\cos\theta\right)r^3\cdot D}{4\eta^2\cdot 10^3}, \tag{2}$$

from which the value of l_0 may be calculated. An extreme case will be represented by such a liquid as water in a capillary which it wets, the external driving pressure on the liquid being one atmosphere. For this case the above equation becomes

$$l_0 = \left(2.5\cdot 10^6 + \frac{400}{r}\right)r^3 \text{ cms.} \tag{3}$$

and the values of l_0 for various values of r are shown in the following table.

P_A = external driving pressure.

r, in mm.	1	10^{-1}.	10^{-2}.	10^{-3}.	10^{-4}.	10^{-5}.
l_0 in mm. (P_A = 1 atmos.) . .	1.3×10^4	13	1.3×10^{-2}	1.5×10^{-5}	3.3×10^{-8}	1.1×10^{-10}
l_0 in mm. (P_A = 0).	2	4×10^{-2}	4×10^{-4}	4×10^{-6}	4×10^{-8}	4×10^{-10}

From this table it is evident that l_0 is entirely negligible for very small capillaries.

For such capillaries we may therefore assume Poiseuille's law which obviously takes the following form, if we neglect for the moment any air resistance,

$$dV = \frac{\pi\Sigma P}{8\eta l}(r^4 + 4\epsilon r^3)dt, \tag{4}$$

where dV is the volume of the liquid which in the time dt flows through any cross-section of the capillary, l is the length of the column of liquid in the capillary at the time t, η is the viscosity of the liquid and ϵ its coefficient of slip, and ΣP is the total effective pressure which is acting to force the liquid along the capillary.

At the end of any time, t, the liquid will have gone a distance, l, along the capillary and the meniscus will have arrived at some point, M, where it is moving with the velocity (dl/dt). To calculate the magnitude of this velocity we proceed as follows: In equation 4 we can put

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[1] Reynolds, Scientific Papers, Volume 2, pp. 563 and 535.

$$dV = \pi r^2 dl, \tag{5}$$

thus giving us the following expression for the velocity

$$\frac{dl}{dt} = \frac{\Sigma P}{8r^2\cdot\eta\cdot l}(r^4 + 4\epsilon r^3). \tag{6}$$

We have now to find an expression for ΣP. The total driving pressure will be made up in general of three separate pressures, the unbalanced atmospheric pressure, P_A, the hydrostatic pressure P_h, and the capillary pressure P_s. P_A we shall take as constant. For P_h we can evidently write (see Fig. 1)

$$P_h = h\cdot g\cdot D - l_s\cdot g\cdot D\sin\psi, \tag{7}$$

where l_s is the linear distance from A to M, D is the density of the liquid and g is the acceleration due to gravity. Finally for P_s we have

$$P_s = \frac{2\gamma}{r}\cos\theta, \tag{8}$$

where γ is the surface tension of the liquid and θ is the angle of contact.

Summing up and substituting in equation 6 gives us the following law for the velocity of penetration,

$$\frac{dl}{dt} = \frac{\left[P_A + g\cdot D(h - l_s\sin\psi) + \frac{2\gamma}{r}\cos\theta\right](r^2 + 4\epsilon r)}{8\eta l}. \tag{9}$$

3. Integration of the Equation.

In equation 9, l_s, ψ, ϵ, and θ will be in general functions of t, and θ will perhaps also be a function of (dl/dt) and of the pressure gradient in the column of liquid. For *any* given known conditions, l_s and ψ could, of course, be determined and expressed as functions of l.

In the illustrations which follow we shall, however, assume ψ, θ, and ϵ, to be constants and on integrating equation 9 for these conditions we find the following integral:

$$\frac{(r^2 + 4\epsilon r)D\cdot g\sin}{8\eta}\psi t + l$$

$$= -\frac{P_A + D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta}{D\cdot g\cdot\sin\psi}\log_e\frac{P_A + Dg(h - l\sin\psi) + \frac{2\gamma}{r}\cos\theta}{P_A + D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta}. \tag{10}$$

The two limiting cases corresponding to $\psi = 90°$ and $\psi = 0°$ offer some special interest. For the former case the integral reduces to

$$\frac{(r^2 + 4\epsilon r)D\cdot g}{8\eta}t + l = \frac{-\left[P_A+D\cdot g\cdot h+\frac{2\gamma}{r}\cos\theta\right]}{D\cdot g}\log_e\left[1 - \frac{D\cdot g\cdot l}{P_A+D\cdot g\cdot h+\frac{2\gamma}{r}\cos\theta}\right]. \qquad (11)$$

For the latter case the integral becomes indeterminate, but a second integration for this case gives us

$$l^2 = \frac{\left(P_A + D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta\right)(r^2 + 4\epsilon r)t}{4\eta}. \qquad (12)$$

It may be noted here that with capillaries open at both ends, $P_A = 0$ and equations 11 and 12 assume the following forms:

$$\frac{(r^2 + 4\epsilon r)D\cdot g}{8(\eta - \eta_A)}t + l = \frac{-(\eta - \eta_A)\left(D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta\right) + D\cdot g\cdot \eta_A l_T}{D\cdot g\cdot(\eta - \eta_A)} \log_e\left(1 - \frac{D\cdot g\cdot l}{D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta}\right) \qquad (11a)$$

and

$$l^2 + \frac{2\eta_A l_T}{\eta - \eta_A}l = \frac{\left(D\cdot g\cdot h + \frac{2\gamma}{r}\cos\theta\right)(r^2 + 4\epsilon r)}{4(\eta - \eta_A)}t, \qquad (12a)$$

where η_A is the viscosity of air and l_T is the total length of the capillary.

4. VERTICAL CAPILLARIES; EXPERIMENTAL.

For a liquid which wets the capillary equation 11*a* becomes,

$$t = \frac{-8(\eta - \eta_A)l}{r^2\cdot D\cdot g} - \frac{8[(\eta - \eta_A)(h + \Delta h) + \eta_A l_T]}{r^2\cdot D\cdot g} \log_e\left(1 - \frac{l}{h + \Delta h}\right), \qquad (13)$$

where Δh is written in place of $2\gamma/r\cdot D\cdot g$. In order to test this equation the experimental arrangement shown in Fig. 2 was employed. The bore of the glass capillary was not examined for uniformity but its average radius calculated from the weight of the mercury thread which filled it was 0.01454 cms. Lengths were measured with a cathetometer and were as follows: $h = 51.38$ cms.; $l = 58.23$ cms.; Δh (the capillary rise) $= 10.01$ and 9.90 cms., two direct measurements as determined by the *static* method at two different points in the tube; and $l_T = 96$ cms. The distilled water employed was from the laboratory supply system and the temperature was room temperature, 30° ± 0.2°. The time of rise through the distance l, as observed with a stop-watch in several successive experiments was 40.2 ± 0.2 seconds. Taking $\eta_A = 0.00018$, $\eta = 0.00800$, $g = 980.1$, $D_{30}^{\circ} = 0.9956$, and calculating Δh and γ from equation (13) we find $\Delta h = 9.90$ and $\gamma = 70.2$ which agrees with the accepted value 71.03 (Landolt-Börnstein-Roth) within the accuracy of the above measurements. No great accuracy is claimed for these measurements which were made for illustrative purposes only.

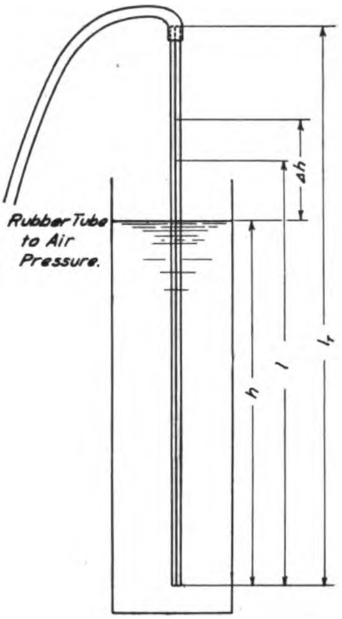

Fig. 2.

5. Horizontal Capillaries; Experimental.

An experimental study of equation 12*a* was made using mercury in open glass capillaries at room temperatures, and varying h, r and l. The results for $l = 95$ cms. are shown in Fig. 3 for two capillaries of different radii. The curves shown in this figure are graphs of the theoretical equation (12*a*) using values of η ($= 0.0152$), γ ($= 440$) and D given in the literature. The value of θ was computed by means of the observed points on the upper curve and found to be 112°. This value was used in plotting both curves.

The data obtained by experiment are indicated by small circles in the figure and it is evident that these experimental values are in good agreement with the theoretical curve. The apparent deviation of some of the points in the neighborhood of $h = 0$ is not significant, as under these conditions sticking friction developed and it was necessary to tap the capillary with the finger in order to keep the meniscus in motion. The observed times of flow for these points were therefore known to be somewhat too long. The measurements were not accurate enough to detect with certainty any variation of θ with h or dl/dt.

A number of experiments with different liquids (benzene, kerosene, alcohols, esters) for which values of η and γ were given in the literature demonstrated also the validity of equation 12*a* with respect to variations in these two variables.

The relation between *l* and *t* was also found to hold within the accuracy of the measurements in numerous experiments with horizontal capillaries wetted by the liquids.[1] In experiments with such liquids it is necessary after each run to elevate the capillary for drainage in order to insure the presence of a thin film of liquid on the wall. If too much liquid is left in the capillary, the observed times of flow will be too short while if the wall is not completely wet the value of θ will be greater than zero. The percentage differences between the observed times of flow with wet and dry tubes respectively, were found to be smaller the smaller the value of r, so that for sufficiently small capillaries the speed of wetting brought about by the diffusion of vapor ahead of the liquid might be sufficient to maintain the angle θ practically zero. Further evidence on this point is, however, needed.

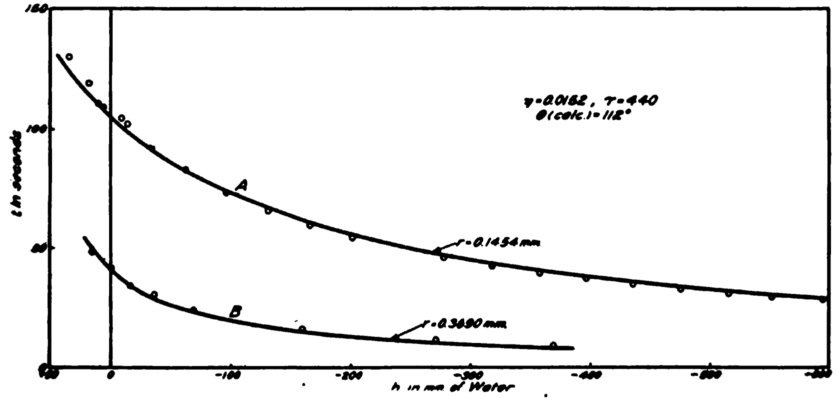

Fig. 3.

6. The Flow of Liquids under Capillary Pressure.

In discussing the rate of penetration of a capillary by a liquid moving under its own capillary pressure we shall consider only the two limiting cases of vertical capillaries and horizontal capillaries, respectively, the equation for the intermediate cases being obvious. The case under consideration is equivalent to assuming that $P_A + D \cdot g \cdot h$ may be neglected in comparison with $(2\gamma/r) \cos \theta$. In this discussion we shall also write $\epsilon = 0$, which is the case for all liquids which wet the capillary and has been shown experimentally to be also true for mercury in glass.

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[1] Bell and Cameron, Jour. Phys. Chem., 10, 659 (1906), deduced the relation, l^2/t = *const.* for a liquid moving through a horizontal capillary under a constant driving pressure and demonstrated its validity by experiments with water, alcohol, and benzene.

Case 1. *Vertical Capillaries.*—For vertical capillaries with small internal surfaces the logarithmic term in equation 10 may be expanded and if we reject all the terms of the series beyond the term containing l^2, equation 10 becomes identical with equation (14) for horizontal capillaries and will be discussed under that head.

Case 2. *Horizontal Capillaries.*—Equation 12 for this case becomes

$$l^2 \doteq \left(\frac{\gamma}{\eta}\frac{\cos\theta}{2}\right)rt, \tag{14}$$

air resistance being excluded or neglected. The corresponding equation for the rate is

$$\frac{dl}{dt} = \frac{r}{\eta}\frac{\gamma}{4l}\cos\theta, \tag{14a}$$

or in words, the rate at which a liquid penetrates any horizontal capillary (or any capillary with a small surface), under its own capillary pressure is directly proportional to the radius of the capillary, to the cosine of the angle of contact, to the ratio of the surface tension to the viscosity of the liquid and inversely proportional to the length already filled by the liquid.

The quantity $\gamma/\eta\,(\cos\theta)/2$ measures the penetrating power of a liquid and will be called the *coefficient of penetrance* or the *penetrativity* of the liquid. Its dimensions are obviously those of velocity. Stated in words, the penetrativity of a liquid is equal to the distance which the liquid will penetrate a capillary tube of unit radius in unit time, when flowing under its own capillary pressure. The presence of the factor $\cos\theta$ makes the penetrativity, in general, a function also of the nature of the material composing the capillary. Thus the penetrativity of mercury into a glass capillary is obviously a negative quantity. The penetrativity of a liquid which wets the capillary is otherwise independent of the material composing the capillary and is simply equal to $\frac{1}{2}$ the ratio of its own surface tension to its viscosity.

The relative penetrativities of two liquids are most easily compared with the aid of a coiled capillary which may be immersed in the liquid during the measurement. Time of flow is the only measurement required in using such a *penetratimeter*.

7. THE RATE OF PENETRATION OF A POROUS BODY BY A LIQUID.

For purposes of calculation we will assume that the penetration of the pores of a body by a liquid in which it is immersed may be taken as equivalent to the penetration of n cylindrical capillary tubes of radii $r_1, r_2, \cdots, r_n$ and we will ask ourselves the question; How much liquid will have entered the pores of the body at the end of the time t? For any one pore the volume which enters will obviously be

$$V = \pi r^2 l = \frac{\pi}{2\eta^{1/2}} t^{1/2} \left(P_E + \frac{2\gamma}{r} \right)^{1/2} r^3 \qquad (15)$$

and hence the total volume which penetrates all the pores will be

$$V = \pi \Sigma r^2 l = \frac{\pi}{2\eta^{1/2}} t^{1/2} \Sigma \left(P_E + \frac{2\gamma}{r} \right)^{1/2} r^3, \qquad (15a)$$

where P_E is the total external pressure behind the liquid.

For a given body under constant driving pressure this equation has the form

$$V = k \left(\frac{t}{\eta} \right)^{1/2} \qquad (16)$$

and for capillaries so small that P_E is negligible in comparison with $2\gamma/r$ this can also be written

$$V = k' \left(\frac{\gamma}{\eta} \right)^{1/2} t^{1/2}, \qquad (17)$$

where k' is independent of the nature of the liquid, that is, the degree of penetration is proportional to the square root of the time of soaking and to the square root of the ratio of the surface tension to the viscosity.

If the pores of the body cannot be taken as equivalent in their behavior to cylindrical pores, equations 16 and 17 would, of course, not be applicable. If the cross-section of a pore changes with its length and especially if the pore contains an enlargement or ends in a pocket, the above equations of course would not apply. Furthermore these equations would in all probability not apply to the filling of micro-pores, that is, pores with diameters approaching the molecular diameter of the liquid. Whether or not, therefore, equation 17 applies to the absorption of a liquid by a porous body in a given case could only be determined by experiment.

Some results obtained by Cude and Hulett[1] on the rate of penetration of charcoal by water give us an opportunity of making a comparison of the above relation. The comparison is shown in Fig. 4. It is evident from this figure that the linear relation holds good within the experimental error for the initial period of penetration. Whether the later deviation corresponds to the complete filling of all except the micro-pores, or whether it is due to the slower filling of enlarged pores or pockets cannot, of course, be determined although the former supposition seems the more probable in this instance.

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[1] Cude and Hulett, Jour. Amer. Chem. Soc., 42, 391 (1920).

It is of some interest to note that if a porous body consists of *n* capillaries of radius *r* and length *l*, each of these quantities may be calculated, if one of them is known.

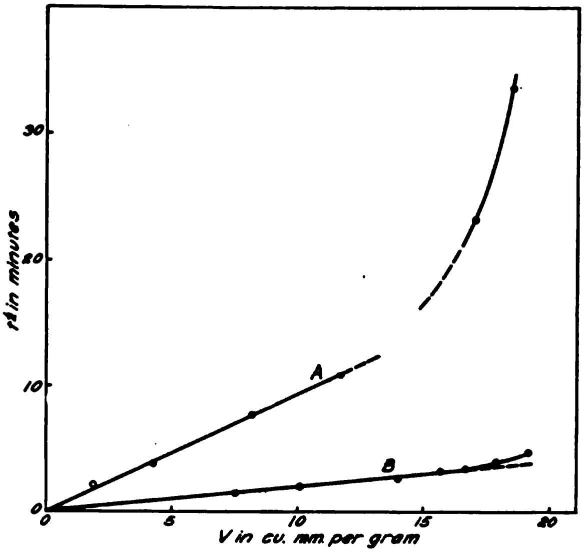

Fig. 4.

The velocity of penetration of charcoal by water. Data by Cude and Hulett. Curve *A* from their Table I., curve *B* from their Fig. 8.

8. The Measurement of Surface Tension by the Dynamic Method.

Horizontal capillaries are perhaps the most convenient to use for this purpose, although, by using a vertical capillary immersed some distance in the liquid and allowing the liquid to drop from a point some distance *beyond* its equilibrium position, it should be possible to obtain very good results. With horizontal capillaries under practically zero head, equation 12*a* shows that the accuracy with which *r* can be measured is practically determined by the accuracy with which η is known. The factors *l* and *t* can be made as large as desired and the value of *r* called for is the average value for the whole tube. In this respect the dynamic method is superior to the static where the height of the capillary rise for a given tube is not under the control of the operator, and where the value of *r* involved is the particular value at the position occupied by the meniscus when equilibrium is reached, a value rather difficult to determine accurately.

The writer is indebted to Dr. E. N. Bunting of this department for valued assistance in the experiments reported in this paper. The writer also recalls with much pleasure his discussions of the subject of capillary flow with his colleague, Dr. Eric K. Rideal.

Department of Ceramic Engineering,
University of Illinois,
August 20, 1920.

THE MASS ABSORPTION AND MASS SCATTERING COEFFICIENTS FOR HOMOGENEOUS X RAYS OF WAVE-LENGTH BETWEEN 0.13 AND 1.05 ÅNGSTRÖM UNITS IN WATER, LITHIUM, CARBON, NITROGEN, OXYGEN, ALUMINUM, AND IRON.

BY C. W. HEWLETT.

SYNOPSIS.

I. *Present Status of the Problem of Absorption and Scattering of X Rays by Elements of Small Atomic Weight.*—The mass scattering and mass absorption coefficients are defined, and a brief summary is given of the results of previous absorption and scattering measurements in the regions of wave-lengths on the short wave-length side of the K radiation of the absorbing substance, in so far as they relate to the present investigation. The current views in regard to the mechanism of absorption and scattering, and some of the theoretical work which has been advanced to account for these phenomena, has been outlined with regard to their bearing on the work reported in this paper.

II. *Absorption and Scattering of Homogeneous X Rays by Water, Lithium, Carbon, Nitrogen, Oxygen, Aluminium, and Iron at various Wave-lengths within the Region Bounded by 0.13 and 1.05 Å. U.*—(1) The total absorption coefficient of homogeneous X rays obtained by passing the X rays from a Coolidge tube through a Bragg X ray spectrometer was measured for the above-named materials at various wave-lengths in the range indicated. The total mass absorption coefficient was found to be proportional to the cube of the wave-length of the X rays over certain regions, but for all the substances where the investigation was complete over the whole range mentioned, the constant of proportionality was different for different ranges of wave-length. (2) *Hydrogen* is apparently an exception to the above statement, for its total mass absorption coefficient appears to be proportional to the nine halves power of the wave-length of the X rays. (3) *The constant which multiplies the cube of the wave-length* to give the true mass absorption coefficient is found to be approximately proportional to the cube of the atomic number of the absorbing element, except for lithium, and this indicates that Moseley's law will not be found to hold for this latter element. (4) *The true mass absorption coefficient for iron* is apparently not proportional to the cube of the wave-length for waves between 0.70 and 1.05 Å. U. No explanation is suggested for this. (5) *The true mass absorption coefficient* for the elements, with the exception of iron, for wave-lengths less than 0.20 Å. U. are smaller than is to be expected from theory if the electron has a diameter of the order of 10^{-13} cm. Compton's modification of the expressions for these absorption coefficients accounts for these experimental results if we assign to the electron a diameter of 0.75 to 0.85×10^{-10} cm. (6) *The lack of this decrease in the coefficients for iron is* attributed to a shrinkage in the diameter of the electrons in the atoms on account of the closer packing of the electrons in this element. (7) *The mass scattering coefficient* of all the substances used is less than that given by

Thomson's theory, but no explanation has been found. (8) *The mass scattering coefficient for hydrogen* is approximately twice that for the other elements, and this is interpreted as further evidence that hydrogen has twice the number of scattering electrons per unit mass as the other elements. (9) *The mass scattering coefficient for iron* apparently increases for wave-lengths longer than 0.70 Å. U. It is suggested as an explanation of this fact that the distance between the scattering electrons in the iron atom is of the order of magnitude of 0.70 Å. U.

III. *Projects for Further Absorption and Scattering Measurements.*—(1) The scattering of X rays should be measured as a function of the angle of scattering. (2) The absorption and scattering of hydrogen should be measured directly.

I. PRESENT STATUS OF PROBLEM OF ABSORPTION AND SCATTERING OF X RAYS BY ELEMENTS OF SMALL ATOMIC WEIGHT.

THE investigation reported below was suggested to the writer by Dr. A. W. Hull of the General Electric Co., Schenectady, New York.

Several investigators have measured the absorption of X rays by various kinds of matter. Much of the earlier work was done with beams of X rays which had been filtered to ensure homogeneity of wave-length, but it appears that in many cases the X rays even after filtering were decidedly heterogeneous. The advent of the X-ray spectrometer has made it possible to obtain highly homogeneous beams of X rays. Experiment shows that X rays of a definite wave-length are absorbed according to an exponential law in passing through matter. If I_0 represents the intensity of a beam of homogeneous X rays as it enters an absorbing substance, and I the intensity of the beam after it has passed through such a thickness of the substance that the mass per square cm. penetrated is m, then the following equation expresses the relation between these quantities:

$$I = I_0 e^{-\frac{\mu}{d} m},$$

where d is the density of the substance and μ is called the total absorption coefficient for the substance. The quantity μ/d is usually called the "total" or "gross mass absorption coefficient."

The beam of X rays in passing through the absorbing matter loses its intensity in two distinct ways. First, the disturbance in passing over the electrons sets them in vibration, and this results in a reëmission of X radiation, by these disturbed electrons, of the same wave-length as that of the beam of X rays. This reëmitted radiation is called "scattered radiation," and it is thought that the electrons outside the nucleus of the atoms are responsible for this scattering, those electrons within the nucleus being held by such enormous forces that their effect, as well as that of the nucleus, is negligible as far as scattering is concerned.

Second, if the beam of X rays is of shorter wave-length than the characteristic radiation of the absorbing substance, electrons are ejected from the atoms of the absorbing matter by the X rays, and these ejected electrons carry with them the energy which they took from the beam of X rays. Later on when another electron falls into the atom to take the place of the ejected one there is emitted a train of X waves with the characteristic frequency of the absorbing substance. In this way the beam of X rays loses energy by "fluorescent absorption" or "true absorption," and the absorbing substance at the same time emits "characteristic radiation." Accordingly the total or gross mass absorption coefficient is usually considered in two parts as follows:

$$\frac{\mu}{d}=\frac{\mu_0}{d}+\frac{\sigma}{d},$$

where μ_0/d is that part which represents the "true absorption" and is called the "true" or "fluorescent mass absorption coefficient," and σ/d is called the "mass scattering coefficient."

Now it has generally been found for regions of wave-length removed from the K and L radiation of the absorbing substance, that the values of the total mass absorption coefficient when plotted against the cube of the wave-length, fall very closely on a straight line which cuts the μ/d axis at some point between 0.10 and 0.20 according to the substance. This is usually interpreted by saying that the true mass absorption coefficient over this region is proportional to the cube of the wave-length, while the mass scattering coefficient is independent of the wave-length and is given by the intercept on the μ/d axis by the straight line. The expression for the total absorption coefficient may then be written

$$\frac{\mu}{d}=K\lambda^3+\frac{\sigma}{d}.$$

K is a constant for a given absorbing substance and has been found to be roughly proportional to the cube of its atomic number.

If the mechanism of scattering is as simple as outlined above we should expect the mass scattering coefficient to be very nearly the same for all substances and for all wave-lengths so long as the wave-length of the X rays used are long compared to the dimensions of the electron, and short compared to the distance between the electrons. For, equal masses of all substances are supposed to contain very nearly equal numbers of scattering electrons, and if the X rays are long compared to the dimenssion of the electron then there can be no interference between the portion of scattered radiation from the different parts of the electron. If the wave length of the X rays is comparable to the dimensions of the electron then this latter statement would no longer hold true and we should expect the mass scattering coefficient to be a function of the wave-length. If the wave-length of the X rays is short compared to the distance between the electrons, then we should expect the electrons to scatter independently of one another, so that as long as this condition is satisfied the distance between the electrons in the atom would have no effect on the amount of scattered radiation. If the wave-length is comparable to the distance between the electrons then the electrons will no longer scatter independently and the amount of scattered radiation will be greater than when they scattered independently. In this latter case there is said to be "excess scattering."

J. J. Thomson,[1] on the basis of the electromagnetic theory and the two further assumptions that the electrons are small compared to the wave-length of the X rays, and that the X rays are short compared to the distance between the scattering electrons, has derived an expression for the amount of scattered radiation which for the purposes of the present discussion may be written in the following form:

$$\frac{\sigma}{d} = 0.4019 \frac{N}{A},$$

where N is the atomic number and A is the atomic weight of the absorbing element.

It is not so easy to account for the view that the true mass absorption coefficient is proportional to the cube of the wave-length. In the same place in which the expression for the scattering was deduced, J. J. Thomson has also deduced an expression for the true absorption, which gives the true mass absorption coefficient proportional to the cube of the wave-length. The additional assumptions made as to the nature of the X ray, however, are rather arbitrary. A. H. Compton[2] has shown by introducing Moseley's law into this expression of Thomson's, (Moseley's law being that the wave-length of a given type of characteristic radiation is proportional to the square of the atomic number), that the "atomic absorption coefficient" is approximately proportional to the fourth power of the atomic number. Calculating from this "atomic absorption coefficient" the true mass absorption coefficient, it appears from this modified expression of Thomson's that the true mass absorption coefficient should be approximately proportional to the cube of the atomic number. But while Moseley's law is known to hold very well for the elements of high atomic number, it does not hold so well for the lighter elements, and consequently we might expect to find the value of

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[1] J. J. Thomson, Conduction of Electricity through Gasses, 2d ed., p. 325.

[2] A. H. Compton, Phys. Rev., Vol. 14, Sept., 1919, p. 247.

K for the elements used in the present investigation only roughly proportional to the cube of the atomic number.

The usually accepted diameter of the electron, is 1.9×10^{-13} cm., which is calculated on the assumptions that the mass of an electron is entirely electromagnetic mass, and that the charge of an electron is a uniform spherical surface distribution. In accordance with this and the views outlined above we should expect the mass scattering coefficient for all X rays producible by ordinary methods, and even for all γ rays, except possibly the very shortest ones about which there still exists uncertainty in regard to the limiting shortness, to be independent of the wave-length so long as the wave-length is not long enough to give rise to "excess scattering." But experiment does not confirm this conclusion.

For very short X rays, and γ rays (below 0.20 Å.U.) the total mass absorption coefficient becomes much smaller for elements of small atomic weight than the calculated value for the mass scattering coefficient alone. In order to explain this marked decrease in the total mass absorption coefficient of very short wave-lengths A. H. Compton[1] has modified the expression given by Thomson, to take account of the relative size of the electron and the wave-length of the X rays. In his papers, Compton considers three types of electron, a rigid spherical electron, a deformable spherical electron, and a deformable ring electron, the latter of which he regards as the most suitable for the purpose in hand. The calculation for the ring electron is only approximate on account of certain assumptions made in order to render the calculation manageable. His expression for the total mass absorption coefficient may be written in the following form

$$\frac{\mu}{d} = \varphi K \lambda^3 + \theta \frac{\sigma}{d},$$

where φ and θ are both functions of the ratio of the wave-length of the X rays to the diameter of the electron. The values of φ and θ lie between 0 and 1 and approach 1 as the wave-length becomes large compared to the diameter of the electron. The other quantities have the same meaning as before. Taking the only available data at very short wave-lengths, which was on aluminium, Compton concluded that if the electron was a ring 1.85×10^{-10} cm. diameter, the marked decrease in the total mass absorption coefficient at very short wave-lengths could be accounted for. For light elements the scattering for very short wave-lengths is comparable to or even greater than the true mass absorption coefficient. Consequently it would seem advisable, in obtaining information to test the validity of Compton's equation, to work with the elements of small atomic number.

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[1] A. H. Compton, PHYS. REV., Vol. 14, July, 1919, p. 20; Sept., 1919, p. 247.

Barkla and White[1] found an abnormally great mass absorption coefficient for aluminum at 0.37 Å. U. and concluded from this that aluminum should give off a characteristic radiation in this region, which they called "J" radiation. Other observers[2] failed to confirm the work or conclusions of Barkla and White. It appeared that an investigation of some of the other light elements in the region from 0.10 to 1.0 Å. U. would be of interest from the standpoint of a possible characteristic absorption on the short wave-length side of the *K* radiation of the absorbing substance.

It is also of interest to know whether the true mass absorption coefficient is proportional to the cube of the wave-length for some of the lightest elements, for if some of these lighter elements should absorb according to another law of simple expression, we should have a new point of view in regard to the mechanism of the true absorption.

II. ABSORPTION AND SCATTERING OF HOMOGENEOUS X RAYS BY WATER, LITHIUM, CARBON, NITROGEN, OXYGEN, ALUMINUM, AND IRON, AT VARIOUS WAVE-LENGTHS WITHIN THE REGION BOUNDED BY 0.13 AND 1.05 Å. U.

During the larger portion of this work the source of X rays was a water-cooled tungsten button anode Coolidge tube. The X rays from the tube were passed into a Bragg X-ray spectrometer constructed by Dr. A. W. Hull[3] and used by him sometime ago. For a description of the spectrometer see F. K. Richtmyer and Kerr Grant.[4] The tube was frequently operated continuously for five hours with one kilowatt input to the tube. For the greater part of the time the regulation of the voltage across the tube and the current through it left nothing to be desired in this direction. A 1,000-volt generator with an automatic voltage regulator supplied the power to the transformer and there was an adjustable inductive regulator in the primary of the transformer, so that any progressive change in the operating conditions could be adjusted. In order to still further render the results free of error owing to slight changes in the intensity of the X ray beam, a set of observations was spread over several minutes, alternating the intensity determinations with and without the absorbing screen in such a manner that the mean intensity without the absorbing screen corresponded to the same time as that with the screen. In order to calculate the mass absorption coefficient it is sufficient to know the ratio of the intensity of the beam after passing through the absorbing screen, to its intensity without the screen. The time required for the gold leaf to move over the same distance on the scale of the electroscope, was determined for the beam of X rays with and without the absorbing screen, and from these time intervals the corresponding rates of motion of the gold leaf were calculated and corrected for the natural leak of the instrument. These corrected rates of motion of the gold leaf were taken as proportional to the intensities of the X ray beam. Since the beam of X rays was homogeneous, this latter assumption is justified.

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[1] Barkla and White, Phil. Mag., Vol. 34, October, 1917, p. 270.

[2] Hull and Rice, PHYS. REV., Vol. 8, September, 1916, p. 326.
Duane and Shimizu, PHYS. REV., Vol. 13, Apr., 1919, p. 288; Vol. 14, Nov., 1919, p. 389.
Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

[3] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.

[4] Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

The zero reading of the instrument was determined several times during the course of the investigation by determining the position of three of the *L* lines of the tungsten spectrum. With the tube in any one adjustment the zero determinations always agreed to within 0.01 degree, and this is as close as an individual setting of the instrument could be made.

With the exception of lithium all the substances used as absorbers were of very high purity, all the substances having been especially selected for this work.

With solid absorbing substances the areas and masses of the screens were determined accurately, and the mass absorption coefficients determined from the mass per square centimeter of the absorbing screen. In order to avoid any error arising from lack of homogeneity of the absorbing screens, the screens were moved along during a set of observations so that the beam of X rays passed through it in eight or ten different places.

For the work on water, cells with parallel sides were made by cutting slots in plate glass of appropriate thickness, and then cementing on the two sides of these slots very thin aluminum on microscope cover glasses. The intensity of the beam of X rays was determined after passing through the empty cell, and then after passing through the cell filled with distilled water.

Liquid oxygen was made by condensing electrolytic oxygen gas at atmospheric pressure in a large test tube emersed in liquid air. A special cylindrical dewar flask was made of very thin lime glass for holding the liquid oxygen. A screen of aluminum was made up by trial that had nearly the same total absorption as did the empty dewar flask. In order to determine the absorption coefficient of liquid oxygen at a given wave-length, three sets of observations were made at this wave-length as follows: (1) The ratio of the intensities of the beam after passing through the aluminum screen, and after passing through the empty flask. (2) The ratio of the intensities of the beam after passing through the aluminum screen and after passing through the flask filled with liquid oxygen. (3) The ratio of the intensities of the beam after passing through the aluminum screen, and after passing through the flask filled with water. It is hardly necessary to remark that the dewar flask was so mounted that it could always be placed in the same relative position with respect to the beam of X rays. The same procedure was followed in working with liquid nitrogen. The liquid nitrogen was made by condensing gaseous nitrogen at atmospheric pressure in a test tube immersed in liquid air boiling at reduced pressure. The densities of liquid oxygen and nitrogen were taken as 1.14 and 0.80 respectively.

The lithium used was made for this work by Mr. J. C. O'Brien of the research laboratory and was obtained by electrolyzing molten lithium chloride. Considerable difficulty was experienced in obtaining pure lithium. This was evidenced by the fact that different specimens of lithium gave widely different values for the absorption coefficient even at the same wave-length. Finally the sample that gave the smallest absorption coefficient was chosen, and the data given was obtained with this sample. To protect the lithium from oxidation the freshly cut surfaces were enclosed in very thin sheet aluminum. To all outward appearances the lithium used was very pure. The salt was marked chemically pure, and the metal was remelted and forced through a small hole under melted paraffin. This fact, that different samples of lithium, all apparently homogeneous and pure, would give values for the mass absorption coefficient differing by 50 per cent. is a marked illustration of how an exceedingly small amount of an element of high atomic weight might easily be detected and measured when mixed with an element of very low atomic weight. Even if the lithium used in this work is impure enough to affect seriously the accuracy of the determinations of the true mass absorption coefficients, it is probable that the mass scattering coefficient is very closely correct. It would take a relatively large amount of any impurity to affect seriously the mass scattering coefficient, since this is so nearly the same for all substances.

The observed values of the gross mass absorption coefficient for the various substances are given in Table I., while the curves show this data graphically for the purposes of the discussion that is to follow.

An inspection of the curves brings out the following outstanding features:

1. The values of the total mass absorption coefficient when plotted against the wave-length of the X rays lie very closely to a smooth curve.

2. The values of the total mass absorption coefficient when plotted against the cube of the wave-length of the X rays lie very closely to straight lines in certain regions.

TABLE I.

Total Mass Absorption Coefficients.

$$\frac{\mu}{d} = K\lambda^3 + \frac{\sigma}{d}.$$

Wave Length Ångström Units.	Lithium.	Carbon.	Nitrogen.	Water.	Oxygen.	Aluminum.	Iron.
.105		.146					
.130		.156		.165	.149	.179	.437
.154		.165	.165	.183	.165	.204	.620
.179		.166		.197	.176	.242	.864
.203			.178				
.210		.176		.204	.187	.288	1.16
.228		.178		.213			
.241		.185		.213		.332	
.252		.188	.195				
.265		.191		.228	.214	.403	2.12
.290		.200				.486	
.301			.225				
.309				.251			
.314	.176				.250		
.319						.577	
.324		.206					3.76
.350			.251				
.373	.195	.229		.301	.302	.825	5.66
.422	.218	.250		.360	.365	1.11	8.12
.471	.228	.280		.444	.440	1.60	11.22
.520	.256	.321		.524	.529	2.03	15.50
.569	.286	.365		.629	.619	2.60	19.40
.618	.318	.413		.759	.802	3.31	24.50
.666	.361	.477		.910	.980	4.08	31.05
.715	.422	.551		1.073	1.126	5.17	37.14
.764		.636		1.310	1.337	6.30	45.65
.812		.729		1.530	1.560	7.58	52.35
.861		.838		1.80		8.96	60.60
.910		.959		2.06		10.38	70.30
.958		1.119		2.43		12.19	79.63
1.006		1.311		2.74		14.03	91.70
1.054		1.470		3.19		16.55	

3. There is no abrupt decrease in the absorption coefficient with increasing wave-length over the region investigated, such as occurs in the region of the *K* and *L* characteristic radiation of the absorbing substance.

4. For carbon, oxygen, water and aluminum the total mass absorption coefficient at wave-lengths below 0.20 Å. U. decreases much more rapidly than is to be expected from the expression $\mu/d = K\lambda^3 + (\sigma/d)$.

5. The peculiarity mentioned in (4) is not observed for iron.

6. The $(\mu/d,\ \lambda^3)$ line for carbon and water bends up at 0.85 and 0.60 Å. U. respectively, becoming straight again with a greater slope for longer wave-lengths.

7. For iron the $(\mu/d,\ \lambda^3)$ line bends down at 0.70 becoming straight again for longer wave-lengths.

These features will be discussed in the order mentioned:

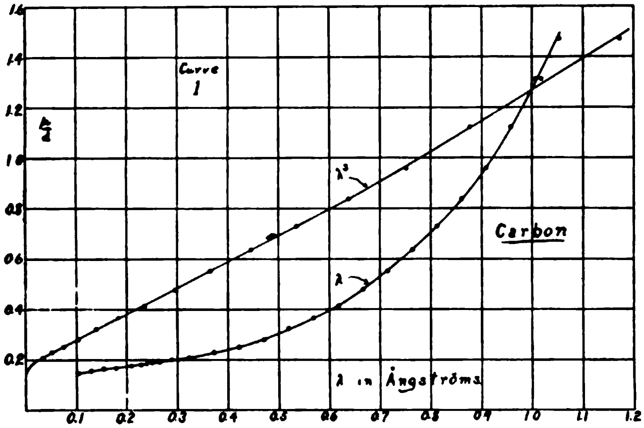

Fig. 1.

1. It is seen that most of the observed values deviate from the curves by less than 1 per cent. or 2 per cent. Consequently we may look upon the values read off from the curves as correct for these particular samples to much better than 1 per cent. as far as they may be affected by subjective error. This does not apply to the points at the extreme ends of the curves where the values read from the curves may be in error by as

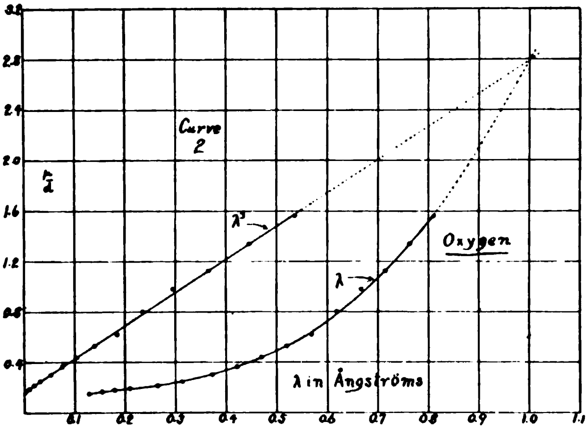

Fig. 2.

much as 2 per cent. or 3 per cent. It is believed that there are no consistent instrumental errors as large as 1 per cent.

2. The relation $\mu/d = K\lambda^3 + (\sigma/d)$ is confirmed over certain regions.

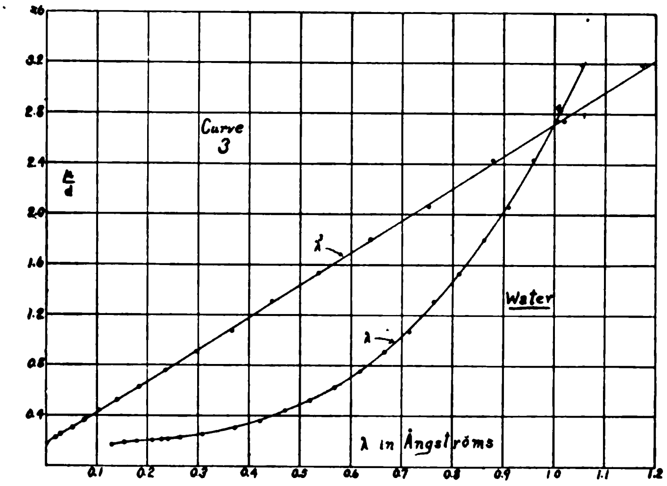

Fig. 3.

All of the curves have a straight portion beginning in the neighborhood of 0.20 Å. U. and extending to 0.50 Å. U. or beyond. For the purpose of comparing the applicability of this expression to the absorption and

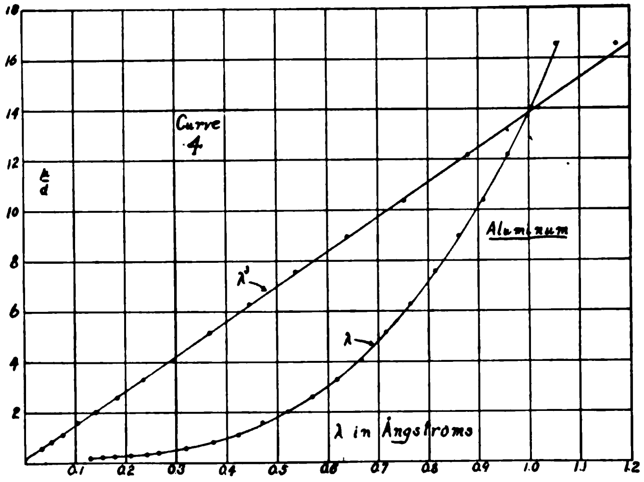

Fig. 4.

scattering of the substances it would seem reasonable to select this portion of each of the curves. Consequently we shall calculate K for each substance from the straight portion of each curve immediately on the long wave-length side of 0.20 Å. U., while the mass scattering coefficients for wave-lengths long compared to the diameter of the electron are to be obtained by finding the intercept of this portion of the curve produced, with the μ/d axis. The mass scattering coefficient determined in this way, and its value calculated from Thomson's expression are given in Table II.

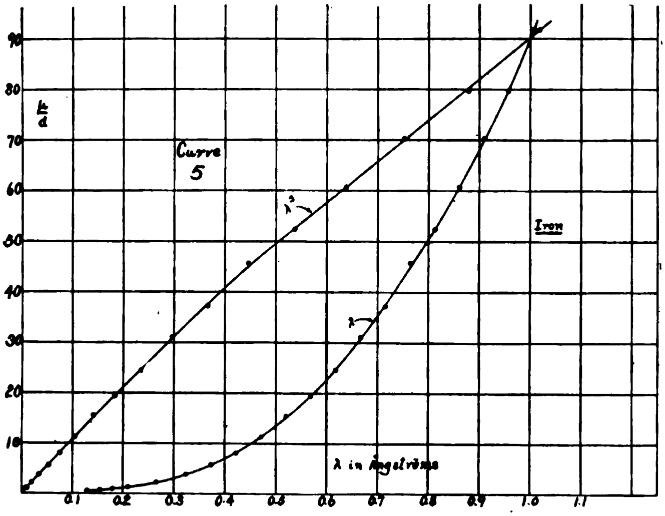

Fig. 5.

TABLE II.

Element.	$\frac{\sigma}{d}$ Observed.	$\frac{\sigma}{d}$ Calculated.	K	$\frac{K}{N^3}$ 10³.
Hydrogen	.309	.3987		
Lithium	.157	.1737	0.702	26.0
Carbon	.175	.2009	1.035	4.79
Nitrogen	.168	.2008	1.98	5.77
Oxygen	.165	.2010	2.644	5.20
Aluminum	.173	.1928	12.54	5.71
Iron	.18	.1871	106.	6.03

This table shows that the observed mass scattering coefficient is considerably smaller than that calculated from Thomson's expression. It should be clearly noted that the observed σ/d given in Table II. is the intercept of the straight portion of the curves from 0.20 to about 0.40 Å. U. For most of the substances the intercept of the curve for longer wave-lengths is different from that given in the table. For instance, the intercept for aluminum between 0.50 and 1.10 Å. U. is 0.12, which agrees with the value of σ/d found by Hull and Rice,[1] and Richtmyer and Grant.[2]

It is interesting to note in this connection that the mass scattering coefficient for hydrogen, calculated from the water and oxygen data is approximately twice that of the other elements. This may be interpreted as further evidence that the number of electrons per unit mass for hydrogen is twice that for the other elements.

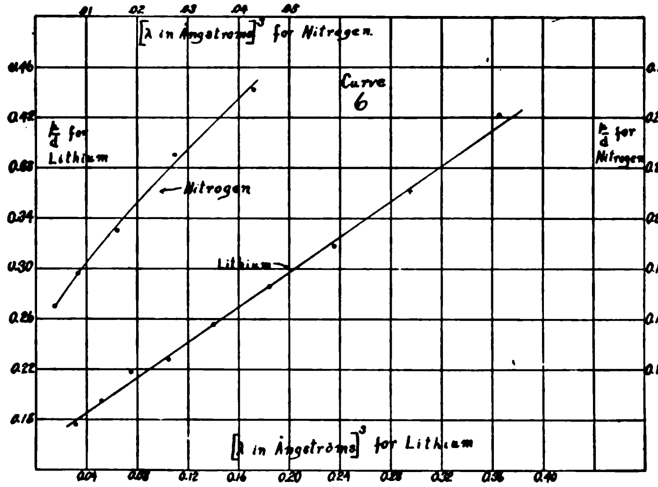

Fig. 6.

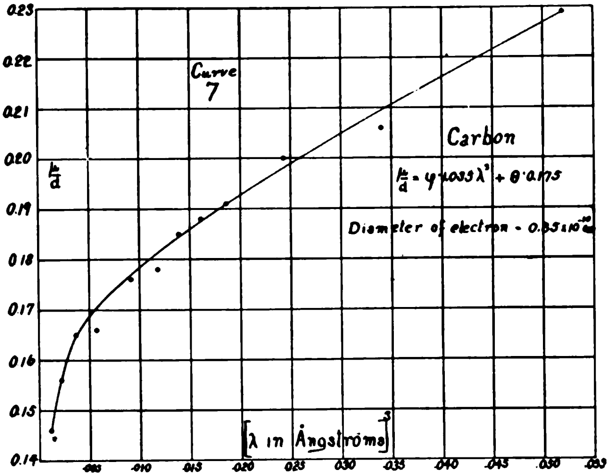

Fig. 7.

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[1] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.

[2] Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

It is very puzzling why the observed value of the mass scattering coefficient should be less than the value calculated from Thomson's expression. It was thought at first that possibly enough of the scattered radiation entered the ionization chamber to account for this discrepancy, but a calculation showed that a great deal less than 0.1 per cent. of this radiation entered the ionization chamber, even in the most favorable case. It is hardly conceivable that the mass of the electrons inside the atom are sufficiently different from the value found for electrons outside the atom to account for this decreased scattering. In fact a calculation shows that if such an increase of mass were due to the velocity of the

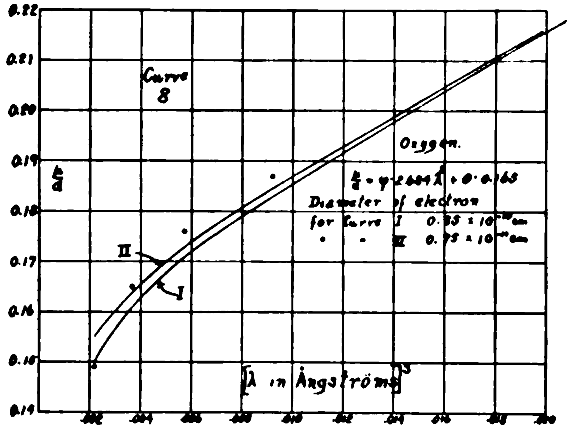

Fig. 8.

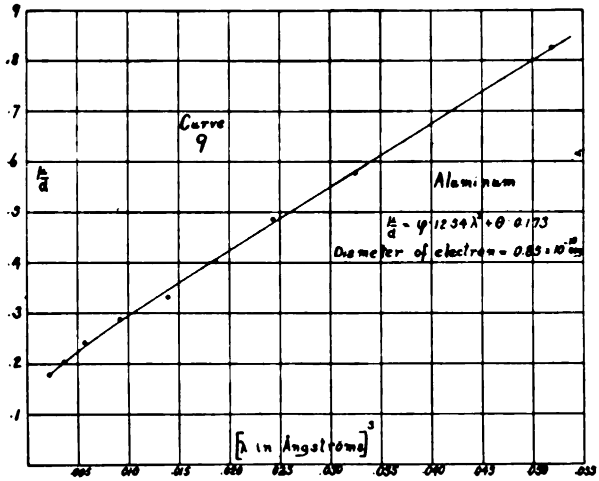

Fig. 9.

electrons in the atom, they would have speeds of the order of 0.4 the velocity of light. It appears as if one of three alternatives must be invoked to account for the discrepancy: (1) The method used does not give the correct value of the mass scattering coefficient. (2) The number of scattering electrons is not strictly proportional to the atomic number. (3) The scattering electrons do not all scatter alike, and there is some other factor which affects the scattering of an electron which has not been considered in the deduction of Thomson's expression.

The value of K calculated in the manner just mentioned is also given in Table II., together with the quotient of this quantity by the cube of the atomic number. For the last five elements in the table this ratio is seen to be roughly constant. For lithium however the value of K is far from what would be expected if Moseley's law applied to it. This last fact is in agreement with a prediction by R. A. Millikan.[1]

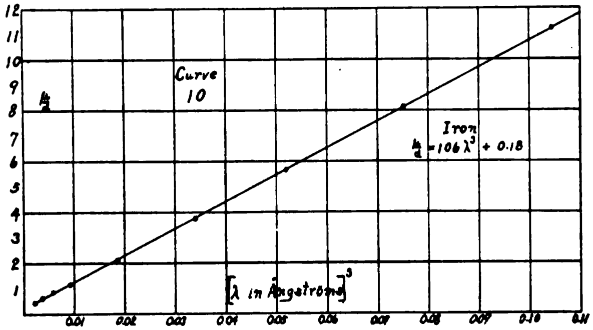

Fig. 10.

3. The absence of any characteristic absorption in the region investigated confirms the work of other investigators[2] in the conclusion that these substances have no characteristic radiation in this region.

4. In order to apply Compton's theory of an electron of diameter of the order of 10^{-10} cm. it is here assumed that the value of σ/d in the equation ($\mu/d = \varphi K\lambda^3 + \theta(\sigma/d)$ is that given as the observed value in Table II. θ as used here is Compton's σ/σ_0, and its value as well as that of φ is taken from the curves on page 254 of Compton's paper.[3] The short wave-length data for carbon, oxygen, aluminum, and iron, are shown in Curves 7, 8, 9, and 10, and the full curves shown, except that for iron, are calculated from the above equation using the values for the

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[1] R. A. Millikan, The Electron, 1st ed., p. 212.

[2] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.
Duane and Shimizu, PHYS. REV., Vol. 13, Apr., 1919, p. 288; Vol. 14, Nov., 1919, p. 389.
Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

[3] A. H. Compton, PHYS. REV., Vol. 14, Sept., 1919, p. 254.

diameter of the electron shown on the curves. It appears that the data for carbon and aluminum are satisfactorily satisfied by assuming an electron whose diameter is 0.85×10^{-10} cm., while it is necessary to assume a still smaller electron namely 0.75×10^{-10} cm. to fit the oxygen data even approximately. These values are somewhat smaller than that given by Compton, namely 1.85×10^{-10} cm. Of course if the value of the mass scattering coefficient given by Thomson's equation is used, a larger electron would give the observed scattering at the very shortest wave-lengths, but then the curve would not fit the data satisfactorily at longer wave-lengths. No success attended the attempt to fit the data in this manner.

5. Since the electrons in the atoms are more closely packed the greater the atomic number, it is not inconceivable, that the electrons may alter in size and be smaller when they are more closely packed. With smaller electrons the wave-length at which φ and θ would be appreciably different from 1 would be shorter, and consequently we should expect the straight line relation to hold for shorter wave-lengths for the elements of high atomic weight than for elements of low atomic weight. The fact that the curve for iron remains straight even for the shortest wave-lengths investigated might be accounted for in this way.

6. The rise in the curves for water and carbon at long wave-lengths is too great to be accounted for by experimental error; apparently the departure from the straight line indicates that the absorption coefficient varies with the wave-length to a greater power than the third at these longer wave-lengths. The data on oxygen was not carried far enough to determine whether or not the same is true for it. A calculation of the absorption for hydrogen from that by water and oxygen indicated that the absorption of hydrogen varies as the wave-length to the nine halves power. The data on water and oxygen are not sufficiently accurate however to establish this relation, and it will be interesting to measure the absorption of hydrogen directly as a function of the wave-length. It is to be noted that the rise of the curve at long wave-lengths cannot be accounted for by an admixture of an impurity of relatively high atomic weight if we assume that the absorption of both the substance and impurity are proportional to the cube of the wave-length. In this case the curve would still have the same slope for short and long wave-lengths.

7. The curve for iron indicates that beyond 0.50 Ångström the true absorption coefficient is proportional to the wave-length to a smaller power than the cube, and at the same time it appears that the scattering must also be a function of the wave-length unless the true absorption coefficient is a more complicated function of the wave-length than

$\mu_0/d = K\lambda^n$. Since the electrons are more closely packed in the atoms of the elements of high atomic weight we should expect to find the scattering become a function of the wave-length at shorter wave-lengths for them, than for the elements of smaller atomic weight.

III. CONCLUSIONS AND PROJECTS FOR NEW WORK ON THE SCATTERING AND ABSORPTION OF X RAYS.

It would appear that the main conclusions to be gathered at present from this investigation are as follows:

1. The total mass absorption coefficient for a number of the elements of small atomic number (the atomic numbers lying between 3 and 26) are proportional to the cube of the wave-length of the X rays over only limited ranges of wave-length.

2. The constant K which occurs in the usually accepted equation for the total mass absorption coefficient is only approximately proportional to the cube of the atomic number. In particular, for lithium, this discrepancy is marked, and indicates that Moseley's law does not apply to this element.

3. The total mass absorption coefficient for hydrogen appears not to be proportional to the cube of the wave-length, but to the nine halves power of the wave-length. This conclusion, however, is not certain on account of the uncertainty involved in calculating the absorption of hydrogen from that of water and oxygen.

4. The mass scattering coefficients for aluminum and the elements of smaller atomic number is considerably less than that given by the expression deduced by J. J. Thomson for this quantity on the electromagnetic theory, and there is at present no adequate account to be given for this discrepancy.

5. The mass scattering coefficient for hydrogen is approximately twice that for the other elements, and this is to be regarded as further evidence that the hydrogen atom contains twice as many scattering electrons per unit mass as the other atoms.

6. The total mass absorption coefficient for aluminum and the elements of smaller atomic weight below wave-length 0.20 Å. U. is considerably less than would be expected from that at longer wave-lengths if the electron is small compared to these short wave-lengths. This decrease in the mass absorption coefficient is accounted for quantitatively on Compton's theory by assuming that the diameter of the electron is from 0.75×10^{-10} cm. to 0.85×10^{-10} cm. in diameter.

It would seem advisable to measure the scattering of some of these elements independently of the absorption. The distribution of the scattered radiation as a function of the angle of scattering from the incident beam, as well as the mass scattering coefficient, has an important bearing on the size and shape of the electron, and the writer plans to attempt some experimental work of this nature in the near future. It would also be interesting to measure directly the absorption coefficient in hydrogen as a function of the wave-length, in order to determine the relation between these quantities. If this relation is different for hydrogen than for the other light elements, as is indicated in this investigation, this fact ought to throw new light on the mechanism of absorption of X rays by matter.

This investigation was carried out during the summer of 1920 at the Research Laboratory of the General Electric Company, Schenectady, N. Y. I wish to express my sincere thanks to Dr. A. W. Hull who at all times aided and facilitated the progress of the work by his helpful suggestions and advice. I am also indebted to several other investigators in the laboratory for their efficient coöperation.

State University of Iowa,
Iowa City, Iowa,
November 1, 1920.

STABILITY CONDITIONS IN VACUUM TUBE CIRCUITS.

BY R. D. DUNCAN, JR.

SYNOPSIS.

Condition for Sustained Oscillations in Vacuum Tube Circuits.—After reviewing the fundamental requirements for such oscillations in any three electrode vacuum tube circuit, the fundamental equation for the plate current of a tube with a linear volt-ampere characteristic: $i_p = (v_p + \mu v_g) R_0$ is differentiated, $-dv_p/di_p$ is put equal to the load resistance in the plate circuit R_1, and thus the equation: $\mu = -(1 + R_0/R_1)dv_p/dv_g$ is obtained as a general relation between the constants of any oscillating circuit: R_1, the internal resistance of the plate-filament path R_0, the amplification constant μ and the derivative of the plate voltage with respect to the grid voltage. This expression is evaluated for five standard types of circuit: Hartley, Colpitts, Meiszner, tuned-plate and tuned grid; and it is experimentally verified for the first two.

1. INTRODUCTION.

In a recent issue of the PHYSICAL REVIEW,[1] there appeared a paper by Heising which discussed mathematically the transient conditions obtaining in a three-electrode vacuum tube circuit of the self-excited type. To those in the radio field who have worked along this line, the paper will be read with great interest, and its importance appreciated.

In the papers[2] which have appeared upon this subject, as well as in the one just referred to, the treatment has been approached more from the standpoint of the grid circuit. In addition, with one exception,[3] the analysis has been based upon the requirements of the individual circuit rather than of the general class.

It has always been felt by the author that it is the more logical procedure to regard a self-excited vacuum tube from the standpoint of the plate circuit, since it is in the latter that the load is introduced, and, hence, where it is desired to expend the major portion of the energy converted by the tube. The relation which the grid circuit bears to the plate circuit is one involving mainly phase relations between the alternating components of the plate and grid potentials; the proper adjustment of these relations determines the stability of the resulting oscillations and the magnitude of power output obtainable. From the plate circuit standpoint, it is possible to derive a perfectly generalized mathematical relation, in a quite simple manner, which under the proper conditions is applicable to any type of self-excited vacuum tube circuit.

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[1] Vol. XVI., Second Series, No. 3, p. 216, 1920.

[2] Bethenod, La Lumiere Electrique, Vol. 35, p. 225, 1916.
Vallauri, L'Elettrotecnia, Vol. 4, Nos. 3, 4, 18 and 19, 1917.
Hazeltine, I.R.E., Vol. 6, p. 63, 1918.
Ballantine, I.R.E., Vol. 7, p. 155, 1919.

[3] See Hazeltine.

In the early part of 1918, the author had occasion to apply the method of analysis, as outlined by Vallauri[1] to a number of circuits (employed in Signal Corps apparatus) which that author had not treated and which, at that time, had not been correctly treated elsewhere. The results of these investigations led to the generalized method of analysis and to the conception of a "generalized" circuit as presented and discussed in the following. In the last part of the paper experimental data is supplied which closely verifies the theoretical deductions.

It is recognized that there exists quite extensive literature on this subject and the present paper is submitted with the view merely of outlining briefly a somewhat different method of conception of the phenomena involved and of directing attention to certain generalized conditions which it is believed have not been sufficiently emphasized heretofore.

2. Fundamental Requirements and Assumptions.

The vacuum tube generator, or more precisely convertor, belongs to the general category of electrical devices which operate as a result of a periodic variation in their internal "constants."[2] Probably, the most familiar example of this class is the electric arc, compared to which the vacuum tube is similar in some respects but quite different in others; the similarity is based on the common property that both devices depend for operation on the conduction of electricity between two electrodes by means of a stream of electrons or ionized particles. Their chief difference is that with the vacuum tube the volt-ampere characteristic is rising while with the arc it is falling; as is well-known, the effect of a falling characteristic is towards instability of operation, which condition when properly utilized, permits of the establishment of sustained alternating currents in a connected circuit in which there are externally applied only continuous E.M.Fs. The effect of a falling characteristic of course, is to introduce into the circuit a "negative" resistance which neutralizes the positive resistance introduced by the load. The vacuum tube, because of its rising characteristic, is inherently stable, and to obtain the condition of instability, or what is equivalent thereto, the effect of negative resistance, it is equipped with a third electrode, the grid, which is so placed with respect to the emitting (filament) and absorbing (plate) elements of the tube, that when properly connected to the output circuit, it exerts an effect upon the conducting properties of the electron path such that, in the dynamic state, the average resistance of the internal plate-filament path is negative throughout an alternation.

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[1] See previous reference.

[2] H. Barkhausen, Das Problem der Schwingungserzeugung.

As fundamental it may be stated that for alternating current power to be available in a circuit which has externally applied only continuous voltages, the average power consumptions during a cycle must be negative; negative power consumption, which demands the introduction of negative resistance in turn requires that the phase difference between the voltage and current lie between 90° and 180°. If the impedance through which the current flows is entirely non-reactive, the value 180° must hold, *i.e.*, the voltage and current are exactly opposing in phase. The volt-ampere characteristic of such a resistance will therefore be linear, and with a *negative* slope equal in value to the resistance. The negative power requirements and phase relations as just stated were set forth a number of years ago by Barkhausen[1] and more recently, as applied particularly to the arc, by Steinmetz.[2]

A further requirement for the generation of sustained alternating currents without the application of external alternating potentials, is that the total effective resistance contained in the circuit must be zero, *i.e.*, if in series with the negative resistance R_1, there is a positive or load resistance R_2 then $R_1 = -R_2$.

As applied to the vacuum tube, the foregoing implies that the alternating components of the plate current and plate voltage will be 180° out of phase[3] and, that during an alternation, the average resistance introduced by the tube into the output circuit is negative, and equal in value to the positive resistance of the load.

The foregoing has dealt, in the main, with the necessary conditions which must obtain in the tube. Considering next the circuit, which consists of a combination of inductance, capacity and resistance, it is evident that an oscillation must exist therein only as a result of a sudden change in circuit conditions. If the circuit is capable of sustaining free oscillations, the period or frequency of the transient will always be that corresponding to the natural period of the circuit, *the latter therefore behaving in all respects as a pure resistance.* If the attenuation of the transient is positive, the oscillations will rapidly die out; if the attenuation is zero, the transient will persist with its initial amplitude, or if negative, will continue to increase in amplitude until limited by other means, or until self destruction of this system results. A transient of constant amplitude indicates that energy is being supplied at a rate equal to the rate of dissipation; a transient of increasing amplitude indicates that the rate of energy supplied is greater than that of dissipation. Sustained alternating currents are established in the circuit, first by the creation of a transient, and second, by so adjusting the tube and circuit relative to each other that attenuation of the transient is negative.

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[1] Barkhausen, Das Problem der Schwingungserzeugung.

[2] Steinmetz, Theory and Calculation of Electric Circuits. Chapter on the Instability of Circuits.

[3] Neglecting the capacity of the tube elements.

On the assumption that the load circuit behaves as a pure resistance, we are led to the conception of a circuit of the type shown in Fig. 1, which may be termed the "generalized" self-excited vacuum tube circuit. Therein are shown only the elements which are affected in the generation of alternating currents. The load resistance, represented by R_2, is connected in series with the external plate-filament circuit; the exact value of R_2 is determined by the type of circuit to which the tube is connected, being a function of the circuit constants and the operating frequency. It will be observed that in Fig. 1, the exact method of connecting the grid to the load circuit is not indicated since this will vary with the type of load circuit and is furthermore permissable since, as it is later assumed, the grid current is zero.

The volt-ampere characteristic of the tube involves three variables, *i.e.*, the plate current, plate voltage and grid voltage. This relation in general is non-linear, curvature being produced by the blocking action of high negative grid potentials, and by high positive grid potentials which cause both electronic saturation in the tube, and the diversion of a portion of the available electrons from the plate to the grid.

When transient conditions are dealt with, *i.e.*, when oscillations are just starting, the amplitudes of the oscillating currents and voltages are quite small and operation is confined to a very restricted region on the characteristic. No appreciable error is then incurred in assuming that the volt-ampere characteristic is linear and considerable simplification is introduced in the analytical operations.

Coincident with the passage of a current from the plate to the filament inside of the tube, there may also be a flow of current from the grid to the filament. This latter in general will be small compared to the plate current and in what follows, is neglected.

The fundamental relation of the tube may then be expressed in the linear form

$$i_p = \alpha v_p + \beta v_g,$$

wherein i_p, v_p and v_g represent respectively the plate current, plate voltage and grid voltage, α the conductance of the internal plate-filament path, and β the mutual conductance.[1] Now $\alpha = 1/R_0$ and $\beta = \mu\alpha = \mu/R_0$, where R_0 is the internal resistance of the plate-filament path, and μ the amplification constant, as defined by Van der Bijl. R_0 and μ are the two factors which characterize the operation of the tube as a convertor. The assumption of a linear volt-ampere characteristic is equivalent to assuming constant values of both R_0 and μ. The assumption of a linear characteristic, also if strictly adhered to, leads to the absurd result that the vacuum tube in its transient state is unstable, since a transient with negative attenuation would continue to increase indefinitely in amplitude. The practical limitation to this, of course, is the curvature of the characteristic, produced, as previously mentioned, by the blocking action of high negative grid potentials and by electronic saturation and absorption of appreciable power in the grid circuit.

3. THEORETICAL.

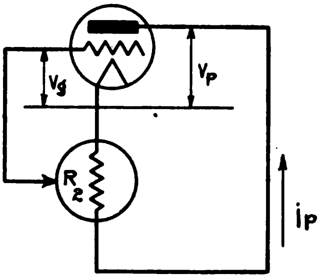

Fig. 1.

Based on the conception of a generalized circuit as shown in Fig. 1, it is possible to derive in a quite simple manner, the so-called conditions for oscillation. In Fig. 1, i_p, v_p and v_g represent respectively the instantaneous values of the alternating components of the plate current, plate voltage and grid voltage. R_2 represents the effective resistance introduced by load or frequency determining circuit. The relation between the three variables is assumed in the linear form

$$i_p = \alpha v_p + \beta v_g, \tag{1}$$

where α and β are as previously defined. From the fundamental requirements it is known that for power to be converted from direct to alternating form by or in a resistance, the slope of the volt-ampere characteristic of the resistance must be negative, *i.e.*, as applied to the vacuum tube, a positive increment in plate current Δi_p must be accompanied by a negative increment in plate voltage Δv_p or vice versa. From the physics of the tube, it is also known that a positive increment in plate current Δi_p must be the result of a similar positive increment in grid voltage Δv_g. Expressed mathematically, we have for a positive increment in grid voltage

$$i_p + \Delta i_p = \alpha v_p - \alpha \Delta v_p + \beta v_g + \beta \Delta v_g. \tag{2}$$

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[1] See Hazeltine.

Subtract equation (1) from (2) and there results

$$(3) \qquad \Delta i_p = -\alpha\Delta v_p + \beta\Delta v_g$$

or dividing through by Δv_p

$$(4) \qquad \frac{\Delta i_p}{\Delta v_p} = -\alpha + \beta\frac{\Delta v_g}{\Delta v_p}.$$

Now from the discussion of the fundamental requirements it was found that the slope of the transient volt-ampere characteristic must be negative or $\Delta i_p/\Delta v_p = -1/R_1$, where R_1 is the absolute value of resistance introduced by the tube during a cycle. Since the total effective resistance in the plate or output circuit must be zero, $R_1 = -R_2$, or

$$(5) \qquad \frac{\Delta i_p}{\Delta v_p} = \frac{1}{R_2}.$$

Equating (4) and (5) and simplifying, remembering that $\alpha = 1/R_0$ and $\beta = \mu\alpha$ we find on solving for μ

$$(6) \qquad \mu = \frac{\Delta v_p}{\Delta v_g}\left(1 + \frac{R_0}{R_2}\right).$$

Equation (6) states mathematically the condition which must obtain between the constants of the tube and those of the circuit in order that sustained alternating currents may be established and exist therein. To apply it to a particular type of circuit it is only necessary to evaluate the factors Δv_p, Δv_g and R_2 in terms of the circuit constants.

Hartley Circuit.

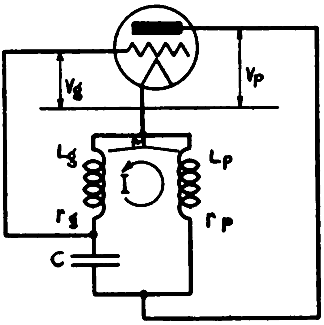

Fig. 2.

This equation corresponds to a somewhat similar expression given by Hazeltine provided that the factor unity may be neglected in comparison to the ratio R_0/R_2 in which case

$$(7) \qquad \mu = \frac{\Delta v_p}{\Delta v_g}\cdot\frac{R_0}{R_2},$$

which is equivalent to Hazeltine's expression.

As an illustration of the method outlined, the theory is applied to a number of standard circuits in use.

$$\Delta v_p = (L_p \pm M)\frac{d\Delta I}{dt},$$

$$\Delta v_g = (L_g \pm M)\frac{d\Delta I}{dt}.$$

$$R_2 = \frac{(L_p \pm M)^2}{C(r_p + r_g)(L_p + L_g \pm 2M)},$$

$$\mu = \frac{\Delta v_p}{\Delta v_g}\left(1 + \frac{R_0}{R_2}\right)$$

$$= \frac{L_p \pm M}{L_g \pm M}\left[1 + \frac{R_0}{\dfrac{(L_p \pm M)^2}{C(r_p + r_g)(L_p + L_g \pm 2M)}}\right].$$

From these equations, it is obvious that the sign of M may be either positive or negative, or its value zero, that is, the mutual inductance may either assist or oppose the two self inductances, L_p and L_g, or the latter may be so placed relative to each other that the mutual inductance or coupling is zero. If $M = 0$

$$R_2 = \frac{L_p^2}{C(r_p + r_g)(L_p + L_g)}$$

and

$$\mu = \frac{L_p}{L_g}\left[1 + \frac{R_0}{\dfrac{L_p^2}{C(r_p + r_g)(L_p + L_g)}}\right].$$

If, in addition, the two self-inductances are made equal in value and identical in construction, $L_p = L_g = L$ and $r_p = r_g = r$. In this case, the effective load resistance is given

Colpitts Circuit.

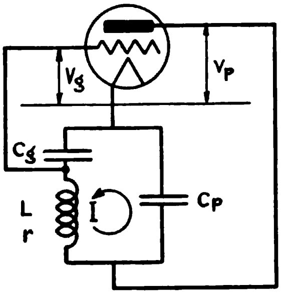

Fig. 3.

Meiszner Circuit.

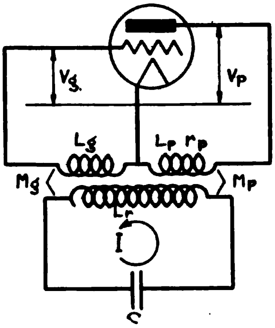

Fig. 4.

$$R_2 = \frac{L}{4rC}$$

and

$$\mu = 1 + \frac{R_0}{L/4rC}.$$

$$T = \frac{4R_0C}{\mu - 1}.$$

If T is the time constant of the inductance where $T = L/r$, then in terms of μ and R_0

$$\Delta v_p = \frac{1}{C_p}\int \Delta I \cdot dt,$$

$$\Delta v_g = \frac{1}{C_g}\int \Delta I \cdot dt,$$

$$R_2 = \frac{LC_g}{C_p(C_p + C_g)r},$$

$$\mu = \frac{C_g}{C_p}\left[1 + \frac{R_0}{\dfrac{LC_g}{C_p(C_p + C_g)r}}\right].$$

In the circuit shown in Fig. 4 it is assumed that there is no mutual inductance or coupling between the two self-inductances, L_p and L_g.

$$\Delta v_p = M_p \frac{d\Delta I}{dt},$$

$$\Delta v_g = M_g \frac{d\Delta I}{dt},$$

$$R_2 = r_p + \frac{M_p^2}{LrC},$$

$$\mu = \frac{M_p}{M_g}\left[1 + \frac{R_0}{r_p + \dfrac{M_p^2}{LrC}}\right].$$

If r_p is negligibly small, then very approximately,

Tuned Plate Circuit.

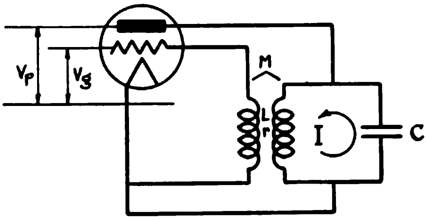

Fig. 5.

$$R_2 = \frac{M_p^2}{LrC}$$

and

$$\mu = \frac{M_p}{M_g}\left[1 + \frac{R_0}{\dfrac{M_p^2}{LrC}}\right].$$

Since the quantities μ, R_0 and $R_2 \simeq M_p{}^2/LrC$, are positive, it is obvious that M_p and M_g must be of the same sign.

$$\Delta v_p = L\frac{d\Delta I}{dt},$$

$$\Delta v_g = M\frac{d\Delta I}{dt},$$

$$R_2 = \frac{L}{rC},$$

$$\mu = \frac{L}{M}\left[1 + \frac{R_0}{\dfrac{L}{rC}}\right].$$

Tuned Grid Circuit.

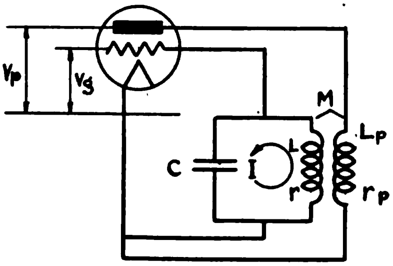

Fig. 6.

$$\Delta v_p = M\frac{d\Delta I}{dt},$$

$$\Delta v_g = L\frac{d\Delta I}{dt},$$

$$R_2 = r_p + \frac{M^2}{LrC},$$

$$\mu = \frac{M}{L}\left[1 + \frac{R_0}{r_p + \dfrac{M^2}{LrC}}\right].$$

If r_p is negligibly small, $R_2 \simeq M^2/LrC$ and

$$\mu = \frac{M}{L}\left[1 + \frac{R_0}{M^2/LrC}\right].$$

Similar to the Meiszner circuit, in the circuits shown in Figs. 5 and 6, it is obvious that the sign of the mutual inductance must be such that the resulting value of μ is positive.

4. Experimental.

The validity of the theory just developed has been tested for different types of circuits and tubes, with resulting close agreement between theory and practice. The results of a number of these tests are given in the following.

The scheme of connections employed for the Hartley circuit is shown in Fig. 7, wherein *T* represents the vacuum tube of tungsten construction, i_{fc}, i_p, i_g represent ammeters, *I* represents a high frequency ammeter of the thermal type, resistance 1.3 ohms. E_p is a local plate battery with short leads running to the test set. L_p and L_g are inductance coils of approximately the same physical dimensions, placed at a distance of 18″ from each other and turned at right angles. *C* is a variable air capacity, Bureau of Standards standard type.

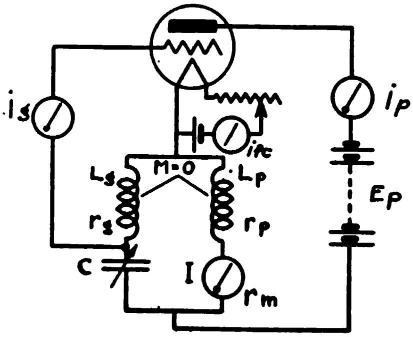

Fig. 7.

The test method adopted is as follows: the tube was placed in the

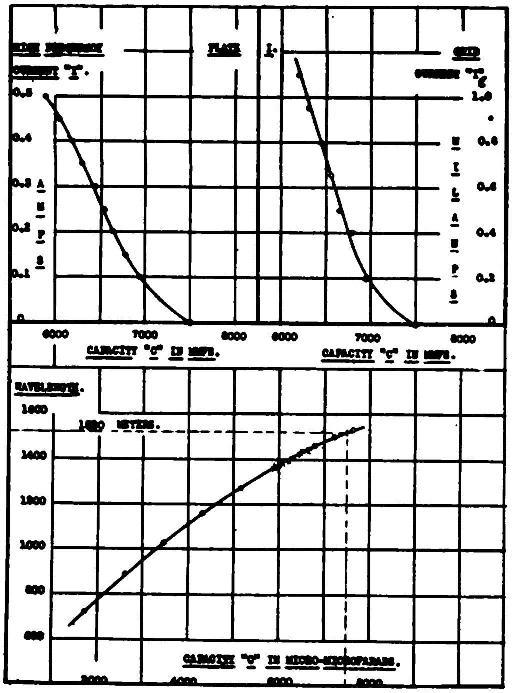

Fig. 8.

circuit and allowed to oscillate for approximately one hour, permitting it to be thoroughly heated and "settled" in its characteristics. The capacity C was then gradually increased from some initial value until oscillations ceased and readings were obtained of the high frequency current I, grid current i_g and wave-length. While still hot, the tube was placed in the standard testing set of the Bureau of Standards and the plate current-grid voltage and plate current-plate voltage characteristics were obtained for the same values of plate and grid voltages and filament current with which the oscillation test was made. The data obtained are shown graphically in Figs. 8, 9 and 10.

In Fig. 8 is shown the manner of variation of the high frequency current I, grid current i_g and the wave-length as a function of the capacity C from which it will be observed that the two former quantities decrease to zero, *i.e.*, the circuit and tube were just incapable of sustaining oscillations when the capacity equalled 7,500 micro-microfarads. The small circles on the wave-length curve at the bottom of the plate represent the points obtained by calibration of the oscillating circuit by means of an external oscillator (the tube under test not functioning), the cross marks

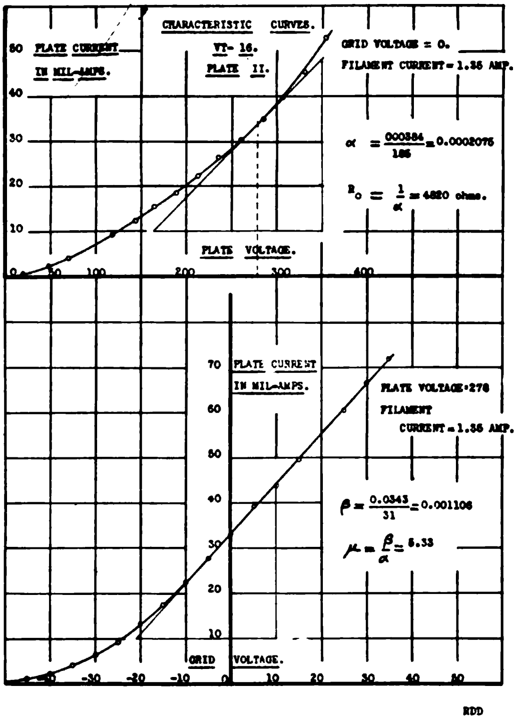

Fig. 9.

represent the values obtained when the circuit was self-excited, *i.e.*, values obtained as the circuit was passing out of oscillation. By extrapolation it was found that the wave-length at which oscillations cease was 1,520 meters.

In Fig. 9 is shown the plate current-plate voltage and plate current-grid voltage characteristics of the tube obtained at the proper filament current, grid and plate voltages. The reciprocal of the slope of the former curve at the operating plate voltage of 278, which is the internal output tube resistance R_0, was found to be 4,820 ohms; the slope of the latter curve at zero grid voltage, which is the mutual conductance, was obtained to be 0.001106 mhos. The amplification constant μ is given by the product of (βR_0) and is equal to 5.33.

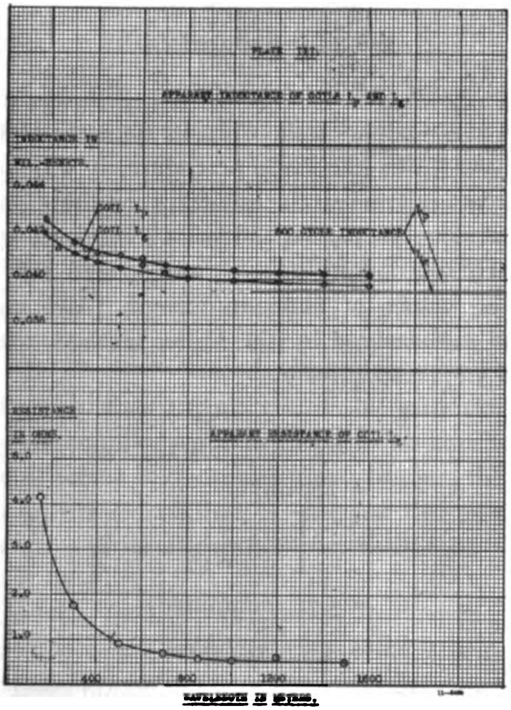

Fig. 10.

In Fig. 10 the apparent resistance and inductance of the coil L_g are shown, plotted as functions of the wave-length. At the critical value of 1,520 meters, $r_g = 0.55$ ohms. Since L_g and L_p were identical in construction, r_p may be taken to have the same value. At the same wave-length the apparent inductance of L_p and L_g are very approximately 0.0402 mh and 0.0398 mh, respectively.

SUMMARY OF DATA.

Wave-length, 1,520 meters, capacity 0.0075 mf. Total resistance including high frequency ammeter, $r_p + r_m + r_g = 2.40$ ohms, $L_p = 0.04020$ mh, $L_g = 0.03980$ mh, $R_0 = 4{,}820$ ohms, $\mu = 5.33$ (measured).

$$R_2 = \frac{L_p^2}{C(L_p + L_g)(r_p + r_m + r_g)}$$

$$= 1{,}123 \text{ ohms},$$

$$\therefore \mu = \frac{L_p}{L_g}\left[1 + \frac{R_0}{R_2}\right]$$

$$= 5.34 \text{ (computed)},$$

$$\mu = 5.33 \text{ (measured)}.$$

A similar test was made with the Colpitts type of circuit with a coated filament tube. Adopting the same procedure, it was found that oscillations just failed to exist when the following condition prevailed: Wave-lengths (just previous to passing out of oscillation), 570 meters $L = 0.040750$ mh, $C_p = 0.008953$ mf, $C_g = 0.002620$ mf, $r = 3.2$ ohms. At the operating values of filament current, plate voltage and grid voltage, R_0, and μ, measured by the Miller method, were found to be, 5,980 ohms and 5.87, respectively.

$$R_2 = \frac{LC_g}{C_p(C_p + C_g)(r_4 + r_m)}$$

$$= 322 \text{ ohms},$$

$$\therefore \mu = \frac{C_g}{C_p}\left[1 + \frac{R_0}{R_2}\right]$$

$$= 5.72 \text{ (computed)},$$

$$\mu = 5.87 \text{ (measured)}.$$

Similar measurements were made upon a second coated filament tube, the measured value of the amplification constant being 28.4 while that computed from theory was 29.1.

OFFICE OF THE CHIEF SIGNAL OFFICER OF THE ARMY,
ENGINEERING AND RESEARCH DIVISION,
WASHINGTON, D. C.,
October 31, 1920.

A STUDY OF THE ERRORS INVOLVED IN SOLVING FOR THE CONSTANTS OF FOURIER'S SERIES BY A CALCULATION METHOD.

By Warren Mason.

Synopsis.

I. *Sources of error* due to measurement and due to higher harmonics.

II. *Error due to measurement* (*a*) Determination due to the placing of the ordinates and the measurement of the ordinates. (*b*) Formula evolved for probable error. (*c*) Error plotted to number of ordinates used, in Fig. 1.

III. *Error due to the assumption that no harmonics above the mth exist* (*a*) Formula derived for error due to the presence of higher harmonics. (*b*) Determination and plotting of the error in a constant ordinate curve. 1. Reason why the error in this sort of a curve is fairly representative. (*c*) Use of Figs. 1 and 2 in determining the best conditions to analyze a wave form. (*d*) Error due to the higher cosine harmonics. (*e*) Formula for the error when the even harmonics are considered.

FOURIER'S series is used to a considerable extent in scientific calculation, for the equations of many wave forms have to be determined. There are several forms of harmonic analyzers on the market, but if there is not sufficient calculation to warrant the purchase of such a machine, analyzation is usually carried out by means of some prepared schedule.

The error in solving for the constants by a schedule system consist of two parts; that due to the errors of measurement, and that due to the assumption that no harmonics above the mth exist, where $(m - 1)$ is the number of equally spaced ordinates used. Measurement with an ordinary ruler can be carried out to about 1/100 of an inch. In measuring any one of the ordinates then there will be two sources of error, one as to the position of the ordinate, and the other as to the length of the ordinate. As the ordinate cannot be placed closer than 1/100 of an inch an error equal to $(1/100\, dy/dx)$ is made in measuring the length of the ordinate. Even if the ordinate were correctly placed, its length could not be obtained closer than the limiting accuracy of measurement. As either error might be positive or negative, the most probable error is the average of all possible combinations. Let the second error be the largest and assume first that it is positive.

$$\begin{aligned} E_2 + E_1 &= \text{maximum error} \\ E_2 - E_1 &= \text{minimum error} \\ \hline 2E_2 \quad &= \text{maximum minimum error} \\ &\text{or average error} = E_2. \end{aligned}$$

If E_2 were negative a similar result would be obtained with the sign changed. If we consider all error simply as error without regard to sign, the most probable error of a combination of two errors, either of whose signs might be positive or negative, is the largest one. The slope of most curves does not exceed one, so the probable error due to the measurement of any ordinate is 1/100 of an inch. When solving for the sin constant of the first harmonic, the maximum error will be

$$.01(\sin\alpha + \sin 2\alpha + \cdots + \sin(m-1)\alpha).$$

α is the distance from the end of the wave to the first ordinate expressed in angle measurement. If the number of ordinates is even, the minimum error is zero, and is obtained by alternating the signs of the errors on successive ordinates. If the error of the first ordinate is negative and that of all the others positive, the total error is (max. error $- 2 \times .01 \sin\alpha$). Now if the errors on all the ordinates except the outer two are alternated in sign, while the error on these two is taken positive, the total error is $2 \times .01 \sin\alpha$. The sum of the errors of both taken together is equal to the maximum error. In a similar manner for any combination of errors, there is always another one such that when it is added to the first one, the sum equals the maximum error. Therefore the average error is

$$\frac{\text{Maximum error}}{2}.$$

Using a sin summation series, the average error for any number of ordinates is given by the expression

$$\text{Average error} = \frac{\sin\left((m-1)\dfrac{\alpha}{2}\right)}{2\sin\dfrac{\alpha}{2}},$$

where m is the number of equidistances between the $(m-1)$ ordinates, and α is one of these equidistances expressed in degrees. As in each case the summation of the terms $(L_1 \sin\alpha +, L_2 \sin 2\alpha +, \cdots +, L_{(m-1)} \sin(m-1)\alpha)$ must be multiplied by $2/m$, the summation of the error must also be. Fig. 1 shows the variation of the probable error due to the errors in measurement plotted to the number of ordinates used in analyzing the curve. This curve only holds for waves that are comparatively flat, for if the wave is very irregular, the error due to the spacing of the ordinates will predominate, and the probable error may be two or three times as great as that shown in Fig. 1. Errors for the higher harmonics due to errors in measurement will be the same as for the first sin harmonic probable error, for they invólve the same quantities in a similar manner.

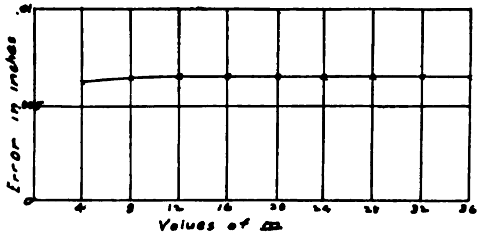

Fig. 1.

The length of the various ordinates L_0, L_1, L_2, $\cdots$, $L_{(m-1)}$, are given by the equations

$$L_0 = A_1 \sin 0 + B_1 \cos 0 + A_3 \sin 0 + B_3 \cos 0 + \cdots,$$

$$L_1 = A_1 \sin \alpha + B_1 \cos \alpha + A_3 \sin 3\alpha + B_3 \cos 3\alpha + \cdots,$$

$$L_2 = A_1 \sin 2\alpha + B_1 \cos 2\alpha + A_3 \sin 6\alpha + B_3 \cos 6\alpha + \cdots,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

Multiplying the first equation through by sin 0, the second by $\sin \alpha$, the third by $\sin 2\alpha$, etc., we have

$$L_0 \sin 0 \;\; = A_1 \sin^2 0 + B_1 \cos 0 \sin 0 + A_3 \sin 0 \sin 0 + \cdots,$$

$$L_1 \sin \alpha \;\; = A_1 \sin^2 \alpha + B_1 \sin \alpha \cos \alpha + A_3 \sin 3\alpha \sin \alpha + \cdots,$$

$$L_2 \sin 2\alpha = A_1 \sin^2 2\alpha + B_1 \sin 2\alpha \cos 2\alpha + A_3 \sin 6\alpha \sin 2\alpha + \cdots,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

Summing up the left-hand portion of the equation we have ($L_0 \sin 0 + L_1 \sin \alpha + L_2 \sin 2\alpha + \cdots$). On the right-hand side of the equation there are three kinds of terms,

$$A_k\Sigma(\sin^2 A + \sin^2 2A + \cdots),$$

$$A_k\Sigma(\sin A \sin B + \sin 2A \sin 2B + \cdots),$$

$$A_k\Sigma(\sin A \cos B + \sin 2A \cos 2B + \cdots),$$

$\sin^2 A$ can be expressed as

$$\frac{(1 - \cos 2A)}{2}.$$

Therefore the value of the first kind of term can be obtained by using the cosine summation formula

$$S = \cos \theta + \cos (\theta + \alpha) + \cos (\theta + 2\alpha) + \cdots,$$

which when summed up becomes

$$S = \frac{\cos\left(\theta + (m - 1)\frac{\alpha}{2}\right)\sin\frac{m\alpha}{2}}{\sin\frac{\alpha}{2}},$$

where m is the number of terms in the series, α the common difference, and θ the first or starting term. In this case

$$\theta = 0, \qquad \alpha = 2A, \qquad \text{and} \qquad m = m.$$

$$\therefore \text{ the summation } = \frac{\cos((m - 1)A)\sin mA}{\sin A}.$$

$$A = \frac{K\ 180^\circ}{m},$$

where K is the order of the harmonic. Therefore $mA = 180^\circ$ or some multiple of it and $\sin mA = 0$. Since $\sin A$ has always a positive value, the expression for the summation of the cosines reduces to zero. As there are m terms however the expression

$$\frac{(1 - \cos 2A + 1 - \cos 4A + 1 - \cos 6A + \cdots)}{2} = \frac{m}{2}.$$

Adding the expression for L_1 to the expression for the last ordinate, that for L_2 to the expression for the next to the last ordinate, etc., and remembering that $\sin\theta = -\sin(-\theta)$ while $\cos\theta = \cos(-\theta)$, the third kind of term is seen to reduce to zero.

From trigonometrical relations

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}.$$

Using the same series to sum up these terms

$$S = \frac{\cos\left((m - 1)\frac{(A - B)}{2}\right)\sin\frac{m(A - B)}{2}}{2 \times \sin\frac{(A - B)}{2}} - \frac{\cos\left((m - 1)\frac{(A + B)}{2}\right)\sin\frac{m(A + B)}{2}}{2 \times \sin\frac{(A + B)}{2}},$$

$(A - B) = \alpha$ or some multiple of α. $(A + B) = \alpha$ or some multiple of α. If $(A - B)$ is odd, $(A + B)$ is odd, and if one is even, the other is even. Since $m\alpha = 180°$, if $(A - B)$ is even,

$$\sin\frac{m(A-B)}{2} = 0, \quad \text{and} \quad \sin\frac{m(A+B)}{2} = 0.$$

When $(A - B)$ is odd, $\sin\frac{m(A-B)}{2}$ is 1 or -1, also when $\sin\frac{m(A-B)}{2}$ is positive, $\cos\left((m-1)\frac{(A-B)}{2}\right)$ is positive and equal to $\sin\frac{(A-B)}{2}$, and when the sin is negative, the cosin is negative also. Therefore the value of the first expression is 0 or $+\frac{1}{2}$. In a similar manner, the value of the second expression is 0 or $+\frac{1}{2}$. Since one term is zero when the other is, and one half when the other is one half, the algebraic sum of the two or the value of the expression reduces to zero. The only time this expression does not hold is when

$$\sin\frac{(A-B)}{2} = 0,$$

and when

$$\sin\frac{(A+B)}{2} = 0$$

for then the expression is indeterminable. Since

$$\sin\frac{(A-B)}{2} = 0, \qquad \frac{(A-B)}{2} = Km\alpha$$

and $(A - B) = 2Km\alpha$, K being some integral number. Likewise since

$$\sin\frac{(A+B)}{2} = 0, \qquad (A+B) = 2Km\alpha.$$

Using again the expression

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

and introducing the additional consideration $(A - B) = 2Km\alpha$, the summation of the series is $m/2$. The summation using the condition, $(A + B) = 2Km\alpha$, is $-m/2$. Therefore

$$(L_1 \sin\alpha + L_2 \sin 2\alpha + L_3 \sin 3\alpha + \cdots)$$
$$= \frac{m}{2}A_1 - \frac{m}{2}A_{(2m-1)} + \frac{m}{2}A_{(2m+1)} - \frac{m}{2}A_{(4m-1)} + \cdots,$$

or

$$\frac{2}{m}(L_1 \sin\alpha + L_2 \sin 2\alpha + L_3 \sin 3\alpha + \cdots)$$
$$= A_1 - A_{(2m-1)} + A_{(2m+1)} - A_{(4m-1)} + A_{(4m+1)} + \cdots.$$

The error then for the first sin harmonic due to the assumption that no harmonics above the *m*th exist is the expression

$$E_{A_1} = -A_{(2m-1)} + A_{(2m+1)} - A_{(4m-1)} + A_{(4m+1)} + \cdots.$$

For the third harmonic the error is given by the expression

$$E_{A_3} = -A_{(2m-3)} + A_{(2m+3)} - A_{(4m-3)} + A_{(4m+3)} + \cdots.$$

Errors for higher harmonics are obtained in a similar manner.

Figure 2 shows the error in inches divided by the height of an average ordinate, for a number of harmonics, plotted against the number of ordinates used, for a constant ordinate curve. It is not necessary to analyze curves approaching constant ordinate curves very often, but the error for one is easily calculated and moreover is fairly representative

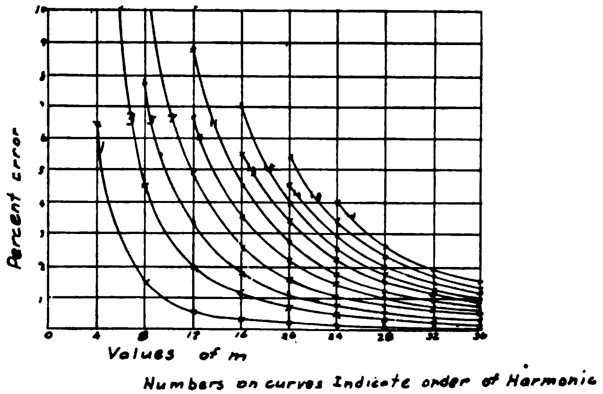

Fig. 2.

of what might be expected on any form of curve. The magnitude of the higher harmonics does not differ much no matter what the shape of the curve and the sign of two of them of nearly the same order, will probably be similar. For this reason the error calculated from a constant ordinate curve will represent fairly well that of any curve that might be analyzed. It is quite evident that the bigger the amplitude of the wave analyzed, the bigger the error in inches for any harmonic will be. Therefore the error for any harmonic may be expressed in per cent. of an average ordinate for the wave form. It is noticed that the higher the order of the harmonic, the larger the per cent. error will be. The reason for this is that the orders of the harmonics causing the error become more widely separated and thus the difference between them is greater.

From the discussion on the error due to measurement, it was found that the error was independent of the amplitude of the wave and depended only on the number of ordinates used. On the other hand, the error due to higher harmonics depends on the amplitude of the wave as well as the number of ordinates. Considering the probable error in this case as the larger of the two, then for analyzing a given curve whose average amplitude is known, for a harmonic of any order, there is a certain number of ordinates beyond which the error does not decrease with an increase in the number of ordinates. This can be determined from Figs. 1 and 2 by determining the place where the error due to both sources is equal. Conversely, if it is desired to analyze a wave for a given harmonic by using a prepared schedule of a definite number of ordinates, there is an average amplitude beyond which it is useless to go, as far as per cent. accuracy is concerned. In addition to determining the above conditions, Figs. 1 and 2 show the probable error for analyzing a wave under any condition.

The errors for cosine terms may be determined in a similar manner and are equal to

$$E_{B_1} = + B_{(2m-1)} - B_{(2m+1)} + B_{(4m-1)} - B_{(4m+1)} + \cdots,$$

$$E_{B_3} = + B_{(2m-3)} - B_{(2m+3)} + B_{(4m-3)} - B_{(4m+3)} + \cdots.$$

In general as the cosine terms are usually smaller than the sin terms,the error will also be smaller.

If there are even harmonics in the wave, the errors for the various harmonics are due to the same higher harmonics and therefore Fig. 2 holds in this case also.

SECONDARY ELECTRON EMISSION FROM COPPER SURFACES.

BY I. GARNETT BARBER.

SYNOPSIS.

Secondary Electrons from a Copper Surface Bombarded by Electrons.—The method used was to measure the current flowing to the bombarded plate as a function of the grid potential. By using sufficiently low pressures the ionization effect was made negligible. (1) *The coefficient of secondary emission* was found to increase somewhat with the energy of the primary electron *up to* 500 *volts* but never exceeded 1.5 per electron; the coefficient was increased by *heat treating the plate*, and was greatly decreased by raising the *temperature of the plate.* (2) *The energy* of the secondary electrons was always less than that corresponding to a fall through 5 volts, yet owing to the roughness of the surface an accelerating potential of about 10 volts is required to release most of the secondary electrons from the plate. (3) As to the *origin* of these rays, the fact that their maximum energy is always less than the energy of the primary rays and the shape of the curves obtained both indicate that the secondary electrons are not reflected electrons.

Reflection of electronic rays with energy below 500 *volts, from a copper surface* seems to be zero, or at least small.

INTRODUCTION.

WHEN α rays, positive rays, or free electrons fall on a metal surface it has been found by various investigators that if the energy of impact is high enough there results a reemission of electrons from the surface. These emitted electrons have been termed δ rays when they are produced by α rays, but in view of their corpuscular nature they will be referred to as secondary electrons throughout this paper no matter what is the cause of their excitation. The number of these secondary electrons has been found by O. van Baeyer,[1] Gehrts,[2] A. W. Hull[3] and others to exceed the number of primary electrons striking the plate when the velocity of impact is sufficiently great. This is evidenced by the fact that with increasing velocities of the primaries a point is reached at which the electron current flowing into the plate begins to decrease, and finally reverses in direction, provided there is present another collecting electrode to which the secondaries may flow. So far as measurements of current alone are concerned when the number of these secondary electrons is less then the number of primaries the effect might as conveniently be explained by assuming a true reflection of a portion of those incident as by assuming a secondary emission. However measurements of the velocity of the emitted electrons appear to show that this velocity is practically independent of the method of excitation,[1, 2] of the nature of the metal, and of the speed of the incident electrons provided this is greater than about 30 volts. Most observers find also that the bulk of the secondary electrons have velocities of about 5 volts while a small proportion may have as great a value as 30 volts but may never exceed this. These facts as to velocity have resulted in the adoption of the point of view that the emission is due to penetration of the atom by the electron and its subsequent absorption so that the atomic stability is in some manner disturbed and a secondary emission occurs. Below the "ionizing potential" of the surface atoms the effect has generally been regarded as a true reflection and this viewpoint was apparently substantiated by the fact that velocity distribution curves found by von Baeyer and Gehrts changed form at a certain potential.

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[1] O. von Baeyer, "Verh. der deutsch. Phy. Gessel.," 10, 1908, p. 903.

[2] A. Gehrts, "Ann. der Phy.," 36, 1911, p. 995.

[3] A. W. Hull, "Proc. Instit. Radio. Engineers," Feb., 1918.

The present paper gives the results of a study of the number of secondary electrons produced per primary for various velocities of impact of the latter on a copper surface which was so arranged that its surface condition and temperature could be changed. Determinations of the velocity were also made in order to see if any relation existed between the maximum velocity of emission and the energy of impact, as well as to see if the conclusions of other observers could be substantiated. The approximate critical velocity below which no secondary electrons are produced, has also been found, and evidence obtained for the view that there is no true reflection of electrons from metal surfaces. These investigations were initiated by Professor Millikan because it was thought especially desirable at the present time, first, to apply modern methods of high vacuum technique to the elimination of all spurious effects due to residual gases which might have inhered in some of the earlier work; second, because with the rapid increase in our knowledge of the ionizing potentials of gases, the relations to these of the ionizing potentials of liquids and solids has become important; and third, because no one had made any conclusive study of the effect of adsorbed gas films on secondary emission, nor of the effect of changing the temperature of the bombarded surface.

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[1] Fuchtbauer, Phy. Zeit., 7, 1906, p. 748.

[2] N. R. Campbell, Phil. Mag., 22, 1911, p. 276; 24, 1912, p. 527.

THE EXPERIMENTAL ARRANGEMENT AND THE METHOD OF MAKING THE OBSERVATIONS.

In order to eliminate any effects which might be due to gases where ionization would mask the effects sought the experiments were performed in very high vacua. In order further to make it possible to determine the exact potential at which secondaries begin to be emitted an equipotential filament was used, whereas most of the other experimenters have been limited by the drop of potential along their hot wire source of primary electrons. A hot platinum tube coated with a Wehnelt oxide served as the source of primary electrons. The use of this oxide was found to be necessary because a pure platinum tube gave too small a supply of thermions to produce currents which were large enough to be accurately measured with a galvanometer, unless very large filament heating currents were used. The platinum tube was 6 cm. long, about 3 mm. in diameter and .01 cm. in wall thickness. It was heated by a coaxial No. 22 tungsten wire which carried all the current. The insulation between the wire and tube was a thin layer of alundum cement. The No. 22 wire was fastened mechanically just beyond the ends of the platinum tube to heavier No. 18 tungsten wire leads. These did not become incandescent for filament currents sufficient to heat the No. 22 wire to a bright red heat. The platinum tube is fastened by a platinum strip to the lead at one of its ends.

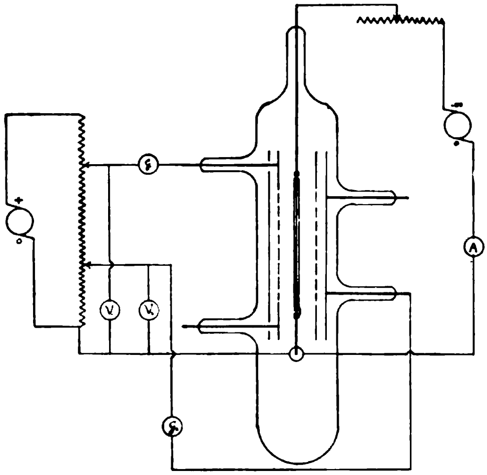

Fig. 1.

The experimental tube which was used in the preliminary experiments is shown in cross section in Fig. 1. The filament is situated in the axis of two coaxial copper cylinders. The outer cylinder which will hereafter be spoken of as the plate, was 10 cm. long and 3 cm. in diameter. The other cylinder, which will be called the grid, is of the same length as the plate cylinder and has a diameter of 2 cm. The grid is perforated symmetrically for a length of 6 cm. with 1 mm. holes whose centers are 8 mms. apart. These holes were made small in an endeavor to eliminate the "stray field" effect. The wall thickness of both the plate and the grid was about .015 cm. Each of the cylinders was supported by two No. 18 dumet wires which screwed into lugs on them, so that the assembling of the tube was easily effected. Platinum leads were used for conveying the filament current through the seals of the surrounding soft glass envelope. The whole tube was mounted vertically, so that any expansion of the filament when it became hot would not let it sag out of center.

The tube through which the exhaustion is effected connects to the side of the glass envelope half way between the ends of the cylinders. The system used for exhausting consisted of a water aspirator, a McNeill oil pump and two mercury condensation pumps in series. This system of pumps connected to the experimental tube through a liquid air trap which served to freeze out any mercury or organic vapors. No stopcocks were used on the high vacuum side of the condensation pumps. The pressure was roughly indicated by Geissler tubes. It could be accurately measured by a Western Electric Co. ionization gauge which was sealed on close to the experimental tube. Both the gauge and the experimental tube were so mounted that an oven could be lowered over them. They could thus have any adhering gas driven from their walls by the process of baking out.

The experimental arrangement that has been described above provided no means of altering the condition of the surface of the plate or its temperature. It may be remarked in this connection that it was found to be impossible to get sufficient energy across to the plate to raise its temperature much by bombardment and thus rid it of any adhering gas. This was because the solid portions of the grid intercepted the greater part of the primary electron stream. Moreover the preliminary arrangement was subject to the objectionable feature that when the grid was positive with respect to the plate, the grid current was a measure of both the secondary emission from the plate and the primary current from the filament intercepted by the solid portions. This latter current was so much larger than the secondary current which it was desired to study that it nearly masked it. Accordingly on the hope of remedying these defects a new tube as depicted in Fig. 2 was designed. It differed from the first tube by having a copper cylinder *S* which was 1.8 cm. in diameter and 10 cm. long. This was perforated symmetrically for 6 cm. of its length with 1 mm. holes. The copper grid-cylinder *g* was 2 cm. in diameter and coaxially situated with respect to the shield *s*. It was perforated symmetrically with 3 mm. holes which were situated directly above the corresponding smaller holes of the shield. These two cylinders were rigidly held together at each end by rings made from the best grade of porcelain. These rings were so arranged that a 4 mm. surface of insulation existed between the cylinders. In order to alter the surface and temperature of the plate it was surrounded by an insulating cylinder of soapstone which had a wall about 2 mm. thick. On this cylinder was wound several turns of No. 36 nichrome resistance wire. All parts were mounted by No. 18 tungsten wires in a Pyrex glass tube as indicated. The filament current was led into the tube by means of very heavy No. 11 tungsten wires. By adopting Pyrex it was found to be much easier to construct the tube than when it was made of soft glass.

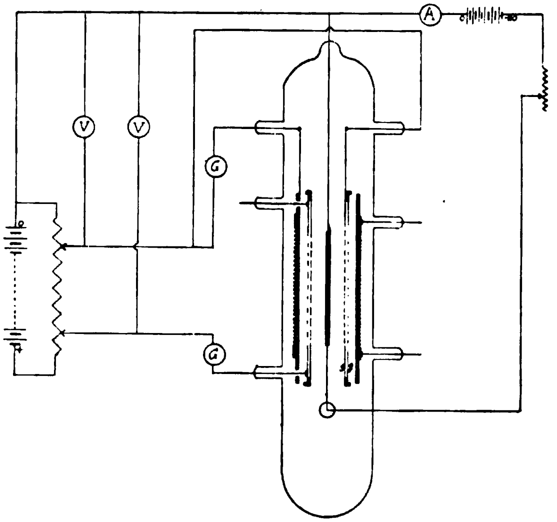

Fig. 2.

The usual method of experimenting was to bake out at about 400° C. for three hours and have the pumps running all the time. The pressure during this whole process was quite low as no visible discharge appeared in the Geissler tubes. After this exhaustion the gauge-filament was glowed and the gauge-plate was bombarded to a cherry red heat to rid it of gas films adhering to its surface. After the gauge-plate was well bombarded in this manner all traces of blue glow disappeared. The tube filament was freed of gases by heating to incandescence. It was not necessary to bake out and bombard every day as it was found in the case of the gauge-plate that, after it was once thoroughly bombarded, it absorbed little gas unless the apparatus was opened to the air. This same thing is undoubtedly true for the glass walls. Accordingly when the system was clamped off and liquid air was kept continuously on the trap, after the tube and gauge had been well exhausted, baked out and bombarded once, it was found that the pressure was still low the next day. It was only necessary to start the pumps and exhaust in order to produce as good a vacuum as had been obtained just after bombardment. As the pressures obtained in these experiments were always of the order of 5×10^{-6} cm. of mercury, a simple calculation shows that the ionization current does not on the average amount to more than one tenth of one per cent of the electron current getting through the holes of the grid to the plate.

The observations consisted of noting the currents flowing to the plate and grid both as to magnitude and direction when various positive potentials with respect to the emitting platinum cylinder were applied to them. Curves were taken in two distinct ways. In the first type of curve, which will hereafter be referred to as a curve taken at constant grid voltage, the positive potential applied to the grid was maintained at an arbitrary fixed value while the potential applied to the plate was varied from zero up to potentials larger than that applied to the grid. In the second type of curve, which will hereafter be referred to as a curve taken at constant plate voltage, the potential applied to the plate is held constant while the potential applied to the grid is varied. The pressure was measured by means of the ionization gauge just before and just after each set of observations and the mean value was taken as correct. The currents recorded below are reduced to amperes from the corresponding galvanometer deflections and sensibilities. The zero of the galvanometer was taken after each reading so as to correct for any "drift" or other cause of its shifting if it were present. The currents were generally measured for increasing potentials and then for decreasing.

For the last tube the shield *S* in Fig. 2 was always maintained at the potential of the plate. Any electrons liberated from the plate were then all attracted to the grid if the latter was positive with respect to the plate. The measurements of the grid current for the last tube had to be discarded as experiments showed that the porcelain insulation allowed a leakage current of the same order of magnitude as that of the secondary emission from the plate to flow between the shield and the grid when it became hot due to heat radiated from the filament. This however was not so serious as it seemed, for the variations in the current flowing to the plate were of themselves able to give most of the information desired on the number and velocity of the secondary electrons. This latter factor might have been better investigated, however, by means of the grid current if the porcelain had retained after the filament was lit the splendid insulating properties it possessed while cold. The shield was not however superfluous as a comparison of Fig. 4 with Fig. 5 below shows that its use practically did away with the "stray field" effect which was present in the first tube. This enabled the method of finding the number of secondaries per primary to be applied to lower velocities of impact of the primary electrons than was possible with the first tube.

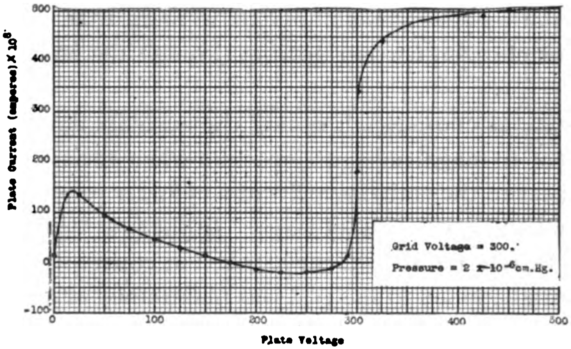

Fig. 3.

After the curves at constant plate voltage and at constant grid voltage had been taken for the untreated surface of the plate the furnace on the tube was slowly heated up until the plate was at a cherry red heat. It was allowed to run this way for 14 hours continuously. The pumps were running all the time. This treatment visibly changed the appearance of the surface of the plate in that it reduced most of the oxide and undoubtedly drove off practically all the adhering gas films. Curves were taken as before for this new surface. This treated surface is not, however, claimed to be a perfectly clean pure copper surface, especially as the tube from which the plate cylinder was manufactured was made of commercial copper.

In order to investigate the effect of temperature on the secondary electron emission it was only necessary to pass a current through the outside furnace. This raised the temperature of the plate and readings analogous to those described above were taken. As it was surmised that the soapstone insulation might break down and cause a leakage current to flow at high temperatures, preliminary measurements of this leakage current were made when the filament at the center of the tube was cold. There are then no primary electrons coming to the plate from the filament so that the whole current flowing in the plate-circuit is due to leakage. These measurements showed that the leakage current was relatively small in comparison with the primary electron current for furnace current less than .5 amperes. Accordingly curves at constant plate voltage were taken at furnace currents of .4, .25 and 0 amperes for various potentials of the plate. It is estimated the corresponding temperatures of the plate are about 110° C., 45° C., and 25° C. For a given plate voltage these readings at various temperatures were taken right after one another, so that the surface of the plate was the same at all the temperatures. In order to correct exactly for the leakage it was only necessary to eliminate the primary electron current coming to the plate by cutting off the filament current. All the applied voltages and the furnace currents are of course left the same as when the filament is lit in this determination of the correction due to leakage.

The Experimental Results Obtained.

A. *Curves Taken at Constant Grid Voltage.*

The form of these curves was the same with both of the tubes. Fig. 3 is typical of the series of curves which was taken under these conditions. The ordinates represent the current in the plate circuit, reckoned positive for electrons passing from the filament to the plate, that is, in the direction which is equivalent to positive electricity flowing from high potential to low across the vacuum. These plate current-plate voltage curves all have the following characteristics.

1. When the plate voltage is zero there is always a small electron current flowing into the plate from the filament. This is due to the primary electrons which do not pass through the very intense field near the boundaries of the holes of the grid but pass through the weaker fields at the center of these holes with a velocity corresponding to the potential of the grid. This velocity is so high that some of them shoot right through the holes into the retarding field between the grid and plate and finally hit the plate with an energy corresponding to their energy of emission from the filament.

2. The plate current assumes larger values as the plate voltage is increased until a maximum is attained. This is believed to be due to the fact that a greater proportion of these primary electrons, which get through the holes of the grid, go to the plate as the retarding field is diminished. It is only when the rate of increase of the primary electron current flowing into the plate in this manner is just compensated by the rate of increase of the electron current due to secondary reemission which flows from the plate to the grid that the maximum is attained.

3. For plate-voltages greater than that corresponding to the maximum of the curve, the current flowing into the plate assumes continually smaller values as the plate-voltage is increased until a plate-voltage slightly lower than the prevailing grid-voltage is reached. The numerical value of the smallest current flowing into the plate is found to become less as the grid-voltage is increased. If the grid-voltage exceeds about 200 volts the current flowing into the plate reverses in direction and assumes small negative values. This means that more electrons are leaving the plate than are hitting it, so that there is some true secondary reemission certainly present.

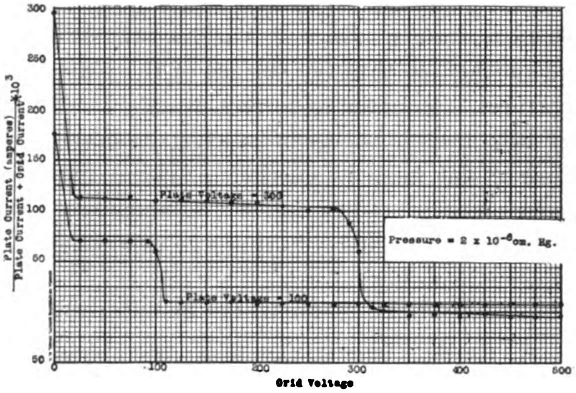

Fig. 4.

4. The current flowing into the plate begins to assume larger values again for plate-voltages greater than a voltage which is about 25 volts less than the potential of the grid. This current increases rapidly in numerical value between plate-potentials that are about 5 volts less and about 10 volts greater than the grid-voltage. After this the rate of increase is much slower and there is the appearance of saturation setting in. This saturation current is a measure of the total number of electrons

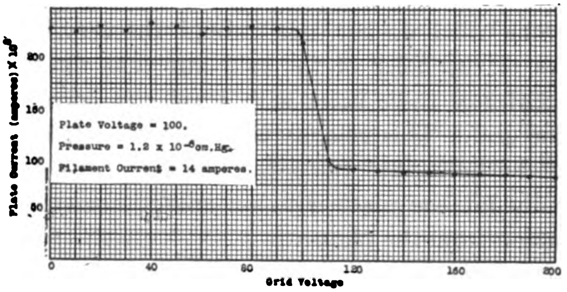

Fig. 5.

getting through the holes of the grid. The difference between it and the lowest value of the current flowing into the plate is an approximate measure of the number of secondary electrons reemitted by the plate. The ratio of this difference to the value of the saturation current is moreover an approximate measure of the average number of secondaries emitted by a primary electron that strikes the plate with an energy corresponding to the grid-voltage. The number of secondary electrons emitted by one primary electron will hereafter be called the coefficient of secondary emission. The curves showed that this coefficient increased with the velocity of impact. For a given velocity of impact a comparison of curves taken under similar conditions before and after treating the surface showed that the coefficient of secondary emission was everywhere increased by the heat treatment.

B. *Curves Taken at Constant Plate-Voltage.*

The forms of the curves taken under this condition are different for the two experimental tubes because of the fact that the "stray field" is largely eliminated in the second tube. This means that varying the grid-voltage for the last tube has practically no influence on the number of primary electrons that shoot through the holes of the shield. Fig. 4 represents two curves taken with the first tube while Fig. 5 and Fig. 6 show two curves typical of those taken with the second tube. The principal facts to be noted in connection with these curves are as follows:

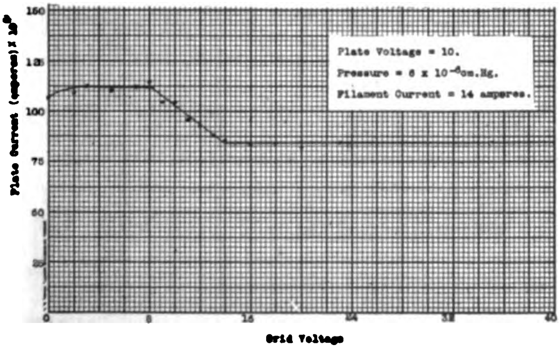

Fig. 6.

1. Increasing the grid-voltage from zero (cf. Fig. 4) causes the ratio of the current flowing into the plate to the sum of the currents flowing into the grid and plate to decrease quite rapidly until a final constant value is attained for a grid potential of 25 volts, provided a potential greater than about 50 volts is applied to the plate of the first tube. Any further increase of the grid voltage does not change this ratio until a

potential slightly less than the applied constant plate voltage is reached. At this voltage Fig. 4 shows that the ratio drops suddenly to a value which is constant for all higher grid voltages. The fact that there exists a constant ratio above 25 volts indicates that the "stray field" is independent of the grid voltage above this point. This constant value is a measure of the number of primary electrons incident on the plate. The second decrease in the ratio is due to the fact that for grid voltages greater than the plate voltage the secondary electrons are attracted to the grid. The difference in the two constant currents is thus a measure of the number of secondary electrons. The ratio of this difference to the original constant plate current gives an accurate measure of the coefficient of secondary emission.

2. The curves taken with the second tube can be extended to much lower plate voltages than was possible on account of the "stray field" with the first tube. Fig. 6 shows how beautifully the method works at 10 volts primary velocity. Table I. gives the coefficients of secondary emission as calculated from a series of such curves before and after treating the surface. The principal limitation that is imposed on obtaining the number of secondaries per primary in this manner is the fact that the strength of the source of primary electrons may not stay constant.

TABLE I.

Coefficient of Secondary Emission.

Plate Voltage.	Before Treating Surface.		After Treating Surface.	
	Increasing Voltage.	Decreasing Voltage.	Increasing Voltage.	Decreasing Voltage.
5	—	—	0	0
7	—	—	.08	—
10	—	—	.21	.23
15	—	—	.30	.29
20	—	—	—	.34
25	—	—	.39	.41
50	.38	.46	.46	.45
75	.59	.58	—	—
100	.64	.64	.68	.61
150	—	—	.85	.86
200	.70	.70	1.00	1.00
250	—	—	1.08	1.10
300	.85	.85	1.26	1.26
400	.91	.92	1.28	1.25
500	—	—	1.23	1.25

It is estimated that the means of the two values of the coefficient of secondary emission in the table are not subject to an error exceeding

3 per cent. This is doing as well as could be expected, since the filament was supplied by the 110-volt power line whose voltage fluctuations sometimes caused the temperature and hence the emission to change by about 5 per cent. The form of the curves and the sharpness with which they fall off indicate however that the coefficient could be obtained to a much higher degree of accuracy with sufficiently constant primary sources.

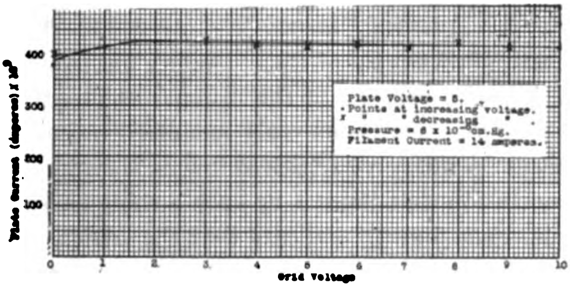

Fig. 7.

3. Figure 7 represents a curve taken when the primary electrons hit the plate with a velocity of 5 volts. It is to be remarked that there is no drop in the current flowing to the plate when the grid voltage exceeds the plate voltage such as Fig. 6 shows is present for energies of impact of 10 volts. *This means that at 5 volts there is neither a true secondary emission nor a reflection of electrons.* This is contrary to what has always previously been considered to be the case.

C. *The Velocities of the Secondary Electrons.*

The curves at constant plate-voltage show that the current does not drop appreciably from its constant value for voltages of the grid less than those of the plate until this voltage-difference is smaller than about 5 volts. Fig. 8 shows how these curves fall off when the velocity of impact of the primary electrons is 300 volts. The principal facts to be noted in this connection are as follows:

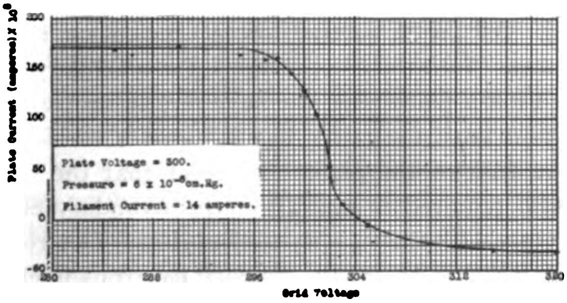

Fig. 8.

1. A comparison of Fig. 6 with Fig. 8 shows that the maximum velocity of emission of the secondary electrons is about 5 volts when the velocity of impact of the primary electrons is 300 volts, while it is 2 volts when the velocity of impact is only 10 volts. This shows that the maximum velocity of emission increases *but slightly* when the energy of impact of the primary electrons is raised. The fact that the velocities of the secondary electrons are small is in accord with the results of other observers.

2. The fact that the velocity of the secondary electrons is always considerably less than the velocity of the primary electrons indicates that the secondary electrons have been truly reemitted rather than reflected. This holds even for primary velocities as low as 10 volts.

3. If this conclusion that there is no such a thing as a "reflection" of electrons is true, Figs. 6 and 7 indicate that the critical velocity for the reemission of secondary electrons lies between 5 and 10 volts. A few attempts at locating this "ionizing potential" of the surface atoms more accurately were unsuccessful because of the fact that the variations caused by the fluctuations of the filament-current were of the same order of magnitude as those differences which it was desired to detect.

4. Fig. 8 shows that it requires an accelerating field of nearly 10 volts to pull the greater portion of the secondary electrons from the plate to the grid, that is, in order to reduce the current flowing to the plate to its final nearly constant value. All of the other curves showed similarly that these accelerating fields were required. The magnitude of these fields decreased however as the velocity of impact of the primary electrons diminished. These facts are readily explained by considering that the surface of the plate is not smooth but rough and irregular. With such a surface the secondary electron liberated by an impinging primary electron may be freed in a "pocket" or depression formed by the atoms of the surface. As the attractive forces exerted by the grid do not readily "penetrate" into these depressions, the secondary electron which is liberated in such a place may not have any attractive forces exerted on it by the grid until the latter has attained a considerable positive potential above that of the plate. The fact that the strength of the accelerating field required to pull all of the secondary electrons to the grid varies directly with the velocity of the primary electrons is explained by the assumption that the faster primary electrons liberate secondary electrons in "pockets" that are deeper or more perfectly shielded from the attraction of the grid than do the slowly moving electrons. It might be thought that the space charge effect of the secondary electrons would also be a factor which would require a considerable grid potential

to overcome it. Calculations which are based on the magnitude of the secondary electron current and the dimensions of the apparatus show however that this effect is probably less than one half a volt. The assumption of the irregular surface is then the most plausible explanation of the effect.

D. *The Effect of Temperature on Secondary Electron Emission.*

The curves at constant plate-voltage that were obtained when the plate was hot were of the same form as those obtained and already described when it was cold. The effect of the correction caused by the leakage current is simply to displace every point of the curve by a constant amount parallel to the current axis. Table II. gives the corrected values of the coefficient of secondary emission as obtained from the corrected curves taken at constant plate voltage for various temperatures of the plate. The table shows the extraordinary fact that the secondary emission appears to be greatly diminished by increasing the temperature of the plate. This was especially true for the primary electrons possessing the highest velocities.

Table II.

Coefficient of Secondary Emission.

	Furnace Current = 0.		Furnace Current = .25.		Furnace Current = .40.	
Plate Voltage.	Increasing Voltage.	Decreasing Voltage.	Increasing Voltage.	Decreasing Voltage.	Increasing Voltage.	Decreasing Voltage.
25	.34	.36	—	.34	.21	.24
100	.67	.68	.71	.71	.50	.51
200	1.09	1.09	.97	.95	—	.56
300	1.26	1.265	.88	.88	—	.31
400	1.41	—	1.12	1.12	.51	.525

This decrease is not due to the magnetic field which is caused by the current in the furnace. This was shown to be the case by applying a magnetic field of the same strength as that which was due to the furnace by means of a large solenoid which was slipped over the experimental tube. Fig. 9 shows a curve obtained when no current was flowing in the furnace and no magnetic field was applied. Fig. 10 shows a curve taken under the same conditions when a magnetic field which was equal in intensity to that which would be set up by passing a current of .40 amperes through the furnace was applied by means of the exterior solenoid. It is to be noted that the coefficient of secondary emission is ***not decreased*** in the second case, as would be the case if the results in Table II. were due to the magnetic field instead of to the temperature of the plate. The main effect of the magnetic field seems to be that it causes the drop in the current to be much less abrupt than when it is not applied. Curves which were taken when the plate was hot showed similarly that the application of the exterior magnetic field did not decrease the coefficient of secondary emission corresponding to the temperature of the plate.

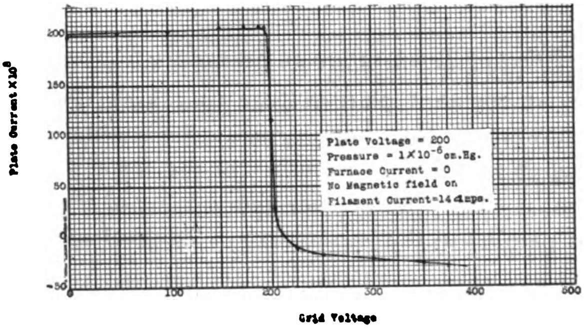

Fig. 9.

DISCUSSION OF THE RESULTS.

O. von Baeyer and A. Gehrts stated in their papers that the two most important sources of error in their experiments were ionization and "tragerbildung." The first error only occurred for velocities of the primary electrons greater than the ionization potential of the residual gas. The second error occurred only with the very slow primary electrons which attached themselves to gas molecules and formed slowly moving ions. These on account of their large mass did not hit the plate hard enough to cause a reemission. The tables given in the papers of both of these authors show this last effect is undoubtedly present. Any of their conclusions for velocities of the primary electrons less than 10 volts

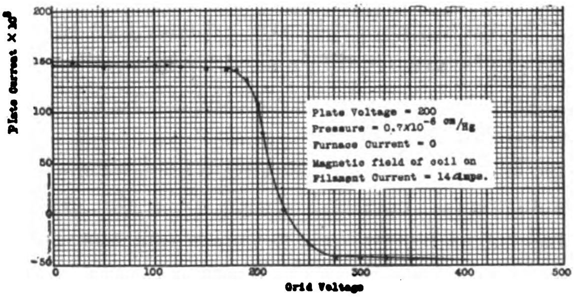

Fig. 10.

are probably then without any great significance as they themselves point out. On account of the high vacuum prevailing throughout the present experiments it is certain that these two sources of error are of negligible importance.

Dr. A. W. Hull in his paper on the dynatron explains the initial rise in his curves taken at constant grid voltage on the ground that there is a voltage drop along the hot filament. As the voltage is increased thermions first reach the plate only from the negative end of the filament, and then gradually from the whole filament. This to be sure explains part of the rise but it can not explain it totally as this same initial rise was found in the present experiments where an equipotential filament was used. The curves in Hull's paper are moreover not extended to plate voltages much higher than the prevailing grid voltage, *i.e.*, to the final saturation values obtained in these experiments. It is accordingly not evident on what grounds the statement is based that as many as twenty secondary electrons may be produced per primary. *It is certain that copper surfaces such as were used in this investigation do not give more than* 1.5 *electrons per primary.* It is expected that molybdenum and other metals will be tested in the future to ascertain to what extent the effect depends on the nature of the metal.

The results found in this investigation do not indicate a maximum emissive power at a primary velocity of about 250 volts. Gehrts found the coefficient of secondary emission diminished beyond this point while the results shown in Table I. of this investigation show that there is a final constant value, or one that increases very slowly.

Summary and Conclusions.

The principal conclusions which may be drawn from this investigation are as follows:

1. A method for obtaining the average number of secondary electrons emitted by a single primary electron when the latter bombards a copper surface has been developed. This method is independent of the geometrical dimensions of the apparatus. No appreciable errors are introduced due to ionization or attachments on account of the high vacua used.

2. The coefficient of secondary emission was found to increase with the velocity of impact for all velocities of the primary electrons which exceeded the ionizing potential of the metallic surface. This coefficient is nearly constant above 300 volts primary velocity for a copper surface from which the adhering gas films and other impurities have been removed by heating.

3. By heating the copper surface in vacuo the surface condition is

changed. The effect of this is to increase the coefficient of secondary emission for any given velocity of impact of the primary electrons above the value it possessed before the heat treatment.

4. The maximum velocity of emission of the secondary electrons varies from 2 volts, when the velocity of the primary electrons is 10 volts, to about 5 volts when the velocity of the primary electrons is 300 volts.

5. There is apparently no reflection or secondary emission of electrons when the velocity of the primary electrons is 5 volts. This fact coupled with the small value of the maximum velocity of emission of the secondary electrons indicates there is no such a thing as true "reflection" of electrons.

6. The approximate critical velocity of the primary electrons necessary to produce a reemission lies between 5 and 10 volts.

7. The coefficient of secondary emission was found to be quite considerably reduced by raising the temperature of the plate. This was especially true for the higher velocities of impact.

In conclusion the author wishes to express his indebtedness to Professor Millikan for suggesting the problem and for continuous assistance during its progress. Mr. Julius Pearson, the mechanic at the Ryerson laboratory, is responsible for several important mechanical suggestions in connection with the design of the tubes. The assistance that Mr. L. E. McAllister has rendered by helping to take the observations of the latter part of this work has been of very great value.

RYERSON PHYSICAL LABORATORY,
August 14, 1920.

THE STRUCTURE OF THE HELIUM ATOM.[1]

By Irving Langmuir.

Synopsis.

Models for the Helium Atom.—(1) *Bohr's model* is unsatisfactory because it gives too great a value for the ionizing potential and is not in accord with the optical and magnetic properties of helium. Now the chemical evidence suggests that each electron in an atom has its own separate orbit, and that these orbits are closely interrelated. Accordingly two new models are considered. (2) In the *double circle model* the two electrons are assumed to move in two separate, parallel, circular orbits. The model, however, is unstable, for the ionizing potential, computed by applying the quantum theory, comes out negative. Another objection is that the magnetic moment is not zero. (3) In the *semi-circular model* each electron is assumed to oscillate back and forth along an approximately semi-circular path in accord with classical mechanics, each being brought to rest at the end of its path by the repulsion of the other. Assuming the maximum angular momentum of each equal to $h/2\pi$, the absolute dimensions of the model were computed. The total energy comes out less than for the Bohr model, and the ionizing potential, 25.62 volts, agrees closely with the experimental value. The magnetic moment is zero.

Application of the Quantum Theory to Coupled Electrons.—The success of the semi-circular model in giving the correct value for the ionizing potential of helium suggests that in the case of coupled electrons the quantum theory should be applied not to the momentum of the individual electrons according to the relation $\int pdq = h/2\pi$, but rather to the momentum which, by being relayed from one electron to another, passes in each direction around the nucleus.

BOHR'S models for the hydrogen atom and the positively charged helium ion are now almost universally accepted. The extraordinary success with which even the fine structure of the lines of the hydrogen and helium spectra have been worked out from these models by Bohr, Sommerfeld and others, is convincing proof that in these cases the structure is definitely known. On the other hand Bohr's models for the helium atom,[2] and for all the molecules considered by him, although they have seemed satisfactory in many respects, have not given quantitative agreement with the spectra or other properties. For example the ionizing potential of helium according to Bohr's model should be 28.8 volts while experiment gives 25.4 ± 0.25 volts.[3] Sommerfeld[4] states that this model is not in agreement with the optical properties of helium. Similarly, the molecular heat of dissociation of hydrogen into atoms should be 63,000 calories according to Bohr's model, while experiment gives about 84,000 calories.[1] From the behavior of the many-lined spectrum of hydrogen in a magnetic field Lenz[2] has concluded that the electrons in the hydrogen molecule can have no momentum about the axis of symmetry. In other words either each electron must have zero momentum, or the momenta of the two electrons must be equal and opposite.

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[1] A preliminary notice giving some of the results of this paper has been published in Science, N. S., *51*, 605 (1920).

[2] Phil. Mag. *26*, 488 (1913).

[3] Frank and Knipping, Phys. Zeit., *20*, 481 (1919).

[4] Atombau, 1st edition, p. 70, Braunschweig, 1919.

From the chemical point of view Bohr's models are satisfactory only in the case of the hydrogen atom. Evidence was given by A. L. Parson[3] that in many atoms the electrons are arranged with cubic symmetry. G. N. Lewis[4] emphasized the importance of pairs of electrons and *octets* or groups of eight electrons. Kossel[5] showed that certain groupings of electrons in atoms, corresponding to the inert gases, are of unusual stability. These theories have been extended in a series of papers by the writer.[6] Similar conclusions regarding the structure and stability of the octet has been reached by M. Born[7] from evidence of an entirely different kind. Landé in a series of papers[8] has developed a mathematical theory of the motions of eight electrons about points in atoms distributed with cubic symmetry. Landé and Born, however, have not given consideration to the fact that the pairs of electrons in the helium atom and in the kernels of other atoms, exhibit a stability even greater than that of the octet.

Bohr's calculations,[9] based on coplanar, concentric, circular orbits gave no indications that a pair of electrons bound by a nucleus would show unusual stability. If $-W$ represents the total energy (kinetic plus potential) of an atom then the most stable model should be that which has the greatest value of W. For Bohr's model of the helium atom, in which the two electrons revolve about the nucleus in a single circular orbit, $W = 6.13W_0$, where W_0 is the value of W corresponding to the hydrogen atom. Bohr's model of the helium ion gives $W = 4W_0$. For a lithium atom, with the three electrons in a single circular ring, W

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[1] Langmuir, Jour. Amer. Chem. Soc., *37*, 417 (1915).

[2] Verh. Deut. Phys. Ges., *21*, 632 (1919).

[3] Smithsonian Inst. Publ., Miscel. Collections, *65*, No. 11 (1915).

[4] Jour. Amer. Chem. Soc., *38*, 762 (1916).

[5] Ann. Physik, *49*, 229 (1916).

[6] Langmuir, Jour. Franklin Inst., *187*, 359 (1919); Jour. Amer. Chem. Soc., *41*, 868, 1543 (1919); *42*, 274 (1920); PHYS. REV., N. S., *13*, 300 (1919); Proc. Nat. Acad. Sci., *5*, 252 (1919); Nature, April 29 (1920).

[7] Verh. deut. Phys. Ges., *20*, 230 (1918).

[8] Verh. deut. Phys. Ges., *21*, 2, 644, 653 (1919); Zeit. f. Physik., *2*, 83, 380 (1920).

[9] Phil. Mag., *26*, 492 (1913).

$= 17.61 W_0$, but if there are two electrons in an inner ring and a single electron in an outer ring $W = 16.02 W_0$. For the lithium ion with two electrons in a single ring Bohr finds $W = 15.13 W_0$. Therefore the work needed to remove an electron from a lithium atom (first model) is $2.48 W_0$, while the work required to take an electron from a helium atom is only $2.13 W_0$. It should thus be easier to ionize helium that lithium vapor. Bohr's theory is even less applicable to beryllium for the model with the greatest value of W is that in which all four electrons revolve in a single ring ($W = 37.04 W_0$). Bohr's original theory would thus indicate that lithium and beryllium (and all subsequent elements) should be inert gases having even greater stability than helium.

The remarkable stability of the pair and the octet is not explainable on the basis of Bohr's theory. The chemical and physical properties of the elements prove that the electrons in atoms are coupled together in a rather complex manner, which seems quite inconsistent with the ordinary properties of the electron. I have tried to summarize the characteristics of this electron coupling in a series of postulates.[1] It is probable that an understanding of these postulates will require more definite knowledge of the mechanism underlying the quantum theory. Landé seems to have made progress in this direction, by postulating coupled orbits. Thus according to Landé, the eight electrons of an octet move in such a way that their positions are always symmetrically located with respect to each of three mutually perpendicular planes which pass through the nucleus. Landé offers no suggestions as to a cause or mechanism of this coupling.

The chemical evidence indicates that there is not only a definite coupling between the electrons in a single shell but also between electrons in different shells. Thus the octet exists only about nuclei which already have stable pairs. The magnetic properties of the elements furnish excellent proof of electron coupling between different shells. Such action is not taken into account in Landé's theory.

The most striking case of electron coupling seems to be that in the helium atom. Since the underlying mechanism must be essentially similar to that of the octet, it would seem that a study of the structure of the helium atom offers the most promising method for the attack on the problem of the mechanism of quantum phenomena.

Bohr's work on the structure of atoms and molecules has been based on the assumption that "the dynamical equilibrium of the systems in the stationary states is governed by the ordinary laws of mechanics" while the quantum laws apply to "the passing of the systems between different stationary states." For each degree of freedom there is an infinite number of solutions of the dynamical problem, but as Sommerfeld has shown, there are as many quantum conditions to be fulfilled as there are degrees of freedom. As a result, there is only a discrete number of possible solutions. These fundamental postulates have received such strong support from the work of Bohr, Sommerfeld and others that there seems to be no reason to doubt their general validity.

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[1] See particularly Proc. Nat. Acad. Sci., *5*, 252 (1919).

According to the theory which I advanced in 1919 (Postulates 1, 3 and 4, l. c.) the electrons in their most stable positions move only within certain limited regions or "cells" within the atom. In the atoms of the inert gases the cells were found to be arranged symmetrically with respect to an equatorial plane, no electrons however lying in this plane. From this viewpoint the two electrons in the helium atom should not move in the same orbit but in separate orbits located symmetrically with respect to the equatorial plane. Let us consider the most probable motions of this kind.

Sommerfeld has shown[1] that in general a single electron in an atom must fulfill three quantum conditions corresponding to its three degrees of freedom. Choosing coördinates like those used on the earth we may describe the position of an electron in terms of its latitude, longitude and distance from the center of the atom. In ordinary helium, with the atoms in their most stable state, we must assume that each electron has only one quantum, and that this quantum corresponds to some one of the three degrees of freedom. Radial motion such as rectilinear motion along the polar axis, is to be excluded[2] since the electrons would have to pass through the nucleus. We have then to consider motion either in longitude or in latitude.

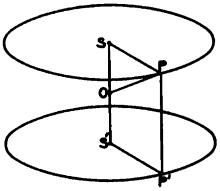

Fig. 1.
Double-circle model for the helium atom.

Motion in Longitude. Double Circle Model.—If the electrons move in longitude only, they will travel along parallels of latitude and rotate in the same direction in circular orbits in two parallel planes equidistant from the equatorial plane, as is shown in Fig. 1. Let r be the distance from the nucleus of the atom (at O) to one of the electrons P; let b be the distance OS or half the distance between the planes of the orbits; and let a be the radius SP of the orbits. Since in the helium atom the nucleus has the charge $2e$, the force of attraction between the nucleus and each electron is $2e^2/r^2$. The component of this force directed towards the other electron is $2e^2b/r^3$ while the repulsive force between the electrons is $e^2/4b^2$. Equating these, we find $r = 2b$ as a condition for dynamical equilibrium; the latitude of the electrons must therefore be 30°.

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[1] Atombau, 1st edit., p. 411 (1919).

[2] Atombau, p. 273.

If ω is the angular velocity of the electron in its orbit, the centrifugal force exerted by it is $ma\omega^2$, where m is the mass of the electron. The component of the attractive force of the nucleus in the direction towards the center of the orbit (PS) is $2e^2a/r^3$. Equating these forces we obtain:

$$m\omega^2r^3 = 2e^2. \tag{1}$$

This is as far as we can proceed by means of the classical mechanics. We still have two unknown quantities and therefore cannot determine the constants of the model.

If we impose the quantum condition that the angular momentum of the electron about the center of its orbit shall be $h/2\pi$ we find

$$2\pi ma^2\omega = h. \tag{2}$$

These two equations, together with the relation $r = 2a/\sqrt{3}$ give:

$$a = \frac{h^2}{3\sqrt{3}\pi^2me^2} = \frac{4}{3\sqrt{3}}a_0, \tag{3}$$

$$\omega = \frac{27\pi^3me^4}{2h^3} = \frac{27}{16}\omega_0. \tag{4}$$

Here a_0 and ω_0 are the values of a and ω for the hydrogen atom as found by Bohr, namely:

$$a_0 = \frac{h^2}{4\pi^2me^2}, \tag{5}$$

$$\omega_0 = \frac{8\pi^3me^4}{h^3}. \tag{6}$$

Since Bohr has shown that the kinetic energy of the electrons is numerically equal to half the potential energy but opposite in sign, we may place the kinetic energy equal to W and thus find from (3) and (4):

$$W = \frac{27}{4}\frac{\pi^2me^4}{h^2} = \frac{27}{8}W_0. \tag{7}$$

Here W_0 is the value of W corresponding to the hydrogen atom:

$$W_0 = \frac{2\pi^2me^4}{h^2} = \frac{e^2}{2a_0}. \tag{8}$$

Taking $h = 6.554 \times 10^{-27}$ erg. second; $e = 4.774 \times 10^{-10}$ e.s. units; and $m = 9.002 \times 10^{-28}$ grams, we find the following numerical values:

$a_0 = 0.5304 \times 10^{-8}$ cm.,

$\omega_0 = 4.134 \times 10^{16}$ radians per second (6.580×10^{15} revolutions per second).

$W_0 = 2.157 \times 10^{-11}$ erg which corresponds to 312,000 g. calories per g. molecule. W_0/e is equivalent to 13.55 volts.

Bohr has shown that for the positively charged helium ion $W = 4W_0$. Comparing this with equation (7) we see that the helium model that we have been considering is unstable as it would lose one of its electrons and form a positively charged helium ion. The ionizing potential would thus be negative in sign and its value would correspond to $3.375 W_0 - 4W_0 = -0.625 W_0$ or -8.5 volts.[1]

This model therefore cannot represent the atom of ordinary helium, although it may well correspond to a possible disturbed state of the atom. There is the same objection to this model as to the Bohr model in regard to the momentum of the atom as a whole about it axis of symmetry, for the work of Lenz seems to show that there should be no momentum in the most stable atoms.

Motion in Latitude. Oscillating, or Semi-circular Model.—Let us consider the case that the electrons have no component of motion in longitude. The motion is thus confined to the plane of a meridian of our system of coördinates. In Fig. 2 let FF' represent the polar axis and the circle $BEB'E'B$ a meridian circle. The intersection of the equatorial plane with the plane of the meridian is the line AOA'. If we imagine two electrons at P and P' symmetrically placed with respect to the line AA', we can conceive of a series of orbits consistent with the ordinary mechanics. We are concerned here with those in which there is as little radial motion as possible. We shall now proceed to show that a motion of the electrons back and forth along the lines CBD and $C'B'D'$ respectively is in accord with the laws of mechanics. This motion corresponds to that of the two electrons in the circular ring of Bohr's model for the helium atom except that the electrons revolve in opposite directions about the nucleus. If we were dealing with electrons having more than one quantum, we should have to consider paths which bear the same relation to elliptical orbits that the paths CBD and $C'B'D'$ bear to the circular orbits of Bohr's model.

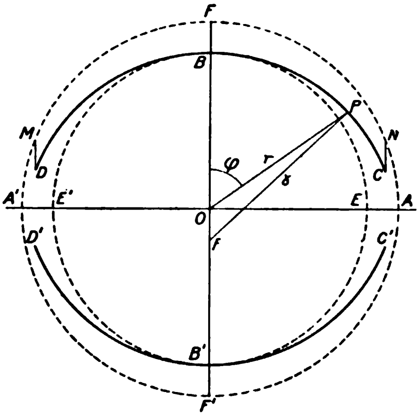

Fig. 2.

Oscillating or semi-circular model for the helium atom.

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[1] By a misprint this appeared as -5.8 volts in the preliminary notice in SCIENCE.

The electron at P is attracted by the nucleus at O and repelled by the electron at P'. Let N be the atomic number of the nucleus at O; it thus has the positive charge Ne. In moving the two electrons from infinity to the positions P and P' the work done by the attraction of the nucleus is $2Ne^2/r$ while the work that must be done against the repulsion of the electrons is $e^2/(2r \cos \varphi)$, where r is the distance PO and φ is the angle BOP. Hence the potential energy E_p of *one electron* is

$$E_p = -\frac{e^2}{r}\left(N - \frac{1}{4 \cos \varphi}\right) \tag{9}$$

and the kinetic energy E_k is

$$E_k = \frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right]. \tag{10}$$

The equations of motion for the electrons are most easily obtained by Hamilton's method. If q is any coördinate, we place $q' = dq/dt$ and define p by[1]

$$p = \frac{\partial E_k}{\partial q'}. \tag{11}$$

Letting q be successively equal to r and to φ we find from (11) and (10)

$$p_r = m\frac{dr}{dt}, \qquad p_\phi = mr^2\frac{d\varphi}{dt}. \tag{12}$$

The Hamilton function H is obtained by expressing $E_p + E_k$ in terms of q and p. Thus (9), (10) and (12) give

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right) - \frac{e^2}{r}\left(N - \frac{1}{4 \cos \varphi}\right). \tag{13}$$

The equations of motion are then found by means of the relation[1]

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q} \tag{14}$$

from (12) and (13) as follows (after dividing by m and mr^2 respectively):

$$\frac{d^2r}{dt^2} = r\left(\frac{d\varphi}{dt}\right)^2 - \frac{e^2}{mr^2}\left(N - \frac{1}{4 \cos \varphi}\right), \tag{15}$$

$$\frac{d^2\varphi}{dt^2} = -\frac{e^2}{4mr^3}\frac{\sin \varphi}{\cos^2 \varphi} - \frac{2}{r}\frac{d\varphi}{dt}\frac{dr}{dt}. \tag{16}$$

Since it is impossible, by the classical mechanics alone, to find the absolute dimensions of this model, it will be convenient to express the above equations in terms of variables which depend only on relative dimensions. It is evident from (16) and from considerations of symmetry that the electrons move with the maximum angular velocity when they are at the mid-points of their paths that is, when $\varphi = 0$. Let us then choose the unit of time so that this maximum angular velocity φ_0' shall be unity. If we let θ be the time expressed in these new units we have

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[1] See Sommerfeld, Atombau, p. 216.

$$\theta = \varphi_0' t. \tag{17}$$

Similarly let us take as our unit of length, r_0 the radius vector BO at the mid-point of the path. Let ρ represent the radius vector at the point P expressed in the new units. Then

$$\rho = \frac{r}{r_0}. \tag{18}$$

By making these substitutions in equations (15) and (16) we are able to reduce them to the form

$$\frac{d^2\rho}{d\theta^2} = \rho\left(\frac{d\varphi}{d\theta}\right)^2 - \frac{1}{\alpha\rho^2}\left(N - \frac{1}{4\cos\varphi}\right), \tag{19}$$

$$\frac{d^2\varphi}{d\theta^2} = -\frac{1}{4\alpha\rho^3}\frac{\sin\varphi}{\cos^2\varphi} - \frac{2}{\rho}\frac{d\rho}{d\theta}\frac{d\varphi}{d\theta}, \tag{20}$$

where

$$\alpha = \frac{mr_0^3(\varphi_0')^2}{e^2}. \tag{21}$$

The total energy of the molecule is $2(E_p + E_k)$ and we may place this equal to $-W$ and thus obtain from (9), (10), (17), (18), and (21):

$$\frac{Wr_0}{e^2} = \frac{2}{\rho}\left(N - \frac{1}{4\cos\varphi}\right) - \alpha\left[\left(\frac{d\rho}{d\theta}\right)^2 + \rho^2\left(\frac{d\varphi}{d\theta}\right)^2\right]. \tag{22}$$

At the mid-point of the path (B in Fig. 2) we may place $\varphi = 0$, $\rho = 1$, $d\rho/d\theta = 0$, and $d\varphi/d\theta = 1$. Since the total energy W is constant we may substitute these values in (22) and obtain

$$\frac{Wr_0}{e^2} = 2N - \frac{1}{2} - \alpha. \tag{23}$$

Combining this with (22) gives

$$\alpha\left[\left(\frac{d\rho}{d\theta}\right)^2 + \rho^2\left(\frac{d\varphi}{d\theta}\right)^2\right] = \frac{2}{\rho}\left(N - \frac{1}{4\cos\varphi}\right) + \alpha - 2N + \frac{1}{2}. \tag{24}$$

This equation together with (19) defines the motion of the electrons in this model. It is necessary to resort to numerical calculations in order to obtain ρ and θ as functions of φ. Table I. gives the results of such calculations. The procedure was as follows:

As a first approximation ρ was assumed constant and equal to unity, so that $d\rho/d\theta$ and $d^2\rho/d\theta^2$ were taken to be zero. Equations 19 and 24 then take simple forms. Placing $N = 2$ for helium, equation (19) gives for $\varphi = 0$, the result $\alpha = 7/4$. By using this value of α approximate values for $d\varphi/d\theta$ as a function of φ can then be calculated from equation (24). By substituting these in (19) approximate values for $d^2\rho/d\theta^2$ can be found. By dividing these by $d\varphi/d\theta$ and by approximate integration with respect to φ we obtain the radial velocity $d\rho/d\theta$ as a function of φ. The end of the path (C in Fig. 2) is determined by finding the point at which $d\varphi/d\theta$ is zero. If the electrons are to return back along the same path it is evidently necessary that $d\rho/d\theta$ and $d\varphi/d\theta$ shall vanish for the same value of φ. Because of the approximate nature of the calculations this condition will not be met and it is therefore necessary to adjust the values of $d\rho/d\theta$ so that they become zero at the point C. By dividing these adjusted values by $d\varphi/d\theta$, and by approximate integration with respect to φ, we obtain ρ as a function of φ. The values of θ as a function of φ are obtained by integrating the reciprocal of $d\varphi/d\theta$ with respect to φ. From the adjusted values of $d\rho/d\theta$ the second derivative $d^2\rho/d\theta^2$ at the point $\varphi = 0$ is calculated. Substituting this in (19) gives a second approximation for α. Using this and the values of ρ and $d\rho/d\theta$ already obtained, we then proceed by (24) and (19) to obtain second approximations for $d\varphi/d\theta$ and $d^2\rho/d\theta^2$ in a similar manner. This process may be repeated until sufficiently accuracy has been obtained.

The numerical calculations were carried out with the aid of a calculating machine to five or six figures. The integrations were done by Simpson's rule or more often by Weddle's rule and special methods were adopted to increase the accuracy in the region close to the end of the path. Four series of approximations were carried out and it is believed that the results which are recorded in Table I. are accurate to about one tenth per cent.

By plotting ρ and φ as polar coördinates as in Fig. 2, it is seen that the path DBC of the electron is very nearly an arc of circle whose center lies at some point F on the line OB'. If we let γ be the distance from F to the moving point P and if ϵ is the distance OF then we have

$$\gamma = \rho^2 + \epsilon^2 + 2\rho\epsilon\cos\varphi. \tag{25}$$

Let us choose the point F so that γ is the radius of curvature of the path DBC at the point B. Then the first and second derivatives of γ with respect to θ will be zero when φ is zero, and for this point we may also place $\rho = 1$;

$$\frac{d\rho}{d\theta} = 0 \quad \text{and} \quad \frac{d\varphi}{d\theta} = 1.$$

We thus obtain

$$\left[\frac{d^2\rho}{d\theta^2}\right]_{\phi=0} = \frac{\epsilon}{1+\epsilon}. \tag{26}$$

TABLE I.

Data for the Motion of the Electrons in the Second Model of the Helium Atom, $\alpha = 2.0920$.

φ Radians.	θ.	$\frac{d\varphi}{d\theta}$.	ρ.	$\frac{d\rho}{d\theta}$.
0.00	0.0000	1.0000	1.0000	0.0000
0.10	0.1001	0.9979	1.0008	0.0165
0.20	0.2006	0.9912	1.0033	0.0325
0.30	0.3020	0.9802	1.0074	0.0480
0.40	0.4048	0.9646	1.0131	0.0628
0.50	0.5095	0.9448	1.0204	0.0765
0.60	0.6168	0.9201	1.0292	0.0887
0.70	0.7271	0.8913	1.0396	0.0992
0.80	0.8417	0.8558	1.0515	0.1074
0.90	0.9612	0.8162	1.0647	0.1131
1.00	1.0881	0.7633	1.0792	0.1146
1.10	1.2245	0.6974	1.0947	0.1114
1.20	1.3784	0.6019	1.1110	0.0995
1.25	1.4659	0.5305	1.1193	0.0885
1.30	1.5712	0.4425	1.1276	0.0701
1.31	1.5957	0.3925	1.1293	0.0650
1.32	1.6224	0.3578	1.1310	0.0591
1.33	1.6520	0.3164	1.1326	0.0521
1.34	1.6864	0.2648	1.1342	0.0434
1.35	1.7300	0.1944	1.1359	0.0318
1.355	1.7596	0.1434	1.1367	0.0234
1.36	1.8110	0.0513	1.1375	0.0085
1.3607	1.8394	0.0000	1.1376	0.0000

By comparing this with equation (19) we find since $N = 2$

$$\gamma_0 = 1 + \epsilon = \tfrac{4}{7}\,\alpha, \qquad (27)$$

where γ_0 is the value of γ corresponding to $\varphi = 0$ (BF in Fig. 2). Placing $\alpha = 2.0920$ this gives $\epsilon = 0.1954$ and $\gamma_0 = 1.1954$. Taking this value of ϵ we can now calculate γ for other values of φ by equation (25). Table II. shows that γ, the radius vector from the point F, is almost exactly constant or in other words the path DBC is nearly an arc of a circle with F as its center.

TABLE II.

Radius Vector from the Point F.

φ....	0.00	0.50	1.00	1.20	1.25	1.36073
γ....	1.1954	1.1954	1.1959	1.1956	1.1953	1.1937

To determine the absolute dimensions of this model we must now apply the quantum theory. The completely successful applications of the quantum theory to atomic structure have been limited to atoms having only one electron. There is therefore no certainty as to how this theory should be formulated for the case of the more complicated atoms. Bohr assumes[1] that "the 'permanent' state of any atomic system is determined by the condition that the angular momentum of every electron round the center of its orbit is equal to $h/2\pi$," but this is supposed to apply only to circular orbits. In the present case the momentum of each electron varies from zero to a maximum so that it might seem at first sight that Bohr's assumption is not applicable. However it is an attractive hypothesis to assume that in the case of coupled electrons, the quantum theory is concerned not with the angular momentum possessed by one electron but rather with the angular momentum which, by being transferred from electron to electron, circulates in each of two directions about the nucleus. The discrete nature of quanta suggests that units of angular momentum have something analogous to independent physical existence. In the model we are considering we may imagine that when the electrons are at the midpoints of their paths, each possesses one quantum of angular momentum. As the electrons move, the velocity decreases, but we may consider that the momentum, instead of disappearing, is transferred from one electron to the other. Thus when the electrons have reached the ends of their paths, the momentum is not zero but each electron has one half quantum of clockwise and one half quantum of counter-clockwise angular momentum. One advantage of this viewpoint is that it indicates a very close analogy between Bohr's model and the model under consideration.

Let us therefore place the angular momentum of the electrons at the midpoint of their orbits equal to $h/2\pi$:

$$2\pi m r_0^2 \varphi_0' = h. \tag{28}$$

Whence by (21) and (5) we find

$$r_0 = \frac{a_0}{\alpha}. \tag{29}$$

The energy of the atom is then found by substituting this and $N = 2$ in (23) and combining with (8):

$$W = W_0\alpha(7 - 2\alpha). \tag{30}$$

From (28), (29), (5) and (6) we obtain

$$\varphi_0' = \alpha^2\omega_0. \tag{31}$$

Placing $\alpha = 2.0920$ gives us the absolute dimensions of the helium atom according to this model:

$$\begin{aligned} r_0 &= 0.4780a_0 &&= 0.2535 \times 10^{-8} \text{ cm.,} \\ W &= 5.8911W_0 &&= 1.2707 \times 10^{-10} \text{ erg.,} \\ \varphi_0' &= 4.3765\omega_0 &&= 1.8094 \times 10^{17} \text{ radians per sec.} \end{aligned} \tag{32}$$

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[1] Phil. Mag., *26*, 875 (1913).

From these results we can now calculate from Table I. the data for the motion of the electrons in absolute units. Thus to express time coördinate in seconds we divide θ by the above value of φ_0'. The angular velocity at any point is found by multiplying $d\varphi/d\theta$ by φ_0'. The radius vector is expressed in cm. by multiplying ρ by r_0 and the radial velocity is obtained by multiplying $d\rho/d\theta$ by $r_0\varphi_0'$ which is the linear velocity of the electrons as they pass the midpoint of their paths (4.587×10^8 cm. per sec.). The time taken for one complete oscillation of the electrons is thus 4.067×10^{-17} seconds, while the number of oscillations per second is 2.459×10^{16}.

The energy of the singly charged helium ion according to Bohr is $W = 4W_0$. The difference between this and the value of W which we have found for our model ($1.8911 \times W_0$) corresponds to the energy required to ionize the helium atom. Expressing this in volts we obtain for the ionizing potential of helium the value 25.62 volts. This agrees with Frank and Knipping's value 25.4 ± 0.25 volts within the probable experimental error. Horton and Davies[1] found the ionizing potential to be 25.7 volts, and K. T. Compton, in a very recent paper,[2] finds 25.5 volts.

In Bohr's model for the helium atom the distance between the electrons remains constant. For this case, we may place in (19) $\cos\varphi = 1$; $\rho = 1$, and $d\varphi/d\theta = 1$, and thus obtain

$$\alpha = N - \tfrac{1}{4}.$$

For the helium atom placing $N = 2$ we find $\alpha = 7/4$. By substituting this value of α in equations (29), (30) and (31) we obtain that same values[3] for r, W and ω that Bohr found for the helium atom,[4] namely:

$$\begin{aligned} r_B &= \tfrac{4}{7}\, a_0 &&= 0.3031 \times 10^{-8} \text{ cm.}, \\ W_B &= \tfrac{49}{8}\, W_0 &&= 1.321 \times 10^{-10} \text{ erg.}, \\ \omega_B &= \tfrac{49}{16}\, \omega_0 &&= 1.266 \times 10^{17} \text{ radians per sec.}, \\ & && \quad (2.015 \times 10^{16} \text{ revolutions per sec.}). \end{aligned} \tag{33}$$

According to Bohr's model, the energy needed for ionization is $W_B - 4W_0$ and the ionization potential is thus 28.79 volts. This is 3.4 volts higher than Frank and Knipping's value and is thus not to be reconciled with their experiments.

We have seen from Table II. that the path of an electron in the oscillating model is nearly an arc of a circle whose radius γ_0 is given by equation (27). Multiplying this by the value of r_0 as given by equation (29) we obtain $(4/7)a_0$ as the absolute value of this radius. By comparison with (33) we see that this is identical with r_B the radius of the orbit of the electrons in Bohr's model. We thus obtain the extraordinary result that notwithstanding the complicated nature of the interaction between the two electrons in the oscillating model, the paths of the electrons consist of unaltered portions of the circular orbits of the Bohr model which are merely displaced bodily along the diameter of the Bohr orbit as indicated in Fig. 2. The circle $AFA'F'$ represents the orbit of the Bohr model. The path CBD of the oscillating model (drawn to scale) is obtained by taking a portion NFM of the Bohr orbit and displacing it by the distance FB in the direction of the radius FO. The distances MD and NC are thus equal to FB, which is 0.1635 of the distance FO. The angle φ corresponding to the end of the path (at C) is 1.3607 radians or 77° 57′ 45″.

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[1] Proc. Royal Soc., A, *95*, 408 (1919).

[2] Phil. Mag., *40*, 553 (1920).

[3] In a similar manner we may obtain Bohr's data for the hydrogen atom from (29), (23) and (31) by placing $\alpha = 1$ and $N = 1$.

[4] Phil. Mag., *26*, 489 (1913).

The above close relationship between the oscillating model and the Bohr model seems to constitute additional justification for the quantum assumption we have made. It is of interest to enquire, however, what results are obtained if the quantum condition is formulated in other ways?

Sommerfeld and others express the quantum condition by the equation

$$\int p dq = nh, \tag{34}$$

where q is a generalized coördinate and p is defined by equation (11). For the kind of model we are considering I have not been able to find any general method for determining what coördinates should be used. Let us try, however, to calculate the ionizing potential of helium by two different quantum assumptions, viz:

1. Let $q = \varphi$, so that we apply (34) to the angular momentum of the electrons about the nucleus. Then p is given by (12) and we then find by (17), (18) and (34), placing $n = 1$:

$$2\pi m r_0^2 \varphi_0' P = h, \tag{35}$$

where P is equal to P_1 defined by the equation

$$2\pi P_1 = \int \left(\frac{d\varphi}{d\theta}\right) \rho^2 d\varphi. \tag{36}$$

The integration is to be carried over a complete cycle. Equation (36) reduces to (28) if $P = 1$.

2. Let q represent the distance between P and B along the path DBC (Fig. 2). The quantum condition then becomes

$$\int mvds = h. \tag{37}$$

where v is the linear velocity of the electron at the point P and s is the distance along the path that separates the point P from the midpoint B. This can be reduced to (35) if we place P equal to P_2 which is defined by

$$2\pi P_2 = \int \left[\left(\frac{d\rho}{d\theta} \right)^2 + \rho^2 \left(\frac{d\varphi}{d\theta} \right)^2 \right] d\theta. \tag{38}$$

From equation (35), by methods similar to those used in the derivation (29) and (30), we obtain

$$r_0 = \frac{a}{\alpha P^2}, \tag{39}$$

$$W = \alpha(7 - 2\alpha)P^2 W_0. \tag{40}$$

The values of P can be found by approximate integration from the data of Table I. but P_2 can also be calculated in terms of α as follows. By comparison of (38) with (10) and (22) we find that the quantity in brackets is equal to $2E_k r_0/\alpha e^2$. By reference to (17) and (23), placing $N = 2$ we then obtain

$$2\pi P_2 = \frac{(7 - 2\alpha)\varphi_0'}{\alpha W} \int E_k dt.$$

But

$$\int E_k dt = 4\bar{E}_k t_c,$$

where $\bar{E}_k$ is the time average of the kinetic energy of each of the electrons, and t_c is the time taken by an electron to move from B to C. Bohr and Sommerfeld have shown that the total energy ($-W$) of any atom is equal to the mean kinetic energy with its sign reversed. Therefore $2\bar{E}_k = W$ and by considering (17) we then find

$$P_2 = \frac{(7 - 2\alpha)\theta_c}{\pi\alpha}. \tag{41}$$

Placing $\alpha = 2.0920$ and $\theta_c = 1.8394$ (Table I.), we thus obtain $P_2 = 0.7881$. By approximate integration of equation (38) by the data of Table I. we find $P_2 = 0.7886$. The closeness of this agreement affords a check on the accuracy of the data in Table I. Substituting the value of P_2 in (40) we find W and thus calculate the ionizing potential as -4.58 volts.

In a similar way we find from (36) $P_1 = 0.7807$ which gives an ionizing potential of -5.55 volts.

The oscillating model for the helium atom is thus not compatible with the formulation of the quantum theory given by equation (34). On the other hand the success in calculating the ionizing potential by assuming the angular momentum at the midpoint of the path to be $h/2\pi$, as well as the remarkable relationships with the Bohr model which were developed, lend strong support to the oscillating type of model and suggest new directions in which the quantum theory may be applied to atoms and molecules containing more than one electron.

The real test for the new model will probably be obtained through a study of the spectrum of helium in the neighborhood of 590 Å. for this spectrum should make it possible to calculate the ionizing potential with very high accuracy. It is also desirable to test the model by calculating other atomic properties such as refractive index, absorption and ionization by X rays. The mathematical difficulties appear rather serious but by no means insuperable. It is hoped that the data of Table I. may prove to be of service in such calculations.

Research Laboratory,
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Schenectady, N. Y.,
November 28, 1920.

ON THE MOTION OF A SPHERE OF OIL THROUGH CARBON DIOXIDE AND A DETERMINATION OF THE COEFFICIENT OF VISCOSITY OF THAT GAS BY THE OIL DROP METHOD.

By Leo Joseph Lassalle.

Synopsis.

I. *Coefficient of Viscosity of CO_2 by the Oil-drop Method.*—The value of e being known, through work in air, to an accuracy of about .1 per cent., the oil-drop method as developed by Millikan has been used in CO_2 for the determination of the coefficient of vicsosity of that gas with the result at 23° C. $\eta = 1.490 \times 10^{-4}$.

II. *Coefficient of Slip between CO_2 and Oil.*—The value of the constant A in Millikan's equation $e_1 = e(1 + Al/a)^{3/2}$ is found in CO_2 to be 0.8249 as against Millikan's value in air 0.864.

III. *Variability of the Constant A for Values of l/a Greater than .5.*—Precisely as in Millikan's work in air, A was found to be constant only up to $l/a = .5$, beyond which it kept increasing as far as it was followed, viz., up to $l/a = 12$.

Introduction.

The behavior of oil drops falling in air has been reported upon by R. A. Millikan.[1] Every precaution possible was taken to assure the highest degree of accuracy. The value of e obtained was shown to be accurate to within 0.2 of one per cent. The law of fall for oil drops in hydrogen was investigated by R. A. Millikan, W. H. Barber, and Y. Ishida.[2] The same precautions as those taken for air were observed.

The work here described was undertaken at the suggestion of Dr. Millikan in order to find the correction factor to Stoke's law for carbon dioxide, and the coefficient of viscosity of that gas by a new method.

The value of $e_1^{2/3}$ is directly proportional to the coefficient of viscosity, η, of the gas used. As is shown later, this relationship furnishes an elegant method of determining η for any gas, and is used in this paper to obtain the viscosity coefficient for CO_2 at 23° C.

The apparatus used was the same one used by R. A. Millikan.[3] The same precautions were taken to give as high a degree of accuracy as possible. All of the time observations were taken with a Gaertner recording chronograph. This instrument records time to one one-hundredth of a second. It was adjusted so as never to be in error by more than two hundredths of a second per minute.

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[1] "On the Elementary Electrical Charge and The Avagadro Constant," Physical Review, N.S., Vol. II., No. 2, Aug., 1913, p. 109.

[2] "The Law of Fall of a Droplet Through Hydrogen," Physical Review, Series 2, 5, 1915, p. 334.

[3] Phys. Rev., N.S., Vol. II., No. 2, Aug., 1913, p. 109.

The following method described by Langmuir was used in generating the CO_2. Using a Kipp generator, $CaCO_3 + 2HNO_3 = CO_2 + Ca(NO_3)_2 + H_2O + HNO_3$ vapor $+ H_2O$ vapor. The gas is bubbled through $NaHCO_3$, which takes up acid vapors but no CO_2; then it passes through P_2O_5 tubes which take up all H_2O vapor.

The HNO_3 was a 50 per cent. pure H_2O solution through which commercial CO_2 had been bubbled for 5 hours before being put into the Kipp generator. The $CaCO_3$ (marble) was broken and boiled 4 hours in a very dilute solution of HNO_3. The vessels into which the gas was introduced were evacuated to a pressure below 0.5 mm. before they were filled. They were then pumped down again to the same low pressure and refilled with CO_2. This process was repeated three times before any observations were taken, thus assuring that not more than one part of air in $(1{,}500)^3$ parts of CO_2 remained in the vessels.

For a complete discussion of the apparatus used and the precautions taken to assure accuracy in the determinations of the constants that enter into the following results see paper by R. A. Millikan.[1]

Part I.

Determination of Coefficient of Viscosity of CO_2.

Oil droplets were obtained for observation by aspirating oil with CO_2 gas under pressure. They were held for a sufficient length of time to observe the time of fall under gravity (t_g) on an average of eighteen times. The average number of changes of charge obtained for each drop was seven for the first thirty-six drops, which are the ones used in the determination of the coefficient of viscosity and the correction factors "b" and "A."

The results of the observations were such as to leave no uncertainty as to the greatest common divisor of $[(1/t_g) + (1/t_f)]$, which will be represented hereafter by $[(1/t_g) + (1/t_f)]_0$, and which is the variable factor upon which the value of $(e_1^{2/3}/\eta)$ and (a) are dependent.

It is significant that these thirty-six drops represent every drop observed where P.D. was constant to as much as 0.4 of one per cent. and where as many as three changes of charge were obtained. In the equation

$$e_n = \frac{4}{3}\cdot\pi\cdot\left(\frac{9}{2}\eta_1\right)^{3/2}\cdot\left(\frac{1}{g(\sigma-\rho)}\right)^{1/2}\cdot\frac{(v_1+v_2)\cdot(v_1)^{1/2}}{F}\cdots, \qquad (1)$$

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[1] R. A. Millikan, Phys. Rev., N.S., Vol. II., No. 2, Aug., 1913, pp. 109–143; also Phil. Mag., XXXIV., p. 13, 1917.

η_1 is the coefficient of viscosity of CO_2 at 23° C., g the acceleration due to gravity, σ the density of the oil, ρ the density of the CO_2, v_1 the speed of descent of the drop under gravity and v_2 the speed of ascent under the action of the electrical field of strength F.

If we now allow e_1 to be the greatest common divisor of all the various values of e_n found on a drop during the observations upon it, and if we let

$$v_1 = \frac{D}{t_g},$$

$$v_2 = \frac{D}{t_f},$$

and

$$F = \frac{\text{P.D.}}{d},$$

where D is the distance the drop falls in time t_g and the distance it rises in time t_f, and d is the distance between the two parallel, horizontal condenser plates, between which a difference of potential in volts, represented by P.D., is maintained by means of storage cells which do not change more than four parts in a thousand during a series of observations on a given drop, then equation (1) may be written in the form

$$e_1 = (300d)\cdot\frac{4}{3}\cdot\pi\cdot\left(\frac{9}{2}\eta_1\right)^{3/2}\cdot\left(\frac{1}{g(\sigma-\rho)}\right)^{1/2}\cdot\frac{\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0\cdot\left(\frac{1}{t_g}\right)^{1/2}\cdot D^{3/2}}{\text{P.D.}}, \qquad (2)$$

where $[(1/t_g) + (1/t_f)]_0$ is the greatest common divisor of $[(1/t_g) + (1/t_f)]$ for the given drop.

Equation (2) may be written

$$\frac{e_1^{2/3}}{\eta_1} = \frac{C\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0^{2/3}\cdot\left(\frac{1}{t_g}\right)^{1/3}}{(\text{P.D.})^{2/3}}, \qquad (3)$$

where

$$C = \left[\frac{9}{2}D\left(\frac{4\pi\times 300d}{3}\right)^{2/3}\cdot\left(\frac{1}{g(\sigma-\rho)}\right)^{1/3}\right]. \qquad (4)$$

This gives a simple method of calculating $e_1^{2/3}/\eta_1$.

a, the radius of the falling drop, is calculated by means of the expression

$$a = \sqrt[3]{\frac{3Fe}{4\pi g(\sigma-\rho)}\cdot\frac{v_1}{(v_1+v_2)_0}}, \qquad (5)$$

which may be written,

$$a = C'\left(\frac{\frac{1}{t_g}\times\text{P.D.}}{\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0}\right)^{1/3}. \qquad (6)$$

The pressure of the gas p is observed directly.

Table I. gives the values of the factors which enter into the determination of $e_1^{2/3}/\eta_1$ and a, as well as into the values of $e_1^{2/3}$, e_1, and $e^{2/3}$. The last three values can only be obtained after η_1 and b have been calculated.

For the present let us assume that the table does not contain $e_1^{2/3}$, e_1, nor $e^{2/3}$, and plot $e_1^{2/3}/\eta_1$ against $1/pa$. Curve 1 shows this relationship.

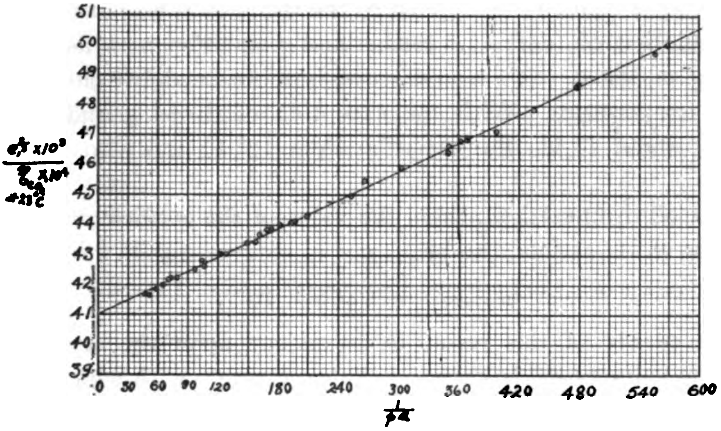

Fig. 1.

It is a straight line which crosses the $e_1^{2/3}/\eta_1$ axis at 41.00×10^{-4}.

If we write

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right),$$

then

$$\frac{e_1^{2/3}}{\eta_1} = \frac{e^{2/3}}{\eta_1}\left(1 + b\frac{1}{pa}\right).$$

It is evident that, for $1/pa = 0$,

$$\frac{e_1^{2/3}}{\eta_1} = \frac{e^{2/3}}{\eta_1}.$$

We are justified in saying that the value of the elementary electrical charge on an oil drop is independent of whether it be in CO_2 or in any other gas, say air. Since the value of $e^{2/3}$ in air is known to be 61.085×10^{-8} E. S. units, with a probable error of less than 0.1 of one per cent.,[1] then at $1/pa = 0$, $e_1^{2/3} = e^{2/3} = 61.085 \times 10^{-8}$;

$$\therefore \frac{e_1^{2/3}}{\eta_1} = 41.00 \times 10^{-4}$$

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[1] This is the value obtained by reducing the number given by R. A. Millikan in Phil. Mag., XXXIV., p. 13, by .07 per cent. to allow for the change in the value of the coefficient viscosity of air from .0001824 to .00018227.

gives

$$\eta_1 = \frac{61.085 \times 10^{-8}}{41.00 \times 10^{-4}} = 1.490 \times 10^{-4},$$

which, because all the calculations were reduced to 23° C., gives the value of the coefficient of viscosity at 23° C.

Wherever the temperature differed from 23° C. during an observation the correction factor to be applied to the value of η_1 was introduced into the calculation of $(e_1^{2/3}/\eta_1)$. Sutherland's equation was used and the variation of log η_1 with temperature was obtained by plotting log η_1 against temperature. With the exception of drops 5 and 15, which differ by about 1° C., the variation from 23° C. was always less than 0.5° C.

By the oil drop method, then, the value of the coefficient of viscosity of CO_2 at 23° C. is found to be

$$\eta_{CO_2 \text{ at } 23° C} = 1.490 \times 10^{-4}.$$

The accuracy of this result is dependent upon the accuracy with which the value of $e^{2/3}$ is known and the accuracy with which the intercept on the $e_1^{2/3}/\eta_1$ axis for CO_2 is known.

$e^{2/3}$ is known with an accuracy of about 0.1 of one per cent. The $e_1^{2/3}/\eta_1$ intercept is known with a probable error of about 0.5 of one per cent. Therefore, the value of η_1 given by these observations should be accurate to about 0.5 of one per cent.

Therefore

$$\eta_{CO_2 \text{ at } 23° C} = (1.490 \pm 0.0080) \times 10^{-4}.$$

Breitenbach[1] obtained

$$\eta_{CO_2 \text{ at } 15° C} = 1.457 \times 10^{-4}.$$

Applying the Sutherland equation to his value there results

$$\eta_{CO_2 \text{ at } 23° C} = 1.494 \times 10^{-4}.$$

If we assume that this investigator's value is too high for CO_2 in the same ratio that his value for air is too high, we get

$$\eta_{CO_2 \text{ at } 23° C} = (1.494 \times 10^{-4}) \frac{1786}{1810} = 1.474 \times 10^{-4},$$

a value 1 per cent. lower than the one obtained in this paper.

Ernst Thomson,[2] using a vibrating disc, obtains a value of η for CO_2 which is 0.0000004 higher than Breitenbach's. However, the main object of his investigation was to find the relationship between the η for two gases and the η for various mixtures of these two gases. He does not claim any high degree of accuracy for his value of the coefficient of viscosity of CO_2.

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[1] Breitenbach, Ann. d. Phys., 5, 1901.

[2] "Ueber die innere Aeibung von Gasgemischen," Inaugural Dissertation, der Konigl. Christian-Albrechts Univ. zu Kiel.

P. Phillips[1] gives a value of η which reduced to 23° C. by Sutherland's equation, becomes

$$\eta_{CO_2 \text{ at } 23^\circ C} = 1.494 \times 10^{-4}.$$

He used the A. O. Rankin device,[2] which is a capillary tube method. The object of his investigation was to determine the variation of η with pressure, going to pressures of eighty atmospheres. No high degree of accuracy is claimed for the determination of the η at atmospheric pressure. The important point sought after was to obtain relative values as pressure varied.

If, however, we take the mean of the values obtained by Breitenbach, Thomsen and Phillips, a value of $\eta_{CO_2 \text{ at } 23^\circ C} = 1.489 \times 10^{-4}$ is obtained. This value is different by less than 0.1 of one per cent. from the one here obtained.

PART II.

The A and b Correction Terms in CO_2.

Table I. gives the values of $e_1^{2/3}/\eta_1$ corresponding to the various values of $1/pa$. $e_1^{2/3}$ is obtained by multiplying $e_1^{2/3}/\eta_1$ by the value of η_1 as found in part 1.

Fig. 2 gives the result of plotting $e_1^{2/3}$ against $1/pa$.

If, now, we write

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right), \tag{7}$$

and let $y = e_1^{2/3}$; $y_0 = e^{2/3}$, and $x = (1/pa)$, we have $y = y_0 + by_0x$. By differentiation, then, the above becomes

$$b = \left(\frac{dy}{dx} \div y\right) = \frac{\text{Slope of } (y, x) \text{ curve}}{y\text{-intercept}}. \tag{8}$$

This gives a simple method of determining the correction factor b.[3] The last column in Table I. gives the values of $e^{2/3}$ calculated by substituting in (7) and solving. The values of e follow directly from those of $e^{2/3}$. It will be noticed that no value of $e^{2/3}$ varies from 61.085 by as much as 0.5 of one per cent.

The value of b, obtained by substituting in (8) is

$$b = \frac{\text{Slope of } y, x \text{ curve}}{y\text{-intercept}} = \frac{(75.37 - 61.085)}{600 \times 61.085},$$

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[1] P. Phillips, Roy. Soc. Proc., Ser. A, 87, pp. 48–61.

[2] Roy. Soc. Proc., 1910, A, Vol. 83, p. 265.

[3] R. A. Millikan, PHYS. REV., II., p. 118, 1913, and XXXII., p. 381, 1911.

TABLE I.

No.	Temp. °C.	P. D. Volts.	t_g (Sec.).	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times \left(\frac{1}{\text{Sec.}}\right)$.	n	$a \times 10^5$ Cms.	p (Cms. Hg.).	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1 \times 10^{10}$.	$e_1^{2/3} \times 10^8$.	$e^{2/3} \times 10^8$.
1	23.27	6036.4	8.09	0.007505	18–32	30.41	69.64	47.22	0.02231	4.895	62.11	60.99
2	23.11	5862.2	10.42	0.008259	12–26	26.81	72.67	51.33	0.02426	4.884	62.02	60.81
3	23.36	5856.1	12.51	0.009099	11–18	24.41	69.69	58.78	0.02770	4.922	62.34	60.94
4	23.05	5897.9	15.52	0.010269	8–18	21.87	71.33	64.09	0.03029	4.945	62.53	61.01
5	22.10	3767.0	19.90	0.007515	11–16	19.25	74.23	69.99	0.03070	4.984	62.86	61.19
6	23.05	5875.2	20.33	0.011833	5–9	19.04	72.47	72.46	0.03424	4.998	62.98	61.26
7	23.35	5864.3	24.27	0.012865	3–10	17.45	73.02	78.49	0.03709	4.989	62.91	61.05
8	23.42	5849.4	33.13	0.015142	2–7	14.88	69.79	96.27	0.04549	5.040	63.34	61.06
9	23.20	5183.0	43.74	0.015595	4–8	12.91	74.43	104.11	0.04920	5.094	63.78	61.30
10	23.13	5192.7	21.31	0.010837	5–11	18.52	50.91	106.04	0.05012	5.059	63.49	60.98
11	23.49	5852.2	58.04	0.020410	1–6	11.18	73.13	122.32	0.05780	5.132	64.10	61.19
12	23.08	5201.6	30.48	0.013180	3–8	15.41	50.48	128.55	0.06074	5.135	64.12	61.06
13	23.18	5189.1	41.71	0.015571	1–5	13.12	51.00	149.45	0.07062	5.200	64.67	61.12
14	23.00	3945.7	30.13	0.010080	4–13	15.43	41.38	156.65	0.07402	5.204	64.70	60.98
15	21.81	3948.4	36.55	0.011260	3–10	13.95	44.34	161.67	0.07640	5.247	65.06	61.20
16	23.21	3997.8	32.97	0.010820	2–5	14.69	40.60	168.50	0.07962	5.275	65.29	61.27
17	23.60	3253.7	54.71	0.011334	11–18	11.40	50.45	173.82	0.08213	5.282	65.35	61.20
18	23.20	3948.6	38.57	0.011630	3–7	13.55	40.92	180.36	0.08522	5.308	65.56	61.25
19	23.22	3926.1	43.90	0.012370	3–11	12.69	41.02	192.11	0.09078	5.322	65.67	61.10
20	23.06	3896.0	21.19	0.008537	6–17	18.29	27.89	196.26	0.09274	5.329	65.73	61.06
21	22.93	3910.5	20.774	0.008551	6–16	18.39	25.98	209.32	0.09892	5.362	66.00	61.03
22	23.05	3922.9	34.21	0.011250	3–7	14.23	27.81	252.76	0.1194	5.482	66.98	60.98
23	23.13	3947.5	37.41	0.012037	2–10	13.53	27.73	266.56	0.1260	5.576	67.75	61.37
24	23.13	3914.8	47.47	0.013643	2–8	11.96	27.70	301.97	0.1430	5.659	68.42	61.21
25	23.24	2603.0	23.599	0.006499	7–20	16.86	17.01	348.69	0.1648	5.752	69.16	60.88
26	22.67	3248.3	17.610	0.007081	10–24	19.45	14.71	349.55	0.1652	5.797	69.52	61.18

Table I (*Continued*).

No.	Temp. °C.	P. D. Volts.	t_g (Sec.).	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times \left(\frac{1}{\text{Sec.}}\right)$.	n	$a \times 10^5$ Cms.	p (Cms. Hg.).	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1 \times 10^{10}$.	$e_1^{2/3} \times 10^8$.	$e^{2/3} \times 10^8$.
27	23.14	2606.2	23.077	0.006521	10–26	16.97	16.27	362.09	0.1711	5.826	69.75	61.12
28	23.01	3905.2	36.624	0.012158	3–11	13.66	19.85	368.94	0.1743	5.830	69.79	61.02
29	22.64	3249.0	21.004	0.007855	6–20	17.72	14.23	396.66	0.1874	5.886	70.23	60.83
30	23.22	2604.1	34.214	0.008206	6–25	13.78	16.70	434.40	0.2053	6.028	71.36	61.03
31	22.70	3252.3	28.61	0.009614	5–12	14.95	14.01	477.56	0.2257	6.168	72.46	61.09
32	23.18	2605.3	40.115	0.009111	5–15	12.63	16.55	478.58	0.2261	6.177	72.53	61.12
33	23.05	2603.4	28.175	0.007890	6–21	14.90	12.07	556.13	0.2628	6.383	74.13	60.93
34	23.04	2601.3	35.045	0.008876	4–13	13.32	13.21	568.49	0.2686	6.443	74.60	61.07

or

$$b = 0.0003898$$

(see Fig. 2).

It should be noticed that the value of b is independent of any theory and is determined by the use of values which are directly measurable.

If, however, it is desired to write (7) in the form

$$e_1^{2/3} = e^{2/3}\left(1 + A\frac{l}{a}\right), \tag{9}$$

then A can be calculated in the same way that b was, provided the values of l/a are known.

Using $\eta = 0.3502mnl\bar{C}$, where the Boltzmann value of K is used so as to give an A that can be compared with that for air obtained by R. A. Millikan in the paper previously referred to, the values of l/a can be calculated.

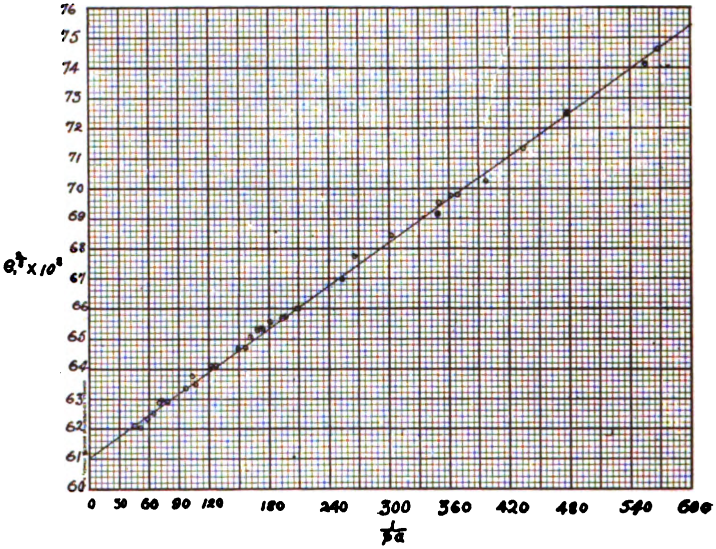

Fig. 2.

From equations (7) and (9) it is evident that

$$A\frac{l}{a} = b\frac{1}{pa}. \qquad \therefore A = b\frac{1}{pl}\cdots. \tag{10}$$

At a constant temperature $1/pl$ is a constant. Therefore, $A = bB$, which means that the $(e_1^{2/3}, 1/pa)$ curve becomes the $(e_1^{2/3}, l/a)$ curve simply by changing the scale of the abscissæ.

By calculating for a given drop and substituting in equation (10) the value of $A/b = B$ is obtained. This is checked by making the calcuation for several drops. In this way B was found to be equal to 2116. This gives for CO

$$A = 0.8249.$$

The value obtained by R. A. Millikan in his work in air was $A = .864$. The difference would seem to be slightly more than the experimental error involved in the determination of the slope of the line from which A is obtained. This error should scarcely exceed two or three per cent. There is then here a somewhat uncertain indication that the coefficient of slip between CO_2 and oil is a trifle less than that between air and oil.

Table II.

No.	Temp. °C.	P. D. Volts.	t_g (Sec.)	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times\left(\frac{1}{\text{Sec.}}\right)$.	n	$a\times10^5$ Cms.	p Cms. Hg.	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1^{2/3}\times10^8$.
35	22.70	3245.0	53.833	0.014186	3–9	10.63	16.70	631.6	0.2985	76.19
36	23.04	2604.8	38.036	0.00971	3–13	12.58	12.02	661.3	0.3125	77.01
37	23.15	2583.0	22.63	0.00771	8–19	16.11	8.160	760.83	0.3595	79.33
38	23.15	2589.5	27.67	0.00920	5–13	14.22	7.638	921.09	0.4352	82.94
39	22.90	1288.5	9.32	0.00274	67–168	24.24	4.210	980.14	0.4632	84.63
40	22.87	1288.4	14.83	0.003817	18–66	18.59	4.588	1172.5	0.5540	90.38
41	23.38	1938.6	24.02	0.00735	7–13	14.57	5.566	1233.3	0.5828	91.05
42	22.85	1286.5	20.81	0.00460	8–56	15.60	5.279	1214.6	0.5739	91.50
43	22.87	1286.0	26.67	0.00560	7–87	13.45	5.365	1386.1	0.6550	96.06
44	22.90	1288.2	20.36	0.00514	10–44	15.15	4.410	1497.1	0.7074	99.18
45	23.35	1933.3	38.55	0.01073	3–12	10.97	5.910	1542.7	0.7290	100.05
46	22.86	1288.8	19.20	0.00522	11–35	15.37	4.110	1583.1	0.7480	102.14
47	22.80	650.3	7.01	0.00169	86–137	24.94	2.450	1636.3	0.7732	106.2
48	22.85	648.5	16.91	0.00298	26–99	15.37	3.356	1938.5	0.9160	115.9
49	22.74	1289.5	27.39	0.00772	5–33	11.99	3.860	2161.6	1.021	117.8
50	22.79	651.4	10.19	0.00254	29–153	19.22	2.290	2271.5	1.073	123.0
51	22.90	653.5	9.58	0.00267	38–129	19.32	2.055	2518.7	1.190	129.5
52	23.20	92.70	9.90	0.000391	240–376	18.91	1.970	2684.7	1.269	131.0
53	22.83	649.5	23.23	0.00530	12–60	11.42	2.638	3319.6	1.569	152.9
54	22.95	656.7	16.06	0.00550	12–44	12.80	1.990	3925.1	1.855	176.0
55	22.79	651.1	42.68	0.01510	2–6	6.58	2.390	6356.7	3.004	250.4
56	23.55	649.8	9.96	0.00809	12–31	13.15	1.060	7171.5	3.389	269.3
57	22.08	156.0	31.26	0.00364	26–77	7.29	1.840	7457.8	3.524	278.4
58	23.14	155.72	13.45	0.00275	37–159	10.59	1.100	8582.6	4.056	307.2
59	22.83	192.50	7.18	0.00117	122–225	14.60	0.798	8584.2	4.056	302.7
60	22.90	155.21	6.87	0.00241	64–122	13.84	0.719	10051.	4.750	352.2
61	24.46	157.00	28.32	0.00507	3–85	6.76	1.450	10205.	4.822	359.9
62	23.22	156.8	13.23	0.00469	17–27	8.94	0.736	12076.	5.706	438.8
63	22.82	652.2	82.18	0.05502	1–2	3.44	2.211	13147.	6.212	476.1
64	22.86	156.72	13.03	0.00571	14–18	8.41	0.841	14142.	6.683	502.7
65	22.80	158.22	36.08	0.01945	3–6	3.99	1.112	22524.	10.64	805.2
66	23.02	92.56	9.87	0.00708	14–27	7.21	0.540	25702.	12.14	904.4
67	22.93	89.97	23.59	0.01103	2–8	4.60	0.824	26552.	12.55	926.4

Part III.

Limits to the Validity of Millikan's Equation in CO_2.

Observations were taken on drops at as large values of l/a as practicable with the method of obtaining drops that was used throughout these observations. It was found very difficult to get drops at pressure below one cm. of Hg. A more direct method of blowing the drops and one which will introduce less gas into the system is desirable in order to go to low pressures.

At low pressures it is not possible to have such high potential differences; also it is not possible to observe on drops of the same size as those used at pressures above, say 2 cm. These two factors tend to counterbalance each other as far as the difficulty of observing on drops with a small number of charges is concerned. If the number is not small the greatest common divisor of the series of speeds begins to be uncertain. However, no drop was used in these calculations where there was doubt as to this greatest common divisor.

Table II. gives the values of the various factors entering into the determinations of $e_1^{2/3}$, $1/pa$ and l/a for drops where the values of $1/pa$ are greater than 600.

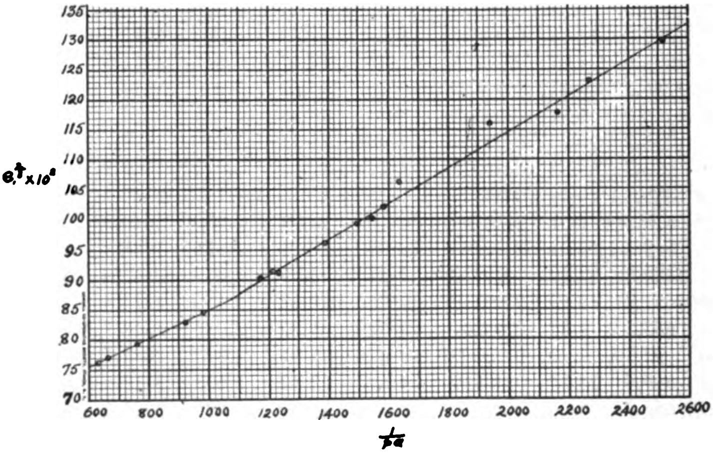

Fig. 3.

Fig. 3 gives the relationship between $e_1^{2/3}$ and $1/pa$, which is also that between $e_1^{2/3}$ and l/a. It gives this relationship from $1/pa = 600$ to $1/pa = 2{,}500$. It will be seen that from 600 to about 1,100 the slope is the same as that for $1/pa = 0$ to $1/pa = 600$. But at about the value of 1,100 the slope clearly begins to change. This behavior is quite like that found by R. A. Millikan in his work with air. He found (Phys.

Rev., II., p. 138) that the linear relation between $e_1^{2/3}$ and $1/pa$ began to break down at about $1/pa = 650$, which corresponds to a value of l/a of about .5. Here the break comes at about $1/pa = 1,100$ which will be seen from Table II. to also correspond to a value of $l/a = .5$.

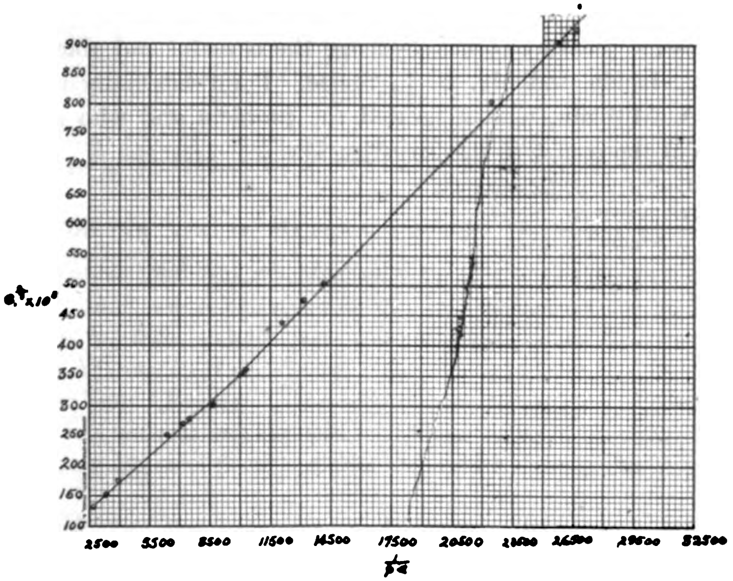

Fig. 4.

Fig. 4 shows the graph of the relations contained in Table II. in the range $1/pa = 2,500$ up to $1/pa = 26,500$, *i.e.*, in the range $l/a = 1$ to $l/a = 12$.

Summary.

I. Using the oil-drop method, the coefficient of viscosity, η, of CO_2 at 23° C. was found to be 1.490×10^{-4}.

II. The correction factors b and A as given in the equations

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right),$$

and

$$e_1^{2/3} = e^{2/3}\left(1 + A\frac{l}{a}\right)$$

were determined for CO_2 and found to be

$$b = 0.0003898,$$

$$A = 0.8249.$$

Applying the correction term to the various drops, 36 in number, values of $e^{2/3}$ were obtained, no one of which varies from 61.085×10^{-8} by as much as 0.5 of one per cent.

III. The relationship between $e_1^{2/3}$ and $1/pa$ was found for values up to $1/pa = 26{,}500$. The value of $A = 0.8249$ holds until $l/a = 0.50$, approximately. The slope of the curve then increases.

The values of l/a for CO_2, at which the change in slope begins is the same as that found by Millikan in the case of air.

It gives me pleasure to acknowledge the cheerful and able assistance of Dr. Y. Ishida and Mr. B. L. Steele. I am especially indebted to Dr. R. A. Millikan, who suggested the problem and who advised me throughout the course of the investigation.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
August 10, 1917.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE ANNUAL MEETING, CHICAGO, DECEMBER 28–30, 1920.

THE twenty-second Annual Meeting (the 106th regular meeting) of the American Physical Society was held in Chicago, on December 28, 29 and 30, 1920, in affiliation with Section B—Physics—of the American Association for the Advancement of Science. The meetings were held in the lecture room of the Kent Chemical Laboratory of the University of Chicago. The attendance was between 200 and 300. On the evening of Tuesday, December 28, there was a Physical Society dinner attended by about 180 members and friends.

The President of the Society, J. S. Ames, presided at all sessions of the Society.

On the morning of Wednesday, December 29, there was a joint session with the Optical Society of America. The members of the American Astronomical Society were also in attendance. The presiding officers were F. K. Richtmyer of the Optical Society and J. S. Ames of the Physical Society. At this session the members of the Optical Society contributed three papers to the program as follows:

Photographic Reproduction of Tone. LLOYD A. JONES.

The Spectral Distribution Required to Evoke the Gray Sensation. IRWIN G. PRIEST.

The Propagation of Light in Rotating Systems. L. SILBERSTEIN.

A special feature of the program was a paper by Prof. A. A. Michelson on "The Application of Interference Methods to Astronomical Measures." There were seven other papers from the program of the Physical Society which are mentioned below.

On the afternoon of Wednesday, December 29, there was the General Interest Session of Section B of the American Association for the Advancement of Science. The chairman of Section B, Prof. J. C. McLennan, being absent, a former chairman, Dayton C. Miller, presided. The retiring chairman of the Section, Prof. Max Mason, gave an address entitled "From Oersted to Einstein." This address will be published in full in *Science*. This was followed by a symposium on recent progress in magnetism, consisting of the following papers:

The Electron Theory of Magnetism. S. J. BARNETT.

Magnetic Susceptibilities. S. R. WILLIAMS.

The Ring Electron. A. H. COMPTON.

The regular annual meeting of the American Physical Society was held on Thursday morning, December 30, 1920, at 11 o'clock. A canvass of the ballots for officers resulted in the elections of the following officers for the year 1921:

For President: Theodore Lyman.

For Vice-President: Charles E. Mendenhall.

For Secretary: D. C. Miller.

For Treasurer: G. B. Pegram.

For Members of the Council, four-year term: K. T. Compton, H. M. Randall.

For Members of the Board of Editors of THE PHYSICAL REVIEW, *three-year term:* G. S. Fulcher, F. A. Saunders, O. M. Stewart.

At the meetings of the Council held on December 28 and 30, 1920, the following elections were made: *Elected to Regular Membership*, F. O. Anderegg, C. A. F. Benedicks, C. W. Heaps, Ludwick Silberstein, F. E. Smith, Wilhelm Stenström, M. S. Van Dusen, E. P. Warner, and A. W. Rowe; *Elected to Associate Membership*, Erik Andersen, Evald Anderson, P. G. Andres, A. A. Bacon, Edward Bennett, A. H. Bucherer, E. K. Chapman, J. F. H. Douglas, H. M. Elsey, J. E. Fox, J. G. Frayne, P. W. Froebes, D. K. Gannett, Pierre Haynes, J. R. Hobbie, Victor A. Hoersch, L. L. Holladay, F. M. Kamrenstine, J. A. Kiefer, A. L. Kimball, Jr., E. H. Lange, E. M. Lowry, L. E. McAllister, B. F. Miessner, Takeshi Nushi, A. L. Ondrak, T. D. Phillips, J. H. Pruette, Ben Rosen, L. Schmiedeskamp, K. F. Sun, V. F. Swaim, C. G. Thompson, L. A. Turner, C. L. Utterback, M. S. Waddell, A. E. Woodruff, B. C. Zimmerman, W. A. Zinzow, C. G. F. Zobel; *Transferred from Associate to Regular Membership*, H. L. Howes, E. O. Hulburt, Enoch Karrer, Benjamin Liebowitz, H. W. Nichols, and H. C. Rentschler.

The secretary reported the death of six members, and the resignation of eleven members. The membership at the present time consists of 7 honorary members, 456 regular members, and 833 associate members, making a total of 1,296.

The regular scientific program consisted of 69 contributed papers, five being read by title, this being the largest program in the history of the Society. The abstracts of all the papers will be found on the succeeding pages. The program was as follows:

The Resistance of a Compressible Fluid at High Velocities. L. THOMPSON.

The Thermal Expansion of Nickel, Monel Metal, Stellite, and Stainless Steel. WILMER SOUDER and PETER HIDNERT. (Read by title.)

Energy Losses in Commercial Hammers. EDWIN MORRISON and ROBERT L. PETRY.

The Dynamics of Capillary Flow. EDWARD W. WASHBURN.

Diathermancy of Ice, Water, and Steam. S. L. BROWN.

The Effect of Cooling on a Cement Pier. ERNEST A. HODGSON.

The Use of the Differential Thermo-couple in Melting Point Determinations. O. H. Gish.

A Seasonal Breakage of the Mainsprings of Watches. S. R. Williams.

Transmission and Absorption of Sound by a Porous Material. Paul E. Sabine.

Effect of Humidity on Sound Absorption. Paul E. Sabine.

The Acoustical Double Resonator and Its Applications. F. R. Watson.

Moving Pictures of Wave Motion. F. R. Watson and A. G. Eldredge.

A Portable Sound Producer of Considerable Range as to Frequency and Intensity. E. A. Eckhardt.

The Sound Field of a Parabolic Mirror. E. A. Eckhardt.

Apparatus for Measuring Sound Intensity. J. C. Karcher.

A Method of Measuring Phase Differences in a Sound Field. J. C. Karcher.

An Acoustic Wave Filter. G. W. Stewart.

Minimum Sound Energy for Audition. F. W. Kranz.

The Acoustics of Large Auditoriums. C. R. Fountain.

The Stability of the Atom as Related to Nuclear Composition. William D. Harkins.

The Separation of Isotopes. R. S. Mulliken and William D. Harkins.

The Abundance of Atomic Species as Related to Rutherford's Theory of Nuclear Structure. William D. Harkins.

The Variation of the Residual Ionization in Air with Pressure. K. Melvina Downey.

Influence of the Earth's Potential Gradient upon the Measurement of the Atmospheric Ionic Density by the Ebert Ion Counter. J. F. Mackell.

The Dynamic Characteristics of Three-Electrode Vacuum Tubes. John G. Frayne.

Secondary Electron Emission from Copper Surfaces. I. Garnett Barber.

Inelastic Collisions of Electrons in Vapors of Certain Compound Molecules. Paul D. Foote and F. L. Mohler.

The Effect of Angle of Incidence on the Reflection and Secondary Emission of Slow-Moving Electrons from Platinum. John T. Tate.

A Comparison of the Thermionic and Photoelectric Work Function from Platinum. Otto Koppius.

The Photo-Electric Properties of the Audion Bulb Filament, Coated with Barium, Strontium and Calcium. Theodore W. Case. (Read by title.)

A Means of Distinguishing Between Intrinsic and Spurious Contact-Electromotive Forces. R. A. Millikan.

Size and Ageing of Ions Produced in Air. Henry A. Erikson.

The Formation of Negative Ions in Air. Leonard B. Loeb.

The Structures of the Helium Atom and the Hydrogen Molecule. Irving Langmuir.

The Cubic Shapes of Certain Ions, as Confirmed by X-Ray Crystal Analysis. Wheeler P. Davey.

The Crystal Structure of Two Rare Halogen Salts. WHEELER P. DAVEY and FRANCES G. WICK.

Multiple Valency in the Ionization by Alpha Rays. T. R. WILKINS.

Additional Experiments in the Nature of the Magnetic Molecule. S. J. BARNETT and L. J. H. BARNETT.

The Application of Interference Methods to Astronomical Measures. A. A. MICHELSON.

Some Peculiarities in Energy Distribution by Speculum Gratings. L. R. INGERSOLL.

The Coloration of Glass by Ultra-Violet Light. J. B. NATHANSON.

A Strained-Glass Elliptic Polarizer and Analyzer. A. DEFOREST PALMER.

A Determination of Young's Modulus and of the Coefficient of Simple Rigidity of Natural Hexagonal Crystals of Selenium. L. P. SIEG, and R. F. MILLER. (Read by title.)

The Blackening of a Photographic Plate as a Function of Intensity of Light and Time of Exposure. P. S. HELMICK.

The Intensity of Light Transmitted Through a Deep Slit. L. P. SIEG, and A. T. FANT.

The Relation of Optical Dispersion and Crystal Structure in Rock Salt and Sylvite. H. H. MARVIN.

Some Diagrams of Lines of Force of Moving Electric Poles. H. BATEMAN.

A High Frequency Resistance Standard. JOHN G. FRAYNE.

The Effect of a Superposed Constant Field Upon the Alternating Current Permeability and Energy Loss in Iron. ALVA W. SMITH.

Effect of Strong Electrostatic Fields on the Vaporization of Tungsten. A. G. WORTHING.

A Low-Voltage Cathode-Ray Oscillograph. J. B. JOHNSON.

A Theory of the Electrical Resistance of Metals. P. W. BRIDGMAN.

Tube Experiments, with the Cathode Region in a Traverse Magnetic Field. L. THOMPSON. (Read by title.)

Piezo-electric Activity of Rochelle Salt under Various Conditions. J. VALASEK.

An Experiment on Electromagnetic Induction and Relative Motion. W. F. G. SWANN.

Vertical Electrical Currents and the Relation Between Terrestrial Magnetism and Atmospheric Electricity. LOUIS A. BAUER.

The Eclipse Magnetic Systems of May 29, 1919. LOUIS A. BAUER.

Anhysteretic Curves of Iron-Carbon Alloys Made with a Braun Tube. C. W. WAGGONER and F. A. MOLBY.

Positive Ray Analysis of Magnesium. A. J. DEMPSTER.

Pressure Shifts in a Calcium Arc. HENRY G. GALE, and L. F. MILLER.

The Luminescence of Zinc Oxide Above the Red Heat. EDWARD L. NICHOLS (Read by title.)

Some Additional Measurements of Wave-Lengths in the Calcium Arc in Vacuo. GEORGE V. MCCAULEY.

Approximate Computations of X-Ray Absorption Frequencies. William Duane.

The Evidence Regarding the So-Called "J" Radiation in the Characteristic X-Ray Spectra of the Elements. F. K. Richtmyer.

The Spectrum of Helium in the Extreme Ultra-Violet. Hugo Fricke and Theodore Lyman.

Character of the Spectra Produced by High Potential Sparks in a Vacuum. Edna Carter.

The Absorption of Light by Mixtures of Inorganic Salts. E. F. George.

Intra-Red Spectra of Isotopes. F. W. Loomis.

The Absorption Spectra of Ortho-Cresolsulphonphthalein. W. R. Orndorff, R. C. Gibbs, M. Scott and S. D. Jackson.

Dayton C. Miller,
Secretary.

The Resistance of a Compressible Fluid at High Velocities.

By L. Thompson.

The difficulties involved in an attempt to describe the motion of rigid bodies in viscous fluids by means of the equations of Hydrodynamics, are well known. Oseen[1] has introduced a system of equations which lead to results at considerable variance with those of classical hydrodynamics, particularly with reference to the essentially different character of the motion of the fluid in the rear, or wake, from that in front of the body, the former not being irrotational even in the limiting case of zero viscosity. In many instances the solution of the problem, with small viscosities, may be reduced to the determination of a potential function.

It is not likely that a complete description of resistance phenomena for velocities approximating that of sound, or higher, in compressible media, will be given by any other than a purely statistical investigation. This report concerns a plan which is being carried out in the case of small bodies (projectiles of different shapes), for a range of velocities obtainable with modern rifles, the object being the simultaneous determination of the total drag and at least two of the three principal factors or partial drags, together with their rates of change, as a function of the velocity and of the density. There is a considerable advantage in the use of small bodies, particularly in the evaluation of the pressure over the surface (which values give by integration, the partial drags, except that due to friction), since this can be accomplished by passing the trajectory through an interferometer field.

The experiments for the total drag are being made with projectiles of .4 inches diameter or less. The procedure is an improved form of an optical method described in an earlier paper.[2] The firing is done through a tube in which the density can be varied, and the observations are made on the time of passing of certain points in the trajectory, secured by the repeated interruption of a beam of light which crosses at these places, being controlled by a proper optical system to give images of a slit source. The beam is focused on a moving film. The shadow images of the projectile obtained, have a very sharp base line, from which the time measurements are made. The time calibration is produced on the record by means of light from a stationary slit, intermittently illuminated at regular intervals of twice the frequency of a freely vibrating control fork. From these observations the space time statistics are obtained with precision. Having $S(t)$ (an empirical relation, by fitting equation to data), the values of $v = S'(t)$ and v' are found and $R(v, \rho)$ or $kv'(S'(t), \rho)$.

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[1] C. W. Oseen, "Zur Theorie des Flussigkeitswiderstandes," Nova Acta R. Soc. Sc. Upsaliensis, Ser. IV., Vol. 4. Beitrage Zur Hydrodynamik, Ann. d. Phys., 41.

[2] L. T. E. Thompson, C. N. Hickman and N. Riffolt, "The Measurement of Small Time Intervals and Some Applications, Principally Ballistic," Proc. Nat. Acad. of Sciences, Vol. 6, No. 4.

Observations over 150 feet at normal air density will give a precision of at least one half per cent. in R (using .30 cal.), with the accuracy possible in the time and length measurements. By use of pressures as great as five or six atmospheres in the tube, and light projectiles (aluminum) of .30 or .22 cal., a distance of 20 to 25 feet is sufficient. The total work done, or kinetic energy change, is such that the velocity decrement is equivalent to that for a length of over 300. feet with heavy projectiles under normal circumstances. This permits better control of conditions for the set of experiments and greater accuracy of measurement. An additional advantage in the use of high density for the work with small projectiles is evident from the form of the expression for the resistance given by dimensional considerations: $R = \rho V^2 L^2 f(\mu/VL\rho, V/V_s)$, where μ is the viscosity, L is a length defining the scale and V_s the velocity of sound. From the values of f determined by experiment with the small bodies, the resistance for larger, similar bodies may be had, providing $\mu/VL\rho$, V/V_s are kept of the same value. Then for the small L, ρ should be large, and comparable velocities will be approximately equal.

The same apparatus can be used for the investigation of motion through solids, by cutting away enough of the solid in the line of flight to allow the beam of light to cross as before, or by the use of slabs of the solid, set up between the various sections of the beam. This is of importance in the ballistics of penetration and has application in plate and projectile testing. Either the resistance function, or simply the loss of kinetic energy on passing through one plate, can be found. The distance of observation need not be greater than one or two feet. This is desirable since the projectile might be deflected from the field by the plate, for long distances. Also the records show the manner in which the projectile is travelling.

KALAMAZOO COLLEGE.

THE THERMAL EXPANSION OF NICKEL, MONEL METAL, STELLITE AND STAINLESS STEEL.

BY WILMER SOUDER AND PETER HIDNERT.

THE increasing use of nickel and nickel alloy wire in spark plugs, of monel metal for high pressure steam valves, stems, seats, etc., of stellite for surgical and household uses and of stainless steel for numerous purposes has created a demand for accurate data on the thermal expansion of these materials. Differential expansions between nickel and porcelain in spark plugs, and between monel metal and steel or brass in steam valves are important factors to be considered by manufacturers of these articles.

The apparatus used was the same as that described in Scientific Papers of the Bureau of Standards, No. 352.

Some of the results are presented in the following table.

TABLE.

Materials.	Average Coefficients of Expansion × 10^6.		
	Room Temp. to 100° C.	Room Temp. to 300° C.	300° to 600° C.
10 samples of commercial nickel (94 to 99 per cent. Ni)	12.9 to 13.5	13.8 to 14.6	15.3 to 17.0
10 samples of monel metal (60 to 69 per cent. Ni)	13.7 " 14.5	14.9 " 15.2	16.6 " 18.4
5 samples of stellite (various grades)	11.0 " 15.6	12.4 " 16.7	14.7 " 18.0
1 sample of stainless steel (C 0.30, Si 0.11, Mn 0.18, S 0.011, P 0.020, Cr 13.10)	10.3	11.3	12.9

A more detailed report giving curves and tables of data is being prepared for later publication.

Bureau of Standards,
Washington, D. C.,
November 1, 1920.

Energy Losses in Commercial Hammers.

By Edwin Morrison and Robert L. Petry.

It is a well-known principle of mechanics that in case a moving object impinges against another object that the total momentum before impact is equal to the total momentum after impact. This law does not permit us to infer, however, that there are no energy losses in impacts. In fact the kinetic energy is less after impacts than before impacts of two impinging objects. It recently became of interest to study the relative values of energy losses from lack of perfect elasticity in different grades of commercial hammers, including railroad track hammers. To make these tests a hammer was substituted for one of the steel spheres in a common laboratory impact apparatus. Three grades of common mechanics' hammers secured from a hardware store have been tested with the following results for the energy losses: first grade 2.3 per cent., second grade 5.3 per cent., third grade 9.3 per cent. A similar hammer from the counter of a five and ten cent store showed a loss of 20 per cent. Railroad track hammers show an energy loss of from 2.3 per cent. to 5 per cent.

Michigan Agricultural College.

THE DYNAMICS OF CAPILLARY FLOW.

By Edward W. Washburn.

THE rate at which a liquid penetrates a small cylindrical capillary of radius r is, according to Poiseuille's law,

$$\frac{dl}{dt}=\frac{Pr^2}{8\eta l}, \qquad (1)$$

where P is the driving pressure, l the length of the column of liquid at the time t, and η the viscosity. For horizontal capillaries or for any capillary with a sufficiently small surface the integral of equation (1) reduces to

$$l^2=\frac{r}{\eta}\,\frac{\cos\theta}{2}\,rt \qquad (2)$$

for a liquid flowing under its own capillary pressure, r being the surface tension and θ the angle of contact.

The quantity, $(r/\eta)(1/2\cos\theta)$, measures the penetrating power of a liquid and may be called its *coefficient of penetrance* or its *penetrativity*.

Experiments with water, mercury and several organic liquids, using both vertical and horizontal glass capillaries of various sizes, verified completely the validity of the integral of equation (1). A dynamic method of measuring surface tension can be based upon equation (1). It is superior to the static method of capillary rise in that the result does not depend upon the radius of the capillary at a single point but upon its average radius.

If a porous body behaves as an assemblage of very small cylindrical capillaries, the volume, V, of liquid which will penetrate it in the time t, is given by

$$V=k\left(\frac{r}{\eta}t\right)^{1/2}. \qquad (3)$$

In the light of this result some data obtained by Cude and Hulett on the rate of penetration of charcoal by water were examined and the degree of penetration was in fact found to be proportional to the square root of the time, for the initial period of penetration (Fig. 1). Later the rate of penetration was much slower than required by equation (3). Whether this slower rate is due

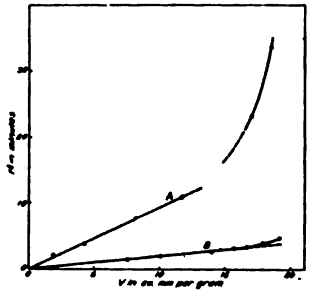

Fig. 1.

to the previous complete filling of all except micropores or whether it is due to the slower filling of expanding pores or pockets could not be determined, although the former supposition seems more probable in this instance.

Department of Ceramic Engineering,
University of Illinois.

Diathermancy of Ice, Water and Steam.

By S. L. Brown.

THE results of experiments by Tyndall, Magnus and others indicate that ice and water do not transmit but a small percentage of the incident radiant heat but these experimenters do not agree about the diathermancy of aqueous vapor.

The plan of the experiment to be described here was to measure the radiation transmitted by ice, water and steam as the temperature of the source of radiation is raised to 1000° C. The radiation from a tubular electric furnace was focused by a concave metallic mirror on a sensitive resistance element which was several feet away from the furnace. A plate of ice or a stream of water or steam was placed between the furnace and the focusing mirror and the percentage of the radiation absorbed by the intercepting screen was measured by the change in the electrical resistance of the element at the focus of the mirror. This method preserves the character of the incident, black-body radiation from the furnace since it is not changed by being sifted through glass or any other material before it reaches the substance under investigation as would be the case when the substance is enclosed in glass or any other more or less transparent material. For example, neither glass nor water are diathermanous to the shorter heat rays, and therefore, any measurements of the total radiation transmitted through water contained in glass would only indicate the transmission through the water of those rays that are not absorbed by either glass or water.

The results show that ice and water are decidedly athermanous to the total black-body radiation for temperatures of the radiating source up to 400° or 500° C. but may transmit 20 or 25 per cent. of the incident radiation when the temperature of the source is raised to 1000° C. Steam does not show any absorption whatever when the precaution is taken not to allow any condensation at the surface of the stream of steam by contact with the cooler air. Condensation on either side of the steam jet was prevented by passing it through an iron tube which was heated to a temperature higher than the temperature of the steam and allowing the radiation from the furnace to pass through the steam and across the tube through lateral openings in the heated tube. Any condensation of steam produced an absorption comparable with the absorption of water. The conclusion is that a jet of dry steam 1.5 millimeters thick certainly does not absorb as much as one per cent. of the total radiation for the temperature range of the radiating source from 500° C. to 1000° C.

Substance	Thickness.	Temperature of Radiating Source.	Per Cent. Transmitted.
Ice	3 mm.	500	1
"	"	600	3.5
"	"	700	6.5
"	"	800	9.5
"	"	900	13.5
"	"	1000	17.5
"	6 mm.	600	2
"	"	700	5
"	"	800	7.5
"	"	900	11.5
"	"	1000	15.5
"	60 mm.	700	2.5
"	"	800	5
"	"	900	8
"	"	1000	11
Water	.5 mm.	300	2.5
"	"	400	5
"	"	500	8
"	"	600	11
"	"	700	14
"	"	800	17.5
"	"	900	21.5
"	"	1000	25.5
Steam	1.5 mm.	500	100
"	"	1000	100

UNIVERSITY OF TEXAS.

THE EFFECT OF COOLING ON A CEMENT PIER.

BY ERNEST A. HODGSON.

THIS paper gives the results obtained at the Dominion Observatory, Ottawa, Canada, from experiments carried on for the past four years with an instrument known as the "Deformation Instrument," which was set up to investigate the rigidity of the earth but which was interfered with by the peculiar behavior of the cement pier on which it was mounted. The deformation instrument is an arrangement of horizontal pendulums in a very sensitive Zöllner suspension, which records photographically the movements of these pendulums. Its sensitivity is such that a tilt of .01 sec. or one inch in 300 miles records as about a millimeter on the record when the pendulums are given a period of 30 sec. If a small electric heater be stationed behind the pier upon which the instrument is mounted, and about 20 cm. from it, the whole will come to equilibrium after some days with the back face near the heater at about 69° F. and the front face at about 61° F. If now the heater be turned off for say 3 hours and then be turned on again, four phenomena appear in the records. First, after about an hour oscillations appear in the record, indicating a periodic tilting which interfers with the period of the pendulum and hence has maxima and minima. (If the heater be left off several days equilibrium will finally be reached, but in the interval the oscillations will continue to show.) Second, on turning on the heater again the light spots indicate a tilting back in the opposite direction, *i.e.*, in the direction to be expected. After this drift has gone on about .01 sec. of arc it pauses. Third, the drift then continues after about ten minutes and maintains this return drift for about an hour at which time it has nearly returned to its old position. This drift after the pause is about 6 mm. varying of course in different experiments. This indicates a tilt of say .065 sec. of arc. Fourth, the drift reverses in direction although the heating is maintained and this reverse goes on until the tilt is greater than it was at the end of the whole period of cooling before the heater had been turned on again. This reverse tilt is of the order of .22 sec. After this the pier top tilts back in the direction one would have expected on heating, but it takes about a day to reach the former position of equilibrium. If the whole experiment is tried in the reverse direction *i.e.*, the whole in equilibrium with the heater off, the heater then turned on for about three hours and then turned off again, the oscillations do not appear. If the heater be left on until equilibrium is again reached the entire curve is always smooth. It is suspected that the anomalous reverses in direction may sometimes occur but this point is not certain. Certainly there are no oscillations in any case in the heating curves. Various possible applications of these phenomena are suggested. Among these emphasis is laid on the hypothesis, suggested by the writer, that a somewhat similar phenomenon may be the source of the micros which obscure seismological records during the winter months. The paper gives full details of the experiments, including a full description of the deformation instrument itself. It will appear shortly as a publication of the Dominion Observatory.

Dominion Observatory,
Ottawa, Canada,
December 25, 1920.

The Use of the Differential Thermo-couple in Melting Point Determinations.

By O. H. Gish.

When the latent heat of fusion is small or the melting process consists of several transformations as is the case with many alloys, ordinary cooling curve methods are inadequate for measurements of the melting point, or a study of the melting process.

Though used to advantage in detecting transformations in solids the differential thermo-couple seems never to have been adapted to such use where transformations occur in the presence of a liquid phase.

The item of chief importance in this method is the design of crucible. A convenient form was made as follows:

The upper half of a cylinder of graphite was turned out in the usual shape for a crucible except that a narrow truncated cone was left along the axis and extending about two thirds way from the bottom to the top. A hole from the bottom of the cylinder extending up through the solid portion and nearly to the tip of the small truncated cone, referred to above, was made to admit the sheath and the thermo-couples it contained. A lid was also provided for the crucible.

With this device a Leeds and Northrup temperature indicator potentiometer was used for the temperature measurements and a low resistance galvanometer was used in connection with the difference couple.

When the E.M.F. of the differential couple produced too great a deflection on the galvanometer the deflection could be reduced by adjusting a counter E.M.F. which with a high resistance in series was applied across the posts of the galvanometer.

This method was found to possess the advantage that melting points could be found without taking a series of readings and plotting curves but simply by reading the potentiometer at the instant the galvanometer in the differential thermo-couple circuit showed a marked disturbance.

Determinations made with this relatively inexpensive apparatus showed a maximum deviation at 800° C. of ± 2° C.

WESTINGHOUSE RESEARCH LABORATORY.

A SEASONAL BREAKAGE OF THE MAINSPRINGS OF WATCHES.

BY S. R. WILLIAMS.

It is a very common belief among jewelers that after thunder showers they must be prepared to replace an unusual number of mainsprings. This they ascribe to the effect of electricity in lightning. If any truth is to be found in such a belief a study of the records which jewelers keep of their repairs should show a seasonal breakage of mainsprings. Such a study was made and a very pronounced effect was found. The seasonal breakage is not, however, due to electricity or magnetism or such causes but due to moisture which starts rust in minute spots either on the surface of the spring or in tiny cracks causing the skin effect of the steel to be weakened and the spring lets go. Oil on the surface of the springs prevents breakage when the springs are placed in a moist atmosphere and leads to a well-established principle that an ounce of preventive medicine is worth a pound of cure.

OBERLIN COLLEGE,
OBERLIN, OHIO.

TRANSMISSION AND ABSORPTION OF SOUND BY A POROUS MATERIAL.

BY PAUL E. SABINE.

USING the method of reverberation, simultaneous measurements of transmission and absorption of sound by different thicknesses of hair felt have been made, covering the entire range of frequencies of the ordinary musical tones.

These measurements indicated that for all frequencies, the logarithm of the reduction in the intensity of the transmitted sound is proportional to the thickness of the material through which it passes. The proportionality factor for the material tested is a linear function of approximately the fourth root of the frequency. An apparent exception was noted in the case of the highest frequency used 4,096 p.p.s. for which the reduction was somewhat less than that called for by this rule. The relation of the incident and the transmitted intensities may be expressed by an equation of the form

$$I_t = AI_0e^{-md},$$

in which *A* for any frequency is a constant whose value is determined by the absorption coefficient of the material for that frequency and *d* is the thickness. The value of *m* is given by an equation of the form

$$m = m_0(f/f_0)^p,$$

in which the constant m_0 and p are probably different for different materials depending on the closeness of the felting.

The values of the absorption coefficient were found to increase with increasing thickness of absorbing material but they approach limiting values which are reached at smaller thicknesses for the higher frequencies. An auxiliary experiment indicated that the change of absorption with change in thickness follows the same law as the reduction of the energy of flow of an air stream through the material. The dissipation of energy of flow through the material at constant pressure approaches a limiting value as the number of layers of the material is increased.

Wallace Clement Sabine Laboratory,
Riverbank, Geneva, Illinois.

Effect of Humidity on Sound Absorption by Plaster Walls.

By Paul E. Sabine.

THE absorption of sound by a massive rigid wall has been shown to be due in large part to the dissipation of energy in the pores of the reflecting surface. Absorption due to this cause increases with the frequency of the sound. Walls of ordinary plaster have been found to show slight increase in absorption with aging due presumably to increased porosity. A marked decrease in the absorption by the walls for sounds of high frequency with large increases of humidity in the room, has been observed. Thus walls of hard gypsum plaster were observed to show a gradual decrease in absorption coefficient amounting to twenty five per cent. of the original coefficient for the tone 4,096 when the relative humidity in the room was increased from 45 per cent. to 85 per cent. Weighings of a small sample of the plaster showed that in a humid atmosphere an amount of water equal to one half of one per cent. of the weight of the sample was taken up. The closing of the minute pores in this way will account for the decreased sound absorption since the latter decreases with decreasing size of the pores.

Wallace Clement Sabine Laboratory,
Riverbank, Geneva, Illinois.

THE ACOUSTICAL DOUBLE RESONATOR AND ITS APPLICATIONS.

BY F. R. WATSON.

DOUBLE resonators of suitable dimensions are much more sensitive in amplifying sound than single resonators.[1] A double resonator consisting of two chambers of unequal volume connected by a short narrow tube has been used by the writer in measuring sound. A Rayleigh disc suspended in the connecting tube allows a quantitative measurement of the energy. By using a delicate quartz suspension and a proper dimensioning of parts, an instrument may be constructed that has the same order of sensitivity as the ear.

Another modification of a double resonator has resulted in an acoustic galvanometer. A telephone receiver is screwed to the open end of the resonator mentioned in the previous description and generates sound when the telephone plate is agitated by an electric current in the receiver. A Rayleigh disc, as before, gives quantitative measurements. Calibration of the instrument shows it will give a response for alternating currents of the order 10^{-8} amperes.

An acoustic relay has also been developed. This consists of two chambers of equal volume with a short connecting tube containing a Rayleigh disc. Each chamber has a telephone receiver screwed to the outer end. If an alternating current of selected pitch is passed through the receivers, the sound generated is amplified by the double resonator and causes the Rayleigh disc to rotate slightly. By suitably mounting the disc with a projecting pin, a record of the telephone action may be obtained either mechanically by an inked trace or electrically by closing an auxiliary circuit.

UNIVERSITY OF ILLINOIS.

A PORTABLE SOUND PRODUCER OF CONSIDERABLE RANGE AS TO FREQUENCY AND INTENSITY.

BY E. A. ECKHARDT.

A SOUND producer which is quite portable, and which has a considerable range both as to frequency and intensity, has been devised. It consists of an electro mechanical output element, the diaphragm of which is cupped in a manner which serves to suppress partial vibrations. The alternating current generator is of the triode type. A wiring diagram is shown in the paper. Two tubes are used, the one serving as an oscillator and the other as an amplifier. The adjustments of the second tube have slight effect on the frequency, but control the amplitude of the alternating current over relatively wide limits. Oscillograms of the current generator and of the diaphragm motion are shown. Constancy of frequency and intensity is discussed.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
December, 1920.

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[1] Lord Rayleigh, Th. of Sound, Vol. 2, sec. 253b.

Moving Pictures of Wave Motion.

By F. R. Watson and A. G. Eldredge.

MOVING pictures of capillary waves on water have been secured showing propagation, interference and reflection of waves. The waves were generated by the apparatus used earlier by the writer.[1] A stream of compressed air is interrupted by a circle of holes in a rotating disc so that puffs of air strike the water surface and generate the waves. To make these visible, light from an arc passes through a second series of holes in the disc and is reflected upward through the glass bottom of the water tank so as to cast a shadow of the waves on a frosted glass. The camera photographs this moving shadow. It is the intention to take further photographs of waves, also of the action of mechanical devices to show wave motion.

University of Illinois.

The Sound Field of a Parabolic Mirror.

By E. A. Eckhardt.

MEASUREMENTS of intensity and phase distribution in the field of a parabolic mirror are described. The mirror investigated was approximately nine (9) feet in diameter, and three (3) feet in focal length. Curves for the following intensity and phase distributions are shown.

1. Intensity distribution across the face of the paraboloid along a line through the focus, the mirror registering on the distant source of sound.
2. Intensity change at principal focus as the paraboloid axis is swept past the distant source of sound.
3. Intensity change at each of a pair of receivers symmetrically disposed relative to the principal focus as paraboloid axis is swept past the distant source of sound.
4. A series of curves as in 2, the paraboloid being progressively reduced in size from one curve to the next.
5. Phase distribution across face of paraboloid along a line through the focus, the mirror registering on the distant source of sound.
6. Phase change at principal focus as paraboloid axis is swept past distant source of sound.
7. Phase difference between a pair of receivers symmetrically disposed relative to focus as paraboloid axis is swept past the distant source of sound.

It is believed that the data obtained will supply a rational basis for the design of listening equipment to be used in conjunction with the paraboloid.

Bureau of Standards,
Washington, D. C.,
December, 1920.

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[1] Physical Review, Vol. 7, p. 226, 1916.

APPARATUS FOR MEASURING SOUND INTENSITY.

BY J. C. KARCHER.

MOST apparatus for measuring sound intensity at any frequency, such as the Rayleigh disc or the Webster phonometer, does not permit of ready change from one frequency to another over a wide range. The apparatus here described is an extension of that devised by Prof. G. W. Pierce in which a telephone receiver is the receiving element, and a crystal detector-galvanometer circuit is the essential indicating element. In order to increase the intensity range of the instrument two stages of triode amplification are introduced, one or both of which may be used according to the requirements of the problem.

The circuits and properties of the apparatus are discussed, and a wiring diagram is given. Calibration curves for a number of frequencies and a number of crystal rectifiers are shown.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
December, 1920.

A METHOD OF MEASURING PHASE DIFFERENCES IN A SOUND FIELD.

BY J. C. KARCHER.

A TELEPHONE receiver becomes a feeble alternating current generator when its diaphragm is acted upon by a continuous sound wave. If the sound is constant as to frequency and intensity the alternating current E.M.F.'s generated will be similarly constant. If a second receiver be placed in the same sound field the E.M.F. generated in it may differ slightly in amplitude and considerably in phase. If the two receivers be connected in series the joint circuit will have an impressed E.M.F. which is the vector sum of its components.

The squares of this vector sum and of the component E.M.F.'s are indicated by the galvanometer readings of the sound intensity measuring device described in another paper. The three sides of the vector triangle are thus obtained and the angle between the components, the phase angle is determined graphically. A few results obtained by this method are discussed.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
December, 1920.

AN ACOUSTIC WAVE FILTER.

BY G. W. STEWART.

AN electric wave filter is well known (vide Chapter XVI., Electric Oscillations and Electric Waves, Pierce, 1920). It consists of a series of sections placed in a transmission line, each section containing an impedance in series in the line and an impedance in shunt across the line, these alternating in sequence. These impedances can be so selected as to transmit without serious attentuation a group of sinusoidal currents between two assigned frequency limits and yet approximately extinguish the currents of neighboring frequencies.

Herewith is presented an acoustic wave filter whose theory is found to be of the same form as that of an electric wave filter that attenuates all frequencies above an assigned value. The formula for such an electric filter is,

$$(2\pi f)^2 = \frac{4}{L_1 C_2},$$

where f is the assigned frequency, L_1 is the inductance in series in the line and C_2 the capacity in shunt.

An analogous acoustic wave filter is found to consist of a cylindrical tube through which the transmission occurs, having a series of side openings equally spaced along the axis, with each opening surrounded by an enclosed volume whose dimensions are large compared to that of the opening. In the actual filters, the single aperture may be really several apertures with centers lying in the same cross-section. In the filter the dimensions are small in comparison with the wave-length for which the transmission is to be extinguished. The attenuation is not caused by dissipation but by the reactions between the sections. The formula for this acoustic wave filter can be shown to be:

$$(2\pi f)^2 = \frac{4S_1 a^2}{lV},$$

where S_1 is the area of the cylindrical tube, a is the velocity of sound in the air, which is the medium used, l is the distance between the apertures in the tube and V is the volume of air in each branch.

The theory of the acoustic wave filter is much more approximate than that in the electrical case, and the interest rests chiefly in the practical success of the acoustic filter. As illustrative of the performance of the filters, the following data are given:

Filter.	Area of Each Aperture.	Number of Sections.	S	l	V	Computed f.	Observed f.
A....	.236(2)	3	.185 cm.^{2}	2.66 cm.	21.4 cm.^{3}	654	650 approximately
B....	.236(4)	3	.185 cm.^{2}	5.0 cm.	42.7 cm.^{3}	323	350 approximately

In each of these filters the transmission between the cut-off frequency and 3,000 d.v. was very small, certainly less than 10^{-7}. This means that the transmitted intensity with a moderately loud incident intensity is inaudible.

In filter A, the transmission from 90 to 500 d.v. was approximately 21 per cent.; in B from 90 to 500 d.v., it was approximately 16 per cent.

That the attenuation is due to the reaction among the sections and is not absorption is indicated experimentally by the fact that the transmission of the desired frequencies is greater for filters of three sections than for filters of one section but otherwise of the same dimensions.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

MINIMUM SOUND ENERGY FOR AUDITION.

BY F. W. KRANZ.

THERE have been experiments made by several different observers to determine the minimum sound energy required for audition, and the variation of the sensitivity of the ear with frequency. The results obtained vary over a considerable range, the values given by Wien[1] showing a much greater sensitivity of the ear and a much greater variation of this sensitivity with frequency than those obtained by Rayleigh,[2] Zwaardemaker and Quix,[3] and others.

A new determination of the ear sensitivity seemed desirable and this was undertaken with a thermo-phone as the source of sound, this being a thin metal strip heated by an electric current. The theory of the thermo-phone has been worked out by Arnold and Crandall[4] and a quantitative expression given for the sound energy produced in terms of the electrical input and the constants of the metal strip. In the present experiment, the thermo-phone element was placed in a small telephone receiver case and held tightly to the ear. The other ear was closed by a dummy head receiver and the experiment was conducted in a padded room, so that external noises were eliminated entirely. A second person opened and closed the electric circuit with a knife switch, and the input was reduced in small steps until the observer became uncertain in his judgments as to when the switch was opened and closed, these judgments being indicated by a movement of a finger. The alternating input was measured with a thermocouple and a micro-ammeter. A direct current of one ampere was used in the thermo-phone which was of platinum, 0.000216 cm. thick. Six frequencies were used, covering the range from 128 to 4096 p.p.s. in intervals of an octave, a vacuum tube oscillator furnishing the alternating currents from which the harmonics were eliminated by means of electrical filters.

The results showed a wide difference in sensitivity for different ears, observations being made on eight ears. Some of the ears showed as great a variation of sensitivity with frequency as is indicated by Wien's data, the energy necessary for audition at 128 p.p.s. being 100,000 times that required at 2048 p.p.s. The smallest rate of energy flow necessary in any observation was about 10,000 times the minimum found by Wien, and was 2.2×10^{-8} ergs per square centimeter per second, the frequency being 2048 p.p.s. The corresponding particle displacement is 0.025 $\mu\mu$.

WALLACE CLEMENT SABINE LABORATORY,
RIVERBANK, GENEVA, ILLINOIS.

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[1] Archiv. für die gesammte Physiologie, V. 97, p. 1–57, 1903.

[2] Phil. Mag., V. 38, p. 370, 1894.

[3] Archiv. für Anat. und Physiol., Supplement, 1902.

[4] PHYS. REV., V. 10, p. 22–38, 1917.

THE ACOUSTICS OF LARGE AUDITORIUMS.

BY C. R. FOUNTAIN.

An account of the acoustics of the New City Auditorium of Macon, Georgia, is given. This is a temporary structure, except for the steel framework, which had been boarded up with rough boards, with cracks in many places. Its dimensions are roughly 200′ × 130′ × 60′ high. Its volume = 47,000 cu. meters.

Attempts had been made to improve the acoustics by hanging large plush curtains on the stage, muslin on the rear wall and wires stretched through the auditorium.

Under these conditions ordinary speaking tones could not be heard even faintly in the center of the building. I expected trouble with reverberation, but found none. Calculation by Sabine's formula for wooden wall, making allowance for open window area represented by cracks, etc., the reverberation came out 13 seconds when empty and 3.5 seconds with an audience of 2,500 on the floor.

This discrepancy may be accounted for in several ways:

1. The boards were thin, the surfaces rough, and cracks were plainly visible in many places.
2. The slope of the roof was such that the reflected waves would concentrate the sound energy at the eaves which were open.
3. The plush curtains on the stage, and the large volume behind them would prevent energy radiated in that direction from ever getting out.
4. The great volume of the auditorium made it impossible to saturate it with an intensity 10^6 times the minimum for audibility.

My problem not being one of reducing reverberation, I prepared to get as much energy as possible directed toward the audience with as few reflections as possible.

An inner stage, 40′ × 20′ × 10′, was constructed of plaster on wood lath.

The result was an almost uniform distribution of energy throughout the auditorium for the sounding body at any point on the stage except the extreme front corners. The intensity was such that a pin dropped on the stage could be heard at any point in the building.

Very large sustained tones gave some reverberation, but hardly as much as is desirable.

The chief points to be considered are these:

1. Walls with cracks in them probably have a much larger total absorption than would be obtained by adding the area of the cracks to the open window value of the boards themselves.
2. Open windows or openings at certain places may increase the absorption much more than their total area.
3. Reflected waves which arrive at a point after most of the direct radiation has gone by may not interfere very much with good hearing, even though they

operate to increase the total reverberation for intense tones to a point that would be very bad in a smaller auditorium.

MERCER UNIVERSITY,
MACON, GEORGIA.

THE STABILITY OF THE ATOM AS RELATED TO NUCLEAR COMPOSITION.

BY WILLIAM D. HARKINS.

IN light atom nuclei the ratio $\frac{\text{total number of negative nuclear electrons}}{\text{total number of positive electrons}}$ is equal to 0.5 for most of the atoms of even atomic number, and only very slightly greater than 0.5 when the atomic number is odd. Since atoms of even atomic number are in general very much more abundant than those of odd atomic number, it may be assumed that the nuclei of the former are more stable, and that, if the above ratio is represented by N/P, then the value of this ratio requisite for the highest stability for a light atom nucleus is 0.5,—that is when one negative electron on the average binds two positive electrons. The most abundant light atoms are those of C, O, Mg^{24}, Si^{28}, S^{32}, and Ca, and the value of N/P in the nuclei of all of these atoms is 0.5.

Of the heavier atoms Fe^{56} seems from present evidence to be by far the most abundant. As the positive charge on the nucleus $(P - N = M)$ increases, the relative number of negative electrons (or the value of N/P) necessary to give the maximum stability also increases, and rises to 0.536 in the extremely abundant Fe^{56}, and finally to 0.614 in uranium. It may be assumed that when the nuclear charge n is high, there as a large self repulsion in the nucleus. Corresponding to this no atom of nuclear charge higher than 28 is at all abundant. The effect of the high positive net charge (n) may be partly counteracted by an increase in the value of N/P. However if the value of N/P becomes abnormally high, the atom becomes unstable with respect to the emission of negative particles, so negative electrons,—beta particles,—are shot out. If N/P becomes abnormally low, then positive (alpha) particles are emitted. The values of $(N/P) - 0.5$ which give the highest stability, give, when plotted against either P or n, an hyperbola.

UNIVERSITY OF CHICAGO.

THE SEPARATION OF ISOTOPES.

BY R. S. MULLIKEN AND W. D. HARKINS.

EARLIER work in this laboratory has resulted in the partial separation of the element chlorine into isotopes, and this work is now being continued by Mr. Hayes. It is the purpose of this paper to consider the relations involved when the separation is carried on by diffusion through a porous wall and by vaporization into a vacuum. Since the principles of the latter method are probably somewhat simpler on the whole, the treatment given will refer to this method, though the same formulæ are found to apply to a considerable extent to the diffusion. The general formulæ for the latter method have already been worked out by Rayleigh.

The rate of vaporization of a liquid into a vacuum is given by the equation of Hertz

$$N \text{ (molecules per cm.}^2 \text{ per sec.)} = \frac{p}{\sqrt{2\pi mRT}},$$

which has been tested by Knudser with mercury, and by Langmuir. The relative rate of evaporation of two isotopes from a mixture is therefore

$$\frac{R_h}{R_l} = \frac{h}{l}\sqrt{\frac{m_l}{m_h}} = K\frac{h}{l},$$

provided the vapor pressures of the two isotopes when separate, are equal, which seems now to be moderately well established. Here h and l are the initial molecular percentages of heavy and light isotope, and m_h and m_l are the respective molecular weights. The composition of the first portion of the *condensate* can be shown to be given by

$$\Delta l = l_c - l = \frac{hl}{l + \frac{100K}{1 - K}} = 0.01\, hl(1 - K) \text{ (nearly)} - \frac{.01hl(m_h = m_l)}{2m_h}$$

nearly if $(m_h - m_l)/m_h$ is small. The atomic weight decrease is approximately

$$-\Delta m = \frac{.00005(m_h - m_l)^2 hl}{m_h}.$$

If a fraction C of the initial quantity of material is evaporated, the above values of Δl and Δm hold for the condensate if multiplied by

$$-\frac{(1 - C)}{C}\log_e (1 - C).$$

For the residue

$$\Delta h = -\frac{hl(1 - K)}{hk + l}\log_e (1 - C)$$

holds if Δh is small. Approximately

$$\Delta h = -\frac{0.0115(m_h - m_1)h_0 l_0 \log_{10}(1 - C)}{m_h}$$

and

$$\Delta m = -\frac{0.000115(m_h - m_1)h_0 l_0 \log (1 - C)}{m_h}.$$

The more general relations will be treated in the final paper.

UNIVERSITY OF CHICAGO.

THE ABUNDANCE OF ATOMIC SPECIES AS RELATED TO RUTHERFORD'S THEORY OF NUCLEAR STRUCTURE.

BY WILLIAM D. HARKINS.

EARLIER papers have given conclusive evidence that the abundance of the elements in the meteorites, and also in the surface of the earth, is a periodic function of the atomic number, with two elements in a period. Periods of this magnitude do not occur in the properties related to the number and arrangement of the outer or planetary electrons, but they do appear in the atomic weights, which makes it evident that the periodicity in question is dependent upon the composition and structure of the nuclei of the respective atoms. There are a number of facts which cannot be summarized here, which indicate that the relative abundance of the different atomic species is dependent to a considerable extent upon the stability of the respective nuclei, and also upon the stability of the electron aggregates from which such nuclei are built.

Since the radioactive atoms in their disintegration emit alpha particles of mass 4 (and charge 2+), and give off no other particles of appreciable mass, so far as is now known, it is a natural assumption that these heavy atoms, and possibly the lighter atoms as well, have nuclei which are built up largely from alpha particles. However, Rutherford, from the results of experimental work on the disintegration of light atoms, comes to the conclusion that he has driven out particles of mass 3 and charge 2 from their nuclei, and on this basis develops the hypothesis that these units of mass 3 are of much more importance than the units of mass 4 in the building of light atoms, though he still considers that the *alpha particles of mass 4 are of more importance in the building of heavy nuclei.*

In this connection it is of interest to take note of the following facts concerning the numerical relations involved: (1) Aston has now investigated the isotopes of 16 elements other than hydrogen, and it may be noted that in 9 of these the most abundant isotope has an atomic weight divisible by 4, while in only two is it divisible by 3. More striking than this is the fact that no such atomic weight is divisible by 3 except when it is also divisible by 4. (2) The analyses of 318 iron and 125 stone meteorites show that there are 7 elements present in appreciable quantities which have atomic weights divisible by 4 (but not by 3), and these 7 elements make up 87 per cent. of all of the atoms in these meteorites. Compared with this the atoms which have atomic weights divisible by 3, but not divisible by 4, make a poor showing, since they are represented by only 3 elements with a percentage of only 0.77 per cent. Thus the atoms of the type $4n$, where n is a whole number, are 113 times as plentiful as the atoms of atomic weight $3n$. (3) While the similar ratio for the surface of the earth is not quite so striking, it shows the presence of about 13 atoms of the type $4n$ to 1 of the type $3n$, or 72 per cent. of all of the atoms are of the former, and only 5.65 per cent. of the latter type. It has already been shown by the writer that the data on the meteorites are the more significant.

The extremely great predominance of atoms of the type $4n$ is shown by the fact that in the meteorites 91.7 per cent. of the atoms have atomic weights equal to $4n$, only 1.26 per cent. equal to $4n + 1$, only 0.77 per cent. equal to $4n + 2$, only 1.46 per cent. equal to $4n + 3$, and while it is true that 5.90 per cent. have atomic weights equal to $3n$, the apparent second place in order of abundance is due to the fact that 5.14 per cent. of this amount is due to an atomic species which belongs also to the $4n$ type, so only 0.76 per cent. can be with certainty attributed to the $3n$ type.

In connection with the above data it may be remarked that a 4 is always equal to a 3 plus a 1, and also that when violent means are used to disintegrate chemical compounds atom groups are often driven off as molecules which have a different form of union than exists in the parent molecule. For example anhydrous oxalic acid contains no HOH group in its molecule, but nevertheless it gives water when it is heated.

The conclusion which the writer draws from the above facts is that while the group of mass 3 is probably of considerable importance in the building of atom nuclei, there is at present no sufficient basis for the abandonment of the theory that light nuclei (atomic numbers 2 to 28) are built up largely from alpha particles.

It should be noted that the isotope of helium of the nuclear formula $(\eta_3{}^+\beta)^{++}$, where η is the positive, and β, the negative electron, as discovered by Rutherford, is an *altogether new* type of nucleus, since in it the ratio of the number of negative (N) to positive (P) electrons is only $\frac{1}{3}$, while in *no known atom* which has more than a transitory existence, *has this ratio a value less than* $\frac{1}{2}$.

University of Chicago.

The Variation of the Residual Ionization in Air with Pressure.

By K. Melvina Downey.

THE present investigation is a continuation of a previous experiment in which a linear relation was obtained between the residual ionization and the pressure when the latter was varied over a range of twenty atmospheres in a sphere one foot in diameter. With the same apparatus, the ionization has been found to increase up to approximately 46.0 atmospheres according to a linear relation, the slope for which becomes somewhat lower at about 27.0 atmospheres. However, a constant value of the ionization was obtained from 46.0 to 56.7 atmospheres (the highest pressure for which a reading was recorded).

If the residual ionization were due to the direct action of a penetrating radiation, it would presumably increase at least at a linear rate. The attainment of an upper limit suggests an ionization arising from an emission from the walls of the vessel, this emission being possibly stimulated by the external radiation, and having a range at least as great as 14 meters at atmospheric pressure.

In this work very careful tests have been made for saturation voltage at the highest pressures. The existence of a saturation voltage was also confirmed by experiments performed under the same conditions with lead-shielded radium bromide placed eight feet from the ionization vessel. In this case, the ionization, initially much larger, was found to increase over the entire range of pressure, but the slope of the ionization-pressure curve was found to change considerably at the high pressures.

It is of interest to note that the increase per atmosphere of the number of pairs of ions produced per c.c. per second is approximately 1.40 for a range from zero to twenty seven atmospheres, while from 27.0 to 46.0 atmospheres it is as low as 0.75. The increase per atmosphere, unlike the total ionization for one atmosphere, is largely free from any alpha radiation or soft beta radiation from the walls of the vessel, since such radiation is completely absorbed at the lower pressures. While it is impossible to discuss the significance of these low values in an abstract, it is of interest to observe that they are lower than the value 2.6 ions per c.c. per second obtained by McLennan for the total ionization at one atmosphere in a vessel of ice situated over Lake Ontario. They are, moreover, about the same as the value deduced by Gockel for the difference between the rate of production of ions one meter above a water surface and two meters below respectively, and attributed by him to the effect of a true penetrating radiation.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF MINNESOTA.

INFLUENCE OF THE EARTH'S POTENTIAL GRADIENT UPON THE MEASUREMENT OF THE ATMOSPHERIC IONIC DENSITY BY THE EBERT ION COUNTER.

BY J. F. MACKELL.

It has been pointed out by Prof. W. F. G. Swann[1] that measurements of ionic density made with the Ebert Ion Counter are subject to error owing to the influence of the charge which is induced on the top of the instrument by the atmospheric potential gradient. It appears that, for the normal sign of the potential gradient, the measured values of the negative ionic densities would be too low while those of the positive ionic densities would be unaltered. According to Swann, the error in the negative ionic density should amount to as much as 26 per cent. for a potential gradient of 70 volts per meter, in the case of an instrument which he examined. In view of some discussion as to the magnitude of the effect when the instrument is shielded with the cap with which modern instruments are provided, experiments have been undertaken to test the effects for this case.

Two similar ion counters were set up with their central systems connected to the respective pairs of quadrants of an electrometer whose outer case was raised to a potential of about 200 volts while the needle was kept at 100 volts. When both ion counters functioned, the effects on the electrometer tended to compensate. The alteration of the effect produced by shielding one of the ion counters with a large earth connected hemispherical dome of wire netting could then be investigated. The effect of one of the ion counters alone could be obtained by disconnecting the other; and, in this way, the effect of the potential gradient could be investigated.

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[1] Terr. Mag. and Atmos. Elec., Vol. 19, p. 205, 1914.

The results are summarized in the table herewith, which represents a large number of observations for the ranges of potential gradient given. N_+ and N_- refer to the measured numbers per c.c. of the positive and negative ions, and ΔN_+ and ΔN_- to the variations of these quantities caused by the potential gradient.

It will be seen that the results are in striking accord with the theory, the error in N_+ being practically negligible while that in N_- is very considerable.

Effect of Potential-Gradient on Measurement of Ionic Content.

Potent. Grad. Volts/Meter.	N_-	ΔN_-	$\left(\frac{\Delta N_-}{N_-}\right)\times 100$	N_+	ΔN_+	$\left(\frac{\Delta N_+}{N_+}\right)\times 100$	$\frac{N_+}{N_-}$	$\frac{(N_+ + \Delta N_+)}{(N_- + \Delta N_-)}$
30– 39	725	151	% 21	1,162	−4	0.4	1.63	1.34
40– 49	701	156	23					
50– 59	655	164	24	1,143	4	0.4	1.67	1.34
60– 69	710	174	25					
70– 79	679	193	28	1,145	0	0.0	1.63	1.28
80– 89	713	212	29					
90– 99	704	228	32	1,173	22	2.0	1.64	1.25
100–109	725	250	35					
110–120	713	257	36	1,156	0	0.0	1.61	1.20

Department of Physics,
University of Minnesota.

THE DYNAMIC CHARACTERISTICS OF THREE ELECTRODE VACUUM TUBES.

By John G. Frayne.

It is an experimental fact that in the case of continuous currents the following relation holds between the plate current and the grid and plate potentials.

$$I_b = F(E_b + kE_c),$$

the b subscripts referring to the plate and the c subscript referring to the grid.

Now in the case of alternating currents and potentials if we postulate that at any instant the following relations hold.

$$I_b' = F(E_b' + kE_c'),$$

where the primes refer to the instantaneous values of current and potentials then the static and dynamic characteristics are really one and the same thing, it being borne in mind that now we are dealing with alternating current phenomena, and that the dynamic characteristic in consequence will need a different interpretation than that derived from direct current measurements. Unless the emission of electrons from the filament is in some way affected by the rapidly changing grid potential (if we are dealing with frequencies of the order of a million or more), and then we might expect the omission to depend on the previous history of the currents and potentials, the current at any instant will only be dependent on the instantaneous plate and grid potentials.

Van der Bijl has worked out an expression for the dynamic characteristic in the case when a pure sine wave is impressed between the grid and filament, and a resistance is placed between the plate and the plate battery. Assuming the parabolic current-potential relation, we have

$$I_b - A(E - RI_b + kE_c + ke \cos pt)^2.$$

In this case $E - RI_b$ is the instantaneous plate potential and $k(E_c + e \cos pt)$ is the instantaneous grid potential. Solving this for I, there results a series in ascending powers of cos pt. As the series converges rapidly, only a few terms need be taken and the different powers of cos pt may be transformed into circular functions of multiples of pt. This enables us to write down the plate current in terms of its fundamental and harmonic components.

Using platinised quartz fibers as resistances the author has measured these different harmonics and found that they agree in general with the theoretical values. This result shows that the assumption that static and dynamic equations are identical must be true. Since the parabolic relation is only approximate it cannot be expected that the higher order of harmonics will approximate closely to the theoretical values, but the general behavior under different conditions can be predicted. It may be remarked here that even when operating on the portion of the characteristic where the harmonics have the greatest amplitude, below the third or fourth they are negligible compared with the fundamental. Measurements of these harmonics have been made, for different positions on the $E_b - I_b$, and the $E_c - I_b$ curves, and for different amplitudes of input potential and for different values of R in the plate circuit.

If the resistance be replaced by an inductance, the resulting value of I_b is not so easily determined, as a differential equation of considerable difficulty is encountered.

The characteristic equation now is

$$I_b = A\left(E_b + kE_c + ke \cos pt - L\frac{dI_b}{dt}\right)^2.$$

By writing I_b as the real part of the expression

$$a_0 + a_1e^{ipt} + a_2e^{2ipt} + a_3e^{3ipt} + \cdots a_ne^{nipt} \cdots$$

the values of the fundamental and first few harmonics have been obtained. The expressions for the harmonics beyond the fourth or fifth become exceedingly complicated. This equation has also been tested experimentally, using an induction coil of very low resistance and the amplitudes of the harmonics as measured experimentally are of the same numerical order of magnitude as those predicted by the above equation. The fundamental and first harmonic check up very closely but as the higher harmonics are reached the discrepancies become greater. It might be stated here that the value of the fundamental is given by

$$I_f = \frac{ke}{R^2 + L^2p^2}\cos(pt - \alpha) \text{ where } \alpha = \tan^{-1}\frac{Lp}{R},$$

and R is the internal output resistance of the tube. This is the same expression as would have been obtained for any alternating current generator, which generated a maximum E.M.F., ke, had internal resistance R and operated into a inductive reactance Lp.

The cases of resistance and inductance, capacity, and a combination of the latter three can easily be derived from the two cases given above. Solutions for these cases with experimental data will be published in the complete paper.

University of Minnesota,
December 10, 1920.

Secondary Electron Emission from Copper Surfaces.

By I. Garnett Barber.

IN measurements made upon the secondary emission of electrons from a copper surface which was stimulated by electronic bombardment it has been found that

(1) There is no reflection of electrons at potentials less than the ionizing potential.

(2) The number of secondaries per primary increases up to about 200 volts from which point on it remains nearly constant.

(3) The number of secondaries per primary never exceeds 1.3 in the case of copper.

(4) The re-emission from the surface can be altered by changing the condition of the surface by heat treatment.

(5) The re-emission is apparently decreased by increasing the temperature of the bombarded surface.

Ryerson Physical Laboratory.

INELASTIC COLLISIONS OF ELECTRONS IN VAPORS OF CERTAIN COMPOUND MOLECULES.

BY PAUL D. FOOTE AND F. L. MOHLER.

EMPLOYING methods described elsewhere the ionization potential of zinc-ethyl was found to be 12 volts and the resonance potential about 7 volts. The ionization potential of zinc chloride is 12.9 volts, and of mercuric chloride 12.1 volts. Carbon monoxide shows a very peculiar behavior. Two ionization potentials exist at 10.1 and 14.3 volts. Besides these inelastic collisions, electrons undergo the following velocity losses without producing ionization: 6.4, 12.1, 13.6, 19.1, 21.9, 24.6 volts. Some of these are due to successive collisions but no simple interpretation can be offered on the basis of two inelastic impacts as no combination of two numbers can represent these data in a reasonable manner. The vapors of potassium and sodium chlorides at 2 mm. Hg pressure are highly ionized preventing measurements of the ionization potential. From the grating energies of their crystalline salts it appears that their ionization potentials should be around 5 volts. Calcium ionizes at six volts and has a vapor pressure of the same order of magnitude as sodium chloride. Yet calcium vapor is not appreciably ionized by heating to 700° C. The marked contrast in the behavior of sodium chloride and calcium vapors is of interest. Hot sodium chloride vapor gives rise to the emission of the D-lines of Sodium.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE EFFECT OF ANGLE OF INCIDENCE ON THE REFLECTION AND SECONDARY EMISSION OF SLOW MOVING ELECTRONS FROM PLATINUM.[1]

BY JOHN T. TATE.

THE reflection and secondary emission of electrons by a metal surface bombarded with slow moving electrons has been investigated by v. Beyer, Gehrts, Campbell, Hull, and others, who have measured the reflection coefficients and the velocity of the reflected and secondary electrons as functions of the velocity of the incident beam. The present paper deals with the effect upon the reflection coefficients of varying the angle of incidence, and with the directional distribution of the reflected and secondary electrons for different angles of incidence.

A narrow beam of electrons from a hot filament was allowed to fall on a plate of platinum capable of rotation about an axis perpendicular to the direction of the beam. The effect of the earth's magnetic field was compensated by placing large Helmholtz coils around the apparatus. All metal parts inside the tube were of platinum. The reflection coefficients were determined by first measuring the current to the plate, i_p, then replacing the plate by a

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[1] Presented at the meeting of the Am. Phy. Society, Dec. 28, 29, 30, 1920.

Faraday cage of special design, having an absorption coefficient of about .95, and measuring the current, if received by it. To within a small correction factor the reflection coefficient R would be given by

$$R = \frac{ip - .95\,ip}{if}.$$

The gas pressure throughout the observations was maintained at less than 10^{-6} mm. although the method was such that consistent results for the reflection coefficients were obtained with the pressure as high as 5×10^{-4} mm.

The reflection coefficients so measured showed the usual variation with velocity of the incident beam. Their magnitude depended very greatly upon the treatment of the surface. Without special treatment of the surface the reflection coefficient was usually about .70 at an incident velocity of 8 volts. "Sputtering" the surface of the plate reduced this value to about .30. Covering the surface with a fine platinum gauze reduced the value to .35.

It was found that the reflection coefficient depended very little, if at all, upon the angle of incidence. In one instance, for example, the reflection coefficient changed from .70 to .71 while the angle of incidence was changed from 0° (normal incidence) to 60°. A larger variation of the angle of incidence was not possible since the plate would no longer subtend the entire incident beam.

The distribution of the reflected and secondary electrons was determined by measuring the current received by a sounding electrode mounted to rotate around the plate. Curves of the current to the sound for different angular positions around the plate were plotted. It was found that for all angles of incidence and for incident velocities up to 50 volts the curves showed a maximum reflection or secondary emission in a direction very nearly straight back on the incident beam, falling off to zero in directions parallel to the surface of the plate. The same result was obtained for surfaces which had been sputtered and for a surface covered with a fine platinum gauze.

These results indicate that the direction of the surface normal with respect to the incident beam plays at most a minor part in the mechanism of both reflection and secondary emission.

Physical Laboratory,
University of Minnesota,
December 5, 1920.

A Comparison of the Thermionic and Photoelectric Work Function from Platinum.

By Otto Koppius.

According to Richardson, the equation governing the emission of electrons from incandescent solids in a vacuum is given by the equation

$$i = AT^{1/2}e^{-b/T},$$

where i is the saturation current at a temperature T from the glowing metal to an auxiliary electrode, A is a constant of the glowing substance which may be associated with the number of free electrons per c.c. of the body in question, T is the absolute temperature, and b is a characteristic constant of the body which is identified with the work necessary to bring one electron out of the body.

The relation governing the emission of electrons from metals in a vacuum under the influence of light was stated by Einstein in 1905 in the form

$$\frac{1}{2}mv^2 = Ve = h\nu - p,$$

where ν is the frequency of the exciting light, h the Planck constant of energy, $h\nu$ is the energy absorbed by the electron from the radiation, p is the work necessary to get an electron out of the interior of the metal, and v is the maximum velocity with which the electron leaves the surface, which can of course be determined by the retarding potential V necessary to just prevent such an escape.

The purposes of the experiments were:

1. To study the variation of the photocurrent from oxide coated and pure platinum filaments with a change in the temperature of the filaments.
2. To see if a constant long wave-length limit for a substance can be determined which is a characteristic of the substance, in particular to test constancy with respect to changes in temperature.
3. To determine the work necessary to get an electron out of the substance photo-electrically, and that necessary to get an electron out of the substance thermionically.
4. To compare the two effects.

It was here proposed to determine both of these effects from the same target in the same tube, since it is well known that the physical conditions of the substance in question affect the values of both b and p very largely. Since the specimen in the work reported here could be kept under identical physical conditions as to pressure, and as to previous history, etc., it was thought that the basis for comparison of the two effects was a more plausible one than the ones usually employed.

The specimen employed were fastened to stout lead-in wires, through which a heating current could be sent. A Faraday cylinder of oxidized copper served to catch the electrons emitted. The temperature of the target was calculated from its change in resistance. The source of ultra-violet light was a Hareus quartz mercury arc. The vacuum usually employed was between $.5 - 1 \times 10^{-6}$ mm. of Hg.

The oxide coated filaments had to be discarded for this comparison work due to the fact that not only the amount of the photocurrent increased with a rise in temperature, but that there was a shift in the long wave-length limit also. The higher the temperature, the longer the waves to which the specimen were photoactive and therefore the smaller the work function as defined in Einstein's equation above. Due to this change, which is most probably due to a change in the surface condition of the specimen, no photoelectric work function could be taken as being characteristic of these oxide-coated filaments, although Wilson had shown that a definite b, or in other words a definite work function could be ascribed to them in the thermionic effect.

In the case of platinum, however, after thoroughly freeing the plate from occluded gases by heating for hours at a red heat and continuous pumping, a long wave-length limit for the photoelectric effect was found which remained remarkably constant for a range in temperature from room temperature to 500° C. as may be seen from the table below:

TABLE I.

Temperature.	Long Wave-length Limit = λ_0.
20° C.	2571 A.
100° C.	2550
370° C.	2568
490° C.	2564

If we take the value of the long wave-length limit at room temperature as 2571 Å., and then compute the work function as follows:

$$p = h\nu = \frac{6.547 \times 10^{-27} \times 3 \times 10^{10} \times 3 \times 10^{2}}{4.774 \times 10^{-10} \times 2.570 \times 10^{-5}},$$

we get $p = 4.80$ volts. Due to the constancy of λ_0 it is believed that we are dealing here with a constant which is an intrinsic property of the platinum itself.

Unfortunately an accident prevented to carry out the thermionic determination of the work necessary to free an electron. But if we agree with Richardson, who in his book makes a critical study of all the determinations of this constant, the work function for platinum comes to $b = 5.00$ volts.

This shows that the two work functions are very near together, although it cannot perhaps be asserted that they are the same. If the theories underlying the two phenomena are correct, then we should expect the work function in the photo-electric effect to come out larger, since presumably the electrons involved in this phenomenon are first detached from the atoms, and then an additional amount of work is necessary to get the electron out of the metal through the surface. In the thermionic effect, the electrons are supposed to be the free conduction electrons in the metal, which, if emitted, need only be brought through the surface force of the metal. This means that the electrons which are detached by light are the ones in the outer orbits, in which the forces due to the attraction of the nucleus and the repelling forces due to the other electrons are nearly equal and hence necessitate a relatively smaller amount of energy to detach an electron than would be the case where the electron finds itself in the inner orbits.

UNIVERSITY OF CHICAGO.

NEW STRONTIUM AND BARIUM PHOTO-ELECTRIC CELLS.

BY THEODORE W. CASE.

THE Photo-electric Effect in the Oxide Coated Audion Bulb.—Upon looking for a photo-electric effect from the oxide coated filaments of high vacuum Audion bulbs, it was observed that the filament showed slight action both heated and cold. More action was found at the plate when *it* was made negative. A close inspection revealed a brownish deposit on the plate, which was found to be the active material.

Strontium and Barium Cells.—With the above as a starting point, the conditions were determined for the production of barium and strontium photo-electric cells separately. It was found that the active material in the audion bulb was a mixture of the elements: barium, strontium, and possibly some calcium. A cell instantly spoils if a trace of oxygen is admitted to oxidize the metal. The oxide of barium and strontium is not perceptibly photo-electrically active.

Calcium Cell.—It was found to be extremely difficult to obtain a calcium deposit on the plate. However, a trace was obtained. Its maximum spectral sensitivity lies well out in the blue.

Possible Physical Reactions.—Both barium and strontium oxide are known to have the property of forming a dioxide under certain conditions of temperature. If the filament to be heated is coated with calcium oxide as a base and then with barium oxide and the cell exhausted; then either the calcium oxide under these conditions reduces some barium oxide, leaving metallic barium to be carried over to the plate, under the influence of an electric field applied between the filament and plate, or some of the barium oxide reduces some of its neighboring barium oxide under slightly different temperature conditions. Another alternative is that barium oxide is carried over to the plate and somewhere in the journey, or after arrival, is reduced by electron bombardment, leaving pure metal. The difficulty here is in accounting for the lost oxygen.

Spectral Sensitivity.—The barium cell has its maximum action in the light red and orange. If coated at a slightly higher temperature, the maximum is shifted to the yellow and green. This may be due to a very slight amount of calcium coming over.

The strontium cell has its maximum in the green. If coated at the maximum temperature, it is shifted to the blue. This may be due to traces of calcium. The above action was determined in the spectrum of a gas-filled tungsten lamp.

Temperature Effect.—A high vacuum barium cell was constructed so that the plate on which the barium was coated could be heated to a red heat, by passing a current through it. The photo-electric current was increased with increase of temperature, with a maximum increase of about one hundred times, just under red heat. A higher temperature ruined the cell, probably due to oxygen being given off the plate and oxidizing the barium. It has not been determined whether this is a direct temperature effect or only a secondary effect. The strontium cell has not been tested for this effect up to high temperatures, but from ten to one hundred degrees Fah. there is not over three per cent. change. For this range of temperature the action of the barium cell increases with temperature approximately 2.5 per cent.

Saturation Voltage Effect.—With either a barium or strontium cell, if the applied voltage be increased, the current increases up to a certain voltage, after which further increase of voltage does not apparently affect the current for any light intensity. If this saturation point be used, the photo-electric action is seemingly instantaneous. However, if a voltage under this saturation point be used, the photo-electric action is sluggish, and requires time to reach its maximum. It behaves like the creeping of a selenium cell.

If a saturated voltage be applied and the barium heated, the action is increased, but is sluggish, as though the saturation point had been shifted and as though a higher voltage was required to instantly take care of all the electrons. At this time it has not been determined whether a new saturation point will be found at a higher voltage.

The Photo-electric Current.—The photo-electric current is proportional to the amount of light, up to 4,000 foot candles, to which the cells are sensitive. It has not been possible to check this for higher intensities as yet. The action of the strontium cells has remained constant, within two or three per cent., for several months. The action of the barium cells remains constant provided a suitable filter be used which will eliminate infra-red radiation. The longer wave-lengths spoil the action of the barium cell. In the case of several of these cells whose action has been decreased sixty per cent. by exposing to strong infra-red radiation, it was found possible to bring their action back to normal again by screening out the infra-red and by exposing to strong light of shorter wave-length from either daylight or a Cooper Hewitt lamp. The current from the average cells made here is around one hundred micro amperes, in average sunlight, the sensitive surface being one and one half by five inches. The above characteristics of these cells make them especially adaptable to be used in conjunction with an automatic potentiometer line recording instrument, such as the Leeds & Northrup temperature recorder. Very interesting daylight curves have been obtained here, using cells with different spectral sensitivity.

Case Research Laboratory, Auburn, N. Y.

A Means of Distinguishing between Intrinsic and Spurious Contact Electromotive Forces.

By R. A. Millikan.

DEFINITE proof has been found of the existence of intrinsic contact electromotive forces as well as of apparent contact electromotive forces which are due to transcient surface charges. The equation contact E.M.F. $= (h/e)(\nu_0 - \nu_0')$ states a relation which is found to hold where only intrinsic contact E.M.F.'s are involved. Its failure in any experimental situation indicates the existence of a surface charge which produces an apparent contact E.M.F., the measure of which is the difference between the right and left sides of the foregoing equation.

University of Chicago.

SIZE AND AGING OF IONS PRODUCED IN AIR.

By HENRY A. ERIKSON.

IF a current of air containing a thin sheet of ions, obtained from polonium, is passed parallel to and between two plates which are at a difference of potential, it is found that the charge received by a narrow strip in the plane of one of the plates varies with the distance along the stream from the point where the ions enter the field. The distance between the plane of the strip and the initial plane of the ions was 4.5 cm.

Plotting the currents received by the strip as ordinates and the distances along the stream as the abscissæ, the curve obtained resembles the probability curve. At a wind velocity of about 500 cm. per second, the curves obtained for the positive and negative ions are quite similar, the maximum of each coming at the same distance. As the air velocity diminishes the maxima of the curves for the positive and negative ions separate, the maximum for the positive coming at a point further along the stream. The downstream side of the positive curve also lifts up and broadens out. The curve for the negative ions undergoes comparatively only a slight change with change of wind velocity. It is found when the wind velocity is kept constant and at a value which gives curves for the positive and negative ions which are similar and which have the maxima at the same distance, that if the ions are produced at a distance from the point of entering the field, the maxima of the positive and negative curves separate, the positive maximum coming later.

The reasonable interpretation of the spreading of the curves, is that there exists in the air ions of different sizes. The separation of the maxima at lower wind velocity or in the case of older ions is due to the aging of the positive ion whereby it loads up and hence moves more slowly in the electric field.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
December 4, 1920.

THE FORMATION OF NEGATIVE IONS IN AIR.

By LEONARD B. LOEB.

THROUGH slight modifications of the Rutherford alternating current method of mobility measurement for the carriers produced in air through the action of ultra-violet light on a metal plate disturbing effects due to stray light and organic impurities present in the results of previous workers were eliminated. The mobility curves obtained from these experiments were accordingly capable of being compared with theoretical curves computed on the basis of equations derived by applying the J. J. Thomson theory of ion formation to the Rutherford alternating current method of measurement. The comparisons show good *qualitative* agreement between the theoretical and the observed curves. This is all that can be expected in view of the difficulty of experimentally fulfilling the simple conditions assumed in the theory. The Thomson constant *n* (the number of collisions with neutral molecules which are on the average encountered by an electron before it succeeds in attaching to a molecule to form the ion), was found to be about 250,000 for air, if the values for the mobility of the electron and its mean free path assumed are accepted. As it has been shown that the electron cannot attach to the nitrogen molecules, and as *n* for pure oxygen was found to be about 50,000, one is justified in assuming that it is to the oxygen molecules in air that the electrons attach to form ions.

The apparent contradiction to this theory of ion formation suggested by the work of Wellisch, is shown to be due to an interpretation by Wellisch which is unwarranted by his results. The actual results of Wellisch are shown to be in excellent agreement with the theory of J. J. Thomson.

Ryerson Physical Laboratory,
November 27, 1920.

THE STRUCTURES OF THE HELIUM ATOM AND THE HYDROGEN MOLECULE.

By Irving Langmuir.

The comparative simplicity of the periodic relationships between the chemical elements, and the Rydberg relation between the atomic numbers of the inert gases, indicates that the problem of atomic structure must be fundamentally much simpler than one would be led to suppose from Bohr's theory. The physical and chemical properties of the elements prove the existence of certain unusually stable configurations of electrons in atoms such as the pair and octet. The stable pair not only characterizes hydrogen compounds and helium atoms but forms the innermost shell of the atoms of all the other elements, and also constitutes the chemical bond that holds atoms together in molecules. The mechanism which is responsible for the remarkable stability and universality of these pairs must be essentially like that which leads to the formation of octets and the groups of eighteen and thirty two which occur in the heavier elements.

Since there appears to be nothing in Bohr's theory to explain such stable groups of electrons, it seems necessary to assume that the electrons in atoms are coupled together in some very definite manner. It is further assumed on the basis of chemical evidence that each electron in an atom has its own separate orbit. There is also much evidence that atoms and molecules of most simple substances have no magnetic moment until brought into a field. On the basis of the classical mechanics I have therefore endeavored to devise models for the simplest atoms and molecules which will be in accord with the chemical as well as the physical properties of the substances. It would seem that the same principles should apply to all elements.

The models proposed for the helium atom and the hydrogen molecule contain two electrons, each of which oscillates back and forth in a nearly semicircular orbit. The resulting magnetic moment is zero. To make these models give results quantitatively in accord with the ionizing potential of helium or the heat of dissociation of hydrogen, it is necessary to modify the method of applying the quantum theory. Instead of applying the theory to the angular momentum of each electron it is applied to the momentum which, by being transferred from electron to electron, circulates about the nucleus. Because of the fact that the quantum theory has not been successfully applied to the structure of atoms whose electrons are coupled together, there seems to be no good evidence against this modification.

RESEARCH LABORATORY
GENERAL ELECTRIC COMPANY.

THE CUBIC SHAPES OF CERTAIN IONS, AS CONFIRMED BY X-RAY CRYSTAL ANALYSIS.

BY WHEELER P. DAVEY.

IN order to account for the chemical properties of the elements, Lewis, Born and Landé, and Langmuir[1] assigned to each element a definite arrangement of electrons. The resulting atomic shapes depend upon the position of the element in the periodic table. The formation of ionic compounds results in a transfer of electrons from the "positive" element to the "negative" element. As a result of this transfer, ions of $\overset{+}{\text{Na}}$, $\overset{+}{\text{K}}$, $\overset{++}{\text{Mg}}$, $\overset{++}{\text{Ca}}$, $\overset{--}{\text{O}}$, $\overset{--}{\text{S}}$, $\overset{-}{\text{F}}$, and $\overset{-}{\text{Cl}}$ each have, according to the theory, an outer shell of eight electrons whose mean positions are at the corners of a cube. The " shape " of these ions is therefore supposed to be that of a cube with rounded corners; $\overset{+}{\text{Rb}}$, $\overset{+}{\text{Cs}}$, $\overset{++}{\text{Sr}}$, $\overset{++}{\text{Ba}}$, $\overset{++}{\text{Ni}}$, $\overset{-}{\text{Br}}$, and $\overset{-}{\text{I}}$ are probably also cubic. $\overset{+}{\text{Tl}}$ and $\overset{++}{\text{Pb}}$ are spheres with flat spots.

It is the purpose of the present paper to consider the shapes of these ions in relation to their arrangement in space in crystals composed of equal numbers of each of two kind of ions, (*i.e.*, NaCl or MgO). The arrangement of these ions in the crystal depends upon their shape, subject to the condition that each ion must be symmetrically surrounded by oppositely charged ions.

Only two kinds of cubic lattices are possible for these compounds:

(*a*) Simple cubic (*i.e.*, NaCl structure). The ions are arranged each on a face-centered cubic lattice. These two lattices interpenetrate so as to produce a simple cubic lattice. The interpenetration of two such lattices to form a diamond lattice would give perfect symmetry, but the cubic shape of the ions would not permit of as close packing as the simple cube, so that such a lattice would not be as stable as the simple cube. From the viewpoint of a kinetic theory of crystal growth, we must therefore assume that an ion newly arrived on the crystal face in a " diamond " position is more likely to leave it again than if it had arrived in a " simple cubic " position, so that the resulting crystal is simple cubic. The diamond itself crystallizes as it does only because of the tetrahedral shape of the C atoms, and their tendency to complete their octets. CaF_2 crystallizes in a modified diamond form in spite of the fact that $\overset{++}{Ca}$ and $\overset{-}{F}$ are both cubes, because this is the closest packing which will give a symmetrical arrangement of two fluorine ions to one calcium ion.

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[1] Lewis, Jour. Am. Chem. Soc., 38, 762, 1916. Born and Landé, Verk. deut. physik Ges., 20, 210, 230, 1918. Langmuir, Jour. Am. Chem. Soc., June, 1919.

All the halides of Na, and K, and MgO, CaO, CaS and BaS have previously been reported in the literature as showing simple cubic structure. To these may now be added RbBr and NiO, having elementary simple cubes of sides respectively 3.465 Å. and 2.07 Å.

(*b*) Body-centered cubic structure. The ions are arranged each on a simple cubic lattice. These two lattices interpenetrate to produce a body-centered cubic lattice. Only one salt composed of cubic ions is yet known which has this structure, — CsCl. As the other halides of Cs have not been investigated it cannot be stated whether this is due to some peculiarity in the shape of the Cs atoms.

The diffraction pattern of TlCl is that of a simple cube of TlCl groups, *i.e.*, the crystal is a body-centered cube of ions. PbS is a simple cube of ions. According to Langmuir's theory $\overset{+}{Tl}$ and $\overset{++}{Pb}$ have only 26 electrons on their outer surfaces, out of 32 necessary to make that shell into a sphere. In other works, their shapes may be regarded as being generally spherical but with six flat spots. These flat spots make the surface simulate a cube, so that the cubic structure of TlCl and PbS is explained.

To summarize: A study has been made of the crystal structure of compounds formed by equal numbers of each of two oppositely charged ions which are cubic according to the theory of Lewis, Born and Landé, and Langmuir. New data added to that at present available in the literature allow this study to cover ions whose atomic numbers range from 8 to 82. The results of these purely physical experiments contribute towards an experimental verification of the theory of atomic shapes, which was originally devised to explain the chemical properties of the elements.

Research Laboratory,
General Electric Company,
Schenectady, N. Y.

The Crystal Structure of Two Rare Halogen Salts.

By Wheeler P. Davey and Frances G. Wick.

The crystal structures of CsCl and TlCl have been studied by the powder method of Hull, with the following results.

CsCl gives a diffraction pattern of a simple cube of side 4.12 Å. The density of the elementary cube shows that one CsCl group is associated with each point of the cubic lattice. CsCl is, therefore, to be thought of as a simple cube of Cs ions with a Cl ion at the center of each cube. This structure is different from that assumed by Bragg[1] from crystallographic data, and the distance observed between Cs and Cl atoms can not be calculated from the atomic dimensions assumed by Bragg.

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[1] W. L. Bragg, Phil. Mag., Aug., 1920.

TlCl gives a diffraction pattern identical with that of CsCl except that this side of the elementary cube is 3.85 Å. Its density shows an atomic arrangement identical with CsCl. The atomic distances are not consistent with Bragg's volume of thallium.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.

MULTIPLE VALENCY IN THE IONIZATION BY ALPHA RAYS.

BY T. R. WILKINS.

USING the Millikan Oil-drop apparatus, the ionization produced by slow alpha rays from polonium has been examined in the case of several gases. It has been found possible to produce doubly charged helium ions and the percentage of doubles produced has been shown to be a function of the speed of the alpha rays. The problem has also been attacked mathematically to rule out what have been classed as "spurious doubles" in earlier work. The percentage of doubles under optimum conditions has run as high as 15 per cent. of the ions caught.

BRANDON COLLEGE,
MANITOBA.

ADDITIONAL EXPERIMENTS ON THE NATURE OF THE MAGNETIC MOLECULE.

BY S. J. BARNETT AND L. J. H. BARNETT.

IN earlier papers it has been shown that the rotation of an electron ring, or a magneton, whose angular momentum bears to its magnetic moment a definite ratio R, about any axis at the angular velocity N revolutions per second is magnetically equivalent to placing it in a magnetic field with intensity H directed along this axis such that $H = 2\pi RN$.

Therefore, if all the magnetic elements in a body are alike, rotating the body at the angular velocity N r.p.s. will produce the same magnetic moment as placing it in a magnetic field with intensity $2\pi RN$ gauss.

For a ring of negative electrons of the familiar type $2\pi R = 4\pi(m/e) = -7.1 \times 10^{-7}$ e.m.u. Hence if a ferromagnetic body owes its magnetic properties to such a ring, its rotation must be equivalent to placing it, at rest, in a magnetic field with intensity $H = -7.1 \times 10^{-7} N$ gauss directed along the axis of rotation.

In two investigations on cold-rolled steel made in 1914 and 1915 by a method of electromagnetic induction we found the above theory verified except that numbers 3.6 and 3.1 were obtained in place of 7.1. Another investigation, made in 1916 and 1917, on steel, cobalt, and nickel, by a magnetometer method, also gave numbers smaller than 7.1.

A new and extended series of experiments has now been made on steel, soft iron, cobalt, and Heusler alloy, by a considerably different magnetometer method, and with additional precautions to eliminate systematic errors due to torsion and eddy currents. In every case a number not greater than about one half of 7.1 has been obtained. The experimental method is still being improved, but it is believed that all systematic errors have already been so far eliminated as to justify the conclusion that one of the two following alternatives must be true: (1) If negative electricity alone is responsible for ferromagnetism, it must exist in some form of orbital ring or magneton for which R has a smaller numerical value than that calculated for orbital rings from the value of m/e attributed to electrons of the familiar type; or (2) positive electricity in orbital rings or in magnetons must be partially responsible, as the rotation of positive magnetons or electron orbits must produce a magnetic moment opposite to that produced by rotating the negative magnetons or orbits, whose effect is preponderant.

Carnegie Institution,
Washington.

On the Application of Interference Methods to Astronomical Measurements.

By A. A. Michelson.

In the number of the *Philosophical Magazine* for July, 1890 (**30**, 1), a method was described for the measurement of the angular magnitude of astronomical objects such as the diameter of planetoids and satellites and the distance between double stars, when these are beyond the powers of the largest telescopes, and the hope was there expressed that it might not be impossible thus to measure the diameter of the fixed stars.

Briefly, the process consists in utilizing only the two portions of a large objective at opposite ends of a diameter. The interference fringes at the focus under these conditions will be a series of equidistant interference bands which are most distinct with a source subtending an infinitesimal angle. For an object presenting an appreciable angle the visibility is less and may become zero—the exact relation being readily expressed for any given distribution of light in the source.

A series of observations was taken on the satellites of Jupiter at the Lick Observatory, in 1891, with results which amply confirmed the practicability and accuracy of the method.

It is clear, however, that as in all probability the stars present an angular diameter less than one hundredth of a second, it would be almost hopeless to make such measurements, utilizing the largest telescope in existence; for it would require a distance between the apertures of at least 10 meters to observe the vanishing of the fringes. While such a large telescope would be entirely out of question, the interferometer arrangements figured in the article referred to may serve the purpose, as there is theoretically no limit to the effective base line and practically only that which depends on the atmospheric disturbances.

With a view to testing the effect of these, a trial was made (August 25, 1919) with the 40-inch refractor at Yerkes Observatory, using two apertures 4 inches by 5 inches at opposite ends of a diameter. The result was very encouraging, the interference bands being remarkably steady, notwithstanding the relatively poor "seeing"—2 to 3 on a scale of 5.

On invitation from Dr. George E. Hale the test was applied (September 18, 1919) to the 60-inch reflector of the Mount Wilson Observatory and then to the 100-inch reflector, and in both cases the experience at the Yerkes Observatory was confirmed. (*Astrophysical Journal*, **51**, 257, 1920.)

In December, 1919, and the months following, Anderson, using this method, obtained measures of the distance and position angles of the components of Capella with great accuracy (*Astrophysical Journal*, **51**, 260, 1920). Upon the author's suggestion, an interferometer beam 20 feet long, provided with movable auxiliary mirrors, was then constructed to test conditions of interference at distances greater than the diameter of the one hundred inch mirror itself. In August, 1920, fringes were obtained with separations of the mirrors as great as 18 feet, the visibility of the fringes for Vega at this distance being as great as that at 6 feet.

Meanwhile, Eddington, Russell, and Shapley had obtained values for the diameter of a number of stars based on estimates of apparent surface brightness, and their results indicated that several of these lay within the range of the 20 foot beam. α Orionis in particular was so large that Merrill investigated it with the apparatus used in the measurement of Capella and found a definite decrease in visibility for the maximum separation of the slits (100 inch aperture), this holding true for all position angles.

Observations were made at the Mt. Wilson Observatory by Mr. F. G. Pease on December 13, 1920, with the outer mirrors of the 20-foot beam at 121 inches separation. No fringes were visible on γ Orionis, while observations on γ Orionis before, and on α Canis Minoris afterwards yielded strong fringes with practically no change of setting. All measures were checked by Dr. Anderson. The seeing was very good on this night, but poor on the following nights, and it appears that better conditions are required than when working with fringes produced by apertures placed directly before the telescope. Nevertheless, observations made December 14–17 indicate that α Ceti, α Tauri and β Geminerum will come within the range of the 20-foot beam.

Assuming that the effective wave-length for α Orionis is λ 5750 its angular diameter from the formula $\alpha = 1.22\lambda/d$ proves to be 0''.047 and with Schlesinger's parallax of 0''.016 its linear diameter turns out to be 271×10^6 miles, or slightly less than that of the orbit of Mars.

The uncertainty of the present measurement is about 10 per cent. The effect of a possible darkening at the limb, which has been disregarded, would tend to make the measured results too small.

UNIVERSITY OF CHICAGO AND MT. WILSON SOLAR OBSERVATORY.

SOME PECULIARITIES IN ENERGY DISTRIBUTION BY SPECULUM GRATINGS.

By L. R. Ingersoll.

In a recent paper (*Astrophys. Jour.*, April, 1920) the writer has described experiments which show that when the directly reflected light (*i.e.*, central image) from a speculum grating is examined it is found markedly deficient in energy of such wave-length as is emerging tangentially from the grating in the first or higher order spectrum. The effect is confined to the polarization with electric vector perpendicular to the rulings and is in general accord with the Rayleigh-Voigt theory. This theory also predicts sharp maxima of energy of this polarization in any spectrum for such wave-lengths as are emerging tangentially in any higher order, while the tangentially diffracted radiation itself should be of much greater intensity for this polarization than for the other.

The last prediction of the Rayleigh-Voigt theory is readily verified by casual examination of almost any speculum grating. As to the maxima in the various

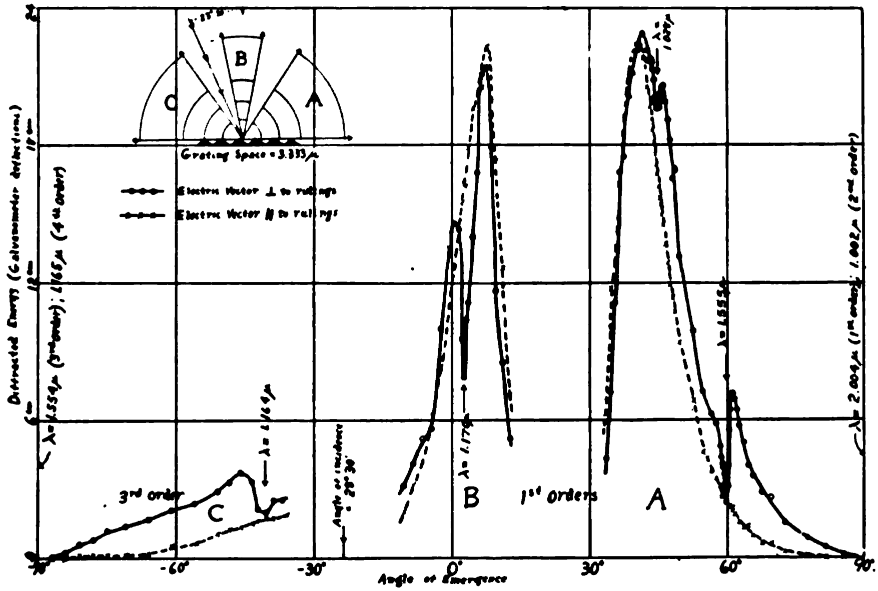

Fig. 1.

Spectrum energy curves, for both azimuths of polarization, as determined with a speculum grating of 300 lines per millimeter.

spectra, however, the writer has carried out experiments on a number of gratings and over a wave-length region from .5μ to 2μ and finds that instead of maxima there are sharp *minima* at such wave-lengths, as the accompanying figure shows. The radiation in this case is incident at an angle of (−)23° 30′ on a grating of 300 lines per mm. It will be noted that the minimum at about 60° corresponds almost exactly with the wave-length emerging tangentially in the third order on the other (negative) side. Similarly the minima in the other first order and in the third order correspond to the fourth order tangential on the negative side, while the agreement of the minimum at $\lambda = 1.028\mu$ with the second order tangential on the positive side is not quite so good.

A simple explanation of these minima can be arrived at if it is noted that the energy which is diffracted nearly tangentially (*i.e.*, between angles of, say, 70° and 90°) is, because of the high dispersion for nearly tangential emergence, coming from a very narrow wave-length region. This excess of perpendicularly polarized energy will accordingly be taken from spectra of other orders and may produce a more or less sharp minimum at this wave-length. The effect seems to show best for wave-lengths somewhat longer than the visible although some special gratings show it in this region also.

In many cases these minima are accompanied by maxima, as appears in the accompanying figure at angle 61°. These might superficially be regarded as in accordance with the Rayleigh-Voigt theory were they not in every case at longer than the predicted wave-lengths. In some cases, particularly for very deeply ruled gratings, the minima themselves are found at longer wave-lengths than the above reasoning would predict. The explanation of this is not clear but probably depends on peculiarities of groove form.

UNIVERSITY OF WISCONSIN.

THE COLORATION OF GLASS BY ULTRA-VIOLET LIGHT.

BY J. B. NATHANSON.

IT has long been a matter of observation that glass globes surrounding arc lamps become colored after long usage. This coloration has been attributed to the ultra-violet radiations coming from the arc and bringing about a certain chemical reaction in the glass. After much experimentation a glass has been produced which it was thought, would not become colored with prolonged exposure to ultra-violet light.

In order to put this matter to an experimental test, a composite globe was made up as follows: An old type of globe as well as a new type globe were each cut into halves, and the composite globe then built up of a half from each globe. Inside of this globe was placed a quartz mercury arc operating at about 87 watts. This arc was placed equally distant from the two different halves of the globe, so that these two kinds of glass would be equally exposed to the ultra-violet radiations. A small piece of tin foil was pasted on the inside of each half of the globe, so as to shield a portion of each glass from those radiations.

The mercury arc was operated for periods of time varying from one to nine hours. After an exposure of 50 hours, the old type globe appeared slightly purplish in color when viewed normally with a white paper backing. With further exposure this purplish color was in creased. At the end of 200 and 400 hours exposure, spectro-photometric data were obtained showing the spectral transmission of this glass. The test spot chosen was located close to the spot covered by the tinfoil. For purposes of comparison, the spectral transmission curve was also obtained for the spot covered by the tinfoil.

The data obtained for the exposed glass, as compared with that for the unexposed portion of the glass, showed a relatively greater absorption for the region 0.51 to 0.57 microns, than for any other portion of the spectrum. For example in the case of wave-length 0.53 microns, the percentage transmission of the glass before exposure, after 200, and 400 hours exposure, are 89.0, 85.0, and 82.7 respectively. As might be expected therefore, the purplish color is due to an absorption in the green portion of the spectrum. There is some absorption in the red, and very little in the violet.

The new type of glass globe showed no trace of coloration after 400 hours of exposure to the ultra-violet radiations. There was no detectable difference between the unexposed and exposed part of the glass.

To produce the coloration observed in the case of the old type of glass globe after 400 hours exposure, an approximate calculation shows that 4.5 Joules of ultra-violet energy are required per square centimeter of glass surface.

The above is published by permission of the director of research, Macbeth-Evans Glass Company.

Carnegie Institute of Technology.

A Strained Glass Elliptic Polarizer and Analyzer.

By A. de Forest Palmer.

THE instrument herein described has been developed primarily for the purpose of detecting and measuring small changes in the ellipticity of polarized light. Two slips of microscope cover glass, 25 millimeters long, 4.7 millimeters wide and .226 millimeter thick, are so mounted that each covers one half of the field of view and that the dividing line between them is horizontal. One of the slips can be strained horizontally by a steel spiral spring the tension of which is regulated by a micrometer screw. The other slip is mounted on adjusting screws that change its position without producing strain. The contiguous edges of the slips are ground and polished to an optical flat. These slips form a half-shade elliptic polarizer and the movable slip is so adjusted that the division line disappears when the intensity in the two parts of the field is the same. A third slip, of the same thickness as the other two but eleven millimeters wide and covering the whole field of view, is mounted back of the half-shade and can be strained in a vertical direction by a steel spiral spring controlled by a micrometer screw. This slip serves as a compensator or analyzer. Both half-shade and compensator are so designed that the stresses are pure tensions and the strains are uniform. The half-shade spring produces a tension of .285 gram's weight per micrometer division and the compensator spring a tension of 2.04 grams' weight per micrometer division.

When the above combination is placed between crossed Nicols, with their planes of polarization at forty five degrees to the vertical, and the observing telescope is focused on the half-shade the field of view will remain dark so long as there is no strain in any of the glass slips. But when the half-shade slip is stressed the light transmitted by it becomes elliptically polarized and one half of the field, say the lower, becomes bright. If the compensator slip is now stressed, it produces an ellipticity in the opposite sense to that of the half-shade, and the lower half of the field becomes darker while the upper half becomes lighter. Consequently the intensity in the two parts of the field can be equalized by properly adjusting the tension on the compensator slip. If the differential order of the half-shade is represented by N_1 and that of the compensator by N_2, it can be proved that balance occurs when

$$N_1 = 2N_2, \tag{1}$$

and that this condition is independent of the relative orientations of the polarizing and analyzing Nicols so long as the axes of strain in the glass slips are at right angles to one another. However, the sensitiveness of the combination is much greater when the planes of polarization of the Nicols are at forty-five degrees to the axes of strain.

The glass slips were calibrated by placing a quarter wave plate between them and the analyzing Nicol with its axes parallel to the planes of polarization of the Nicols. Under these conditions the differential order of the slips can be computed from the angle through which the analyzing Nicol must be turned to produce extinction. Accuracy in this method can be attained only with very intense and nearly monochromatic light. For this purpose a 1000-watt gas-filled tungsten lamp was used with a light filter giving maximum intensity at wave-length 584 $\mu\mu$. The half-shade and compensator slips were calibrated independently by this method and the values thus obtained were adjusted, by a series of balance observations, to satisfy equation (1). N_1 and N_2 were found to be linear functions of the tension expressible by the relations

$$N_1 = 2.00\,T_1 \times 10^{-6} \quad \text{and} \quad N_2 = 6.112\,T_2 \times 10^{-6}, \tag{2}$$

where T_1 and T_2 are the tensions on the slips expressed in micrometer divisions. The maximum tension used on the half-shade was 1,100 divisions and that on the compensator 182 divisions. Consequently the largest value obtained for N_1 was .0022 and that for N_2 was .00111.

If the light incident on the half-shade is elliptically polarized, balance will occur when

$$N_1 = 2(N_2 + N_3), \tag{3}$$

where N_3 is the differential order corresponding to the ellipticity of the incident light. Any value of N_3, or any change in this value, within the range of the instrument can be easily determined by equations (2) and (3). The final balance can be obtained by varying either N_1 or N_2, but greater sensitiveness is usually obtained by varying N_2. When N_3 is greater than the range of the instrument it must be partially compensated by a suitably chosen mica plate placed with its axes parallel to the axes of strain in the glass slips.

For the purpose of making an estimate of the sensitiveness and accuracy of the instrument, the micrometer screw controlling N_1 was set arbitrarily and a number of independent determinations of the tension T_2 necessary to produce balance were made. The values of N_1 chosen were about equally spaced throughout the range of the instrument. The maximum deviation of a single observation of T_2 from the mean of its series was one micrometer division; the average deviation was .31 division. These deviations correspond to errors in N_2 of 6×10^{-6} and 2×10^{-6} respectively. Consequently the limiting sensitiveness of the instrument corresponds to a phase difference of about five millionths of a wave-length of the light used and to a path difference of about 3×10^{-9} millimeter.

Brown University,
December 13, 1920.

THE BLACKENING OF A PHOTOGRAPHIC PLATE AS A FUNCTION OF INTENSITY OF LIGHT AND TIME OF EXPOSURE.

By P. S. Helmick.

EXPRESSING the blackening of a photographic plate in the form[1] $D = 1/a \operatorname{Log}_e [b - (b - 1)e^{-t \cdot 10^{A+B \operatorname{Log}_{10} I + C \operatorname{Log}_{10}^2 I}}]$ an algebraic and a graphical method are shown for determining the parameters a, b, A, B, and C.

The "p" of Schwarzschild's expression ($I \times t^p =$ const. for equal blackening)[2] is equal to $(B + C \operatorname{Log}_{10} I)^{-1}$.

To test this method, exposures were made with light of 450 $\mu\mu$, 550 $\mu\mu$, and 650 $\mu\mu$ upon Slow, Rapid, and Very Rapid plates. With variations in times of exposure from 1 sec. to 15 min., and variations in intensities from $10^{-2.0}$ watts/sq. m. to $10^{-4.3}$ watts/sq. m. the formula fits the experimental facts fairly well for densities between 0.2 and 5.0.

Physical Laboratory,
The State University of Iowa.

A DETERMINATION OF YOUNG'S MODULUS AND OF THE COEFFICIENT OF SIMPLE RIGIDITY OF NATURAL HEXAGONAL CRYSTALS OF SELENIUM.

By L. P. Sieg and R. F. Miller.

TO obtain the five constants necessary for the description of the elastic nature of a hexagonal crystal one would have to obtain, as shown by Voigt,[3] at least three specimens; one with the principal crystallographic axis parallel to the greatest dimension, one with it at 45°, and one cut transversely. The crystals used in the present work were made by sublimation a few years ago by a former colleague, F. C. Brown. These crystals are beautifully regular hexagons with their length parallel to the principal axis. As it appeared impossible to obtain specimens of sufficient size oriented in the second two directions, mentioned above, it was not possible to determine the five constants, so merely two of the gross constants; viz., Young's modulus and the simple rigidity, were obtained. The values of these as determined are 5.8×10^{11} dynes/cm.^{2}, and 0.64×10^{11} dynes/cm.^{2}, respectively. The large ratio of these two; viz., 9.1 is unusual, and disturbing, for theory, and results for most common substances, indicate that this ratio should not be far from three.

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[1] An empirical modification of Hurter and Driffield's original formula. See Jour. Soc. Chem. Ind., 9, 455, 1890.

[2] Astrophys. Journ., 11, 89, 1900.

[3] Ann. d. Phys., 16, p. 408, 1882. Also 29, p. 604, 1886.

The advantage is pointed out of using elements in crystalline form whenever possible, for the determination of elastic constants, as the results thus obtained should show more agreement among themselves, and should be more useful in the study of the structure of matter than results from heterogeneous mixtures of crystalline and amorphous states, as found in drawn wires, could ever possibly be.

STATE UNIVERSITY OF IOWA,
December, 1920.

THE OPTICAL DISPERSION AND THE CRYSTAL STRUCTURE OF ROCK SALT AND SYLVITE.

BY H. H. MARVIN.

LARMOR has developed the dispersion formula

$$\frac{n^2 - 1}{n^2 + a} = \frac{K - 1}{K + a} + \frac{c_1}{\lambda^2 - \lambda_1^2} + \frac{c_2}{\lambda^2 - \lambda_2^2} + \cdots$$

connecting the refractive index n of a transparent medium with the wave-length λ of radiation propagated through it, and has deduced the value $a = 2$ for the constant in the first term in the case of isotropic substances. This value has been confirmed by the author by consideration of dispersion data of CCl_4, $SiCl_4$, $TiCl_4$, $SnCl_4$, and $SbCl_5$. Maclaurin has found, however, that values $a = 5.51$ and $a = 1.04$ are required to fit the formula to the dispersion data for rock salt and fluorite, and has surmised that the deviation from the value 2 is due to the crystalline nature of these substances.

Calculations now in progress show that the dispersion data for rock salt and sylvite may be fitted by the Larmor-Maclaurin formula with the value $a = 6$, and a greater value might be used. The formulæ show remarkable similarity. The mean experimental values, $K = 5.92$ for rock salt and $K = 4.85$ for sylvite, are used for the dielectric constants. Three resonance wave-lengths, λ_1, λ_2, and λ_3, are needed in each case; the values are approximately 0.113 μ, 0.154 μ, 51.7 μ for rock salt, and 0.118 μ, 0.159 μ, 61.6 μ for sylvite.

The formula may be deduced from electron theory. The medium is considered as an aggregation of oscillators which are set into forced vibration by an electromagnetic wave. Each oscillator, when displaced, is acted on by a force arising within the atom, which accounts for the resonance wave-lengths. There is also a force due to oscillators of other classes in neighboring atoms, which acts to increase the displacement. The force exerted on an oscillator of class 1 by those of class 2 is given by a term

$$-\frac{4\pi}{a+1}\cdot e_1 N_2 e_2 (x_1 - x_2),$$

in which N_2 gves the concentration of oscillators of class 2, e_1 and e_2 the charges, and x_1 and x_2 the displacements.

The value $a = 6$ in the dispersion formula requires the value 1.8 for the proportionality factor $4\pi/(a + 1)$. Larmor deduced the value $4\pi/3$ by " smoothing out " the structure of the medium so that the atom under consideration lies in the center of a cavity surrounded by oscillators uniformly distributed. This is the case for any cavity of symmetrical shape.

In the crystal structure of rock salt or sylvite, each Cl atom has for its closest neighbors six Na (or K) atoms located at the vertices of a regular octahedron. These are surrounded by a similar concentric shell containing only Cl atoms, and so on. If these atoms are considered to be simple doublets, the proportionality factor is zero. The value 1.8 required may be obtained by supposing that about 0.43 part of the oscillators are distributed, and the remainder are penned up in the atoms. The required value can also be worked out by assuming that the oscillators having ultra-violet resonance wave-lengths are the electrons of the outer shells of Langmuir atoms, provided the radius of the shell is about one fourth the distance between the centers of the closest atoms, and that the electrons are constrained to move along meridians on the shell when the electric force acts along the polar axis.

The latter assumption is supported by the calculation from the values of the constants c_1 and c_2 in the dispersion formula that the number of oscillators belonging to resonance wave-length λ_1 is almost seven times the number belonging to λ_2 in both crystals. W. L. Bragg has recently pointed out that the Cl atom probably completes its outer shell of eight electrons by borrowing one from Na or K. The borrowed electron might easily occupy an exceptional position, and have a different resonance frequency from the other seven electrons in the shell.

Brace Laboratory of Physics,
University of Nebraska.

The Intensity of Light Transmitted through a Deep Slit.

By L. P. Sieg and A. T. Fant.

As indicated by Rayleigh,[1] practically every slit is a " Deep " slit, and a " Thin " slit can be obtained only with the greatest difficulty. Rayleigh used for the latter a fine scratch in a thin silver film deposited upon glass. In the present work experiments were made with a series of slits with steel jaws, varying in depth from that of thin safety razor blades to approximately 2.5 cm. The intensity of the transmitted light was experimentally determined as a function of the width and depth of the slits, and of the wave-length of the light. For narrow slits, the narrowness depending on the depth and wave-length, practically complete polarization, with the electric vector parallel to the length of the slit, was noted, but exact measurements were deferred to a later work. On the basis of diffraction, multiple reflections, and the alteration of the ratios of the two electric vectors due to the latter, a simple theory was developed, the results of which agree well with the experimental results. The chief fact derived from the experiments is that for narrow slits the total amount of light transmitted is not, even approximately, proportional to the opening, and that therefore the use of such slits for photometric purposes will lead, unless proper corrections are made, to erroneous values for the intensity of the transmitted light.

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[1] Roy. Soc. Lond. Proc., A, 89, 1913–14, p. 194.

THE STATE UNIVERSITY OF IOWA,
December, 1920.

THE LINES OF ELECTRIC FORCE OF A SYSTEM OF ELECTRIC POLES MOVING ALONG THE SAME STRAIGHT LINE.

BY H. BATEMAN.

LET the line along which the electric poles move be taken as axis of x. The lines of force may be found with the aid of the following theorem. If ξ denotes the distance from the origin at time τ of one of the poles, the field vectors in the electromagnetic field produced by the pole may be expressed in the form

$$\begin{aligned} E_x &= -\frac{\partial(\alpha, \beta)}{\partial(y, z)}, & H_x &= \frac{1}{c}\frac{\partial(\alpha, \beta)}{\partial(x, t)}, \\ E_y &= -\frac{\partial(\alpha, \beta)}{\partial(z, x)}, & H_y &= \frac{1}{c}\frac{\partial(\alpha, \beta)}{\partial(y, t)}, \\ E_z &= -\frac{\partial(\alpha, \beta)}{\partial(x, y)}, & H_z &= \frac{1}{c}\frac{\partial(\alpha, \beta)}{\partial(z, t)}, \end{aligned} \tag{1}$$

where $\alpha = \text{const.}$, $\beta = \text{const.}$ are the equations of a line of electric force, and

$$\alpha = \frac{ce}{4\pi}\frac{x - \xi - v(t - \tau)}{v(x - \xi) - c^2(t - \tau)}, \qquad \beta = \tan^{-1}\frac{y}{z},$$

$v = \xi'(\tau)$ is the velocity of the pole at time τ, e, the electric charge associated with the pole, c the velocity of light. The time τ is given as a function of x, y, z and t by means of the relation

$$[x - \xi(\tau)]^2 + y^2 + z^2 = c^2(t - \tau)^2, \qquad \tau \leqq t.$$

For a number of poles the expressions (1) still hold but

$$\alpha = \Sigma\frac{ce_s}{4\pi}\frac{x - \xi_s - v_s(t - \tau_s)}{v_s(x - \xi_s) - c^2(t - \tau_s)}, \qquad \beta = \tan^{-1}\frac{y}{z}.$$

The linesofforce for anumber of such poles may be derived from those for a single pole by a graphical method similar to that used in the case of electrostatic fields. The lines of force in a number of cases are shown in the diagrams.

CALIFORNIA INSTITUTE OF TECHNOLOGY.

A HIGH FREQUENCY RESISTANCE STANDARD.

BY JOHN G. FRAYNE.

A HIGH frequency resistance standard should possess negligible inductance and capacity and be free from " skin effect." The ideal resistance would be a short straight wire of small diameter, for its inductance would be negligible, the capacity between it and the other parts of the circuit could be made as small as desired, and the " skin effect " would be negligible if the diameter were sufficiently small. However, no ordinary wire will possess a greater resistance than 10 ohms (say) per centimeter, and in order to obtain a pure resistance of say 5,000 ohms, so much wire will be necessary, that inductance and capacity effects are certain to be introduced.

A spluttered quartz fiber seems to meet all the above requirements. Its high volume resistivity makes it possible to use a very short length for even a resistance of the order of 10,000 ohms. In this case the whole wire is " skin," and the " skin effect " is infinitesimally small.

Platinised quartz fibers about 10 centimeters long and in the neighborhood of one hundredth millimeter diameter, mounted on hard rubber bases and immersed in acid-free paraffin oil were found to be the best resistances from the standpoint of usefulness and reliability. In the case of thin metallic films it is known that they exhibit a negative temperature coefficient, and in the case of these comparatively heavy fibers the temperature coefficient appears to be practically zero. The fact that platinum will not oxidize easily also tends to preserve the constancy of the resistance.

From Lord Rayleigh's formula for resistance of wires at high frequencies,

$$R_n = R_0\sqrt{\frac{p\mu\pi d^2}{8\rho}},$$

where $p = 2\pi x$ frequency, d = diameter, μ = magnetic permeability and ρ = volume resistivity of wire, it can be seen that the small diameter and the high resistivity of platinum in the then film condition make the skin effect negligible. Actual calculations show the value of R_n/R_0 to be about 1.001 for a frequency of about 10^6 cycles per second.

The following table shows the constancy of the resistances over a period of two months.

1920.

Specimen No.	May 24.	May 30.	June 10.	June 21.	July 6.	July 18.
I	464	464.3	465	465.5	465	466
II	963	962.5	962.5	964	963.5	963
III	629.8	630.4	631.6	632.4	633	633

The temperature of the oil bath was practically constant when these measurements were made. A slight fluctuation is noticeable, but the true resistance can always be measured on an ordinary Wheatstone Bridge. These measurements were made during a period when the fibers were being constantly used in connection with experimental research on vacuum tubes. Currents as high as 65 milamperes were carried by the fibers which did not appear to show any apparent deterioration or change in resistance as a result. Their apparent ruggedness and their ability to carry such large currents make it seem possible that they could be of immense value in high frequency measurement work.

UNIVERSITY OF MINNESOTA,
December 10, 1920.

THE EFFECT OF A SUPERPOSED CONSTANT FIELD UPON THE ALTERNATING CURRENT PERMEABILITY AND ENERGY LOSS IN IRON.

BY ALVA W. SMITH.

BY the use of a suitable bridge, measurements have been made upon the apparent inductance and resistance of a coil containing an iron core when this core is under the action of a constant field. From these data, the alternating current permeability, iron loss and a quantity—the direct current induction multiplied by the alternating current permeability—which is proportional to the useful mechanical force per unit current when the iron is used as the core of a polarized electromagnet, were calculated.

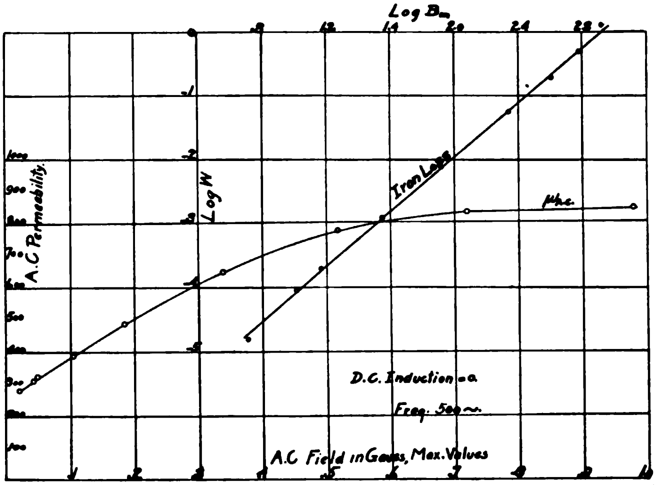

Fig. 1.

The test specimen was in the form of a ring built up from laminations cut from transformer plate. All of the results herein given were obtained with an alternating current of 500 cycles, giving fields ranging from 0.01 to 1.0 gauss.

Fig. 1 represents the alternating current permeability as a function of the alternating field with no superposed field. From an initial value of 250, the permeability rises to a maximum for a field which is approximately equal to that for which the direct current permeability reaches its maximum, but the alternating current permeability has a maximum value of about one fifth of

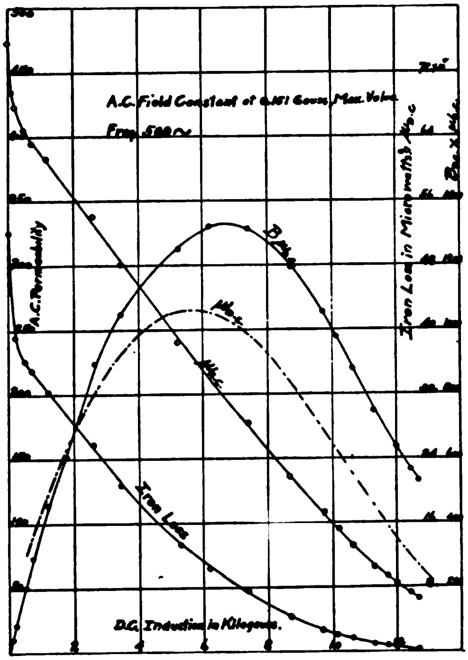

Fig. 2.

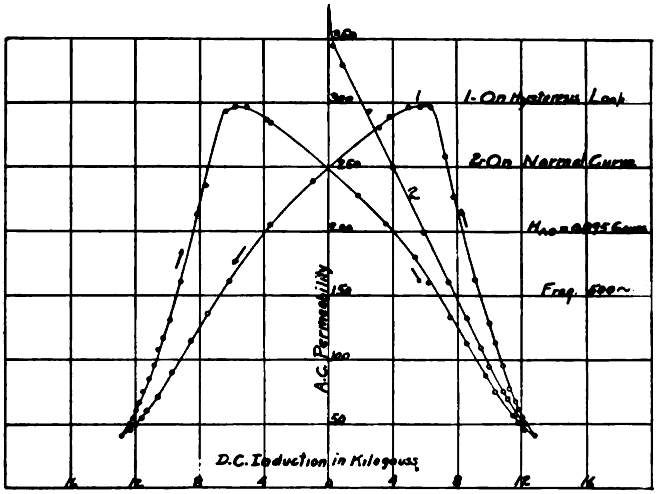

Fig. 3.

the maximum direct permeability. From the insert, in which the logarithm of the iron loss is plotted against the logarithm of the maximum alternating current induction, the iron loss is found to be proportional to the 2.16 power of the induction. A ballistic test on the same specimen gave the value 2.13 for this exponent.

The results represented by the full-line curves of Fig. 2 were obtained from measurements with constant effective alternating current for different superposed constant inductions on the normal magnetization curve. This is a typical set taken from a number of determinations with different values of alternating current. The alternating current permeability and iron loss diminish with increasing direct current induction. Beyond an induction of 12 kilogauss the permeability is independent of the alternating field. $B\mu_{a.c.}$ reaches a maximum which comes near the maximum direct current permeability given by the dotted curve. It is of interest to note that the iron loss is relatively low at this point.

The A.C. permeability for various points on the hysteresis loop is shown in curve 1 of Fig. 3. For comparison, A.C. permeabilities on the normal curve are plotted in curve 2 of Fig. 3. The curve of iron loss is exactly similar to curve 1. Up to an induction of about two kilogauss, the permeabilities and iron loss on the normal curve exceed any value reached on the hysteresis loop. On the other hand $B\mu_{a.c.}$ attains a maximum on the descending branch of the hysteresis loop, and for an induction near the remanent value, which is about 50 per cent. above the maximum value of $B\mu_{a.c.}$ on the normal curve. But at this point the iron loss on the hysteresis loop is more than twice as great as on the normal curve.

OHIO STATE UNIVERSITY,
COLUMBUS, OHIO.

EFFECT OF STRONG ELECTROSTATIC FIELDS ON THE VAPORIZATION OF TUNGSTEN. II.

BY A. G. WORTHING.

THIS abstract covers work done in continuation of that reported on at the November meeting in Cleveland. The procedure, unchanged except in minor details, consists in noting the rate of increase in resistance of an incandescent filament in vacuo, first when in a weak electrostatic field such as to prevent thermionic emission, and second when in a strong field similarly directed. A difference in the two rates of increase is taken as evidence that vaporization is influenced by electrostatic fields.

In the work previously reported, a field strength of 800,000 volts/cm. at the surface of the filament, resulted in no appreciable change in the rate of increase of resistance with time. Since then improvements in the kenotrons—principally the reduction in filament diameter from 0.060 mm. to 0.020 mm.—have resulted in considerably greater field strengths the maximum being about 3.2×10^6 volts/cm.

The variation of resistance with time during a test, in which a field strength of 2.7×10^6 volts/cm. and a filament temperature of 2500° K. were employed, is shown in the accompanying figure. The filament was operated throughout at 0.100,000 ampere. A steady temperature distribution was obtained after two hours of preliminary burning. Voltage readings were begun at 11 hr. 38 min. At 12 hr. 23 min. the strong electrostatic field was applied, at 2 hr. 0 min. it was removed, etc. With one exception, the variations shown are typical of those found in each case for field strengths above 1 × 10^6 volts/cm. Continued operation of the filament under each of the two conditions of burning led to a straight line relation between voltage and time, and hence also between filament resistance and time. Each change from a strong to a weak field caused a sudden increase in voltage readings, followed by a rather slow asymptotic change superposed upon the final straight line change with respect to time. Reverse changes, though less evident, occurred when the strong field was applied.

A positive effect of the electrostatic field on the rate of vaporization is shown in the figure by (1) the change in slope from 2.55 × 10^{-3} ohms/hr. for the weak field to 1.15 × 10^{-3} ohms/hr. for the strong field and (2) the literal shifts in the straight lines representing the resistance changes after prolonged burning in the weak field. These effects diminished with diminishing field strength in such a manner as to indicate the practical disappearance of the effects at somewhat less than a million volts per cm. On the other hand, tests with one

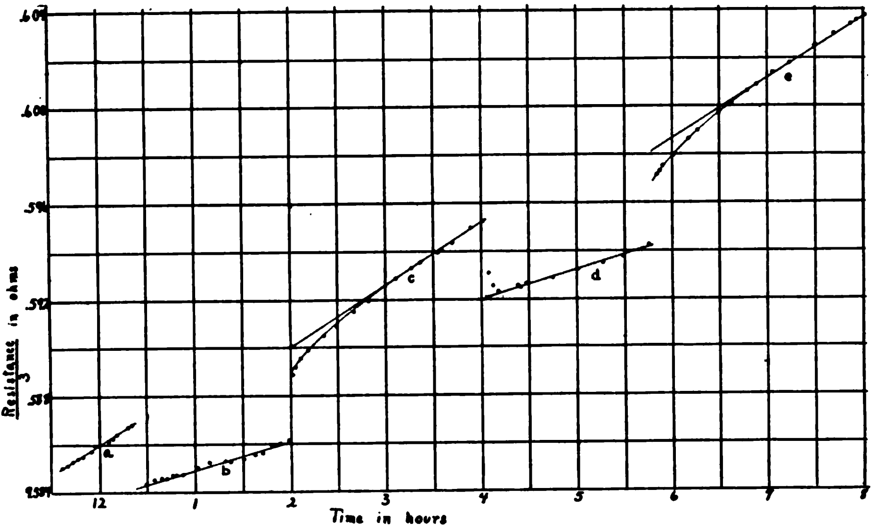

Fig. 1.

Effect of a strong electrostatic field on the vaporization of tungsten.

Kenotron No. 2.

Filament diameter 0.020 mm.

Coil diameter, 3 mm.

Potential difference between coil and filament.

a c e 30 volts.

b d 15,300 volts.

Field strength at surface of filament.

b d 2.7 × 10^6 volts per cm.

Rates of increase in resistance.

a c e 2.55 × 10^{-3} ohms per hour.

b d 1.15 × 10^{-3} ohms per hour.

kenotron—the one used in obtaining the data shown in the figure—with a field of 3.2×10^6 volts/cm. showed neither of the two positive indications noted above; even though, when the strong field was applied or removed, the sudden and the slow asymptotic changes were more evident than when the lower field strengths were employed.

Work on this problem is being continued.

NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO,
December, 1920.

A LOW VOLTAGE CATHODE RAY OSCILLOGRAPH.

BY J. B. JOHNSON.

ATTEMPTS to make a low voltage cathode ray oscillograph with a thermionic cathode operating in a high vacuum have met with two great difficulties. These are the charging up of the fluorescent screen and glass walls of the tube, and the spreading of the beam caused by the mutual repulsion of the electrons. If, however, there is a small amount of gas in the path of the electron beam, the positive ions produced by the passage of the electrons eliminate both of these difficulties. The slowly moving positive ions build up a space charge within the beam which effectively neutralizes that of the electrons, so that the beam remains narrow and a sharp spot is produced. The ionized gas also permits the escape of the electrons from the fluorescent screen back to the anode and prevents the accumulation of disturbing surface charges.

We have made tubes on this principle having a Wehnelt cathode and operating on 500 volts. The advantages of this device over the common form of Braun tube lie in higher sensitivity, ease of operation, and reliability of performance.

With gas present steps have to be taken to guard against arcing and the injurious effects of positive ion bombardment on the cathode. This is done by making the volume of gas surrounding the electrodes very small, for which purpose the cathode and anode are enclosed in a glass tube 3 cm. long and 7 mm. in diameter. The cathode is a short oxide-coated filament bent into hairpin shape. The anode is a thin metal tube 1 cm. long and .4 mm. in diameter, one end of which is about 1 mm. from the top of the filament loop, the other end opening into the main tube towards the fluorescent screen. Between the cathode and the anode is a metal shield with a small aperture which serves to keep down the current to the outside of the anode. The small glass tube containing the electrodes is mounted within the main tube at the end opposite the screen. The gas in the tube may be the vapor from a small globule of mercury, although other gases at a pressure of about .005 mm. work equally well.

Since a comparatively large current can be obtained from the cathode without spreading of the beam, the fluorescent spot is quite brilliant, being easily seen in daylight. A pattern traced by the spot can be photographed with an ordinary camera in from one to ten seconds. These tubes have been used to study the dynamic characteristics of the audion at radio frequencies, and are convenient indicating instruments for general laboratory use.

Research Laboratories of the
American Telephone and Telegraph Co. and
Western Electric Co., Inc.

A Theory of the Electrical Resistance of Metals.

By P. W. Bridgman.

A PREVIOUS theoretical discussion of the effect of pressure on resistance suggested most strongly that in metallic conduction the electrons pass through the substance of the atoms, and that the mechanism by which resistance is produced is intimately connected with the amplitude of atomic vibration. This view is here extended and given quantitative form. The classical expression for conductivity

$$\sigma = \frac{e^2}{2m} \cdot \frac{nl}{v}$$

is retained; the number of free electrons is supposed to remain constant, their velocity is taken to be that of a gas particle of the same mass and temperature, and their mean free path is supposed to be many times the distance between atomic centers. The variations of path are then computed in terms of the variations of amplitude, and thus the variations of resistance are obtained and checked with experimental results. It is shown that the theory in this form explains Ohm's law, gives the correct temperature coefficient and the most important part of the pressure coefficient, avoids the difficulty of the classical theory with reference to the specific heats, indicates a vanishing resistance at low temperatures, leaving open the possibility of super-conductivity, and retains the classical expression for the Wiedemann-Franz ratio.

A large number of new data for the effect of pressure on the resistance of a number of metals have recently been obtained, particularly for certain metals in both the liquid and the solid forms. In addition to the quantitative checks above, the new form of the theory is entirely consistent qualitatively with all the new data; in fact many of these new results seem to demand uniquely this conception of metallic conduction.

Harvard University.

Tube Experiments, with the Cathode Region in a Transverse Magnetic Field.

By L. Thompson.

A TYPE of discharge tube, devised by R. H. Goddard to be used for the investigation of high vacuum spectra, has been described.[1] Two parallel and opposite cathode plates, separated one half inch or more, are so placed that the tube can be conveniently inserted in a magnetic field with plates perpendicular to the field. A bright cylindrical region appears between the cathodes when the field is applied, as a consequence of the great concentration of the ionizing particles and the resulting intense ionization. The path of the particles is helical, with changing pitch, in a space limited by the electric and magnetic intensities in the neighborhood of the plates.

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[1] Physical Review, 1917.

As the magnetic field is weakened, the dark sections which surround and are very close to the cathodes, spread radially, the cylinder shortening and increasing in diameter, and eventually a dark circular center appears, of diameter varying with the value of H. For particles moving at the periphery of the dark circle, the usual relations are:

$$Hev = mv^2/r + Ee \qquad \text{and} \qquad Ve = mv^2/2.$$

If it is assumed that these which are moving circularly or spirally in the dark sections have insufficient velocity to ionize the gas, or to produce radiation, the value of V corresponding to the limiting circle (assumed thus to be a characteristic potential, ionizing or radiating) may be found if E is known. Or for a gas of known V, the electric field might be explored in the vicinity of the cathodes (varying H) and the values of V for other gases possibly then determined by comparison. Calculations for mercury vapor, with the tube used and a vacuum of the order of .0001 mm. give a value of 120 volts for E, assigning the value 10 volts to V. It is evident that the second term in the first equation cannot be neglected, as the radius of the cylinder varies considerably with the difference of potential across the tube. A low value of E might be expected in view of well-known results with high vacuum discharge in a magnetic field, and also the effect of space charge.

Piezo-electric Activity of Rochelle Salt under Various Conditions.

By J. Valasek.

In a previous communication, the writer has shown that the variation of piezo-electric activity with applied electric field is approximately the same as that of the derivative $\partial D/\partial E$ of the curve relating the charge D and the electric field E of the crystal used as a condenser. The latter relation being in the form of a hysteresis loop, it follows that the relation of activity to applied field depends on the direction of variation of the electric field. At zero field, accordingly, the piezo-electric response depends for a time on the previous electrical treatment of the crystal. This after-effect dies off exponentially with the time. The release of the "absorbed charge" which is though to represent a recovery from hysteresis in the polarization of the crystal follows the same kind of curve and may explain the piezo-electric fatigue.

At − 70° C. the piezo-electric activity is negligibly small. If the temperature is raised slowly ($\frac{1}{2}$ to 1° C. per minute) the activity stays small until

− 30° C. is reached. It begins to rise at − 20° C., increasing very rapidly, and reaching a maximum at about − 5° C. Then it decreases again until + 23° C. is reached where it rises to a second sharp maximum from which it diminishes slowly, becoming very small at + 50° C. The magnitude of this second maximum varies with the temperature at which heating begins.

The charging throws of the crystal used as a condenser show the same variation except that, of course, they do not tend to zero at the lower temperatures. Just in the middle of the second maximum, the crystal begins to conduct and this soon far outweighs the condenser effect. Experiments indicate that Ohm's law holds, at least approximately. Calling to mind Kelvin's theory of piezo-electricity and J. J. Thomson's theory of conduction this would indicate that the piezo-electric doublets become loosely bound and are broken up by the applied field.

When the crystal dries in phosphorus pentoxide, a white layer or dehydration slowly penetrates into the crystal causing a change in its piezo-electric behavior. The curves of activity against electric field which are already displaced from zero in the field axis, shift still farther therefrom, the maxima diminishing in magnitude. This effect is in the same direction as, and may be entirely due to, the difference of the dielectric properties of the crystal and the dehydrated layer.

Physical Laboratory,
University of Minnesota,
December 10, 1920.

An Experiment on Electromagnetic Induction and Relative Motion.

By W. F. G. Swann.

In a recent paper,[1] the writer has discussed the relation of the moving line of force theory to the Maxwell-Lorentz theory. One of the results following from the conclusions reached in the paper relates to the case of a sphere rotating in a magnetic field about an axis parallel to the field. It was shown that the alteration of potential of a shell surrounding the sphere is entirely determined by the effect of the motional intensity resulting from the motion of the sphere in the magnetic field, and is given by

$$V = \frac{2\pi Bna^2}{3c} \cdot \frac{q_{12}}{q_{22}},$$

where B is the induction, n the frequency of rotation, a the radius of the sphere, c the velocity of light, q_{22} the capacity of the insulated system (including the electrometer), and q_{12} the coefficient of capacity between the rotating sphere and the insulated system.[2]

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[1] Phys. Rev., Vol. 15, pp. 365–398, 1920.

[2] One of the a's in the former paper has been replaced by q_{12} which differs from a except when the surrounding shell has a radius large compared with a.

The significance of this result lies partly in the fact that it is true even though the rotating sphere is of iron, where, in view of the rotation of the iron, one who thought entirely in terms of moving lines of force would, presumably, have doubts as to what portion of the total induction B was to be considered as operative.

An experiment carried out to test the above conclusion resulted in the following data, giving excellent agreement with the theory. The values of $2V$ are in volts $\times$ 10^{-2}, except for a factor nearly unity affecting both columns equally.

Results of Experiments on an Iron Sphere 3.2 Cms. in Radius.

No. of Obsns.	Speed R. P. M.	B C. G. S.	$2V$ (Volts $\times 10^{-2}$) Observed.	$2V$ (Volts $\times 10^{-2}$) Calculated.
8	3,260	3,860	1.02	1.02
8	3,370	4,540	1.24	1.25
8	3,280	5,060	1.36	1.36
9	3,250	6,000	1.57	1.60
4	4,830	3,860	1.59	1.52
4	4,840	4,540	1.80	1.80
4	4,850	5,060	2.01	2.00
4	4,900	6,000	2.35	2.40
2	5,600	6,000	2.73	2.75
4	6,430	3,860	2.08	2.03
4	6,460	4,540	2.45	2.40
4	6,430	5,060	2.70	2.66
5	6,430	6,000	3.17	3.16

DEPARTMENT OF PHYSICS,
UNIVERSITY OF MINNESOTA.

VERTICAL ELECTRIC CURRENTS AND THE RELATION BETWEEN TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY.

BY LOUIS A. BAUER.

TO what extent the magnetic forces as observed on the surface of the earth can be referred to a potential, is a subject of paramount interest. The solution of the problem is of great importance both as regards the constitution of the so-called permanent magnetic field of the earth and the systems giving rise to the manifold variations to which the terrestrial magnetic field is continually subject.

Any electric currents circulating above or below the earth's surface in concentric layers, *i.e.*, parallel to the surface, will give rise to magnetic forces which may be represented by a potential. Electric currents, on the other hand, cutting the earth's surface, produce, in general, a mixed magnetic system; the horizontal components of such currents produce a magnetic potential, whereas, the vertical components cause magnetic forces which cannot be referred to a magnetic potential. As known, the test of the existence of a potential is that the line-integral of the magnetic force taken around a reëntrant curve or circuit on the earth's surface shall vanish. If the line-integral does not vanish, and its departure from zero cannot be explained by error of observation, or local magnetic disturbance in the region of the circuit, then the existence of a non-potential is revealed; from the magnitude and sign of the integral we may then determine the strength and direction of the electric currents passing perpendicularly through the area inclosed by the circuit.

The author published investigations concerning vertical currents in 1897 and again in 1904, based on the computations of line-integrals, taking as circuits of the earth parallels of latitude and using data scaled from various world magnetic charts. The method of computation eliminated, or reduced to a minimum, the effect of systematic errors in these charts. A rather interesting and suggestive distribution of vertical electric currents over the earth was found for the region investigated, 60° North to 60° South. The average strength of the vertical currents turned out to be one fortieth of an ampere per square kilometer, which is about eleven thousand times that found from atmospheric-electric observations.

At the Washington meeting in April, 1908, of the Society, the results were communicated of a third investigation on vertical currents, dependent this time upon the 1905 magnetic charts for the United States, which is the largest land area over which a detailed and carefully-conducted magnetic survey has been made. The results of line-integrals computed for various large circuits within the United States again revealed apparently vertical currents on the order of ten thousand times that from atmospheric-electric measurements.

The results of the present investigation are based on line integrals computed for circuits of the earth along parallels of latitude from 60° North to 60° South, as dependent upon data scaled from the most recent magnetic charts constructed with the aid of the new data obtained by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, both on land and at sea. Some corrections have been applied to the data thus scaled from the charts on the basis of the observations obtained on the present cruise of the *Carnegie*. The line-integral was also computed for the *Carnegie's* circumnavigation cruise in the Sub-Antarctic regions, December, 1915, to April, 1916.

The results from the new computations were shown in slides exhibited. The circulation of the present vertical electric currents agrees in general with that obtained by the author previously. If the asymmetries about the equator are eliminated by taking mean results of corresponding parallels north and south, we may say that there would be downward currents of negative electricity in higher latitudes and upward in lower latitudes. The average strength of these currents, for the region considered, would be about one thirty-fifth of an ampere per square kilometer—once more on the order of ten thousand times that from atmospheric-electric observations. The actual current-density in certain regions may be as much as one eighth ampere per sq. km.

In order to correlate the vertical currents observed in atmospheric electricity with those resulting from the line integrals of the magnetic force, it will be necessary not only to account for the outstanding factor of 10^4 but also to explain the variation in sign. The air-earth conduction current of atmospheric electricity is assumed to consist of currents of positive electricity streaming downwards all over the earth, whereas the currents obtained from the magnetic observations, as shown by the slides, proceed downward over certain regions and upward over others. The integral of the latter currents may vanish if taken over the entire earth, whereas the atmospheric-electric current corresponds to a continual replenishing of the earth's negative charge amounting to about 1,000 amperes. Further research will be necessary before more definite information can be given as to possible correlations between the two sets of currents.

The variations in vertical currents during the solar eclipse of May 29, 1919, as obtained from the line integrals of magnetic force, agreed qualitatively with the variations in the vertical current shown by the atmospheric-electric observations during totality at Sobral, Brazil. However, once more, the results from the two sets of observations, magnetic and electric, do not agree quantitatively, the outstanding factor being on the order of 3×10^3.

CARNEGIE INSTITUTION,
WASHINGTON.

THE ECLIPSE MAGNETIC SYSTEMS OF MAY 29, 1919.

BY LOUIS A. BAUER.

WITH the aid of the magnetic data observed by the various expeditions of the Department of Terrestrial Magnetism during the solar eclipse of May 29, 1919, and by coöperating observatories, an analysis was made of the systems of forces which produced the observed magnetic effects.

The vectors showing the combined effect in declination and horizontal intensity invariably were found to point approximately towards a focus located in the vicinity of the shadow cone and moving with it over the earth as the eclipse progressed from station to station.

With the aid of the vertical-intensity effects, it was found that the eclipse magnetic systems were composed of an external and an internal system of forces. The analysis in general shows that the eclipse magnetic systems of May 29, 1919, have characteristics analogous to those exhibited by the systems causing the solar-diurnal and the lunar-diurnal variations of the earth's magnetism.

It was furthermore found that the observed magnetic effects could not wholly be referred to a potential; there was an appreciable outstanding portion which clearly showed the existence of vertical electric currents passing through the surface of the earth in the region of visibility of the eclipse. The line integrals of the magnetic force were computed for circuits of various dimensions, inclosing Sobral, Brazil, where atmospheric-electric observations were made by the expedition of the Department of Terrestrial Magnetism. The slides shown exhibit the results of these computations. There was a drop during totality of the eclipse in the vertical electric current resulting from the magnetic observations, corresponding with the drop in the vertical current shown by the atmospheric-electric observations. The strength of the vertical current resulting from the magnetic observations was about 3,000 times that from the atmospheric-electric measurements.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.

Anhysteretic Curves of Iron-Carbon Alloys Made with a Braun Tube.

By C. W. Waggoner and F. A. Molby.

The anhysteretic character of magnetization curves in which the alloy is subjected to longitudinal magnetic fields during the time the static hysteresis curves were being made, was the subject of a former paper.[1] Certain errors in the method used led the present writers to repeat the study of the hysteresis of a series of iron-carbon alloys under different conditions.

1. Hysteresis curves were made without the longitudinal 60-cycle A. C. field and then curves with the A.C. field in which the A.C. field was introduced and then removed before each observation in the step by step method. In all cases the A.C. field was equal to or greater than the D.C. magnetizing field. Curves taken in this latter manner show no hysteresis even in the case of samples containing 1.13 per cent. carbon.

2. Curves were taken of the complete loop of magnetization with continuously superimposed A.C. magnetization as well as the varying D.C. excitation. Except in the case of very high carbon steels there seems to be no residual or hysteresis area. In the high carbon samples there is a bare trace of this residual area but the reduction in area of the static loop with the applied 60-cycle A.C. field was far greater than that found by using the method described in the previous paper referred to above.

West Virginia University.

Positive Ray Analysis of Magnesium.

By A. J. Dempster.

Using the method described in the Physical Review for April, 1918, p. 316, magnesium has been analyzed into three isotopes with atomic weights 24, 25 and 26. The components at 25 and 26 are present in approximately equal amounts, and the one at 24 is about six times as strong as either of the others. The average atomic weight agrees closely with that given by the chemists.

University of Chicago.

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[1] Waggoner and Freeman, Gen. Elec. Rev., Vol. XXI., 1918, p. 143.

PRESSURE SHIFTS IN A CALCIUM ARC.

BY HENRY G. GALE AND L. F. MILLER.

SINCE the discovery by Goos[1] of the so-called pole-effect in arc spectra, considerable doubt has been felt as to the significance of pressure shifts of spectral lines. Some results on the pole-effect in a calcium arc[2] were obtained at the Ryerson Laboratory several years ago. Measurements of pressure-shifts of the same lines in a similar arc have just been completed.

In both sets of experiments the arc was formed between electrodes of metallic calcium. The poles were about 9 mm. in diameter and the current was kept at 4 amperes on a 110-volt D.-C. circuit. The following table gives the new pressure-shifts as measured by Mr. Miller, and the pole-effect change in wave-

TABLE.

Series Group.

	λ	Press Shift.	Pole-Effect. +	Pole-Effect. −
p	6,499.8	.002	.005	.000
	6,493.9	.001	.003	.000
	6,449.9	.001	.003	−.003
	4,302.6	.000	−.001	.000
	4,283.1	−.001	−.003	.000
SI_2	5,041.9	.015	.024	−.003
	4,527.1	.027	.036	.008
SI_3	4,878.3	.031	.045	.000
	4,355.4	.068	...	...
t_4	4,586.1	.030	.040	−.004
	4,581.6			
	4,578			
t_5	4,098.6	.044	.055	−.013
	4,095.0			
	4,092.7			
T	4,318.8	−.001	−.003	.001
	4,299.1	−.001	−.003	.000
	4,289.5	−.000	−.001	.001
P_2	3,737.1	.008	.012	.001
	3,706.1			
P_1	3,181.4	.013	.008	.003
	3,179.4	.013	.008	.001
	3,158.9	.014	.010	.000
H	3,968.6	.001	.002	.000
K	3,933.8	.001	.002	.000

	λ	Press Shift.	Pole-Effect. +	Pole-Effect. −
$T1_4$	4,456.1	−.001	−.003	.001
	4,454.9			
	4,435.1			
	4,435.1			
	4,425.6			
$T1_5$	3,644.8	−.029	−.037	.005
	3,644.5			
	3,631.1			
	3,630.8			
	3,624.1			
$T1_6$	3,361.9	−.054	−.050	.011
	3,350.2			
	3,344.4			
$T2_3$	6,162.4	.012	.017	.000
	6,122.4			
	6,102.9			
$T2_4$	3,973.8	.016	.024	−.003
	3,957.2			
	3,949.1			
$T2_5$	3,487.7	.041	.030	−.003
	3,474.9			
	3,468.6			
$T2_6$	3,286.2	.046	.034	−.005
	3,274.8			
	3,269.3			

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[1] Astrophysical Journal, 38, 141, 1913.

[2] Astrophysical Journal, 43, 161, 1916; 44, 65, 1916.

Non-Series Group.

λ	Press Shift.	Pole-Effect.		λ	Press Shift.	Pole-Effect.	
		+	−			+	−
6,471.8	.002	.004	.000	5,513.1	−.023	−.032	−.003
6,462.7	.001	.003	−.002	5,349.6	.005	.007	−.003
6,439.3	.001	.000	−.001	5,270.4	.004	.018	.000
6,169.8	.012	.017	.000	5,265.7	.003	.016	.000
6,169.3	.013	...	...	5,264.4	.010	.014	.000
5,857.7	.022	.032	.001	5,262.4	.010	.016	.000
5,603.1	.006	.011	−.003	5,261.9	.011	.016	.000
5,601.1	.006	.011	−.003	5,189.0	−.003	−.007	.003
5,598.6	.003	.008	−.003	4,685.4	−.009	−.012	.002
5,594.6	.007	.011	−.001	4,307.9	−.001	−.001	.000
5,590.3	.005	.009	−.003	4,240.5	...	...	...
5,588.9	.005	.008	−.004	4,226.9	.018	.005	.000
5,582.1	.005	.009	−.003				

length as measured by Mr. Whitney in 1916. The pressure-shifts were the changes in wave-length observed for a change in pressure from 5 cm. to one atmosphere. The middle of a horizontal arc, about 4 mm. long, was projected, with an enlargement of about four diameters, on the slit of a 21 ft. concave grating spectroscope.

The table gives the series classification, the wave-length, the pressure-shift, pole-effect from center of arc to + pole, and center to − pole.

The agreement between pole-effect and pressure-shift points strongly to the intimate relation between these effects, as has been suggested frequently.

It is not possible to say whether the change in the character of the arc produced by a change in pressure causes a "pole-effect" change in wave-length at the middle of the arc, or whether the pressure change *per se* causes the shift. In any case, the table gives the changes at the middle of a 4 ampere arc when the pressure is changed by approximately one atmosphere.

University of Chicago.

The Luminescence of Zinc Oxide above the Red Heat.

By E. L. Nichols.

THIS oxide, when pure, is not photo-luminescent nor does it respond to X-rays. It is, however, susceptible to excitation by the hydrogen flame[1] with the development of two bands:

1. A red band which appears when the oxide is heated to 568° and which on further heating to about 700° is displaced by a yellow green band.
2. A yellow-green band.

The latter disappears at about 940°.

This is a special case of the effect described in recent papers. It is mentioned here because measurements of the brightness of luminescence with an optical pyrometer show:

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[1] Nichols and Wilber, "On Flame Excitation of Luminescence," Physical Review, (2), XVII., Feb., 1921.

1. That the luminescence of the zinc oxide reached a maximum of brightness at a temperature somewhat above 800° when it was about 11.4 times as bright as copper oxide at the same temperature and 115 times as bright as zinc oxide of the same temperature when not excited to luminescence.

2. That the relative emissivity of zinc oxide referred to that of copper oxide as a convenient standard falls to a minimum at the temperature of greatest luminescence and rises rapidly as the luminescence disappears with increasing temperature.

3. That the electric resistivity of zinc oxide, as shown by certain unpublished data obtained by Professor C. C. Bidwell and to which he permits me to refer in this connection, undergoes a sudden change within this temperature range of luminescence indicative of some molecular transformation.

This last named relation is deemed to be of special significance in that it may indicate that under conditions where molecular bonds are undergoing a rearrangement substances are rendered susceptible to the excitation of luminescence.

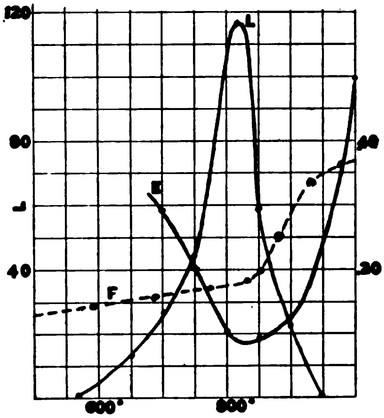

Fig. 1.

In the accompanying diagram curve *L* shows the brightness of zinc oxide under flame excitation compared with its brightness when unexcited. Curve *E* is the relative emissity and *F* is a function of the resistance of the oxide which is linear everywhere except for the temperature range beginning at about 850°.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
December, 1920.

SOME ADDITIONAL MEASUREMENTS OF WAVE-LENGTHS IN THE CALCIUM ARC IN VACUO.

BY GEO. V. MCCAULEY.

IN order to check the grating measurements of Crew and McCauley[1] and verify the differences existing between their values and those of Holtz,[2] measurements were undertaken with the Fabry and Perot interferometer in conjunction with a Hilger fixed arm spectroscope.

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[1] Astrophysical Journal, 39, 29, 1914.

[2] Zeitschrift für Wissenschaftliche Photographie, 12, 101, 1913.

The same source as was used by Crew and McCauley was employed at about one centimeter of mercury pressure. Measurements were made on 46 Ca lines between 4283 λ and 6500 λ. On 44 of these lines there exists a mean divergence from Crew and McCauley's values of .0067 Å.; while the mean divergence from the measurements of Holtz on these same lines is .0168 Å.

One new line 4581.466 Å. was observed.

Corning Glass Works,
Corning, New York.

APPROXIMATE COMPUTATIONS OF X-RAY ABSORPTION FREQUENCIES.

By William Duane.

THREE general laws—the acceleration law, the angular momentum law and the frequency law—form the basis of Bohr's theory of radiation. These laws do not determine the distribution of electrons in the various orbits. In order to calculate radiation frequencies this distribution must be known.

A number of scientists have shown that it is possible to choose the distribution of electrons in the orbits in such a way that the equations deduced from the theory represent some of the observed frequencies of X-rays with considerable accuracy.

At the meeting of the A.A.A.S. in December, 1919,[1] I presented a set of computations of the K critical absorption frequencies characteristic of the chemical elements. These frequencies have been measured for almost all of the chemical elements from magnesium (atomic number 12) to uranium (atomic number 92). In making these computations I did not choose the distribution of electrons to fit the frequencies. I used what may be called the chemical distribution. The computed values differ from the observed values by less than the corrections due to the mutual influence of electrons in different orbits on each other. In these calculations the orbits were supposed to lie in planes through the nucleus. In the present paper computations of the same kind are presented for the case in which most of the orbits lie in parallel planes that do not pass through the nucleus. This gives a volume distribution of electrons.

Since the early history of electron theory the numbers of electrons in the various orbits (or groups into which the electrons in an atom may be subdivided) have been supposed to correspond to the intervals between the atomic numbers of the chemically inert gases. The atoms of these gases have groups that are completely filled up. The atomic number intervals are

$$2,\ 8,\ 8,\ 18,\ 18,\ 32.$$

As is well known the intervals may be written

$$2 \times 1^2,\ 2 \times 2^2,\ 2 \times 3^2,\ 2 \times 4^2.$$

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[1] Science, May 21, 1920.

In making the computations I have interpreted this to mean that the number, n, of electrons in a completed orbit equals the square of the quantum number, τ, associated with that orbit, $n = \tau^2$; and further that there are two orbits in each group. Each completed group, therefore, contains $2\tau^2$ electrons. Thus there are two orbits with the quantum number $\tau = 1$, each of which contains one electron, the plane of each orbit passing through the nucleus. These two electrons must revolve in opposite directions or in planes inclined to each other, otherwise they would travel in the same orbit. Corresponding to the second atomic number interval there are two 2 quantum orbits in parallel planes that do not pass through the nucleus, and each orbit contains 4 electrons. These, also, must revolve in opposite directions, else they would combine (for elements of high atomic number) to form a ring of 8 electrons. Outside of these electrons there are two other 2 quantum orbits corresponding to the third atomic number interval, each containing 4 electrons and so forth. This arrangement gives a volume distribution of electrons quite similar to the layers and shells of the Lewis-Langmuire static atom.

In order to compute a K critical absorption frequency we calculate the amount of energy required to remove either one of the 1 quantum electrons to a point outside of the atom. This can be done by using the equations of Bohr's theory to calculate the energy of the atom with the two 1 quantum electrons in place, and then to calculate the energy when one of them has been removed. The difference between the two equated to $h\nu$ gives the required frequency, ν.

The largest term in the computation represents the energy required to remove the electron against the attraction of the nucleus. From this must be subtracted a correction term (amounting to 15 or 20 per cent.) due to the influence of the electrons. To compute this I have used an approximate expression for the force exerted by one ring of electrons on the ring parallel to it and of the same size. To calculate the influence of orbits on each other that are not of the same size would require an enormous amount of labor. To estimate the effect on the calculated frequencies of this influence I have assumed it to be about the same as it would be, if the orbits lay in the same plane. The correction to the value of ν due to this influence averages 2 per cent. for the different atomic numbers.

With these approximations the computed values of the K absorption frequencies for 14 chemical elements ranging in atomic number from 12 to 92 equal the observed values to within 1 or 2 per cent., as the lantern slide indicates.

The fairly close agreement between theory and experiment becomes very striking, when we remember that none of the quantities entering into the computations comes from X-ray measurements alone. The equations do not contain a single undetermined constant. These quantities are (a) the Rydberg fundamental frequency, which is known with very great accuracy, and (b) the electron's charge, Planck's action constant and the velocity of light. The three quantities (b) enter into the equations only as a correction term due to the change of the electron's mass with its velocity.

Various corrections have been omitted in the computations—for instance the magnetic influence of the electrons on each other, the question as to whether the electron must be removed to an infinite distance from the atom, or only to its periphery, the question as to whether some of the electrons may not be forming bonds with other atoms, and may, or may not, under these conditions contribute to the energy changes, etc.

It appears that both the coplanar distribution of electrons previously reported on and the present volume distribution enable us to calculate the *K* absorption frequencies fairly well. There are also other distributions, corresponding to different interpretations of the atomic number intervals, that work almost, if not quite as well as these. An important point in the theory is that there are 2 electrons with 1 quantum, 16 electrons with 2 quanta, 36 electrons with 3 quanta and 32 electrons with 4 quanta.

By arbitrarilly shifting electrons about in the outer orbits it would be possible to make the theory agree with the facts to almost any degree of accuracy. This, however, would not mean very much.

Computations of the *L* and *M* absorption frequencies present very great difficulties.

In coplanar orbits elliptic *L* orbits are difficult to admit. They would interfere with the *K* orbits. In the volume distribution, however, there is room for orbits of some such shape as that of ellipses. They would not be plane orbits, but orbits in space of three dimensions. The three dimensions may give an explanation of the three critical absorption frequencies observed in the *L* series. Plane orbits that may be either ellipses or circles explain only two of these frequencies.

Harvard University.

THE EVIDENCE REGARDING THE SO-CALLED "*J*" RADIATION IN THE CHARACTERISTIC X-RAY SPECTRA OF THE ELEMENTS.

By F. K. Richtmyer.

In a previous communication[1] it was shown that, contrary to the observations of Barkla and White,[2] there were no discontinuities in the X-ray absorption of aluminum and water at wave-lengths, $\lambda = .37$ Å. and $\lambda = .39$ Å. respectively, and hence that there is no basis for the conclusion that these elements have "*J*" radiations at these wave-lengths. This agrees with results obtained by Duane and Shimizu[3] on the radiation from an aluminum target.

After reporting the above, the present writer's attention was called to a paper by Williams[4] who concluded that the discontinuity in the absorption curve for Al was at $\lambda = .49$ Å., instead of at .37 Å. as found by Barkla and

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[1] Richtmyer and Grant, Phys. Rev., XV., 547 (1920).

[2] Barkla and White, Phil. Mag., 34, 270 (1917).

[3] Phys. Rev., XIII., 289 (1919).

[4] Proc. Roy. Soc., 94, p. 576.

White. Williams also concluded that a discontinuity in copper existed at .448 Å. A very careful investigation of the absorption curves of these two elements at these wave-lengths fails to reveal any discontinuities as great as one per cent. The apparent low precision in Williams' work does not seem to justify his conclusion.

Still later Dauvillier[1] reports a discontinuity in the absorption curve of aluminum at $\lambda = .358$ Å. Using his own and Barkla and White's data on carbon and oxygen he predicts that there should be a "*J*" emission of boron at $\lambda = .42$ Å., although, using boron as a target he fails to reveal any characteristic radiation in the region $.2 < \lambda < 1.0$ Å. Dauvillier concludes that although the "*J*" absorption exists, the "*J*" radiation is too faint to measure. A discontinuity for bromine was observed at $\lambda = .227$ Å.

Dauvillier refers to a paper by de Broglie[2] in which reference is made to an absorption band in silver at $\lambda = .178$ Å. If, according to Dauvillier, the square root of frequencies of these *J* radiations for Ag, Br and Al be plotted against *N* a curve similar to Mosley's curves for the *K* and *L* series results, passing through one of the gamma rays of Rad. *C* as observed by Rutherford and Andrade.

Such a graph would predict *J* absorptions in Cu and Mo at .250 Å. and .196 Å. respectively.

The present writer has studied the absorption of all of the above named elements, with the exception of bromine, over the critical ranges where *J* radiations have been observed or would be predicted by a relation similar to Mosley's for the *K* series. It has been shown[3] that the mass absorption coefficient, μ/ρ, is a linear function of the cube of the wave-length. Data gives a precision of approximately 1 per cent. *In no case* is there a break in the curve between μ/ρ and λ^3 at the critical wave-lengths mentioned. *It may therefore be concluded that within* 1 *per cent., there are no discontinuities in absorption suggestive of J radiations.*

The break at $\lambda = .358$ in the energy curve of an X-ray tube with an aluminum filter interposed, as found by Dauvillier, is obviously due to the *K* absorption of the iodine in the ionization chamber, as pointed out by Duane and Shimizu.

CORNELL UNIVERSITY.

THE SPECTRUM OF HELIUM IN THE EXTREME ULTRA-VIOLET.

BY HUGO FRICKE AND THEODORE LYMAN.

FORMER work on the spectrum of helium has shown that but two lines, one at 1,640 Å.U. and the other near 1,215 Å.U., can be attributed with any certainty to this gas in the Schumann region. A number of observers have found a resonance potential in helium corresponding to a wave-length of about 600 Å.U.; the present paper deals with an attempt to find this line.

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[1] Ann. de Physique, 13, p. 49 (1920).

[2] Journal de Ph., Vol. 6, p. 161 (1916).

[3] Richtmyer and Grant, loc. cit., also Richtmyer, Proc. Am. Phys. Soc., PHYS. REV., Feb., 1921.

A small vacuum spectroscope was employed containing a grating of but 20 cm. radius; a continuous current of about 30 milliamperes flowing through a quartz discharge tube was used; the electrode consisted of a hollow cylinder similar to that described by Paschen. The region investigated lay between 0 and 1,200 Å.U.

Under these conditions the strongest line in the region occurs at 585 Å.U., it is accompanied by five or six fainter lines almost certainly due to impurities.

It is interesting to note that the wave-length 585 corresponds with the resonance potential determined by Franck and Knipping rather than with the results of other observers.

Harvard University.

The Absorption of Light by Mixtures of Inorganic Salts.

By E. F. George.

The object of the present work was:

1. To investigate the effect of one salt upon the absorption of light by another salt with which it is mixed in an aqueous solution.

2. To test the validity of Jones' "Solvate Theory" by the results of the above investigations.

3. To measure the absorption of alcoholic solutions of certain organic salts.

4. To study the effect of temperature upon absorption.

A Hilger spectrometer with a Nutting photometer box was used. In most cases the extinction coefficient was computed and plotted against the wave-length.

The following conclusions are drawn:

1. The effect of one salt in changing the absorption of light by another salt seems to be greater in chlorides than in nitrates or sulphates.

2. The temperature effect appears to be greater in chlorides than in nitrates or sulphates.

3. Ostwald's theory that the colored ion is alone effective in the absorption of light seems to break down, as Jones has already pointed out.

4. The ratio of the absorption of the mixture to the sum of the absorptions of the component parts, when plotted against the wave-length, has a minimum value somewhere between $\lambda = 5,100$ and $\lambda = 6,100$ Å.U.

5. The change in absorption appears to be greatest in solutions containing one or more of the magnetic elements. This may possibly mean that some of the effect is due to frequency changes arising from magnetic forces.

6. Certain salts which have little tendency to form hydrates very often cause greater changes in absorption than some other salts which have a great avidity for water. This is diametrically opposed to the "Solvate Theory."

These investigations were carried out at Ohio State University at the suggestion and under the supervision of Professor Alpheus W. Smith.

WEST VIRGINIA UNIVERSITY.

CHARACTER OF THE SPECTRA PRODUCED BY HIGH POTENTIAL SPARKS IN A VACUUM.

BY EDNA CARTER.

PHOTOGRAPHS were obtained of the "hot spark" spectra of calcium, magnesium, cadmium, titanium and iron. They were made for the most part with a 20 cm. focus concave grating. The iron spectrum was photographed in the region λ 3,800 to λ 6,600 by using an æscoline filter and films bathed in the Wallace three-dye solution.

The spectra of calcium, magnesium and cadmium resemble closely the usual spark spectra of these elements. In the titanium spectrum many arc lines are retained which are faint or absent in the spark in air, but the enhanced lines are also present in considerable intensity. Although the iron spectrum seems to be very much like the arc spectrum of iron in general appearance, still the low temperature lines are absent, the lines which are stronger at the poles of the arc are relatively intense and certain enhanced lines such as λ 5,018 and λ 5,316, which do not appear in the arc, are found in the "hot spark" spectrum. Of all these metals the iron required the highest potential gradient between the electrodes for the production of this vacuum spark.

In general, the spectra obtained from this source seem to be intermediate between the arc and the spark spectra and to resemble most closely the spectra produced by cathode ray bombardment in a high vacuum.

VASSAR COLLEGE,
MT. WILSON SOLAR OBSERVATORY.

INFRA-RED SPECTRA OF ISOTOPES.

BY F. W. LOOMIS.

IT is shown that only minute differences are to be expected in the visible and ultra-violet spectra of isotopes, since these spectra are caused by vibrations of electrons in the outer shells of the atoms and are only affected by the masses of the nuclei to the extent that they enter into the expression, $m = m_1m_2/m_1 + m_2$, for the effective mass of the vibrating electrons. Such differences, of the order of 0.004 Å., have recently been found by Aronberg and by Merton between the wave-lengths of corresponding lines in the spectra of lead isotopes. Much greater differences should occur in the infra-red absorption bands of such isotopic substances as HCl, since in this case the mass of the nucleus is much more nearly equal to the vibrating mass, the hydrogen nucleus, and has a correspondingly greater effect on its effective mass. The frequency with which the hydrogen nucleus vibrates towards and away from the chlorine atom is proportional to $1/\sqrt{m}$. Aston has shown that chlorine consists of isotopes of atomic weights 35 and 37. The corresponding values of m, the effective mass of the hydrogen nucleus, are 35/36 and 37/38, which differ by 2 parts in 1,330. Consequently the vibration frequency of HCl^{35} should be 1 part in 1,330 greater than that of HCl^{37} and lines in the absorption spectrum of HCl should occur as doublets separated by 1/1,330 of their wave-length. Since the average atomic weight of Cl is 35.46, the atoms of Cl^{35} must be over 3 times as numerous as those of Cl^{37} and the doublet lines of longer wave-length should be the fainter.

The most accurate measurements of the HCl infra-red absorption spectrum were published by Imes in the Astrophysical Journal, Vol. 50, p. 251. The "fundamental" band at $3.46\,\mu$ was not sufficiently resolved to show the satellites but each line of the "first harmonic" band at $1.76\,\mu$ shows a satellite on the long wave side separated by an average interval of 14 ± 1 Å. The theoretical separation is $1.76\,\mu/1{,}330 = 13$ Å. Clearly these satellites are the lines due to the heavier isotope of chlorine. Imes also published curves on HBr but not sufficiently resolved to show the expected doublets. His curves for HF would show doublets if any appreciable quantity of isotopes were present. The absence of doublets confirms Aston's conclusion that fluorine is pure.

New York University.

The Absorption Spectra of Ortho-Cresolsulphonphthalein.

By W. R. Orndorff, R. C. Gibbs, M. Scott and S. D. Jackson.

MEASUREMENTS covering a range of natural frequencies from 40×10^{13} to 120×10^{13} show that ortho-cresolsulphonphthalein and other related compounds have in neutral aqueous solutions two absorption bands and that the addition of either acid or alkali results in the disappearance of one of the bands and the appearance of two new absorption bands one on each side of the position of the band that disappears. The other band of the neutral solutions seems to be modified by the addition of acid, but with the addition of alkali it also disappears and a new band with lower frequency appears. In the case of dilute *alkaline* solutions the new type of absorption is not stable but reverts more or less rapidly to the two band absorption found in the corresponding neutral solution. These changes and reversions indicate that in the neutral aqueous solutions the carbinol and hydrate forms of the phthalein are present and that on the addition of either acid or alkali a salt having a quinoid structure is formed.

Cornell University.

NEW BOOKS.

An Enquiry Concerning the Principles of Natural Knowledge. By A. N. WHITEHEAD. Cambridge: The University Press, 1919. Pp. xii + 200. Price, $4.50.

This is a philosophical consideration of the traditions and data of science, followed by a detailed development of what the author terms "the method of extensive abstraction," which deals in a generalized and systematic manner with the relations of time and space.

Elements of Vector Algebra. By L. SILBERSTEIN. New York: Longmans, Green and Co., 1919. Pp. 1 + 42. Price, $1.60.

A brief outline or synopthical presentation of vectors. It is intended as a companion volume to the author's Method of Tracing Rays.

Mesures en Radioactivité. By W. MAKOWER AND H. GEIGER. Paris: Gauthier-Villars et Cie, 1919. Pp. vii + 174.

A translation of Makower and Geiger's book on Practical Measurements in Radioactivity.

Ions, Electrons, and Ionizing Radiations. By JAMES ARNOLD CROWTHER. New York: Longmans, Green and Co., 1919. Pp. xii + 276. Price, $4.00 net.

This volume covers more ground than might be inferred from the title. There are chapters on the passage of current through gases; on the properties and charges of ions; on the phenomena of the discharge tube; on cathode rays and positive rays; on the emission of electricity by hot bodies; on photoelectricity; X Rays; α, β and γ rays; radioactivity and the electron theory of matter.

The Realities of Modern Science. By JOHN MILLS. New York: The Macmillan Company, 1919. Pp. xi + 327. Price, $2.50.

This is a treatise chiefly on physics, adapted to the general reader. The earlier chapters are historical; then follow discussions of the constitution of matter; of the electron; of force in its relation to energy; of the concepts of temperature, etc.

The latter half of the volume deals largely with topics molecular and electronic as becomes a modern popular wrok on physics.

The Present Position of the Theory of Ionization. A General Discussion held by the Faraday Society: London: Published by The Faraday Society, 1919. Pp. 1 + 178. Price 12/6.

Contains papers by Arrhenius, Acree, McBain and Coleman, Bowsfield, Ratan Dhar, Sand, Partington, Porter, Newberry, Prideaux, Milner, Dawson, and Ghosh.

The Luminiferous Ether. By Frank W. Very. Boston: The Four Seas Company, 1919. Pp. 1 + 55.

The author discusses the relation of the ether to the electron and the atom and to a universal interstellar medium.

The Radiant Properties of the Earth from the Standpoint of Atmospheric Thermodynamics. By Frank W. Very. Boston: The Four Seas Company, 1919. Pp. 1 + 84.

In this paper the author emphasizes the important part which the radiation of the atmosphere has in the radiant properties of the earth, a factor which has very generally been excluded or ignored by previous writers.

Encyclopedie Scientifique. Bibliothèque de Photographie. Applications de la Photographie Aërienne. By L. P. Clerc. Paris: Doin and Fils, 1920. Pp. iv + 350.

Deals with the reading and interpretation of photographs from the air; the taking, mounting and study of stereograms; measurements of distance and area by means of aërial photographs and map making from photographic data.

Thèses Présentées A La Faculté des Sciences de L'Université de Paris pour Obtenir Le Grade de Docteur ès Sciences Physiques. I. Recherches Spectrométriques sur Les Rayons X. By A. Dauvillier. Paris: Masson et Cie, 1920, Pp.

Spectrometric measurements of X-rays taken in connection with oscillographic observations of the discharge through the tube have led the author to the discovery of intimate relations between the cathode rays and the X-rays which they produce and have made it possible to standardize X-rays in terms of tension, current, distance and duration of exposure.

Electric Oscillations and Electric Waves with Application to Radiotelegraphy and Incidental Application to Telephony and Optics. By George W. Pierce. First Edition. New York: McGraw-Hill Book Company, Inc., 1920. Pp. ix + 517. Price, $5.00 net.

This is a mathematical treatise on the fundamental theory of electric oscillations and electric waves with special reference to wireless telegraphy but of much broader application. After several chapters of general introductory theory we come to free oscillations in a single circuit and in coupled circuits followed by resonance relations in receiving stations and the general properties of artificial lines and electrical filters

The discussion of electric waves forms a book by itself, independently treated.

Silvanus Phillips Thompson. His Life and Letters. By JANE SMEAL THOMPSON AND HELEN G. THOMPSON. New York: E. P. Dutton and Company, 1920. Pp. ix + 372. Price, $7.50.

A life of one of pioneers in electrical engineering and applied physics by his wife and daughter. It is full of personal touches that will appeal especially to those who had the good fortune to know Professor Thompson.

The Photographic Researches of Ferdinand Hurter and Vero C. Driffield. Edited by W. B. FERGUSON. Great Britain: The Royal Photographic Society, 1920. Pp. xii + 374.

In this memorial volume the Royal Photographic Society of Great Britain has assembled the scientific papers of Hurter and Driffield, with an admirable biography of these two workers.

Hurter and Driffield joined forces as early as 1876 in their effort "to make photography a quantitative science." Their partnership lasted to the end of the last century. Their results are now classic.

Second Series. *April, 1921.* *Vol. XVII., No. 4*

THE

PHYSICAL REVIEW.

THE THERMIONIC CURRENT BETWEEN PARALLEL PLANE ELECTRODES; VELOCITIES OF EMISSION DISTRIBUTED ACCORDING TO MAXWELL'S LAW.

BY THORNTON C. FRY.

SYNOPSIS.

Thermonic Current between Parallel Plane Electrodes.—The electrical equations applying to this problem are developed without neglecting the distribution of initial velocities, which in the first place is allowed to be entirely general. *Maxwell's distribution of velocities* is then considered in detail and a complete solution obtained.

Curves from which to compute the space current when Maxwell's distribution applies are presented, together with an illustrative example of their use. Curves are also included showing the *deviation of the current-voltage relation from the 3/2-power law;* the variation of the *minimum potential between the electrodes* with plate voltage, and also the variation with plate voltage of *the distance between the cathode and the point at which this minimum potential occurs.*

1. INTRODUCTION.

A SIMPLE method of computing the thermionic current between parallel plane electrodes under circumstances such that the 3/2-power law published by Childs[1] and later by Langmuir[2] does not apply seems highly desirable. An attempt made several years ago to solve this problem led to a method of solution which was at once direct and general. It is the aim of this paper to present this method of solution as concisely as possible; and to apply it to a particular problem with a view to deriving quantitative results of a typical sort.

The solution requires the evaluation of a difficult integral which appears in equation (4). This evaluation was carried out, with the result shown in Figs. 2 and 4, by the use of the Integraph of Abdank-Abakanowicz. This integraph, which is not so well known in this country as its relatively low cost and very great accuracy seem to justify, is so constructed that when a pointer is passed along a curve $y = f(x)$ a ruling pen attached to the machine automatically draws the curve representing the indefinite integral, $\int f(x)dx$. Its accuracy when properly operated is such that successive integrations of the same function will seldom vary from one another by more than the width of the line drawn by the ruling pen.

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[1] PHYS. REV., 32, p. 492, 1911.

[2] PHYS. REV., 2, p. 350, 1913.

2. Mathematical Formulation.

It is obvious that in any case of thermionic emission in which voltage saturation has not been reached there must be a region of negative potential gradient adjacent to the cathode. If this were not true every emitted electron would be drawn to the anode, and consequently no increase of current would result from an increase in the anode voltage. It therefore follows that the voltage curve must have the characteristic form shown in Fig. 1. A convenient notation denotes the potential at an arbitrary distance x from the cathode by V, the minimum potential by V' and the distance of this minimum potential from the cathode by x'. The region between O and x' may be called the region α, the remainder of the distance to the anode being denoted by β. If attention is fixed upon a particular electron its emission velocity may be denoted by v_0, its velocity at x' by v' and its velocity at an arbitrary point x by v. The number of electrons emitted per second per unit area with this particular velocity v_0 may be called $n(v_0)$ and the total emission N. Then

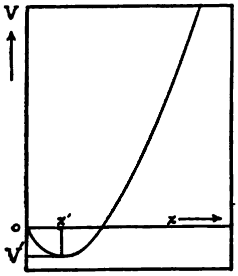

Fig. 1.
Characteristic Form of Voltage Curve.

$$N = \int_0^\infty n(v_0)dv_0.$$

In a current of electrons all of which travel with the speed v, the space charge is given by $e[n(v_0)/v]$, $n(v_0)$ being the number of them emitted per unit time. Hence where velocities differ the space charge is given by the equation

$$\rho = e\int \frac{n(v_0)}{v}dv_0,$$

the integration being performed over all the velocities of emission which are of such a magnitude that the electrons pass the point in question.

If this point lies in the region β all electrons pass it which had emission velocities greater than $\sqrt{2V'e/m}$, that is, high enough to pass the region of adverse gradient. Hence, denoting this critical emission velocity by v_0',

$$\rho = e\int_{v_0'}^\infty \frac{n(v_0)}{v}dv_0. \qquad (1)$$

The space charge at a point x in α is not quite so simply expressed, since it is made up of those electrons which pass on to the anode and also of those which, although possessing enough energy when emitted to pass x, are brought to a stop before reaching x' and turn back. There are two equally dense streams of these latter electrons passing in opposite directions, each of which contributes to the space charge. If V is the potential of the point whose distance from the cathode is x, those electrons which are emitted with lower velocities than $\sqrt{2Ve/m}$ do not reach x, and those which are emitted with velocities higher than $v_0' = \sqrt{2V'e/m}$ do not return after passing. Hence,

$$\rho_\alpha = 2e\int_{\sqrt{\frac{2Ve}{m}}}^{v_0'} \frac{n(v_0)}{v}\,dv_0 + e\int_{v_0'}^{\infty} \frac{n(v_0)}{v}\,dv_0. \tag{2}$$

These formulæ assume that all the electrons are shot out normally to the cathode. They are equally true, however, whatever the direction of emission may be, provided v_0 is understood to signify the *normal component* of the emission velocity and $n(v_0)$ the number of electrons shot off per second with this *normal component*. When later in the paper the Maxwell distribution of velocities is introduced the normal component only, and not the complete velocity, will be dealt with.

The current to the cathode is given by

$$\frac{i}{e} = \int_{v_0'}^{\infty} n(v_0)\,dv_0. \tag{3}$$

The only other equations necessary to determine the solution of the problem are the equation of energy,

$$v^2 = v_0^2 - \frac{2Ve}{m}, \tag{4}$$

and Poisson's equation,

$$\frac{\partial^2 V}{\partial x^2} = -4\pi\frac{\rho}{k}, \tag{5}$$

k being the dielectric constant.

Equations (1) and (5) of section 2 result in the differential equation:

$$\frac{d^2V}{dx^2} = -\frac{4\pi e}{k}\int_{v_0'}^{\infty} \frac{n(v_0)}{v}\,dv_0,$$

which formulates the potential distribution in the region β. A first integral of this equation is found in the customary manner by multiplying it, on both sides, by $2(dV/dx)$, and integrating the right-hand side under the sign of integration, the result being,

$$\left(\frac{dV}{dx}\right)^2 = \frac{8\pi m}{k}\int_{v_0'}^{\infty} n(v_0)(v - v')\,dv_0. \tag{6}$$

This result applies only to the region β. By using (2), (4) and (5) in exactly the same way a similar result may be obtained for the region α, provided the limits of integration, which are variable in this case, are properly dealt with when the order of integration is changed. This gives:

$$\left(\frac{dV}{dx}\right)^2 = \frac{8\pi m}{k}\left[\int_{v_0'}^{\infty} n(v_0)(v - v')dv_0 + 2\int_{\sqrt{\frac{2Ve}{m}}}^{v_0'} vn(v_0)dv_0\right]. \qquad (7)$$

So far, it has not been necessary to know the form of the function $n(v_0)$, but in order to proceed further it is desirable to introduce it at this time. The most logical expression is Maxwell's distribution law for the number of gas molecules having the normal velocity v_0 which pass a given plane in a second of time. This expression is

$$n(v_0) = \frac{\pi N v_0}{2\bar{v}_0^2}\,\epsilon^{-\left(\frac{\sqrt{\pi}v_0}{2\bar{v}_0}\right)^2}, \qquad (8)$$

in which $\bar{v}_0$ represents the average velocity of emission,

$$\bar{v}_0 = \frac{1}{N}\int_0^{\infty} n(v_0)v_0 dv_0.$$

It is somewhat simpler to use v instead of v_0 as the variable of integration in (6) and (7). When this is done and $n(v_0)$ is given the value (8) the equation

$$\left[\frac{dV}{dx}\right]^2 = \frac{4\pi N m}{k\bar{v}_0^2}\left[\int_0^{\infty} v^2\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv \right.$$
$$\left. - \int_{\sqrt{\frac{2e}{m}(V'-V)}}^{\infty} vv'\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv \pm \int_0^{\sqrt{\frac{2e}{m}(V'-V)}} v^2\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv\right] \qquad (9)$$

is obtained, in which the upper sign is to be used in the region α and the lower sign in the region β. The integrals involved in this expression are of well-known types and may easily be evaluated either in terms of algebraic functions or in terms of the normal error function. Denoting this latter function by

$$\text{erf}\,(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx,$$

(9) may be expressed as

$$\left(\frac{d\eta}{d\xi}\right)^2 = \epsilon^{\eta} - 1 \pm \left(\epsilon^{\eta}\,\text{erf}\,\sqrt{\eta} - \frac{2}{\sqrt{\pi}}\sqrt{\eta}\right), \qquad (10)$$

where the quantities ξ and η are related to x and V by the formulæ

$$\xi = \epsilon^{-\frac{\pi e V'}{4\bar{v}_0^2 m}}\sqrt{\frac{2\pi^2 e^2 N}{km\bar{v}_0^3}}\,(x - x'),$$

$$\eta = \frac{\pi e}{2\bar{v}_0^2 m}(V - V').$$

It is to the introduction of these quantities ξ and η that the generality of the solution which we obtain must be attributed. They are both dimensionless, but, as the defining equations show, they are proportional to distance and potential difference respectively, both being measured from the point of minimum potential. Hence for any one state of the system the curve which represents η as a function of ξ will also represent V as a function of x; the only difference being that for the latter purpose the units of measure along the axes, and the location of the origin of reference must be changed. For this reason it is probably most convenient physically to think of ξ and η as length and potential respectively, remembering, however, that with each change in such quantities as cathode temperature and anode voltage the unit of measure must be readjusted.

From a purely mathematical point of view the reason it is desirable to use them rather than x and $\bar{v}$ is that they reduce equation (9) to the form (10) in which, aside from themselves, only absolute constants occur. Equation (9) defines one relation between V and x for every set of values of the constants $\bar{v}'$, $\bar{v}.$, etc., and therefore to obtain a complete solution for it by graphical means would require the construction of a large number of curves. On the other hand equation (10) has only one solution which may be found once for all. The labor of obtaining a complete answer to our problem is, therefore, materially reduced by the use of the new symbols.

3. Solution of Equation (10).

The equation (10) applies to the region between the cathode and the point of minimum potential when the upper sign is used and to the region beyond the point of minimum potential when the lower sign is used. In either case, however, the boundary condition to be applied is that $V = V'$, and hence $\eta = 0$ when $x = x'$, and hence $\xi = 0$. The right-hand side of this equation has been computed for a sufficient range of values of η. These values are shown in the accompanying table, where it is denoted by $\phi(\eta)$.

The equation (10) may now be written in the form

$$\xi = \int_0^\eta \frac{d\eta}{\sqrt{\phi(\eta)}}, \tag{11}$$

which represents the general solution of the problem under discussion provided the integral can be evaluated. This could easily be accomplished by the use of the integraph if it were not for the fact that the integrand becomes infinite at $\eta = 0$. However, no mechanical device is ever capable of handling infinities, and it is necessary to perform the

TABLE I.

Values of the Functions $\phi(\eta)$.

η	$\phi(\eta)$.		$[\phi(\eta)]^{-1/2}$.	
	Upper Sign.	**Lower Sign.**	**Upper Sign.**	**Lower Sign.**
.1	.129	.0808	+2.79	−3.52
.2	.293	.1488	1.85	2.59
.3	.489	.2107	1.43	2.19
.4	.716	.2681	1.06	1.93
.5	.977	.3214	1.01	1.76
.6	1.271	.3726	.887	1.64
.7	1.607	.4213	.789	1.54
.8	1.983	.4688	.710	1.46
.9	2.408	.5118	.644	1.40
1	2.878	.5575	.590	1.34
1.2	4.002	.6377	.500	1.25
1.4	5.384	.716	.431	1.18
1.6	7.116	.790	.375	1.12
1.8	9.24	.862	.329	1.08
2	11.85	.931	.291	1.04
2.5	21.27	1.09	.217	.962
3	37.13	1.24	.164	.901
3.5	62.85	1.38	.126	.855
4	105.68	1.52	.097	.813
4.5	176.41	1.63	.075	.781
5	293.07	1.75	.058	.758
9		2.56		0.625
16		3.65		0.525
36		5.86		0.413
64		8.09		0.352
100		10.34		0.311
225		15.9		0.251
400		21.6		0.215
900		32.8		0.174
1600		45.2		0.149
2500		56.4		0.133

integration for small values of η in some other way. This is done by noting that when η is small

$$\phi(\eta) = \eta \pm \frac{2}{\sqrt{\pi}}\eta^{3/2} + \frac{1}{2}\eta^2.$$

Substituting this value in (11) and performing the integration the approximate relation

$$\xi \doteq 2\sqrt{\eta} \mp \frac{1}{\sqrt{\pi}}\eta + \left(\frac{3}{\pi} - \frac{1}{2}\right)\eta^{3/2} \tag{12}$$

is obtained.

This curve is shown plotted as curve (1) of Fig. 2. Curve (2) is $[\phi(\eta)]^{-1/2}$ for the region between the cathode and the point of minimum potential while curve (3) represents the same function beyond this point. Curves (4) and (5) are the integrals of (2) and (3) as drawn by the integraph, so constructed as to meet the approximation curve (1) near the origin. From the fact that (4) and (5) join on to (1) smoothly, it is evident that the approximation given by (12) is sufficiently accurate for all physical purposes.

4. Discussion of Results.

There are three distinct types of potential distribution curves, for the case of pure electron discharge between plates. The first of these is met when the space current is saturated; that is, when every electron which frees itself from the cathode is pulled across to the anode. It is at once obvious that the potential gradient in this case cannot be negative; hence this curve must have a positive slope throughout its entire range. It is typified by curve (1) of Fig. 3. As the plate voltage is increased, it straightens out more and more, and the angle at which it meets the cathode becomes more and more acute. Similarly, as the plate voltage decreases toward the critical voltage which will just saturate the current the curve rounds off, and the angle at which the cathode is met becomes more and more nearly a right angle. When the plate voltage is made just sufficient to saturate the current a limiting distribution of potential is met that separates the first type of curve from the second. For this distribution the gradient at the surface of the cathode is zero, while at all other points of space it is positive. Such a curve is shown by the dotted line (2) of Fig. 3.

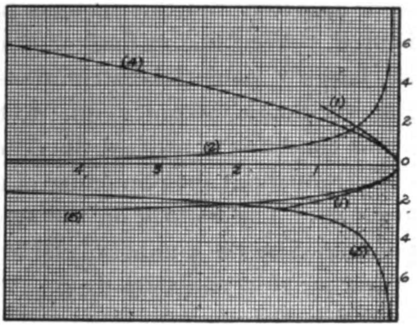

Fig. 2.

Voltage Distribution Curve, Approximation Curve, and Derivatives.

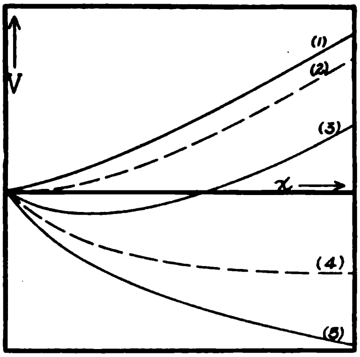

Fig. 3.

Three Types of Voltage Distribution.

For all distributions of potential belonging to the first class, the space current is constant and determined solely by the emissivity of the cathode. Whatever the design of the tube or the voltage applied, so long as it exceeds the saturation voltage, this current remains constant. When, however, the voltage is reduced below the limiting value of this class the gradient at the cathode, and for some distance x' beyond, becomes negative. At x' it changes sign and thereafter remains positive, as shown in Fig. 3 (3).

At x' there is a minimum on the potential distribution curve. The value of V' at this minimum point, and the distance x' between it and the cathode vary with the anode potential. When this is so high that nearly all of the emitted electrons are drawn across both V' and x' are very small; but as it is lowered more and more V' becomes greater and greater, and moves further and further from the cathode. During these changes of plate voltage the current is determined solely by the number of electrons which are capable of passing the restraining voltage V', the relation between them being that given in equation (3).

When the anode voltage is made zero the potential throughout the space between the plates is everywhere negative, and this remains true as the anode voltage is still further reduced. A second limiting condition, which separates the second and third classes of potential distributions is reached when the anode potential is so low that x' becomes equal to the distance between the plates. When this condition is reached the potential gradient at the anode is zero; while throughout the remainder of the space between the electrodes it is negative. The voltage on the anode is then V', and the space current is determined by the number of electrons emitted with sufficient energy to overcome this potential drop. This limiting condition is shown in Fig. 3 by a dotted line (4).

When the potential on the anode is still further reduced, the gradient becomes everywhere negative, and curves of type (5) result. When potential distributions of this type occur the space current is determined solely by the number of electrons which are able to move against the adverse potential $-V$.

The second condition is the one most frequently met in practice, and is the only one to which consideration is given in this paper. The curve obtained in Fig. 2 when drawn for a sufficient range of values of ξ and η as in Fig. 4 applies to all distributions of this type and makes possible computations regarding them. In order to make use of it most conveniently, however, it is desirable to express ξ and η in terms of quantities which are not so difficult to determine as V' and $\bar{v}_0$. The current i, the saturation current i_s and the potential change $\overline{V}_0$ which would give to an electron an energy equal to the average energy of those shot out from the cathode are satisfactory in this respect, and it is not difficult

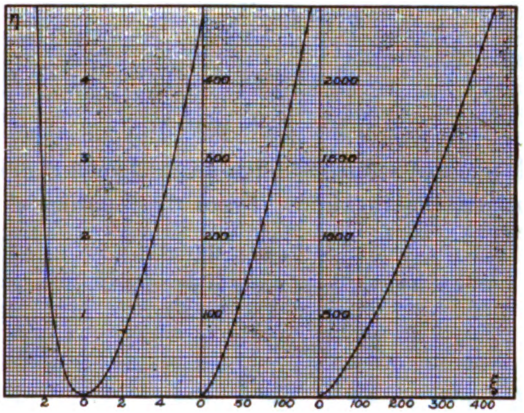

Fig. 4.

The ξ, η Curves.

in the light of the preceding equations to see that they are related and the quantities $\bar{v}_0$ and V' by means of the equations

$$i_s = Ne,$$

$$\overline{V}_0 = -\frac{2m}{e}\bar{v}_0^2,$$

$$i = i_s\epsilon^{V'/\overline{V}_0}.$$

The last of these equations furnishes the relation between V' and i, which can be written in the form

$$V' = \overline{V}_0 \log_e \frac{i}{i_s}. \tag{13}$$

Upon introducing these new symbols in the equations defining ξ and η and inserting the numerical values of such of the quantities as are universal constants, they become

$$\begin{aligned} \xi &= 820\frac{i^{1/2}}{\overline{V}_0^{3/4}}(x - x'), \\ \eta &= \frac{V - V'}{\overline{V}_0}, \end{aligned} \tag{14}$$

where current and potential are expressed in amperes and volts.

In order to illustrate the procedure to be followed in making use of Fig. 4 a particular example may be considered. For this purpose take a pair of electrodes ½ cm. apart, one of which is emitting electrons in such a fashion that[1] i_s is 0.16 amperes and $\overline{V}_0$ is 0.3 volts. In this case if ξ' and η' are the values of ξ and η at the cathode equations (14) become

$$\begin{aligned} \eta' &= \log\frac{i_s}{i}, \\ V &= 0.3(\eta - \eta'), \\ \xi &= \xi' + 1010\sqrt{i}. \end{aligned} \tag{15}$$

In Table II. a set of values of i has been chosen ranging from 0.1 per cent. to 100 per cent. saturation. From these values η' has been found by the first of equations (15) and then ξ' taken from the curve of Fig. 4. Knowing the values of ξ', it is easy to compute the value of ξ corresponding to each value of i. Using these values of ξ, η is picked off from Fig. 4 and entered in the table. Finally V is obtained by substituting in the second of equations (15) the values already found for η and η'.

TABLE II.

The Application of Fig. 4 to a Specific Problem.

i/i_s.	i amps.	η'.	ξ'.	$1010\sqrt{i}$.	ξ.	η.	V Volts.	X' cm.	V' Volts.
0.001	0.00016	6.91	−2.4	12.8	10.4	13	2	0.094	−2.1
0.002	0.00032	6.21	−2.4	18.0	15.6	23	5	0.067	−1.9
0.005	0.0008	5.30	−2.4	28.4	26.0	47	13	0.042	−1.6
0.01	0.0016	4.61	−2.40	40.4	38.0	80	23	0.030	−1.4
0.02	0.0032	3.91	−2.35	57.1	54.8	133	39	0.021	−1.2
0.05	0.008	3.00	−2.24	90.4	88.2	263	78	0.012	−0.9
0.1	0.016	2.30	−2.09	128	126	435	130	0.008	−0.7
0.2	0.032	1.61	−1.88	180	178	695	208	0.005	−0.5
0.4	0.064	0.92	−1.56	255	253	1130	338	0.003	−0.3
0.6	0.096	0.51	−1.22	313	312	1515	453	0.002	−0.2
0.8	0.128	0.22	−0.86	361	360	1840	550	0.001	−0.1
1.0	0.16	0.00	−0.00	404	404	2150	645	0.000	−0.0

The relation between V and i is represented graphically in Fig. 5, where it has been plotted to logarithmic scales in order to facilitate comparison with the 3/2-power law, which gives the accompanying straight line.

In the process of obtaining this curve it has been necessary to find the values of ξ' and η', from which x' and V' can easily be obtained by the use of the relations $V' = -0.3\eta'$ and

$$x' = -\frac{\xi'}{2020\sqrt{i}}.$$

Accordingly these quantities, which have a certain amount of interest in themselves, have been entered in the last two columns of Table II. and represented by Figs. 6 and 7. Of course the absolute magnitudes of x' and V' corresponding to any given voltage are very largely dependent upon such things as the electrode separation and the saturation current,

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[1] This corresponds roughly to a tungsten cathode at 2400° K.

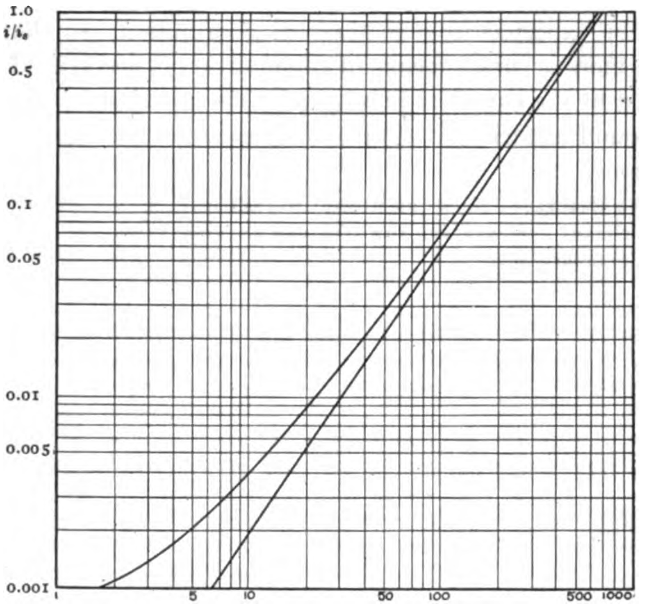

Fig. 5.

The Relation of Space Current to Plate Voltage: Comparison of Maxwell Distribution and 3/2-Power Law.

so that Figs. 6 and 7 can be regarded as typical only in the sense that curves of the same general shape are obtained for other values of these quantities.

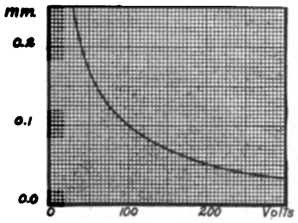

Fig. 6.

The Relation between Plate Voltage and Distance of Minimum Potential from Cathode.

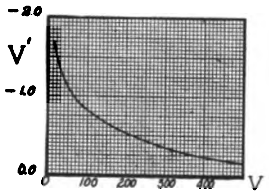

Fig. 7.

Relation between Minimum Potential and Plate Voltage. The Scales are in Volts.

5. Conclusion.

Perhaps the most interesting comment which may be made on these results concerns the magnitude of the variations from the 3/2-power law introduced by the finite velocities of emission. These variations are frequently quite large, and, as is evident from Fig. 5, may be close to 50 per cent. for voltages as high as 40 or 50 volts. At still lower voltages the 3/2-power law need not even give a rough approximation to the values to be expected from a Maxwell distribution; so that at these voltages a more accurate means of computation is needed. This means is furnished by the curves of Figs. 2 and 4, the use of which, it is believed, will be found sufficiently simple and accurate for most laboratory computations.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE &
TELEGRAPH COMPANY AND THE WESTERN ELECTRIC COMPANY, INC.,
February, 1920.

FLAME EXCITATION OF LUMINESCENCE.

By E. L. Nichols and D. T. Wilber

Synopsis.

Luminescence Excited by Contact with the Hydrogen Flame.—It has been discovered that many substances may be excited to fluorescence by contact with a hydrogen flame. The conditions for this excitation are described and it is shown that this luminescence differs from photo-luminescence and also from mere temperature radiation.

Many pure oxides and sulphides not luminescent under the action of light (notably the oxides of calcium, magnesium, silicon, aluminum, zirconium and zinc and the sulphides of zinc and calcium) and certain of the phosphorescent sulphides are shown to respond to flame excitation.

The range of temperatures through which flame excitation occurs is perfectly definite for each substance and the upper limit is sometimes several hundred degrees above that at which photo-excitation becomes extinct.

The spectra, like those of the phosphorescent sulphides, are broad banded and when subjected to spectro-photometric measurements they are found to consist of numerous equidistant overlapping components. These components, in the case of substances capable of photo-luminescence are identical with the components of the fluorescence spectrum excited by light but are of different relative intensities.

The existence of a definite well-defined upper limit of temperature above which bodies are neither phosphorescent nor fluorescent has long been accepted in the science of luminescence. Lenard and Klatt[1] in an early paper estimated roughly the temperatures of extinction of the afterglow and of fluorescence for the phosphorescent sulphides. The "upper momentary-range" in their scheme of temperature relations lies between the extinction temperature of phosphorescence and the still higher temperature at which fluorescence ceases.

The precise boundaries of this region were left undetermined, but their tables show fluorescence to be in general very dim or entirely gone at 400°. In the case of Ca-Bi; Sr-Cu; Sr-Zn and Sr-Bi they were still able to detect traces of it when the substance was at a dull red heat. Phosphorescence usually disappears at a much lower temperature (200° to 300°).

There are however instances of visible radiation at somewhat higher temperatures which are suggestive of luminescence and the study of one of these, in particular, was the starting point of the present investigation.

When the oxygen of an oxy-hydrogen blast lamp used in the production of the lime light is turned off and the hydrogen flame is allowed to play over the surface of the slowly cooling lime cylinder, a greenish glow may sometimes be seen in the regions reached by the flame. This furtive glow which occurs at temperatures corresponding to a very low red heat, or below, must have been familiar to many of the early users of the projection lantern. It has however never been studied or described, so far as we are aware.

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[1] Lenard and Klatt, Annalen der Physik, XV., p. 425 (1904).

Attempts to produce this effect in somewhat modified form led to a number of striking results. In our first experiments air-slaked lime from old lime light cylinders which had broken down under exposure to the atmosphere was inserted in a hydrogen flame.

For convenience the powder was placed on a fine wire gauze and rubbed into the mesh. It was immediately noted that when the flame was applied from above a yellowish green glow occurred in those parts of the surface of the oxide which were within certain zones of the flame.

When the flame was applied underneath the gauze, heating it to about the same temperature no glow appeared on the upper surface but only on such portions of the oxide as protruded through the mesh and were *directly bathed by the flame.* When the hydrogen jet was below but the gas was ignited only above the gauze the glow appeared in a ring shaped zone at the outer base of the flame. When the flame struck through the gauze or was purposely ignited below the gauze the glow almost or entirely disappeared on both the upper and lower sides of the gauze.

By repeating these observations with such modifications of material and method as suggested themselves we were soon able to establish the following facts:

1. *The effect is not an ordinary temperature radiation*: it exists only in a comparatively narrow range of temperatures within which it rises, reaches a maximum and disappears.

2. It sometimes greatly exceeds in intensity the black body radiation for the same temperature.

3. It is not producible by heating to the required temperature either electrically, as on a heated strip of metal, nor by radiation or conduction, as in a furnace. When a flame is used the glow occurs within the flame only.

4. In the case of photo-luminescent substances the lower limit of temperature of flame excitation is often, but not always, above the point at which ordinary photo-excitation ceases.

5. The active range in general extends above that at which thermo-luminescence occurs.

6. *Not all flames are effective*; in fact the hydrogen flame—and to a

lesser degree flames containing hydrogen such as the colorless flame from ordinary illuminating gas—is so far as known the sole source of this type of excitation. Flames from alcohol, ether and carbon disulphide were found to be *in-active.*

7. The effective region may be described as the outer portion of the reducing zone of the flame. This zone may be readily and very strikingly located and observed by allowing the flame to impinge upon a red hot copper surface. The coating of oxide on the copper is then reduced to metallic form throughout a perfectly defined and sharply bounded region within the flame; the reduced area being distinguished by the marked change in reflecting power and emissivity. The identification of the region of activity with the edge of this reducing zone is therefore very simple.

Temperatures were estimated in these experiments by placing the substance in the form of a fine powder on one of the end faces of a massive cylindrical block of copper about 30 mm. in diameter and 20 mm. thick. A narrow slot in this face received a platinum wire, the copper being hammered down so as to grasp the wire and make it a part of the surface. This copper-platinum contact formed one junction of a thermo-electric element by means of which the temperature of the face of the block could be measured. As we have been unable to find data giving directly in millivolts the E.M.F. of such a copper-platinum junction we insert here for the benefit of those who may have occasion to use this very convenient means of measuring temperatures to 1000° C.,[1] the results of our calibration.

Table I.

Millivolts and Temperature Differences of Copper-Platinum Thermo-element.

M.V. = millivolts: *t* = temperature of hot junction in degrees centigrade—the cold junction being at 0° C.

t.	M.V.	*t.*	M.V.	*t.*	M.V.	*t.*	M.V.
25° C.	0.210	275° C.	2.640	525° C.	7.180	775° C.	14.440
50°	0.425	300°	2.850	550°	7.780	800°	15.075
75°	0.640	325°	3.330	575°	8.400	825°	15.770
100°	0.855	350°	3.750	600°	9.020	850°	16.490
125°	1.069	375°	4.170	625°	9.900	875°	17.320
150°	1.282	400°	4.610	650°	10.820	900°	18.220
175°	1.497	425°	5.060	675°	11.640	925°	18.580
200°	1.707	450°	5.600	700°	12.410	950°	18.940
225°	1.916	475°	6.110	725°	13.120	975°	19.360
250°	2.235	500°	6.076	750°	13.820	1000°	19.760

To ensure snugness of contact between the luminescent substance and the copper, the face of the block was variously grooved and slotted and in some instances the powder was applied by smoking the block in fumes of a burning metal or over an electric arc, into which the oxide or some salt of the substance under investigation had been introduced. Above 550° temperatures could likewise be determined with the Morse thermo-gauge using a calibration of that pyrometer, previously made, which agreed perfectly with the indications of the thermo-element. The copper block was bored through axially and the hole, one centimeter in diameter, afforded a surface which could be coated and which gave a fair approximation to black body conditions. It was sometimes advantageous to compare the appearance of coatings thus enclosed with those on the open end of the block.

Much attention was given to the difficult question of the difference of temperature between the upper surface of the coatings and the surface of the metal on which they were mounted. Obviously the face of a poorly conducting layer would be cooler than the hot block when out of the hydrogen flame and hotter than the underlying metal when in the flame and it was important to estimate this difference or to know that it could be neglected.

That the difference was small in most cases where the precise temperature was in question we were able to show in many ways.

If for example a coating of one of the sulphides which glows strongly with a green or blue color at a temperature just below the red heat be scattered loosely and with varying thicknesses on the brass gauze or the copper block and excited in the manner already described it is easily possible to render the thickest parts of the coating red hot and non-luminescent while thinner regions within the flame, owing to cooling effects of the metal beneath are below the red heat and strongly luminescent. If now the thinner regions of the coating coincide in color and brightness with the very thinnest and therefore with the underlying metal one may feel assured that temperature measurements of the metal surface will apply up to a certain thickness of coating and even estimate approximately at how many hundredths of a millimeter thickness the departure of the temperature of the surface of the metal will become noticeable. It was found that for thicknesses under 1/10 mm. the departure was not of a size to appreciably vitiate our estimates but that at greater thicknesses it rapidly became significant.

In general estimates of temperature were made with the massive copper block, heated by an electric furnace or a Bunsen flame. Its mass and conductivity were such that the small hydrogen flame applied from above had no measurable effect on the surface, *i.e.*, no effect that could be noticed with the Morse gauge. The only difficulty therefore was of local heating of the upper layers of the powdered coatings. The above mentioned estimate of the permissible thickness of coatings was confirmed by observations of the heating power of the flame on layers of black oxides the radiating power of which rendered it possible to determine their temperatures directly with the pyrometer. Thus we were able to assure ourselves that for the thicknesses of coating used when temperature measurements were involved the temperature of the surface of the copper coincided with sufficient exactness to that of the luminescent coating. The most satisfactory direct check, on the whole, consisted in substituting for the luminescent powder a coating resembling it as nearly as possible in all other physical characteristics and to observe its appearance in and out of the flame and different zones of the flame. Since under the conditions of our experiments the brightness of these regions was appreciably the same it seemed altogether reasonable to assume that the luminescent surfaces in the study of which we were engaged had approximately the same temperature as the copper block or metal gauze upon which they rested.

This conclusion was indirectly verified in subsequent studies of kathodo-luminescence at high temperatures in which it was found that the temperatures at which calcium oxide and other substances ceased to be active under kathode bombardment were the same as the extinction temperature estimated in the present investigation.

Experiments with Electrostatic Fields.—Deeming it highly probable that the effect in question involved ionization or the production and movement of charged particles, electrostatic fields were brought to bear with lines of force transverse to the stream lines of the flame and also parallel to the same. Although potential differences of many kilovolts were thus applied *no result whatever could be observed.*

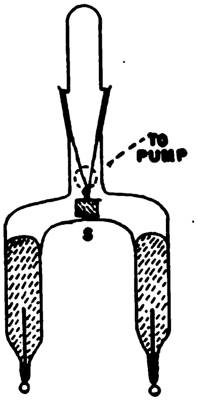

Fig. 1.

Photo-excitation at High Temperatures.—To test as thoroughly as possible the suggestion that flame excitation may be due to ultra-violet radiation within the flame itself very powerful photo-excitation was applied to certain of our substances at high temperatures. To this end a mercury arc lamp was made of the form shown in Fig. 1. Above the arch of the inverted U-tube which contained the arc a vertical tube T was inserted. This had a glass stopper to which a body (S), to be subjected to radiation from the arc, could be suspended so as to hang within the path of the discharge. Various substances which were excited by the flame were thus treated; *e.g.*, calcium oxide, zirconium oxide, etc. These remained completely inert whereas a synthetic ruby under like conditions was exceedingly brilliant until it reached its temperature of extinction when it was suddenly quenched. It seemed highly improbable that any photo-excitation by the hydrogen flame could approach that of this mercury arc in intensity.

Combustion a Necessary Factor.—This fact is very prettily brought out in an experiment based upon a method described by Simonini[1] and used by him in the study of the luminous properties of gas mantles.

A glass tube about 1 cm. bore and 20 cm. in length was coupled to the nozzle of an oxy-hydrogen blast-lamp as in Fig. 2. A small amount of calcium oxide was distributed within the tube from end to end, coating the inner walls thinly.

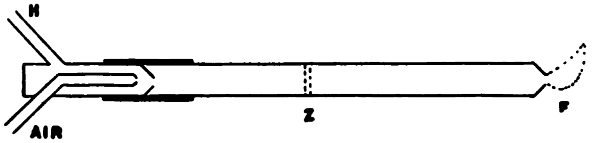

Fig. 2.

When hydrogen was run through the tube and ignited at the free end (*F*) there was no excitation of the calcium oxide even though the tube was heated to a dull red. The addition of air admitted through the oxygen intake likewise produced no effect, whatever the temperature, until the supply of oxygen within the tube caused the flame to "strike back." When this occurred the progress of the flame through the tube could be traced by the luminescence of the oxide.

By adjustment of the supply of oxygen the zone of combustion (*Z*) could be brought to rest within the tube and its position was then clearly marked by a narrow ring of luminescent powder.

From these observations it appears:

1. That luminescence is not produced by the mere presence of hydrogen, at temperatures up to the red heat.
2. That hydrogen and oxygen mingled as above have no effect.
3. That where combustion occurs luminescence is produced but then only in the narrow zone where the reaction is in progress.

In the hydrogen flame we have, besides the combination with oxygen, the production of H_1 or H_3 or both as has been shown by Langmuir,[2] by Wendt[3] and others.

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[1] A. Simonini, Trans. Illuminating Eng. Soc., 1909, p. 648.

[2] Langmuir.

[3] Wendt.

It might reasonably be suspected, that disturbances incident to return of these unstable forms to H_2 would produce the phenomenon of luminescence in substances with which the gas was in contact. If so the atomic hydrogen would presumably be swept along with the flow of gas in the experiment just described and the luminescent zone extended, with decreasing intensity for some distance from the boundary between oxygen and hydrogen within the tube. That, however, was not the case and it was found by numerous modifications of the experiment that the active zone was equally sharp and well defined whether combustion occurred by igniting a jet of hydrogen in oxygen or vice versa. The luminescence always occurs however *within the hydrogen* next to the boundary between hydrogen and oxygen.

Experiments with hydrogen filled tubes at various pressures, within which it was attempted to produce atomic hydrogen by powerful electric discharges both direct and alternating and to expose our luminescent oxides to this gas when heated to a wide range of temperatures, were all equally without result.

The evidence at present therefore appears to be against the assumption that the presence of H_1 or H_3 in contact with the luminescent materials is the cause of the effect which we have called flame excitation.

Active and Inactive Materials.

Some of the materials sensitive to flame excitation, in addition to the old lime already mentioned are:

(*A*) Calcium oxide, from various other sources, zirconium oxide, magnesium oxide, silicon oxide, aluminum oxide, zinc oxide.

(*B*) Certain phosphorescent sulphides such as Sidot blende and Balmain's paint and sulphides of the Lenard and Klatt type.

Not all the Lenard and Klatt preparations are active. Out of forty-four tested the twelve given in the following table responded strongly; a few others dimly.

(*C*) In addition to the above, white saphires, corundums, synthetic rubies and crystals of calcite, fluorite, kunzite, and topaz responded to the flame excitation; as did boric acid, telluric acid, and very pure samples of cadmium phosphate, calcium carbonate, calcium sulphide and of zinc sulphide all incapable of photo-excitation. On the other hand all dark-colored substances, tried, such as the oxides of copper, lead, iron, nickel, manganese, cobalt, cadmium, tungsten., etc., were inert. So likewise were the oxides of bismuth, antimony, barium and strontium; samples of the oxides of erbium, cerium, lanthanum, and thorium; materials from a freshly ignited Welsbach mantle and from a Nernst filament and specimens of uranium glass, didymium glass, fused quartz, etc.

TABLE II.

List of Lenard and Klatt Sulphides that Responded Strongly to Flame Excitation.

Compound.	Excitation.	Color and Brightness.		Spectrum.
No. 3 CaS, Bi, Na_4BO_4	Photo-	Blue-violet	Very bright	Green to violet
	Thermo-	Deep blue	Very bright	
	Flame-	Blue-white	Very bright	
No. 5 CaS, Mn, NaF	Photo-	Yellow	Dim	Red to green (no violet)
	Thermo-	Absent	——	
	Flame-	Yellow-green	Very bright	
No. 9 SrS, Bi, K_2CO_3	Photo-	Blue-green	Bright	Green to blue (no red)
	Thermo-	Very dim	——	
	Flame-	Blue-green	Bright	
No. 27 CaS, Sb, Na_2F_2	Photo-	Blue-green	Medium	Green to blue (no red)
	Thermo-	Dim	——	
	Flame-	Blue-green	Very bright	
No. 29 CaS, Pb, NaF	Photo-	Green	Very dim	Green to blue (no red)
	Thermo-	Absent	Absent	
	Flame-	Pale green	Very bright	
No. 32 CaS, Cu, LiF	Photo-	Blue	Bright	
	Thermo-	——	Absent	
	Flame-	Blue-green	Medium	
No. 34 CaS, SrS, Bi, NaF	Photo-	Blue	Bright	
	Thermo-	——	Absent	
	Flame-	Deep violet	Moderate	
No. 36 CaS, SrS, Bi, Na	Photo-	Blue	Medium	Blue to green (no red)
	Thermo-	——	Very dim	
	Flame-	Blue-green	Bright	
No. 38 CaS, SrS, Bi, Na	Photo-	Blue	Medium	
	Thermo-	——	Absent	
	Flame-	Blue-green	Bright	
No. 39 CaS, SrS, BaS, Cu, Bi, LiF	Photo-	Yellow-green	Medium	
	Thermo-	——	Absent	
	Flame-	Pale yellow	Medium	
No. 43 CaS, Bi, Cu, CaF	Photo-	Blue-green	Bright	
	Thermo-	Blue-green	Very bright	
	Flame-	Blue-white	Bright	
No. 44 CaS, Bi, Sb, Na	Photo-	Blue	Medium	Blue to green (no red)
	Thermo-	——	Absent	
	Flame-	Pale green	Very bright	

Calcium tungstate, one of the most brilliant of fluorescent substances under the action of X rays is quite inert when subjected to the hydrogen flame as is artificial willemite, which is perhaps the brightest of photoluminescent bodies.

THE ACTIVE OXIDES.

Calcium oxide whether obtained from the slaked lime mentioned above or from calcite crystals or by burning metallic calcium gave the same strong yellow glow in the hydrogen flame and the various preparations were active through the same range of temperature. Complete extinction occurred for all these samples at about 690° C.[1] Since the oxide from these varied sources was equally and similarly active it does not seem probable that the effect depends on the presence of small amounts of an accidental impurity as has been shown[2] to be the case in the photoluminescence of Franklin Furnace calcites.

Such admixtures however may modify the character of the glow. Thus a specimen of calcium oxide to which a trace of a bismuth salt had been added, with subsequent heat treatment, showed a brilliant *blue-green* fluorescence instead of the usual *yellow*, when excited by the hydrogen flame.

Of *zirconium oxide* the preparations available were some old disks from a Linnemann[3] "zircon-light" apparatus[4] and a purified white oxide in powdered form from the Cornell Department of Chemistry.

In the hydrogen flame the Linnemann disks gave a remarkable deep red and in cooler portions a pale blue-green fluorescence. The white powder showed the blue-green only.

The red fluorescence is probably due to some impurity but apparently not to the boric acid used as a binder. At least admixture of the latter with the pure oxide failed to impart even a suggestion of a ruddy tone to the glow in flame or in vacuo.

To estimate the active range of luminescence, part of a disk was pulverized and strewn on the surface of the cold copper block in the usual manner.

The observed phenomena were as follows:

1. Pale blue green fluorescence which began very definitely when the block was heated to 76° C. It remained visible until the upper limit of temperature was reached at 372°. Throughout this entire range of

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[1] To be more precise it should be stated that the luminescence spectrum consists of two or more overlapping bands. One of these, lying chiefly in the red-yellow, begins to respond to flame excitation at a lower temperature than the other and vanishes at about 670°. The other band which is yellow-green becomes visible only at higher temperatures and does not disappear altogether until the oxide reaches a temperature of 725°. Details of the experiments on which this statement is based will be given in forthcoming paper.

[2] Nichols, Howes and Wilber, PHYSICAL REVIEW (2), XII., p. 35 (1918).

[3] Linnemann, Dingler's Journal, CCLX., p. 218

[4] The disks according to Erdmann (Chimie, p. 139) were prepared by moistening zirconium oxide—"Zirkonerde"—with a solution of boric acid, compressing and baking. Such disks were used by Rubens and others as sources of infra-red radiation.

temperatures there was no suggestion of the red luminescence but on heating to a dull red we saw:

2. The deep red luminescence already described.

Lower limit...440°
Upper limit...720°

On account of the ruddy color there was some difficulty in distinguishing with certainty between this luminescence and ordinary red heat, but its temperatures of appearance and vanishing were quite fixed and definite and the color was quite different from that of a red hot body, being more nearly of the approximately monochromatic tint which one gets through ruby glass.

Silicon oxide whether in the form of a precipitated powder, or a smoked coating sublimed in the arc from metallic silicon or from quartz exhibits a pale green-white, nearly white fluorescence in the hydrogen flame. Quartz crystal, not crushed, is difficult of excitation and fused quartz appears to be inactive.

To determine the temperature range a very thin coating of the precipitated silica was rubbed into the surface of the copper block. Inactive at room temperature, the fluorescence came very suddenly into view on heating the block. The threshhold was very sharp and definite; so much so that a drop of cool water applied to the surface of the block two centimeters away would immediately extinguish the luminescence within the flame on the side nearer the drop by the slight local fall in temperature thus temporarily produced. The range of temperatures was found to be

Lower limit... 85°
Upper limit...367°

Aluminum oxide in the form of a fine powder from the department stock room was notable for the very low temperature at which it began to glow. The temperature range for the pale green fluorescence was:

Lower limit... 52°
Upper limit...692°

At higher temperatures this coating when in the active zone of the flame presented a curiously mottled appearance there being patches of reddish hue others green-blue and others nearly white. This strange mosaic might have been ascribed to a lack of homogeneity in the material used but for the fact that another coating obtained from the smoke of burning aluminum was similarly variegated under kathode bombardment. It seems to be a general property of this oxide and is under detailed investigation.

Aluminum oxide was found to be susceptible to flame excitation likewise in crystal form. A synthetic ruby gave a greenish glow without a trace of the well-known deep red photo- and kathodo-luminescence. A white sapphire known to be synthetic fluoresced a very delicate pale green in the active zone of the flame; although it like the ruby glowed red under kathodo excitation.

Magnesium oxide obtained by smoking the copper block in the fumes of burning magnesium wire showed a fine white flame fluorescence:

Lower limit	76°
Upper limit	680°

Magnesium carbonate, the ordinary compressed pharmaceutical preparation, showed a similar white fluorescence and since the limits were practically the same as for the oxide (76° to 670°) it may well be that there is partial decomposition of the carbonate to oxide as in the case of the calcite referred to in a subsequent paragraph.

At temperatures much higher than the above, blended with the brilliant white selective radiation from magnesium oxide there is another stage of luminescence, blue-violet in color, the exact temperature relations of which are difficult and have not yet been determined.

Zinc oxide like the oxides of zirconium and magnesium exhibits two stages of flame luminescence but in this case the lower stage is somewhat difficult to bring out. The color of the luminescence is an intense red. It begins at 568° and above 700° [704°] it goes over into a higher stage characterized by a blue-green glow which persists to 948°. The total range for both bands is thus: Lower limit, 568°, upper limit, 948°. The green glow is brilliant and so strikingly different in hue from the temperature radiation on which it is superimposed, that it admits of very definite quantitative study.

An account of these measurements and of observations on the more difficult higher stage luminescence of magnesium oxide are to be presented in a separate paper.

The Phosphorescent Sulphides.

These substances differ from the oxides just considered in that they are strongly photo-luminescent at ordinary temperatures owing to the artificial or sometimes accidental admixture of some active element, such as, bismuth, copper, lead, or antimony.

The solid solvent of those which respond to the flame is usually, calcium sulphide; sometimes strontium sulphide; never, so far as at present known, barium sulphide. Zinc sulphide in the form of Sidot blende is also highly sensitive to flame excitation.

A phosphorescent sulphide differs from the oxides already described in that at room temperature, it usually reacts to the ultra-violet radiation from the hydrogen flame even at a considerable distance.

When warmed by the radiation, even though not in contact with the flame, it shows thermo-luminescence. Then, having become dead to all photo-excitation it will still respond to flame excitation if brought into the active zone and heated there to the proper temperature.

That the flame excitation of these sulphides is not due to a conversion of the surface layer into the oxide is indicated by the color of the luminescence which corresponds to the kathodo- or photo-luminescence of the sulphide and not to the flame effect as observed in the oxide.

Spectrophotometric measurements, moreover, have been made by Dr. H. L. Howes. These were extended to three Lenard and Klatt sulphides the spectra of which had been previously studied[1] and it was found that the spectra of these, under flame excitation, were composites of the same groups of narrow components which make up their spectra in the case of photo- or kathodo-luminescence.

The response of these substances to light after cooling indicated that no permanent decomposition occurred under the action of the flame. At higher temperatures, as in the electric arc, the sulphides were, however, converted into the corresponding oxide and the flame luminescence became that characteristic of the oxide.

While a so-called active element in solid solution is generally conceded to be necessary to the production of a phosphorescent sulphide, this does not seem to be essential in flame excitation. What was found for the oxides in this respect, applied to the sulphides also. A sample of zinc sulphide which was so pure as to be quite inert under the iron spark, both as regards fluorescence and phosphorescence, responded brilliantly with a yellow-green glow in the hydrogen flame at a red heat. The effect became distinctly visible at 560° and was strong at 616°; much brighter indeed than the luminescence under like conditions of specimens of Sidot blende especially prepared with a view to phosphorescent properties. This same pure zinc sulphide responded strongly under kathode rays at room temperature but was without persistent afterglow.

Attempts to determine the upper limit of the luminescence were less definitely successful. Above 900° the glow was feeble but discernible and at 934°, the highest temperature attained in this run, it had not altogether vanished. We were now almost at the upper limit for ZnO as described in a previous paragraph and it seemed almost certain that portions of the sulphide had been converted into the oxide.

Calcium sulphide, when pure, is inert alike under photo-excitation and kathode bombardment. A sample which fulfilled these criteria of freedom from admixtures was found however to be excited by the hydrogen flame through an unusually wide range of temperatures. When placed in a very thin layer on the surface of the copper block and gently heated it showed flame luminescence quite definitely when the block was heated to 60° C. The color was *bluish-green.* The intensity increased in brightness with rise of temperature and below the red heat there was a rather sudden change to a much brighter yellow glow—a lemon yellow and very luminous. This continued with growing and then waning intensity until a cherry red heat of the block, when it vanished. Measurements of the highest temperature at which the luminescence could just be perceived were made with the Pt-Cu thermo-junction, and nearly simultaneously with the Morse thermo-gage, with the following result:

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[1] Nichols, Proc. of Am. Philosophical Society, LV., p. 496 (1916).

Vanishing temperature by pyrometer	744° C.
Vanishing temperature by thermojunction	735° C.

An attempt to pick up this temperature again as the block slowly cooled gave

Pyrometer	731°
Thermojunction	730°

Taking the average of these four estimates we have 737° C. as the temperature of the surface of the block and the approximate temperature of the coating. The yellow glow increased rapidly as the block cooled further and was apparently at its brightest somewhat below 500°. The transformation to the blue-green occurred at 330° according to the indications of the thermo-junction. At this temperature the very thinnest parts of the coating which were nearest the metal and therefore cooler had changed while the high points were still a bright yellow.

Calcium sulphide may therefore be added to the list of those substances having two stages of flame luminescence. The temperature ranges were as follows:

Lower stage	60° to 330° (blue-green).
Upper stage	330° to 737° (bright yellow).

Calcite, Fluorite and other Crystals.

Of the crystals thus far tested for flame luminescence calcite and fluorite especially merit detailed mention.

A crystal of calcite in the flame is at first inactive. Then its edges begin to glow brightly, framing the otherwise dark crystal in lines of light, and the light very gradually extends over the faces. Examination shows that decomposition of the carbonate precedes luminescence. This occurs first along the exposed edges and later over the faces of the crystal. The glow seems to depend on the presence of this film of oxide.

The obviously localized and superficial glow produced by flame excitation is altogether different from the phenomenon of thermo-luminescence, as produced by the same flame, when applied to certain calcites or to kunzite or fluorite. In thermo-luminescence the effect is seen to pervade the entire body of those portions of the crystal which have attained the requisite temperature; in flame excitation proper it is confined to those portions of the *surface* which are within the active zone.

Several specimens of fluorite were examined. Of these some showed the characteristic blue-violet fluorescence under the action of the iron spark. Others, which were mostly of a yellowish body color did not respond visibly to photo-excitation. All however were more or less thermo-luminescent and all showed the same *pale greenish white* flame-luminescence.

The temperature limits observed in the case of one of the almost white, clear, transparent crystals were as follows:

Lower limit.	58° C.
Upper limit.	310° C.

Upon heating to a much higher temperature this crystal began to be converted into the oxide along its edges. Upon cooling it developed a very bright fluorescence under excitation of the flame having the yellow color characteristic of calcium oxide, which was picked up at about the usual temperature, *i.e.*, 690°.

The significant feature in the behavior of these fluorites is that while they differ in color and in their response to photo-excitation—differences usually ascribed to the presence or absence of admixtures of the rare earths, etc.—they are all much alike, if not identical under flame excitation; from which, as in various previous experiments, we infer that flame excitation does not depend on foreign admixtures.

A similar result is obtained in the case of aluminum oxide when crystallized either as synthetic ruby or white synthetic sapphire, or as native sapphire from North Carolina, or as corundum. All of these crystals showed the same *pale green* fluorescence as the powdered oxide described in the paragraph on the active oxides. *These crystallized forms* however all gave the red fluorescence of the ruby when tested in the kathode tube because presumably of the presence in all of traces of chromium.

How general an effect flame-luminescence is among crystals and also among compounds other than the oxides and sulphides remains to be determined. As has already been indicated in the list of substances thus far found active, boric acid and telluric acid respond to the flame. Crystals of the latter compound even during the process of being decomposed by the flame glow with a vivid green light.

From our present evidence it appears probable that only those materials will become luminescent under the action of the hydrogen flame which are capable of the proper sort of rapid reduction and subsequent oxidation already described and that the active range of temperatures in each case is that within which the above reaction occurs.

SUMMARY AND CONCLUSIONS.

1. Flame excitation occurs when certain substances are brought into the zone of a hydrogen flame between the regions of reduction and oxidation.

2. It occurs, so far as known, in certain oxides and sulphides; in a few phosphorescent preparations and in certain crystals and inorganic salts.

3. Many substances strongly luminescent under the action of light, such as calcium tungstate, willemite and many of the Lenard and Klatt sulphides do not respond to flame excitation.

4. The effect does not appear to depend on the presence of traces of some activating elements. Such admixtures may modify the character of the fluorescence; they often inhibit it altogether. Thus pure cadmium phosphate responds to flame excitation, but the addition of manganese, which renders it strongly photo-luminescent with a well-known red phosphorescence, destroys its susceptibility to excitation by the flame.

5. The spectra obtained by flame excitation are characteristic fluorescence spectra consisting of two or more overlapping bands. Each band is made of numerous components forming series of equal frequency intervals and these components in all the cases thus far investigated are identical with the components of the fluorescence spectrum of the substance when photo-excited.

6. Flame excitation occurs through a definite range of temperatures. The temperature limits thus far studied are as follows:

TABLE III.

Substance.	Lower Limit.	Upper Limit.	Color of Band.	Substance.	Lower Limit.	Upper Limit.	Color of Band.
SiO_2.....	85°	367°	White	ZnO.......	568°	704°	Red
MgO....	75°	680°	Blue-green		704°	948°	Green
Al_2O_3....	55°	692°	Pale green	ZnS.......	0° ±	123°	Yellow
CaO.....	0° ±	600°	Red		211°	575°	Blue-green
	40°	725°	Green	CaS.......	60°	330°	Blue-green
ZrO_2....	76°	372°	Blue-green	CaF.......	58°	310°	Yellow-green
	440°	720°	Red	$Cd(PO_4)_2$..	50°	733°	Yellow

These values are to be regarded as only approximate. The fluorescence in all cases is doubtless complex, there being at least two broad bands. These bands have their own independent ranges of temperature and the effect observed is usually a composite. In the case of CaO the two temperature ranges greatly overlap. In that of ZnO they barely overlap. The bands of ZrO_2 and ZnS are quite distinct as to temperature. The phenomenon with the sulphides is complicated by the probability of a change into oxides at the higher temperatures. Both CaS and ZnS exhibit bands above the temperatures of the table which coincide as to color and upper limit with those of the corresponding oxide. These have not been included.

7. Excitation of these substances to fluorescence does not occur by heating in air or hydrogen nor in vacuo nor by contact with a jet of hydrogen while hot unless the hydrogen be ignited and conditions are favorable for free oxidation.

8. Luminescence does however occur by kathode bombardment throughout essentially the range of temperatures indicated under section (6) but the effect is not identical since the bands are not excited in the same proportions as in the flame. X rays also excite many of the substances in question at high temperatures.

9. The after glow, in so far as the same has been studied follows the law of vanishing phosphorescence. Details will appear in a subsequent paper.

10. The action of strong static fields having been found without effect; also the use of powerful photo-excitation at high temperatures and the substitution of other flames for hydrogen, the explanation put forth in an early paragraph is tentatively adopted, *i.e.*, that flame excitation is a dual process in which reduction and oxidation rapidly succeed each other in an unstable layer within the flame between the region where reduction is continuous on one side and oxidation on the other. The alternative suggestion that it is due to disturbances arising from the formation of H_1 or H_3 and their subsequent recombination to H_2 is not positively excluded.

Numerous experiments and measurements not given in the present paper are already completed and ready for publication and others are in progress.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
October, 1920.

THE SPECTRAL STRUCTURE OF THE LUMINESCENCE EXCITED BY THE HYDROGEN FLAME.

By Horace L. Howes.

Synopsis.

Luminescence Spectra Excited by the Hydrogen Flames.—Calcium oxide and the Lenard and Klatt phosphorescent sulphides Nos. 3 and 13 were studied by the aid of a spectrophotometer. The broad-banded spectra were found to consist of the same components, individually overlapping and forming overlapping series of constant frequency intervals. Series and individual components were identified with those entering into the spectral structure of these substances when excited to fluorescence by light, but the relative intensities of the bands were found to differ with the mode of excitation.

THE discovery of the luminescence of many substances when partially bathed in the hydrogen flame but at a temperature below red heat has very recently been reported by Dr. E. L. Nichols.[1] Since the hydrogen flame luminescence was found to be very brilliant when air-slaked lime was presented to the outer layer of the flame this substance was first studied by the author. The specimen was found to fatigue or disintegrate under the flame hence a fine brass gauze was filled with the lime and slowly rotated at a constant speed before the flame. The size of the flame was kept as constant as possible by the use of a reducing valve and a gas regulator. The impossibility of maintaining the flame sufficiently constant necessitated the use of the statistical method and each spectrophotometric curve represents an average of many trials, properly weighted. Fig. 1 represents the distribution of brightness in the visible spectrum of the air-slaked lime. The luminescence before dispersion was of an orange-yellow color and the distribution of color between wave-lengths of .61 μ and .53 μ is indicated in the curve, the maximum lying near .56 μ. The subsidiary crests and shoulders, marked by short vertical lines, although of widely varying intensities and forms are not without a systematic arrangement. When the reciprocals of the wave-lengths of the crests are studied they are readily found to lie in two series of a constant interval of 42 units, designated as the "*A*" or "*B*" series. These wave-lengths and reciprocals are given in Table I. together with the wave-lengths obtained from the luminescence by excitation with the iron spark of natural calcite. The spectral analysis by the iron spark excitation was done by Nichols, Howes and Wilber with the aid of an elaborate photometric control.[1] The positions of the bands are the same under the two different modes of excitation and the structure of the calcite and air-slaked lime must contain the same luminescing system. Because of the nature of the crests it is not considered possible to locate the positions of the bands within ten Ångström units of the proper places hence the agreement found in Table I. is conclusive. The more interesting fact is that although the general form of the spectral envelope is quite different from that obtained by the iron spark excitation yet the partially resolved crests are in no way shifted by change in mode of excitation. The form of the envelope indicated in Fig. 1 corresponds

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[1] E. L. Nichols, 1920.

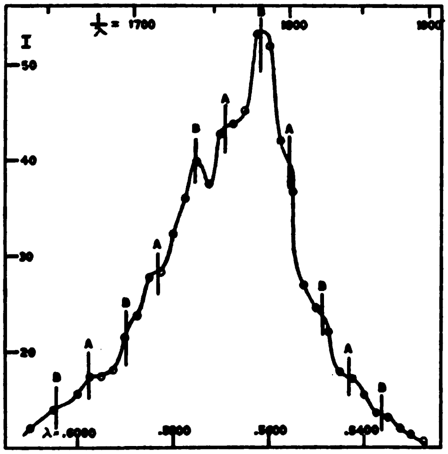

Fig. 1.

TABLE I.

Luminescence of Lime.

Series *A* (Interval = 42).				Series *B* (Interval = 42).			
Band *A*.	Hydrogen Flame, λ.	Iron Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.	Band *B*.	Hydrogen Flame, λ.	Iron Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.
A_4......	.5975	.5977	1673	B_4.......	.6045	.6046	1654
A_5......	.5833	.5831	1715	B_5.......	.5894	.5896	1696
A_6......	.5689	.5692	1757	B_6.......	.5755	.5754	1738
A_7......	.5563	.5559	1799	B_7.......	.5617	.5618	1780
A_8......	.5432	.5432	1841	B_8.......	.5488	.5488	1822
				B_9.......	.5365	.5365	1864

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[1] Nichols, Howes and Wilber, "The Photo-luminescence and Kathodo-Luminescence of Calcite," PHYSICAL REVIEW (2), Vol. 12, p. 365, Nov., 1918.

more nearly to that for the calcite under the iron spark excitation at 200° C. than at 20° C. See Fig. 13, page 364; Nichols, Howes & Wilber (loc. cit.). This is naturally so, since the hydrogen flame warmed the specimen to a temperature of approximately 350° C. Temperature changes the relative intensities of the crests but the "*A*" and "*B*" series are independent of both temperature and mode of excitation for their frequencies.

The Lenard and Klatt sulphide known as No. 3 is a calcium compound containing both bismuth and fluorine as active elements. Since the spectral structure of this preparation had previously[1] been studied under the excitation of the zinc spark it was thought that an interesting comparison could be based on the resemblance between the zinc spark and hydrogen flame excitation. The spectrum had been known to exhibit a strong band in the blue and a broader but much dimmer band in the orange. See Fig. 4, p. 265, E. L. Nichols (loc. cit.). Under hydrogen flame excitation at a very moderate temperature the blue band was found in approximately the same location but the orange band was of much greater proportionate intensity. To produce the orange band the mounting was rotated more slowly to allow a more favorable temperature. In some trials the specimen was mounted in the gauze without rotating but in such cases every precaution was taken to maintain the temperature constant but below red heat. A comparison of the location of the crests of the complex orange band under the two different modes of excitation is given in Table II. One new band was found at .6360 μ. The agreement is as good as could be expected and no systematic deviation is evident. The bands are undoubtedly in the same position and

TABLE II.

Luminescence of Calcium Bismuth Sulphide, No. 3.

Hydrogen Flame, λ.	Zinc Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.	Frequency Interval, $\Delta(1/\lambda) \times 10^3$.
.6365	——	1575	39
.6208	.6200	1614	39
.6048	.6049	1653	39
.5912	.5910	1692	39
.5785	.5781	1731	39
.5650	.5650	1770	39
.5530	.5528	1809	39
.5415	.5411	1848	39
.5300	.5300	1887	

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[1] E. L. Nichols, "Spectral Structure of the Phosphorescence of Certain Sulphides," Proceedings of the American Philosophical Society, Vol. 55, p. 496, 1916

are spaced by a constant frequency interval of 39 units. Again the spectrophotometric curve is not similar to that obtained by the zinc spark excitation, as can be noted by a comparison of Fig. 4, page 265 of the paper last cited with Fig. 2 of the present paper. Evidently a similar vibrating system has been excited by the new method at a slightly higher temperature since the same bands, having the same constant frequency interval of 39 units, are present.

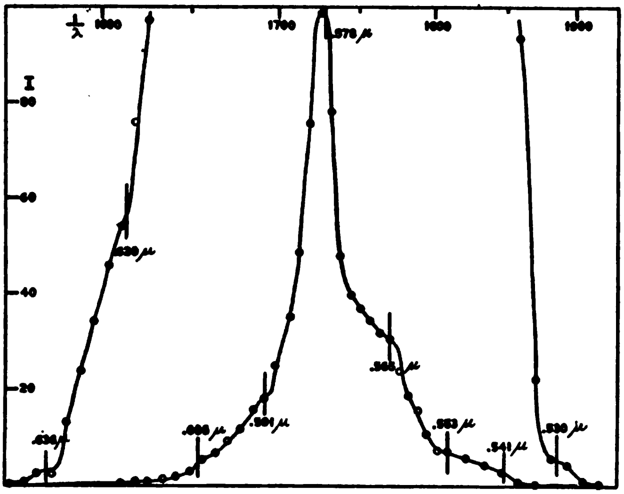

Fig. 2.

To make sure that the identity of band structure was not accidental another sulphide of Lenard and Klatt containing strontium and bismuth was excited by the hydrogen flame. This compound, known as No. 13, had been previously studied by Nichols.[1] The undispersed light was of a delicate blue color and so dim that a considerable study of the method of mounting was necessary before sufficient radiation could be obtained to render spectrophotometric measurements possible. The light from the 4-candle-power tungsten lamp was reduced greatly by interposing three or more sheets of white porcelain between it and the comparison slit. The spectrophotometric field was almost colorless and the time required for eye accommodation about thirty minutes. Since the range of intensities was unusually great, the curve shown in Fig. 3 is broken into five sections to facilitate the location of the partially resolved crests. With the exception of crest .494 μ and .480 μ the presence of partially hidden bands is conclusive. Table III. gives the wave-lengths of these bands together with those of the same specimen excited by the zinc spark.

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[1] E. L. Nichols, loc. cit.

There is again evidence of an identical underlying system of component bands under the two different modes of excitation. The distribution of intensities is again different and two new bands were discovered which fortunately fit the series intervals. The new bands are .5414 μ and .5080 μ.

As a digression from the field of comparison of the hydrogen flame excitation with that of the metallic spark an attempt was made to compare two Lenard and Klatt preparations which have the same solid solvents and active metals but a different flux under the same hydrogen flame. The formula given for the Lenard and Klatt sulphide No. 13 is $SrSBiNa_2SO_4$ and the series has been given in Table III. The formula given for the Lenard and Klatt Sulphide No. 9 is SrSBiK Phosphate;

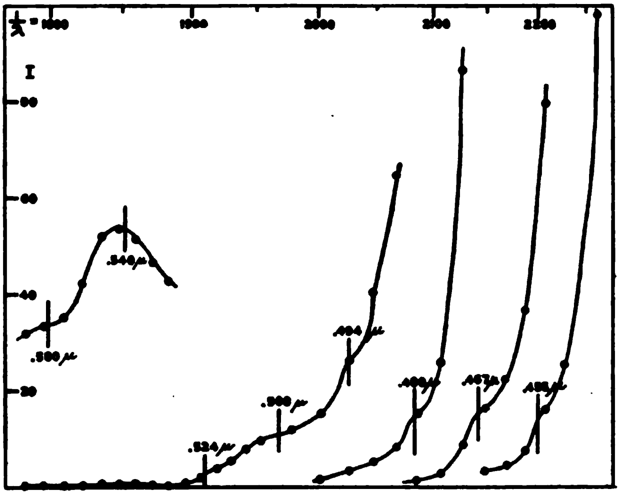

Fig. 3.

Table III.

Luminescence of Strontium Bismuth Sulphide, No. 13.

Hydrogen Flame, λ.	Zinc Spark, λ.	Frequency Unit, 1/λ × 10³.	Frequency Interval, Δ 1/λ × 10³.
.5565	.5562	1793	58
.5414	——	1851	58
.5235	.5238	1909	58
.5080	——	1967	58
.4938	.4938	2025	58
.4800	.4801	2083	58
.4668	.4670	2141	
.4546	.4547	2199	

that is, both contain strontium sulphide with bismuth for the element active in luminescence but the flux is a sodium sulphate in the first preparation and a potassium phosphate in the second preparation. The problem was to ascertain the effect of change of flux on the band series. It was thought that the flux might produce little or no effect on the location of the bands. The hydrogen flame was applied to preparation No. 9 and a beautiful dull blue glow appeared. The spectrophotometric curve, not presented in this paper, was obtained with some difficulty from six independent explorations and found to be somewhat similar to that obtained for specimen No. 13. Table IV. gives the wave-lengths and frequency units for comparison with Table III. The interval for No. 9 is 51 frequency units while that for No. 13 is 58 frequency units and it is observed that none of the bands occupy the same positions in both spectra. Band .5235 μ of No. 13 and .5230 μ of No. 9 are the most nearly together. The nature of the flux, then, determines in part the disposition of the bands, although the presence of the same base is the predominant factor.

TABLE IV.

Luminescence of Strontium Bismuth Sulphide with Potassium Phosphate Flux, No. 9.

λ	$1/\lambda \times 10^3$.	$\Delta(1/\lambda) \times 10^3$.
.5525	1810	51
.5373	1861	51
.5230	1912	51
.5094	1963	51
.4965	2014	51
.4843	2065	51
.4726	2116	

A somewhat parallel phenomenon is observed when the luminescence series of lime (Table I.) is studied in comparison with the series obtained from the Lenard and Klatt sulphide No. 3. (Table II.) Both contain calcium as a base; the active element in the lime is unknown while that in the Lenard and Klatt sulphide is bismuth together with fluorine. The frequency interval in both series found in the spectrum of the lime is of 42 units and that of the calcium bismuth preparation is 39 units. It is evident that the strontium or the calcium when used as the base of the compound largely determines the frequency interval.

PIEZO-ELECTRIC AND ALLIED PHENOMENA IN ROCHELLE SALT.[1]

By J. Valasek.

Synopsis.

Electric Hysteresis in Rochelle Salt.—On the doublet theory of dielectric action, the dielectric displacement D, electric intensity E, and the polarization P are analogous to B, H, and I in the case of magnetism. Rochelle salt shows an electric hysteresis in P analogous to the magnetic hysteresis in the case of iron. The loops obtained are displaced from the origin by an amount which gives a measure of the *permanent polarization in the natural state.* The moment per unit volume in the natural state is of the order of 50 e.s.u./cm.^{3} under ordinary conditions, this being about 13.3×10^{-21} e.s.u./cm.^{3} per molecule.

Piezo-electric Response of Rochelle Salt in an Electric Field.—Mechanical force and electric field are to a certain extent equivalent in producing a change in the piezo-electric moment. Measurements of the piezo-electric response in various applied fields therefore give curves of the same nature as the curves of dD/dE against E. Accordingly they show maxima displaced from the origin by the effect of permanent polarization.

RECENT work on the piezo-electric and allied properties of Rochelle salt has brought to light a number of peculiarities in the electrical reactions of this substance to applied forces and electric fields. A survey of these effects is given by Capt. H. O. Wood in a report dated March, 1919. In the first place, observations by Cady[2] show that the piezo-electric response, or quantity of electricity set free by a pressure is not proportional to the force, as is demanded by the commonly accepted theory of piezo-electricity. Initially the response is more nearly proportional to the square of the force, subsequently increasing less and less rapidly and finally tending toward more or less of a saturation value. Investigating the action of the crystal as the dielectric in a condenser Anderson[3] has found that instead of the usual relation $Q = CV$ between the charge and the applied electromotive force, there exists a more complicated relation much like that described above as holding between piezo-electric response and applied force. The capacity of the condenser and hence also the dielectric constant of Rochelle salt thus appears to be a function of the applied electromotive force. Moreover Anderson found that the ballistic galvanometer charge and discharge throws were unequal for fields of one sign but smaller and equal for fields of the opposite sign. In explanation he suggests that an electro-striction effect enters into and modifies the action of the crystal as a condenser. It appears however, that these results can be accounted for by a hysteresis in the crystal similar to ferromagnetic hysteresis, in a manner suggested by Professor W. F. G. Swann, as follows:

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[1] Presented at the meeting of the American Physical Society in Washington, April 23–24, 1920

[2] W. G. Cady, Report to National Research Council, May, 1918.

[3] J. A. Anderson, Reports to National Research Council, March and April, 1918.

On the doublet theory of dielectric action, the dielectric displacement D, electric intensity E, and the polarization P (moment per unit volume) are analogous to B, H, and I in the case of magnetism. They are connected by the relation $D = E + 4\pi P$, and the charge Q of a parallel plate condenser of plate-area S is equal to $Q = S(E + 4\pi P)/4\pi = (S/4\pi)D$. The alteration in the charge density on the plates of the condenser is thus a measure of the alteration in the dielectric displacement D. A hysteresis in P and consequently in D would show itself in measurements of the charge on the condenser. A permanent polarization, such as is demanded by Lord Kelvin's theory, would be made measurable by a displacement of the center of the loop with respect to the origin. When this original polarization is disturbed, a restoring force is brought into play and it appears from experiments on fatigue of Rochelle salt crystals that the restoration of the permanent polarization takes place quite gradually. Suppose then, that the original condition of the crystal is represented by some point on, or near, the upper right-hand part of the hysteresis loop. A field tending to decrease the natural polarization will bring into play a larger hysteresis effect, causing the charge and discharge throws to be unequal to an extent depending on the rapidity with which the restoring force acts. For throws in the opposite direction, however, there will be less of a hysteresis effect and the throws will be more nearly equal. When these conditions were imposed on iron in a magnetization experiment, curves like Anderson's were obtained.

In the apparatus set up to verify this hysteresis effect provisions were made for the regulation of temperature and humidity as well as applied force and electric field. Rochelle salt is often very sensitive to slight changes in the surrounding conditions. The variations in the charge on the electrodes of the crystal were measured by a Leeds and Northrup high-sensitivity ballistic galvanometer. The crystal and all directly connected parts were enclosed in a glass container as shown in Fig. 1. This allowed the apparatus to be evacuated or sealed off as desired in order to preserve a constant atmospheric condition. A solenoid with an iron plunger connected to a glass rod was used to apply pressure to the crystal, which was mounted on a hard rubber block directly below. The weight of the plunger and the effect of residual magnetism were compensated by a spring and screw attachment which could be adjusted from the outside by means of a ground glass tap. To provide for a uniform application of force, the end of the glass rod was furnished with a universal joint arrangement of hard rubber which came into contact with the upper end of the crystal plate. The crystal itself was supported on a hard rubber block which was attached by metal strips to an inverted cone. This cone was clamped by a sleeve to a projection on a glass tube as shown in the figure.

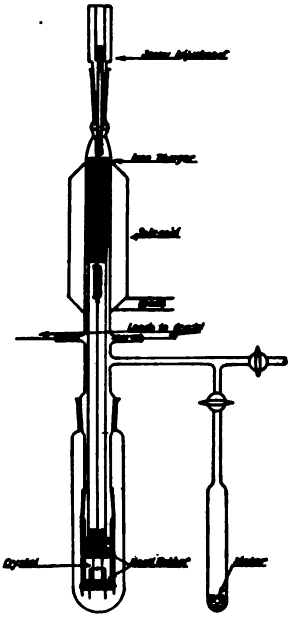

Fig. 1.

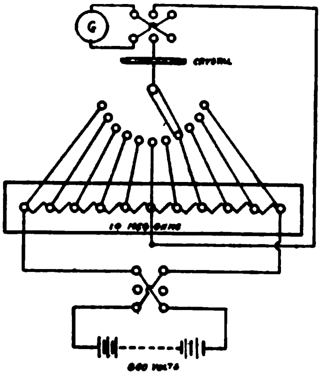

Fig. 2.

For regulating the humidity a bulb containing some water was connected to the glass container. During most of the experiments this bulb was kept packed in ice, giving a constant absolute humidity in the apparatus. The lower portion of the chamber containing the crystal was immersed well into a large Dewar jar which provided heat insulation and also served as means for the regulation of temperature by water baths and freezing mixtures. The temperatures were measured by copper-constantan thermocouples whose junctions were soldered to the tinfoil electrodes on the two faces of the crystal. Two of the thermocouple wires also served as connections to the ballistic galvanometer and to the high voltage potentiometer which regulated the applied electric field.

This consisted of a resistance of ten megohms subdivided into ten equal parts. The various subdivisions were connected to a commutating switch and to the crystal and galvanometer as shown in Fig. 2. The electromotive force usually employed was 800 volts, so that the potential applied was varied from − 400 to + 400 volts in steps of 80 volts each. The potentials used were measured by an electrostatic voltmeter.

The crystal plates were cut from a 100-gram crystal of Rochelle salt which was kindly furnished by Dr. Whitney, of the General Electric Company, to whom the writer wishes to express his appreciation. The plates were cut in the manner described by Cady, namely by a wet thread operating over two pulleys and kept moist by running through some water. The faces were dressed down and smoothened by a moist piece of ground glass. The orientation was such as to give the maximum response. The faces were accordingly perpendicular to the $\breve{a}$ axis of the crystal, the edges making angles of 45° with the $\bar{b}$ and $\dot{c}$ axes. A compression of the ends of the slab so cut results in a shear in the *bc* directions. Such a shear produces a maximum of piezo-electric polarization in the direction at right angles to its plane. Denoting the polarization by p_1 and the shear by Y_z we have according to Voigt[1] $p_1 = -\delta_{14}Y_z$. The charge q on a crystal of length l, breadth b, and thickness d, will be: $q = -\delta_{14}F(l/2d)$, where F is the total force on the end of the crystal. The piezo-electric modulus δ_{14} here concerned may have any value between practically zero and about 40×10^{11} e.s.u./dyne. It varies in a capricious manner in response to a number of conditions such as temperature, humidity, and previous history as to electrical and mechanical treatment. These latter fatigue effects, described by Cady,[2] are not permanent but persist often for many hours. The effect of humidity is to make the response greater. Preliminary observations seem to indicate that for changes in temperature the response is small above 35° or 40° C. and that it increases as the temperature is lowered, rising to a maximum at about 0° C. and then slowly decreasing again.

Investigating the behavior of the crystal as a condenser, the conclusions as to a hysteresis in D were verified, the method being that given by Ewing.[3] The hysteresis loops were unsymmetrical as was to be expected if the crystal had a natural polarization. The exact nature of the loops varied with the moisture and temperature conditions. Fig. 3 shows the results of observations taken at a temperature of 23° C. and a relative humidity of 30 per cent. Fig. 4 shows a loop obtained at 0° C., with the

[1] W. Voigt, Lehrbuch der Krystalphysik, p. 818.

[2] W. G. Cady, Report to National Research Council, May, 1918.

[3] Ewing, Magnetic Induction in Iron and other Metals, pp. 358–60.

surfaces of the crystal thoroughly dried. The plate had been kept under cover with P_2O_5 for 15 hours before these observations were made. After 24 more hours under these conditions there was, however, very little hysteresis and only a very small piezo-electric activity, apparently due to the excessive dryness. The moment per unit volume in the natural state varies but is of the order of 50 e.s.u./cm.[2] under ordinary conditions.

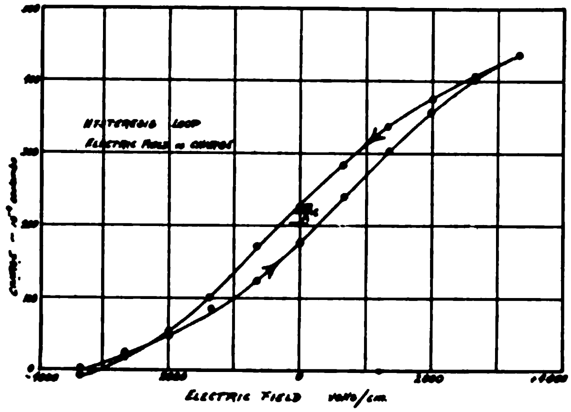

Fig. 3.

It was also observed that the response to pressure underwent a hysteresis with respect to changes in the temperature when the absolute humidity was kept constant by the method described. This hysteresis accordingly may be partly or wholly due to slow changes in the moisture content at the various temperatures concerned. Further work on this point is under way. That such variations in moisture held by the crystal

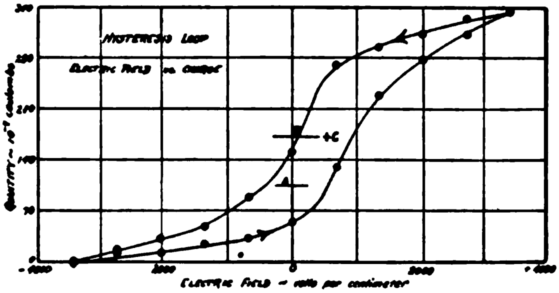

Fig. 4.

actually occur was shown by periodic weighing of two crystal plates which were kept respectively under moist and dry conditions. The changes in weight during the first four days amounted to 5 per cent.

Curves showing the relation between electric field and piezo-electric response were obtained for a number of values of the applied force (Fig. 5). All of the curves show a maximum for an applied steady field of a certain sign, while they fall off gradually in the opposite direction. These curves may be qualitatively explained on the basis of the hysteresis loops already discussed. It is quite obvious that the change in electric moment, which is the quantity measured, depends on crystalline strain.

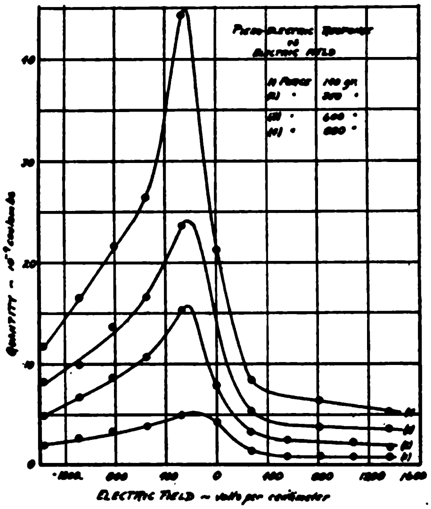

Fig. 5.

Since an electric field produces a piezo-electric stress, one may expect that a mechanical force applied on top of this field is equivalent to a certain extent to a change in applied field. If this equivalence were strictly so, the curves obtained would be just the derivatives of a branch of the hysteresis loop, namely dD/dE versus E. The similarity is apparent, and it is seen moreover, that the curves of Fig. 5 are analogous to the susceptibility curves in the case of magnetism.

Anderson[1] has obtained a set of curves somewhat similar in character by observations made on the charge and discharge of a crystal condenser when under pressure. These results may likewise be attributed to the equivalence of force and electric field in producing a change in the piezo-electric moment. Here, however, the curves are derived from the relation of D and F, which is very much like that of D and E considered above.

The center C of the loop is found by a consideration of symmetry, and may be assumed to represent the condition of no polarization. If the natural condition of polarization is assumed to lie half-way between the two branches of the loop at zero field, then the value of the permanent polarization is given by AB, Figs. 3 and 4. Values so obtained give 16 e.s.u./cm.^{2} and 85 e.s.u./cm.^{2} in the two cases respectively. Using 282.2 as the molecular weight of Rochelle salt, 1.76 as its density, and 1.66×10^{-24} grams as the mass of the Hydrogen atom, it follows that there are 3.76×10^{21} molecules per cubic centimeter. The corresponding molecular moments are then 4.26×10^{-21} e.s.u. and 22.6×10^{-21} e.s.u. respectively. If it is assumed that only one electron takes part in the creation of the natural polarization, the moment arm becomes equal to 8.9×10^{-12} cm. and 24.7×10^{-11} cm. in the two cases mentioned above, or at least one thousandth of the diameter of the hydrogen atom. The number of electrons in each molecule may, however, be taken as the sum of the atomic numbers of the elements involved ($NaKC_4H_4O_6 \cdot 6H_2O$) giving a value of 140. These may be equally affected in the natural condition of the crystal, which would indicate a separation of 6.4×10^{-14} cm. and 3.4×10^{-13} cm. respectively, which is the order of the diameter of the electron.

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[1] J. A. Anderson, Reports to the National Research Council, March and April, 1918.

The writer takes this opportunity to thank Professor W. F. G. Swann for his continued interest in this research.

University of Minnesota,
December 4, 1920.

DAMPED ELECTRIC OSCILLATIONS.

BY R. B. ABBOTT.

SYNOPSIS.

Historical.—Logarithmic and linear damping in electric circuits. A summary of previous work is given with reference to important original papers.

Damped Electric Oscillations; solutions of differential equations for four types of oscillations: (1), *simple harmonic;* (2), *logarithmically damped;* (3), *linearly damped;* (4), *combined logarithmically and linearly damped.*—The mathematical treatment is based upon the analogy between the dynamical and electrical differential equations of motion. A simplification results by this method in the solution of the fourth case.

IN 1842 Joseph Henry discovered the oscillatory character of the discharge of a condenser. In 1853 Lord Kelvin[1] published the law of logarithmic damping for oscillatory currents. However, it was not until 1900 that rectilinear damping was discovered. The discharge of a condenser through a spark gap was investigated by F. Richarz[2] and W. Ziegler in 1900; they viewed the image of a spark gap in a revolving mirror; Fig. 1 is from their published drawing. In 1904, J. Zenneck photographed the spark with a Braun tube oscillograph and obtained photographs of the turning points of the oscillations in the line of deflections. Fig. 2 shows the turning points plotted parallel to the time axis. In 1906 Heyweiller[4] attempted a mathematical solution for rectilinear damping.

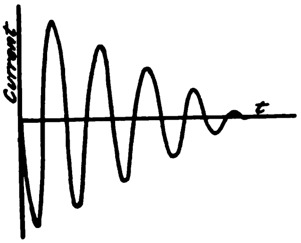

Fig. 1.

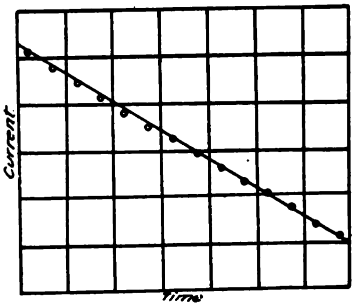

Fig. 2.

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[1] Phil. Mag., 1853, Ser. 4, Vol. 5, p. 393.

[2] Ann. d. Phys., 1, p. 468, 1900.

[3] Ann. d. Phys., 13, p. 822, 1904.

[4] Ann. d. Phys., 19, p. 646, 1906.

H. Barkhausen[1] published a solution in 1907, based upon the theory that the back E.M.F. across the spark gap, for the ideal case, is $e = a$ for the first half of an oscillation and $e = -a$ for the second half. The change in sign of the back E.M.F. at the end of each half cycle introduces a discontinuity, such that it becomes necessary to get a new solution for each half period; this is equivalent to an alternating change of origin. The E.M.F. circuital equation is stated as follows:

$$L\frac{di}{dt} + Ri = \frac{q}{C} \mp a,$$

where q is the charge on a condenser of capacity C, which discharges through an inductance L and resistance R (spark gap and circuit).

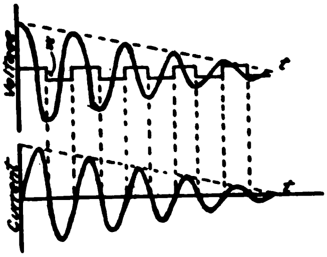

Fig. 3.

The equation reduces to

$$L\frac{d^2E}{dt^2} + R\frac{dE}{dt} = -\frac{1}{C}(E \pm a),$$

where

$$i = -\frac{dq}{dt} = -C\frac{dE}{dt}.$$

The two cases considered were: first, that where $R = R$; second, that where $R = 0$. The first case is the general one; the second case is that of simple harmonic motion with change of origin each half period.

The solution for the general case, given for the first half cycle is:

$$E - a = (E_0 - a)e^{-\frac{Rt}{2L}} \cos wt,$$

$$i = (E_0 - a)\sqrt{\frac{C}{L}}e^{-\frac{Rt}{2L}} \sin (wt + \theta),$$

$$W = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

The solution for the case where $R = 0$ is

$$E - a = (E_0 - a) \cos wt,$$

$$i = (E_0 - a)\sqrt{\frac{C}{L}} \sin wt,$$

$$W = \sqrt{\frac{1}{LC}}.$$

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[1] Phys. Zeit., 8, p. 624, 1907.

In 1908 and 1911 Roschansky[1] published his work with the Braun tube. Photographs of the spark voltage characteristic are given with various metals as electrodes. Drawings showing the characteristics in the two extreme cases are given here. Fig. 4 shows the potential wave form of the copper group, while Fig. 5 shows that of the magnesium group. The curves are said to hold for radio frequencies.

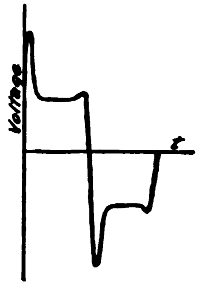

Fig. 4.

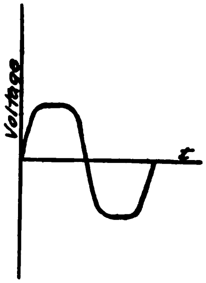

Fig. 5.

In 1911 E. L. Chaffie[2] published his work with the Braun tube, showing photographs of the current wave form at the spark. The photographs are remarkable and show the true linear decrescence of the current amplitude. Fig. 6 is a drawing of the spark gap damping curve which he gives. He says in regard to the form of the curve: "When the discharge is a pure arc, the separate oscillations are regular and practically sinusoidal, and the damping is linear. If the discharge is classed as a spark discharge, although the damping is linear, the separate oscillation loops are distorted as will appear presently."

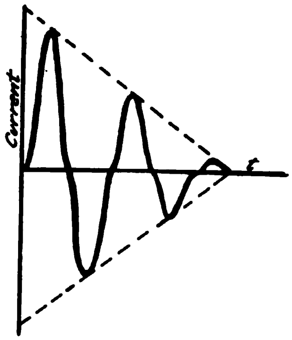

Fig. 6.

In 1915, A. Blondel[3] and Carbenay published their work with pendulums which were damped by constant bearing friction and by friction proportional to the velocity. They developed and applied a solution for pendulums from the same principles of harmonic motion which Barkhausen had used for electrical circuits. Their results show that the principle of origin change in harmonic motion is correctly applied to pendulums. (See Fig. 7.)

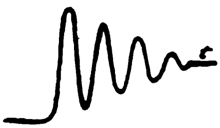

Fig. 7.

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[1] Phys. Zeit., 9, p. 627, 1908.
Ann. d. Phys., 36, p. 281, 1911.

[2] Proceedings American Academy of Arts and Sciences, 47, p. 267, 1911–1912.

[3] La Lumiere Electrique, p. 194, Nov., 1915; p. 216, Dec., 1915; p. 241, Dec., 1915.

We know that slow motion pressure friction is fairly constant and that it always opposes motion; it must, therefore, be represented by the plus and minus sign. The equation expressing the oscillatory motion of a pendulum is given as:

$$A\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + C\theta = \mp K.$$

If only the pressure friction is present $B = 0$ and

$$A\frac{d^2\theta}{dt^2} = -C(\theta \pm K').$$

This equation represents S.H.M. with alternate change of origin. These investigators used a magnetic pendulum with a small mirror and a beam of light reflected on to a photographic plate. Constant friction was produced by the bearing pressure, and a strong magnetic field at right angles to motion gave damping proportional to the speed. In this way accurate tracings of the motion were made which show logarithmic and linear damping and combinations of both. Fig. 6 is taken from their published photographs. Application of this law is made to meters with pivot friction and air damping.

The next notable contribution to the subject was presented by John Stone[1] in 1914. This paper deals with the subject from a new viewpoint. Stone's method of attacking the problem is to build up an expression for the current which will give a linear decrescence of amplitude without bringing in a discontinuity. (See Fig. 8.)

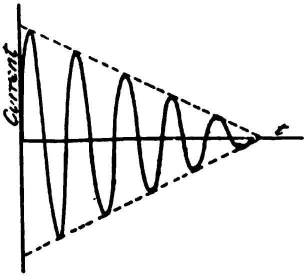

Fig. 8.

The expression for an amplitude of linear decrescence which he gives is $(A - Bt)$ and the current at any time t is given by $i = (A - Bt) \sin wt$. The assumptions are that A, B, and w are not functions of t, but constant quantities. The assumptions in regard to the spark resistance are that it is the instantaneous value of e/i and is made up of two components, one a dissipative resistance, the other a thermoelectric counter E.M.F.

The first paper deals with the case where the conductor resistance is zero, and all of the energy of the condenser charge is dissipated at the gap. The expression $i = (A - Bt) \sin wt$ for the current is applied to the circuital equation for the discharge of a condenser of capacity c through an inductance L and a spark gap of a so-called "resistance" r

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[1] Proceedings I. R. E., 2, p. 307, 1914.

(undetermined). The circuital equation is written

$$L\frac{di}{dt} + ir = \frac{q}{C}, \qquad \text{where} \qquad i = -\frac{dq}{dt}.$$

The result follows:

$$r = \frac{2BL}{A - Bt}, \qquad w = \sqrt{\frac{1}{LC}}$$

and spark gap E.M.F. = $2BL \sin wt$.

A complete discussion of the wave forms of the current and the gap E.M.F. is given in the paper.

In 1915[1] Stone published his second paper dealing with a solution for the general case. This paper is based upon the results of the previous one together with Kelvin's solution for logarithmic damping. The case is that of linear and logarithmic damping taking place at the same time. The conductor resistance R_1 is considered separate from the spark resistance r, in the circuital equation which follows:

$$L\frac{di}{dt} + R_1 i + ri = \frac{q}{C}, \qquad \text{where} \qquad i = -\frac{dq}{dt}.$$

If r (spark resistance) is zero the current is, from Kelvin's logarithmic theory,

$$i = I_1 e^{-R_1 t/2L} \sin wt,$$

$$w = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{2L}\right)^2}.$$

If both resistances are present in the circuit, Stone leaves the amplitude I undetermined and expresses the current in the same form as he did in the first paper, that is $i = I \sin wt$, where the value of I is to be determined by substitution in the above circuital equation. The value of r (spark resistance) is assumed to have the general value as determined in his former paper, that is inversely proportional to the current amplitude, or

$$r = \frac{d}{I} \text{ where } d \text{ is constant.}$$

By substitution of these conditions in the above circuital equation and by some rather complicated mathematical transformations Stone arrives at an expression for the current; it is given as follows:

$$i = Q_0 \frac{w_0^2}{w} e^{-a_1 t} \sin wt - \frac{2a_0 w}{a_1 \sqrt{a_1^2 + 4w^2}} \left\{ \sin\left(wt + \tan^{-1}\frac{a_1}{2w}\right) \right.$$
$$\left. - \frac{\sqrt{a_1^2 w^2 + (a_1^2 + 2w^2)^2}}{w\sqrt{a_1^2 + 4w^2}} e^{-a_1 t} \sin\left(wt + \tan^{-1}\frac{a_1 w}{a_1^2 + 2w^2}\right) \right\},$$

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[1] Proceedings I. R. E., 4, p. 463, 1916.

where

$$w_0 = \sqrt{\frac{1}{LC}}, \quad w = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{2L}\right)^2}, \quad a_1 = \frac{R_1}{2L} \quad \text{and} \quad a_0 = \frac{d}{2L}.$$

Oscillograms show that odd harmonics are present in the spark gap voltage, no doubt, due to the presence of ionization in the spark gap gases so that the current and voltage relation departs from Ohm's law. Quoting Stone: "To sum up, the proportionality of the instantaneous values of the gap voltage to the gap current, may notably depart from constancy, in any particular circuit, without sensibly modifying the frequency or mode of subsidance of the oscillations except perhaps near the end of a train of oscillations."

Outline of a Solution by Analogy.

The whole subject of " Damped Electric Oscillations " can be treated by the method of analogy. The parallelism between the dynamical and electrical equations for simple harmonic motion and logarithmically damped motion is well known. It is proposed to show: first, that a parallelism exists between the electrical and dynamical equations for linearly damped motion and for combined linearly and logarithmically damped motion; second, that the solutions in the last two cases named above are easily obtained, by analogy and that a greater simplification is obtained in the solution of the latter case than has hitherto been obtained.

The dynamical equations will be developed first and the electrical equations last, together with their solutions.

Oscillatory Motion and Dynamical Equations.

Oscillatory motion is most easily described in terms of the projection of a radius vector r, rotating with constant angular velocity w, about a center c. Let the radius vector make an angle θ with the X axis, passing through the center c such that $\theta = wt$, where t is the time from the instant when the radius vector coincides with the positive X axis. The motion may be described as taking place in either X or Y, thus: the displacement $x = r \cos wt$ and $y = r \sin wt$. (See Fig. 9.)

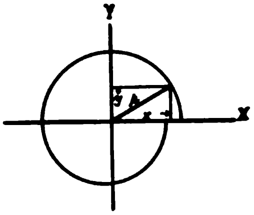

Fig. 9.

The point P at the end of r traces out some curve such as a circle, ellipse or some form of spiral. If there is no dissipation of energy during periodic motion, the auxiliary curves will be of the spiral family: the auxiliary curve for damping proportional to the speed being the logarithmic spiral and for constant damping, the rectilinear (Archimedes') spiral.

The differential equations expressing the acceleration in free periodic motion such as the cases above mentioned, are of the second order with coefficients which are either constants or expressible as functions of the time. If p and q are such coefficients, the general equation is written as follows:

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0.$$

By reference to the auxiliary curves, the values of p and q can be determined for all classes of periodic motion, and their differential equations can be written. The types of motion which have been mentioned will be considered in the following paragraphs.

Circular Harmonic Motion (Case 1).

In this case r is constant and the auxiliary curve is a circle. The well-known differential equation is

$$\frac{d^2y}{dt^2} + w^2y = 0. \tag{1}$$

Logarithmic Damping (Case 2).

In this case r decreases at a rate proportional to itself. The rate is expressed as follows:

$$\frac{dr}{dt} = -k_1r.$$

By integration

$$\log\left(\frac{r}{r_0}\right) = k_1t.$$

Solving for the radius vector, $r = r_0e^{-k_1t}$; therefore the auxiliary curve is a logarithmic spiral. The differential equation is

$$\frac{d^2y}{dt^2} + 2k_1\frac{dy}{dt} + (k_1^2 + w^2)y = 0. \tag{2}$$

Rectilinear Damping (Case 3).

The radius vector r decreases at a constant rate $-k$.

$$\frac{dr}{dt} = -k.$$

By integration $r - r_0 = -kt$. The radius vector $r = r_0 - kt$, therefore, the auxiliary curve is a rectilinear spiral.

Writing $y = (r_0 - kt)\sin wt$, the differential equation is found to be

$$\frac{d^2y}{dt^2} + \frac{2k}{r_0 - kt}\frac{dy}{dt} + \left[\frac{2k^2}{(r_0 - kt)^2} + w^2\right]y = 0. \tag{3}$$

Logarithmic and Rectilinear Damping Combined (Case 4).

The radius vector is proportional to the radius vector of each case separately and is proportional, therefore, to their product when the cases are combined; it is equal, therefore, to a constant times their product. The constant may be taken as unity in our case. The radius vector is

$$r = (r_0 - kt)e^{-k_1 t}.$$

Writing $y = (r_0 - kt)e^{-k_1 t} \sin wt$, the differential equation is found to be

$$(4) \quad \frac{d^2y}{dt^2} + 2\left[k_1 + \frac{k}{r_0 - k_1 t}\right]\frac{dy}{dt}$$
$$+ \left[2\frac{k^2}{(r_0 - kt)^2} + 2k_1\left(\frac{k}{r_0 - kt}\right) + k_1^2 + w^2\right]y = 0.$$

Analogous Electrical Equations.

When the current follows the simple harmonic law for the discharge of a condenser of capacity C through an inductance L, the differential equation is

$$(5) \qquad \frac{d^2i}{dt^2} + \left(\frac{1}{LC}\right)i = 0.$$

The term $1/LC$ corresponds to W^2 in the parallel dynamical equation (1).

Likewise, when the current follows the law of logarithmic damping, due to a resistance R in the discharge circuit, the differential equation is

$$(6) \qquad \frac{d^2i}{dt^2} + 2\left(\frac{R}{2L}\right)\frac{di}{dt} + \left(\frac{1}{LC}\right)i = 0.$$

The term $R/2L$ corresponds to K_1 and the term $1/LC$ corresponds to $k_1^2 + w^2$ in the parallel dynamical equation (2).

Similarly, the differential equation for the current, when it follows the law of linear damping, can be shown to be analogous to the dynamical equation (3), provided, that the usual manner of expressing the E.M.F., circuital equation

$$L\frac{di}{dt} + ir = \frac{q}{C}$$

holds in this case. Here, q is the quantity of electricity in the condenser of capacity C discharged through an inductance L and a spark gap, r being an undetermined function such that ir expresses the back E.M.F. of the spark gap.

It is only necessary to differentiate the above equation to get the analogous electrical equation. The equation is

$$\frac{d^2i}{dt^2}+\frac{r}{L}\frac{di}{dt}+\left[\frac{1}{L}\frac{dr}{dt}+\frac{1}{LC}\right]i=0.$$

Thus by analogy, one can choose r such a function of the time as to make the solution of this equation correspond to linear damping. For, let r/L correspond to $2k/(r_0 - kt)$ in (3); its value will be $2k/(I_0 - kt)$ in this case, where I_0 is the initial current amplitude and K is the rate at which the current amplitude decreases; also letting $1/LC$ correspond to W^2, the analogy is completed because

$$\frac{1}{L}\frac{dr}{dt}=\frac{2k^2}{(I_0=kt)^2},$$

which corresponds to $2k^2/(r_0 - kt)^2$ in (3).

The complete analogous electrical equation, therefore, writing

$$\frac{r}{2L}=\frac{k}{I_0-kt}$$

and

$$2\frac{k^2}{(I_0-kt)^2}=2\left(\frac{r}{2L}\right)^2$$

is

$$\frac{d^2i}{dt^2}+2\left(\frac{r}{2L}\right)\frac{di}{dt}+\left[2\left(\frac{r}{2L}\right)^2+\frac{1}{LC}\right]i=0. \tag{7}$$

The solution of this equation is

$$i=(I_0-kt)\sin\left(\sqrt{\frac{1}{LC}}\right)t.$$

When the current undergoes both linear and logarithmic damping simultaneously, its differential equation can be shown in the same manner to be analogous to the dynamical equation (4), provided that the E.M.F. circuital equation be written for the mutual case as follows:

$$L\frac{di}{dt}+Ri+ri+E=\frac{q}{C},$$

where r has the same value as determined in the preceding case and E, a mutual term, is an undetermined function. Thus, by differentiation the resulting equation becomes analogous to the dynamical equation (4), if one writes

$$\frac{dE}{dt}=Zi.$$

The equation thus derived is

$$\frac{d^2i}{dt^2}+\left(\frac{R}{L}+\frac{r}{L}\right)\frac{di}{dt}+\left[\frac{1}{L}\frac{dr}{dt}+\frac{Z}{L}+\frac{1}{LC}\right]i=0.$$

By analogy one can now choose Z/L such a function of the time as to make the solution of the equation correspond to combined logarithmic and linear damping. Thus, let

$$\frac{Z}{L} = 2\left(\frac{R}{2L}\right)\frac{k}{I_0 - kt}$$

to correspond to $2k_1(k/(r_0 - kt))$ in (4). Writing

$$\frac{r}{2L} = \frac{k}{I_0 - kt}$$

as in the previous case,

$$2\left(\frac{R}{2L}\right)\frac{k}{I_0 - kt} = 2\left(\frac{R}{2L}\right)\left(\frac{r}{2L}\right)$$

and the analogous electrical equation is

$$(8)\quad \frac{d^2i}{dt^2} + 2\left(\frac{R}{2L} + \frac{r}{2L}\right)\frac{di}{dt} + \left[2\left(\frac{r}{2L}\right)^2 + 2\left(\frac{R}{2L}\right)\left(\frac{r}{2L}\right) + \left(\frac{1}{LC}\right)\right]i = 0.$$

The solution of this equation, therefore, is

$$i = (I_0 - kt)e^{-\frac{Rt}{2L}}\sin\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right)t,$$

which is simpler in form than any hitherto derived for the combination case. Equation (8) reduces to any one of the three previous cases as follows:

first, when R and r are zero it reduces to (5), namely, simple harmonic oscillations;

second, when r is zero it reduces to (6), namely, logarithmically damped oscillations;

third, when R is zero it reduces to (7), namely, linearly damped oscillations;

fourth, as it stands it is a combination of the three cases.

It is of importance to determine the resulting E.M.F. expression for the spark. This is seen to be expressed as $ir + E$ from the E.M.F. equation already considered. It is now necessary to determine E. The rate of change of E has already been found, by analogy, to be $dE/dt = Zi$, where Z was found to be

$$Z = 2LK\frac{k}{I_0 - kt}.$$

The value of E can be found by integration as follows:

$$E = \int Zidt = 2k_1kL\int e^{-k_1t} \sin wtdt,$$

$$E = -\frac{2k_1kL}{w^2 + k_1^2} e^{-k_1t} (k_1 \sin wt + w \cos wt).$$

The total E.M.F. across the gap, expressed as $ri + E$, is now determined in terms of the circular functions and is,

$$ir + E = 2kL \frac{w}{\sqrt{w^2 + k_1^2}} e^{-k_1t} \sin\left(wt - \tan^{-1}\frac{k_1}{w}\right).$$

This expression gives the gap E.M.F. as a sinusoid with a logarithmically decaying amplitude. Moderately large amplitudes in the odd harmonies of the gap E.M.F. need not necessarily affect the current curve to an appreciable extent, so that, from a practical standpoint these results, based upon the above assumptions, do have considerable value so far as the current relations are concerned.

This, however, does not exactly fit the facts as the analyses of both current and gap E.M.F. wave forms which have been made to determine the harmonics present, show. These show that the odd harmonics are present in both current and gap E.M.F., especially in the latter, and produce distortions in the wave forms. Before a complete solution is possible for the circuits, parts of which are made up of ionized gas, one must use some function expressing the departure from Ohm's Law for such circuits.

WASHINGTON STATE COLLEGE,
December, 1920.

SOME PECULIARITIES OF POLARIZATION AND ENERGY DISTRIBUTION BY SPECULUM GRATINGS.

By L. R. Ingersoll.

Synopsis.

Intensity Minima in Spectra of Speculum Gratings.—A series of intensity minima, resembling absorption lines, are to be found in the continuous spectrum of a speculum grating at such wave-lengths as are being tangentially diffracted in other orders. They are limited to the azimuth of polarization with electric vector perpendicular to the rulings and appear best in such gratings as show strong spectra in several orders. While they appear in the visible spectrum for a deeply ruled grating, in general they may be observed only for longer wave-lengths—say between .8 μ and 2 μ. Although simple energy considerations furnish a fairly satisfactory explanation of the phenomena, it is highly desirable that the Rayleigh-Voigt grating theory be modified and extended to fit these and similar facts or else that a new theory be developed.

Practical Importance to the Spectroscopist.—It is highly desirable that work on the Zeeman effect or other polarization phenomena with speculum gratings be arranged—as may be easily done—so as to avoid this effect. The same may be said of infra-red absorption measurements.

It is a well-known fact that grating spectra show certain peculiarities not exhibited by prismatic spectra, such as ghosts, abnormalities of polarization and some others. To these there may now be added—for speculum gratings—another peculiarity in the form of a series of marked minima of intensity in the continuous spectrum, superficially resembling absorption lines. These are confined to the azimuth of polarization in which the vibration (electric vector) is perpendicular to the rulings, and are as a rule much more prominent in the early infra-red region—wave-length from .8 μ to 2 μ—than in the visible spectrum. This probably accounts for their not having been generally observed before.[1] Another reason is that the effect is not likely to show very well in a grating of the sort which would ordinarily be chosen for spectroscopic work, *i.e.*, one having, among other properties, that of concentrating its energy in one spectrum, while it is marked in gratings which give bright second and higher orders as well as the first.

The phenomenon was not wholly unlooked for when the problem was taken up, but was not just what was expected. In a previous paper[1] the writer has described experiments which show that the directly reflected light or central image from a speculum grating is markedly deficient in light of such wave-length as is, in the first or higher order spectrum, emerging tangentially from the grating. The effect is confined to the azimuth of polarization already mentioned, viz., with electric vector perpendicular to the rulings, and is in general accord with the Rayleigh-Voigt grating theory.[2] This theory goes on to predict that, for this same azimuth of polarization one should find in the first order spectrum, say, at such wave-lengths as are being tangentially diffracted in the second or higher order spectra, sharp *maxima* of energy. Wood[3] has found such an effect in the case of a special grating ruled on silver. The phenomena noted here are, with one exception, just the reverse of this, viz., *minima* instead of maxima.

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[1] The dark bands observed by Wood (Phil. Mag. (6), 4, 396, 1902, and 23, 310, 1912) for an abnormal speculum grating and particularly for gratings ruled on silver, while unquestionably closely related to these minima cannot, from the description, be certainly identified with them.

A rather simple spectro-bolometric apparatus served very well in the measurement of this effect. Light from a special tungsten strip filament lamp[4] was focused by a concave mirror (of approximately 30 cm. focal length and 8 cm. aperture) on a slit at the principal focus of another mirror, from which the light then fell in a parallel beam on the grating. The diffracted light passed through a large (5 cm. aperture) Wollaston double image prism of calcite, which separated the beam into the two azimuths of polarization, perpendicular and parallel to the rulings, respectively. These spectra were focused by another mirror on the two strips of a special bolometer so that in this way the energy in each state of polarization could be investigated independently. A sensitive 8-coil Thomson galvanometer at a distance of approximately 3 meters served in connection with the bolometer to measure the energy at any given wave-length, which energy is expressed, in this paper, in terms of the galvanometer deflections. To avoid the effect of overlapping spectra a 25° glass prism was placed between the lamp and first concave mirror. The latter was adjustable so that the slit could be illuminated by a block of radiation, perhaps .2 μ wide in wave-length, taken from any part of the resulting (impure) spectrum.

The illuminating system of lamp, prism, two mirrors, and slit, was rigidly mounted and arranged so that it could be rotated with the grating about the axis of the spectrometer. In studying the energy distribution the incidence was fixed at any desired angle, and the whole system then rotated so that the angle of diffraction of the radiation falling on the bolometer could have, in turn, any value up to 90°.

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[1] Astrophys. Jour., 51, 129, 1920.

[2] Proc. Roy. Soc. (A), 79, 399, 1907. Gött. Nachr. Math. Phys. Kl. (1911), p. 40.

[3] Phil. Mag. (6), 23, 310, 1912.

[4] These lamps are now obtainable from the Nela Laboratory. They are invaluable in spectroscopic work where an intense constant source of this nature is desired.

Gratings.—Through the kindness of Professor Michelson and Professor Gale four gratings were specially ruled by Mr. Fred Pearson of Ryerson Laboratory and furnished for this investigation. They all had ruled surfaces approximately five centimeters square, and were of the following spacings: No. 1, 8709 lines to the inch—$S = 2.916\ \mu$; No. 2, 7620 lines per inch—$S = 3.333\ \mu$; No. 3, 10160 lines per inch—$S = 2.500\ \mu$; No. 4, same spacing but ruled with at least ten times the pressure, giving a comparatively very deep cut. No. 5 was a Hopkins grating of 15,000 lines per inch ($S = 1.693\ \mu$), with a quite extraordinary concentration of energy in one spectrum.

The results are shown graphically in Figs. 1 to 4. These are in reality energy curves plotted with angle of diffraction as abscissa instead of wave-length, as in the ordinary case. No correction of any sort has been applied. The plus and minus signs in connection with the angle of incidence or diffraction are used to indicate the two sides of the grating normal.

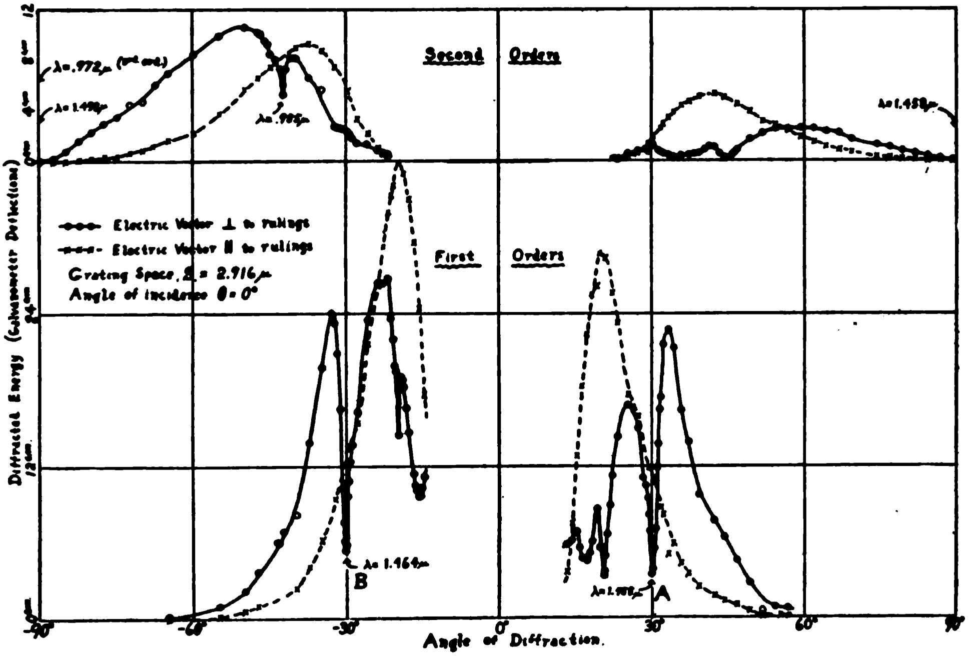

Fig. 1.

Energy curves (of a tungsten filament) for both azimuths of polarization, as determined with a speculum grating (No. 1) of 8,709 lines per inch. Light incident normally.

The minima at or very near angle 30° in Fig. 1 are very marked. Now since sin $30° = 1/2$, the same wave-length occurs here in the first order as is emerging tangentially in the second: how closely these wave-lengths check is indicated. There are also secondary minima in the neighborhood of 20° and 15° for wave-lengths which are tangential in the third and fourth orders respectively, though the agreement is not so good in this case.

Referring now to the upper part of the figure, showing the second order, it will be remarked that there is a considerable amount of energy in this order and it is particularly to be noted that as the angle of emergence approaches 90° the proportion of radiation of perpendicular polarization to parallel becomes larger and larger. This is in accordance with the Rayleigh theory and is a fact which can be readily observed with almost any speculum grating by examining the tangentially emergent light with a nicol prism.

A simple and fairly satisfactory explanation of these minima may be based on this last mentioned fact, viz., the relatively large amount of perpendicularly polarized radiation coming obliquely from the grating (although to account for the excess of energy of this state of polarization we must, for the present, rely entirely on the Rayleigh theory; no purely physical reason can, as yet, be adduced). For this excess must be made up by a deficiency at this same wave-length region in other spectra and since the dispersion at nearly tangential emergence is very great—the change in the sine of the angle between 80° and 90° is only one eighth that between 40° and 50°—the obliquely diffracted radiation comes from a very limited spectral region. This accounts for the (usually) narrow minima found at this wave-length in other orders.[1]

A rather critical test of this reasoning was applied by tilting the grating slightly so that the incidence was about 2 1/2° instead of zero. The wave-length tangential in the second order on one side would then be somewhat shorter and on the other, somewhat longer than before. The minimum in the first order should then appear split into two, neither as pronounced as the original one, and the two might furthermore be expected to be of different intensities, corresponding to the difference between the two second order spectra. These predictions are all seen to be amply verified in Fig. 2.

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[1] It might be thought that this point could be made more evident by plotting energy in terms of wave-length rather than angle of diffraction, in which case there should appear a sharp maximum at tangential emergence which would correspond exactly to the minimum in some other order at the same wave-length. This does not show satisfactorily, however, because there is so much loss of energy at or near tangential emergence. That is to say, at 70° light may come from all parts of a furrow, whereas at 89° the shielding of one furrow by another is such that the only light emerging comes from the tips.

At other angles of incidence and for other gratings the minima may have markedly different shapes, as Fig. 3 shows. A casual study of these curves will show the correspondence between the wave-lengths of the minima and the radiation tangentially emergent in some other order. While in most cases this order is higher than the one under investigation, a number of instances have been found in which the minimum in the first order on one side corresponds to tangential emergence in the first order on the other. It will be noted that a characteristic of all the minima is a sharp and rapid rise of intensity on the side of longer wave-lengths (*i.e.*, when this wave-length in the second order, say, has passed entirely from the grating) while the rise on the short wave-length side may be very much delayed. In some cases indeed it does not occur at all and the minimum degenerates into a sharp change in intensity.

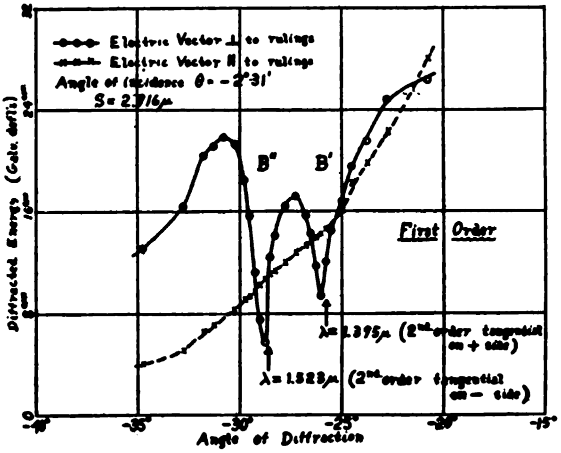

Fig. 2.

Detailed study of minimum *B* (Fig. 1) for an incidence of −2°31′.

A rather remarkable case of this is shown in Fig. 4. At first sight it would be thought that this is conclusive evidence for the Rayleigh theory, but a little closer investigation causes one to doubt this and indeed to consider it as merely a special case of the preceding phenomena. It will be noted that the detailed study of this small maximum, for the three angles of incidence, shows that the *sharp side*, and not the peak of the minimum, corresponds exactly to the above theory. Why the energy should fall off rapidly on the right-hand side as it does, giving to the whole the appearance of a fairly sharp maximum, cannot be explained save by remarking that it is undoubtedly due to the groove form, which doubtless accounts in turn for the extraordinary concentrating power of this grating.[1]

Visual Observations.—As already remarked, most gratings show this effect very much better in the infra-red than in the visible spectrum. An exception, however, is grating No. 4 which has very deep furrows and which gives visible dark bands in excellent agreement with the above reasoning. As a matter of fact the agreement is very much better than in the case of the infra-red minima for this same grating. The dark bands observed by Wood, as already mentioned, are doubtless also manifestations of this same general phenomenon and may indeed be identical with the minima found here, although his data does not afford a quantitative check in this case.

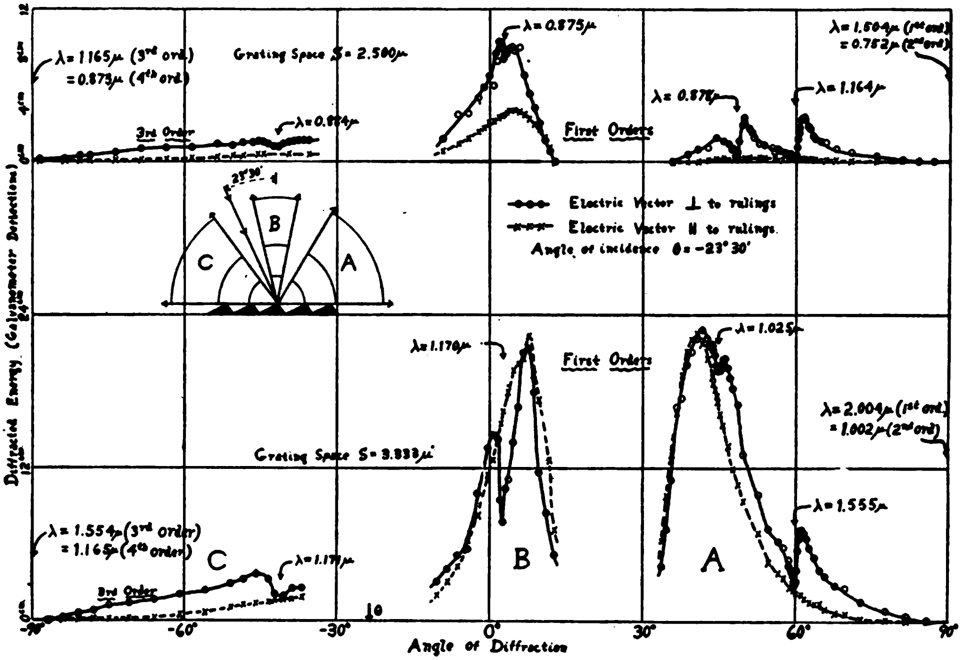

Fig. 3.

Energy curves (of tungsten filament) for both azimuths of polarization, as determined with speculum gratings (Nos. 2 and 3) of 7,620 and 10,160 lines per inch. Angle of incidence, −23°30'.

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[1] It is perhaps a fair question to ask if the phenomena observed by Wood (Phil. Mag. (6), 23, 315, 1912) were not identical with this, a fact which, of course, could only be cleared up by careful investigation, if possible, as to whether the *center* or *edge* of the sharp maximum he observed checks with the theory.

Discrepancies.—While the foregoing explanation of the phenomena as described seems fairly satisfactory, at least to the writer, there are certain other effects observed in this connection which must at present remain totally inexplicable. It will be noted in the curves that in some cases the wave-length at the point of the minimum is slightly longer than that tangentially emergent in the other order. While in the figures shown the discrepancy does not exceed 2 per cent., there are cases in which this is even more.

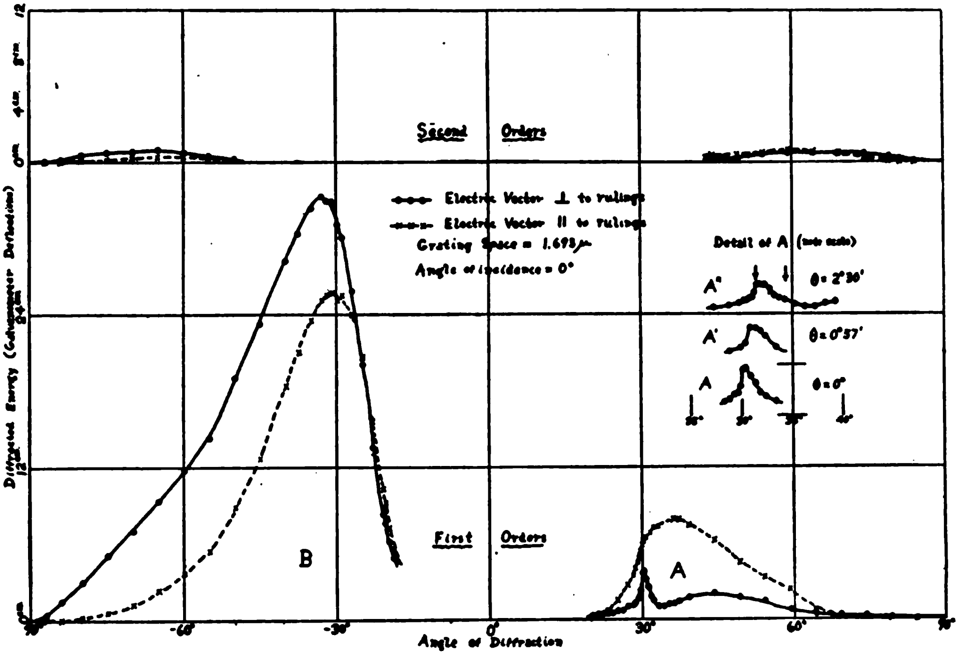

Fig. 4.

Energy curves for a grating (No. 5) of 15,000 lines per inch. Light incident normally.

Considerable study and discussion[1] of this point has failed to yield any very satisfactory explanation, but the outstanding phenomena will be briefly described in order to aid anyone in fitting an explanation to the facts. They are: (*a*) The discrepancies are, when they exist, always in the same sense, *e.g.*, the wave-length at the minimum is always *longer* than that tangentially emerging in the other order. If it were shorter it could be easily explained. (*b*) The disagreement is greater, as a rule, when the light is incident normally or nearly so. In some cases a grating which shows a discrepancy of 2 or 3 per cent., at normal incidence will give results checking to a small fraction of a per cent., when the incidence is increased to 30° or so. (*c*) The discrepancy is most prominent in deeply furrowed gratings. (*d*) In the one grating No. 4 which shows both infra-red and visual minima the discrepancy is as already remarked, much greater in the infra-red than in visual measurements,—the agreement indeed, in the latter case, frequently reaching one part in a thousand.

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[1] Professor Sparrow suggests in explanation of this, an interaction between the light vibrations and the conduction currents they occasion in the furrowed metal on reflection. Professor Wood believes that this is another point of evidence to be added to his results (Phil. Mag., July, 1919) on reflection from films of granular or furrowed surfaces which seem to demand for explanation some new principle in optics. It is probable that the best method to clear this up is to use either very long infra-red wave-lengths or short electromagnetic waves with gratings which can be ruled on a correspondingly large scale.

It may be remarked on this latter point that the phenomena in the infra-red region under investigation doubtless depends on the furrow as a whole, whereas in the visual results probably only corresponding facets of the furrows, or possibly the thin edges, come into account. Wood found that the slightest rubbing of the gratings ruled on silver fundamentally altered the polarization effects he observed visually, while the writer in investigating for longer wave-lengths the phenomena for the central image found that they were changed in no way, even when the grating was polished with Vienna lime or rouge. We may accordingly then believe that when the furrow exceeds a certain depth or degree of roughness, as in grating No. 4, it can no longer act as a whole even on the longer wave-lengths and hence the results in this region are somewhat confused. The visual results, however, if concerned with only some small but similar part of each furrow may be quite distinct and regular and this seems to be the case. It may be added that for this deeply cut grating some trace of these polarization abnormalities are observable in the other azimuth of polarization.

Conclusion.—The minima of intensity for polarization (electric vector) perpendicular to the rulings which occur either in the central image or in any spectrum at such wave-lengths as are tangentially emergent in some other spectrum are probably characteristic of all speculum gratings. They are, however, prominent only in gratings cut with a poor diamond point so that considerable light is diffracted in all of the orders. While simple conservation-of-energy reasoning will account for the effects in a general way, a new grating theory, or substantial modification of the Rayleigh-Voigt theory, is undoubtedly needed to explain comprehensively these and similar phenomena. From a practical standpoint it is eminently desirable in working on polarization phenomena, *e.g.*, the Zeeman effect, with a speculum grating, to avoid the region of such wave-lengths as might show the effect described here. In studying infra-red absorption spectra it would also be well to have this point in mind, for while the minima would be only half as prominent with unpolarized light they might still give rise to false absorption observations. A slight change in the angle of the grating would, of course, at once show such measurements as spurious.

In conclusion the writer wishes to express his indebtedness to the Rumford Fund for assistance in conducting this investigation.

Physical Laboratory,
University of Wisconsin,
January, 5 1921.

CONDUCTIVITY OF INSULATING MATERIALS NEAR THE BREAKDOWN VOLTAGE.[1]

BY J. E. SHRADER

SYNOPSIS

Conductivity of Insulating Materials up to Breakdown Voltages.—(*a*) In the *method of measurement* used, the recording instruments were protected from injury in case of puncture, by placing an insulated hot cathode rectifier in series with the sample being tested. This enabled the current through the sample to be limited and also controlled by controlling the temperature of the cathode of the rectifier. When with a given applied voltage the current did not reach a constant maximum value upon raising the temperature of the cathode, the voltage had reached the breakdown point. (*b*) *These classes of insulating materials* were found whose conductivity, respectively, (1) obeys Ohm's law throughout; (2) obeys Ohm's law up to near breakdown and then increases at an accelerated rate; and (3) increases over the whole range of voltage at an accelerated rate to breakdown. In general, material in class 3 have the greatest dielectric strength; these in class 1, the least. Results are given only for these materials, untreated cement paper, paraffined fish paper, and black treated cloth, which were selected as representative, respectively, of the three classes.

MOST of the data on the conductivity of insulating materials have been obtained from observations of tests made at comparatively low potential stresses. It is quite well known that the behavior of all insulating materials under varying voltages is not the same. Some obey Ohm's Law while others increase in conductivity with voltage at a greater rate than in accordance with Ohm's Law.

Any data to be of service should be taken under the voltage conditions to which the materials are subjected. If material is to be used not merely for its resistance to the passage of a current but to stand high electrostatic stress and prevent breakdown then this material should be studied under stresses up to and in the neighborhood of actual breakdown.

To test insulating materials over a range of voltage up to the breakdown without danger to the current measuring instruments the following scheme shown by the diagram, Fig. 1, was devised. The D.C. voltage of 12,000 volts is obtained from the rectification of the A.C. voltage from the secondaries of two transformers *AB* and *BC*, by the hot cathode rectifier *E*. The primaries of the transformers are connected in parallel to an auto transformer by which the voltage is varied. The secondaries are connected in series and the terminals *A* and *C* are connected to the anodes of the two-anode rectifier *E*. The middle point *B* is connected through a second rectifier *F*, and the sample *H* to the cathode of the rectifier. The cathode of this rectifier is heated by the stepdown transformer *I* connected to a 110-A.C. source. A capacity *G* consisting of ten One-M.F. condensers connected in series is connected across the sample to maintain a constant voltage. The current through the sample is read by the galvanometer *J* which is provided with an Ayrton Shunt. The voltage across the sample is read by two electrostatic voltmeters *K* and *L*, one reading to 3,500 and the other to 15,000 volts.

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[1] Paper read before the Physical Society at Cleveland, Nov. 25–27, 1920.

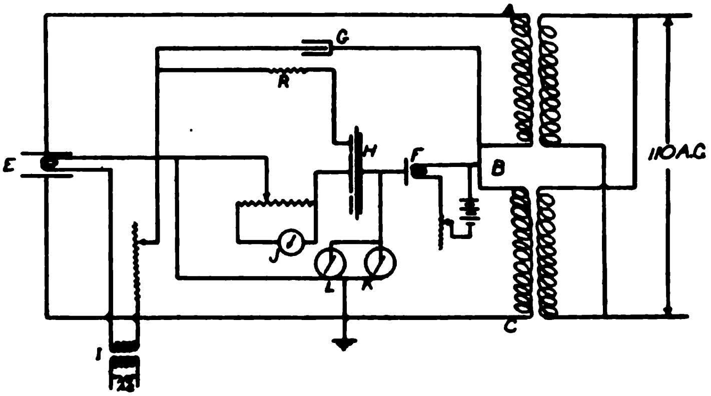

Fig. 1.

The new feature in this arrangement is the insertion of the hot cathode rectifier *F* between the high tension electrode of the sample and the middle point *B* of the transformers. This rectifier with its battery and regulating resistance is well insulated. This rectifier or valve tube as it is used in this case serves as a protective device for the galvanometer. With the transformers excited no current can pass through the sample until the cathode of the rectifier *F* is heated. The value of the current which can pass through the tube is dependent upon the temperature of the filament, which is controlled by the variable resistance. Then for any given testing voltage the temperature of the cathode is gradually increased, observing at the same time the current indicated by the galvanometer. When there is no further increase of the current upon further increasing the temperature of the cathode of the rectifier it is known that the current is limited by the insulation and not by the valve. By this method one has complete control at all times of the value of the current which may flow through the instruments. If the current does not reach a steady value upon opening the valve it is a good indication that the sample is near the point of breakdown. Even in the case of actual breakdown, with the valve in the circuit, the current is limited to such a value that damage to the instruments is prevented.

For a study of the conductivity of insulating materials up to the point of breakdown three samples from the same sheet of material were prepared. As might be expected by one familiar with insulating materials, there is considerable variation in conductivity among these samples and this is shown by the curves which follow. The tests were made with a guard ring to prevent surface leakage to the measuring instruments. Close contact of the electrodes with the sample was obtained by the use of powdered graphite. The lower electrode was a round shallow tray filled with powdered graphite and smoothed off with a straight edge to a depth of about two millimeters. Upon this was placed the sample to be tested. Two concentric cylindrical glass rings were fastened with shellac upon the upper surface of the sheet material. The space between the rings occupied by the guard ring and the surface inside the inner ring were covered with the powdered graphite upon which were firmly pressed the guard ring and the central electrode respectively. The resistance of the graphite layer itself was negligible being only a few ohms. The contact, especially for an unpolished surface, was better than a contact of mercury since the powder could be rubbed into the small surface irregularities. That this was true was actually proven by trial.

For the materials tested the current came to practically a steady value in two to five minutes after the application of voltage. Hence the values used in this work were obtained after a steady reading of the galvanometer was observed.

Tests were made on a variety of commercial insulating materials, observations being made from comparatively low voltage to breakdown voltages for the particular sample. From the results obtained all these materials may be divided into three classes:

1. Materials whose conductivity varies directly with voltage according to Ohm's Law.

2. Materials whose conductivity varies according to Ohm's Law up to the nieghborhood of breakdown after which the conductivity increases at an accelerated rate to breakdown.

3. Materials whose conductivity over the whole range increases at an accelerated rate to breakdown.

Of quite a number of materials tested three have been chosen which illustrate the three classes. Fig. 2, untreated cement paper, is representative of the first class. This shows a straight line relationship between conductivity and voltage up to breakdown. Fig. 3, paraffined fish paper, illustrates the second class. Here the straight line relationship holds up to the neighborhood of breakdown where there is a marked increase in the slope of the curve after which breakdown soon occurs. Fig. 4, for black treated cloth represents the more numerous materials of the third class. This curve shows that there is an increasing slope to the curve from low voltages up to the point of breakdown.

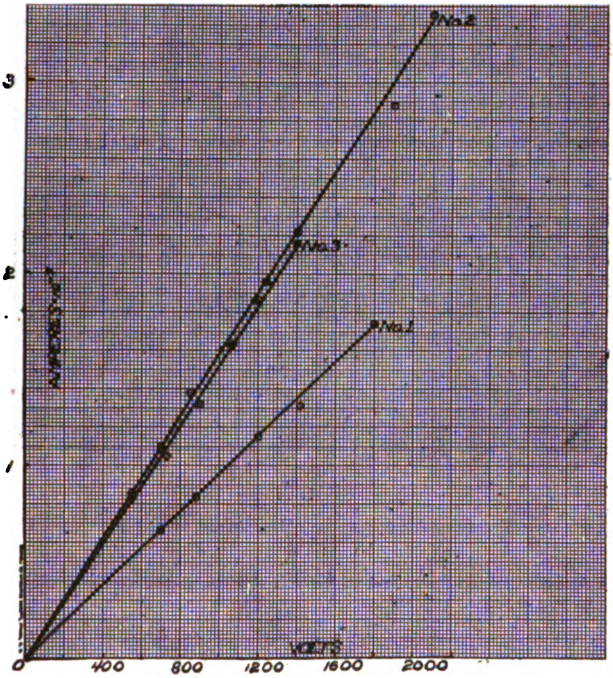

Fig. 2.

Untreated cement paper. Thickness =.015 in.

From a comparison of calculated specific resistances of the various materials tested there is no apparent relation between specific resistance and breakdown voltage. It is quite apparent that the curves for Class 1, give no indication of breakdown. In Class 2, the breakdown point is clearly indicated by the upward slope just before breakdown. In Class 3, we might say that breakdown is liable to occur after the slope of the tangent to the curve exceeds an arbitrary angle determined from experience. From a comparison of the voltage gradient at breakdown, the results show that the materials in class 3 have the greater dielectric strength and those of Class 1, the poorest. Class 1, has the characteristics of a conductor in that the resistance is independent of voltage while Class 3, has no true resistance but only apparent resistance which is a function of the applied voltage. In class 1 the current is no doubt due to the free electrons in the material under the directive force of the applied potential. In Class 3, the number of free electrons is a function of the applied potential. At the point of breakdown, then the number of free electrons is enormously increased, in Class 1 breakdown takes place very suddenly, while in Class 3, the change is more gradual.

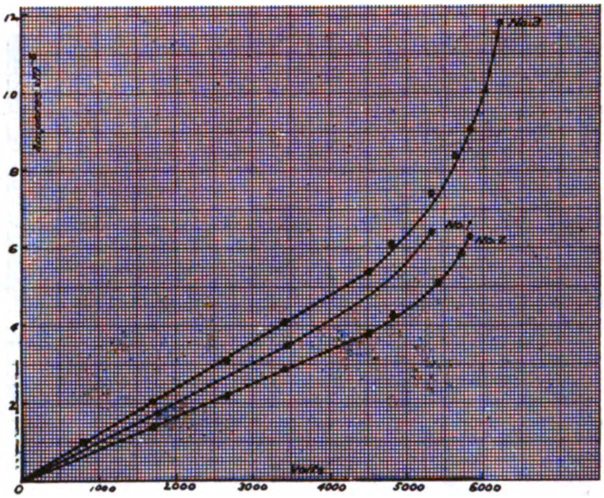

Fig. 3.

Paraffined fish paper. Thickness =.015 in.

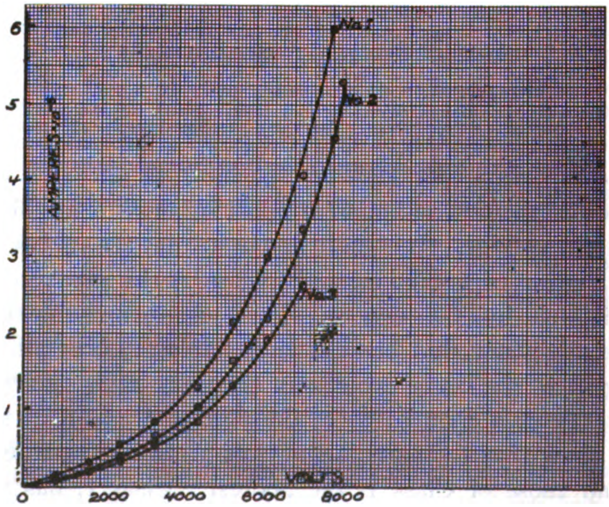

Fig. 4.

Treated cloth (black) No. 1010. Thickness =.011 in.

While no claims are made that the work here reported is complete, considerable useful information has been obtained as to how insulating material behaves at potentials near the breakdown. The method devised may be useful also for further work in investigating the nature of breakdown of insulating materials.

Research Laboratory,
Westinghouse Electric and Manufacturing Co.,
East Pittsburgh, Pa.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE STANFORD UNIVERSITY MEETING, JANUARY 22, 1921.

THE 107th meeting of the American Physical Society was held in the lecture room of the Department of Physics, Stanford University, January 22, 1921, at two o'clock. Professor Fernando Sanford presided. About fifty persons were in attendance.

The following program was presented:

The Balmer Series of Hydrogen and the Quantum Theory of Line Spectra. R. T. BIRGE.

Charts as an Aid in Teaching Electricity. F. J. ROGERS.

Damped Electrical Oscillations. R. B. ABBOTT.

A Thermoelectric Motor. PAUL KIRKPATRICK.

A Wave Composition Model. JOSEPH G. BROWN.

Diurnal Variations in Terrestrial Magnetism and in the Electrical Potential of the Earth. FERNANDO SANFORD.

Sunspots and Terrestrial Magnetic Storms. FERNANDO SANFORD.

Note on the Fahrenheit Scale. FLORIAN CAJORI.

Nature of Ionization in the Point to Plate Discharge. EVALD ANDERSON.

An Explanation of X-Ray Diffraction Patterns from Rolled Metals. C. T. DOZIER.

The Concentration of Monochromatic X-Rays by Crystal Reflection. ELMER DERSHEM and C. T. DOZIER.

At the conclusion of this program, Professor D. L. Webster showed and explained his high potential X-ray outfit.

Thirty-six persons were present at the dinner served at Stanford Union at six-thirty o'clock.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

THE BALMER SERIES OF HYDROGEN, AND THE QUANTUM THEORY OF LINE SPECTRA.

BY RAYMOND T. BIRGE.

THE purpose of this paper is to make a rigid comparison between the experimental results for the Balmer series of hydrogen, and the theoretical developments of the quantum theory of line spectra. In particular, Merton's experimental results for the separation, half-width, and relative intensity of the H_α and H_β doublets are examined in the light of the recent Bohr-Kramers' work on the intensity of the fine structure components, as estimated on the basis of the "selective" principle.

It is concluded that, by assuming a general electric field of 100 volts per cm., all of Merton's experimental results check quantitatively with theory. The frequencies of all the lines of the Balmer series are then fitted into the Bohr relativity formula, after the fine structure has been analyzed and the "circle" line computed, using the Sommerfeld theory to determine the position of the fine structure components, and the Bohr-Kramers' theory to determine the relative intensities. The Rydberg constant for hydrogen is calculated to be 109,677.7 ± 0.2, and the constant for a nucleus of infinite mass 109,736.9 ± 0.2.

The paper concluded with a brief discussion of Merton and Nicholson's observations on the appearance of the Balmer series in mixtures of hydrogen and helium, at relatively high pressure.

University of California.

Lecture Room Wall Charts.

By F. J. Rogers.

LECTURES in physics, particularly in electricity and mganetism, demand more or less profuse illustration by plots and diagrams. This may be secured by blackboard diagrams, lantern slides, or wall charts. The latter have marked advantages, chief of which is the fact that they continue silently to proclaim their messages so long as they hang on the wall.

I have had constructed about fifty charts illustrating all branches of electricity and magnetism. They are uniformly four feet square, and when stored are neither folded nor rolled but are hung compactly on a rack one behind another so that any one can be removed sideways without disturbing any of the others. One or two colors besides black are used when this will contribute to clearness of exposition. To the same end, explanatory words and phrases are printed on the charts so that the meaning of the chart may be conveyed with the minimum of explanation. It has been found that letters an inch square or somewhat less, are amply large enough to be readily visible to a class of a hundred.

The content of the message conveyed by a chart varies more or less continuously between two extremes. In the case of one extreme the chart is expected to present a single fact or principle rather concretely, precisely, and with reference to a real experiment. As an example consider the chart, "Lines of Force of a Magnet." A long slim magnet was laid on a sheet of paper and its field carefully plotted over the whole area, four feet square. The effect of the earth's field was cancelled. The result at first glance is quite familiar, but a second glance shows that many lines near the poles have a double curvature, which I have never seen represented in any published diagram. As a second illustration consider the chart representing "Electric Flux-densities for Equal Voltage Gradients" and "Voltage Gradients Required to Produce Equal Flux Densities." Along with the diagrams representing flux densities for condensers in parallel and voltage gradients for condensers in series are given numerical data for common dielectrics.

Charts representing the other extreme are intended to illustrate a series of related facts, principles, and experiments, as fully as may be done in a single chart. As an example let us consider the chart, "The Effect of Temperature on Resistance." This chart represents graphically the resistance of ten metals, three alloys, carbon, and electrolytes at temperatures from – 180° C. to 500° C. All are assumed to have a resistance of 100 at 0° C. Most pure solid metals are represented by lines very close together, with iron and nickel increasing in resistance considerably faster with temperature than the others. The alloys and carbon are represented by lines which are nearly horizontal while electrolytes are shown to change at a very much greater rate in the opposite direction. The changes of resistance upon melting of Hg., Sn., Pb., Bi., and Zn. are shown. On the charts are the following brief statements: "Melting a metal doubles its resistance, Bi and Sb exceptions." "Glass and many salts become conductors at red heat or below." "All insulators become conductors above 2000° C."

The chart "Vacuum Tube Rays" may be taken as another example of charts with a multiple message. A diagram of tube with a pierced cathode has properly placed a single loop carrying a current, beyond the cathode is a "magnetic screen" and beyond that an electromagnet. The "cathode rays" represented by a series of dots, "Electrons" are curved by the "weak magnetic field" and strike the "Anti-cathode," from which radiate "X-rays" represented as a series of concentric wave crests called "ether waves." Passing through the pierced cathode and slightly deflected by the "strong magnetic field" on the "canal rays" or "positive atoms" which are represented by a series of dots much larger than those representing the cathode rays. On the same chart is a diagram of a "Coolidge tube" showing the "hot cathode" in series with a battery and the tungsten anti-cathode. "Cathode rays" are represented as striking the anti-cathode in a "highest vacuum" and "X-rays" are represented as radiating from the anti-cathode and through the glass walls of the tube.

STANFORD UNIVERSITY.

DAMPED ELECTRICAL OSCILLATIONS.

BY R. B. ABBOTT.

THE subject of "Damped Electric Oscillations" can be treated by the method of analogy. The parallelism between the dynamical and electrical equations for simple harmonic motion and logarithmically damped motion is well known. A parallelism exists between the electrical and dynamical equations for linearly damped motion and for combined linearly and logarithmically damped motion. The solutions in the last two cases named above are easily obtained, by analogy and a greater simplification is obtained in the solution of the latter case than has hitherto been obtained.

Oscillatory motion is most easily described in terms of the projection of a radius vector r, rotating with constant angular velocity w, about a center c. Let the radius vector make an angle θ with the X-axis, passing through the center c such that $\theta = wt$, where t is the time from the instant when the radius vector coincides with the positive X-axis. The motion may be described as taking place in either X or Y, thus: the displacement $x = r \cos wt$ and $y = r \sin wt$.

The point P at the end of r traces out some curve such as a circle, ellipse or some form of spiral. If there is no dissipation of energy during periodic motion, the auxiliary curves will be of the spiral family: the auxiliary curve for damping proportional to the speed being the logarithmic spiral and for constant damping, the rectilinear (Archimedes') spiral.

The differential equations expressing the acceleration in free periodic motion such as the cases above mentioned, are of the second order with coefficients which are either constants or expressible as functions of the time. If p and q are such coefficients, the general equation is written as follows:

$$d^2y/dt^2 + p\,dy/dt + qy = 0.$$

By reference to the auxiliary curves, the values of p and q can be determined for all classes of periodic motion, and their differential equations can be written.

In the case of logarithmic damping, r decreases at a rate proportional to itself.

$$\frac{dr}{dt} = -kr,$$

from which $\log r/r_0 = kt$ and $r = r_0e^{-kt}$. Therefore the auxiliary curve is a logarithmic spiral. In the case of rectilinear damping, r decreases at a constant rate,

$$dr/dt = -k_1,$$

from which $r = r_0 - k_1t$.

In the case of combined logarithmic and rectilinear damping, it is easily shown that $r = r_0 - k_1te^{-kt}$.

In each case substitute $y = r \cdot \sin \theta$ in the general equation. By comparison with the electrical cases, the constants can be determined in terms of C, L, and R.

Washington State College.

A Thermo Electric Motor.

By Paul Kirkpatrick.

THE reaction of thermo-electric currents to a magnetic field is familiarly shown in the radio-micrometer. If the suspended system of the radio-micrometer were free to turn without constraint and the radiant impulses communicated to its juncture intermittently the coil might be caused to rotate continuously. Such an instrument, which may be called a thermo electric motor, has been constructed.

The armature possesses a number of thermo couples disposed in a circle about the intended axis of rotation. An opaque shade screens half of these while the remainder are exposed to a radiation, say of the sun. Currents traverse a net of wires connecting the couples and, being mounted in the field of a permanent magnet, the system starts to rotate. The successive eclipses of the junctures serve the purpose of commutator and brushes and the forward rotation is maintained.

In the present model there are six silver-German silver junctions and six conductors in the drum armature. The axis of rotation is vertical and the armature carried on a single jewel. The magnet poles are shaped closely about the armature and a fixed iron core protrudes into the latter. The motor will operate under the action of sunshine or any artificial radiation of proper intensity.

University of California.

A Wave Composition Model.

By J. G. Brown.

A MODEL designed to show the simultaneous production of transverse and longitudinal waves was shown at a previous meeting of the Physical Society. The present model, based upon the same principle, shows the composition of two transverse waves in the same plane and also in planes at right angles.

Each component wave is produced by the rotation of a shaft having cranks set at uniform distances and with constant difference of phase along the entire shaft. By acting upon rods, these shafts convey their motions to two rows of balls, the motions being combined in the same direction in the upper row and at right angles in the lower row.

The wave-length of the back component is fixed at 18 inches, but the front component can be varied by changing shafts. Wave-lengths of 18 inches, 12 inches and 9 inches are provided, giving ratios of 1 : 1, 1 : 2 and 2 : 3.

The shafts are connected by chain and sprockets and the relative frequencies are determined by the size of the sprockets. Ratios of 1 : 1, 1 : 2, 2 : 3 and 3 : 4 are provided.

Relative velocities are determined by the combination of wave-lengths and frequencies. The direction of the wave components may be made the same or opposite by means of a reversing gear.

The phase of the wave components can be fixed by the positions of the shafts when the gears are enmeshed.

The model is especially designed for laboratory use, although it will show certain things very well to large groups.

Some of the phenomena which can be illustrated are the following:

1. Variation of amplitude with phase in the interference of similar plane polarized waves.
2. Standing waves. Reflection at a dense medium and at a rare medium.
3. Elliptically and circularly polarized waves.
4. Beats between similar waves.
5. Plane polarized white light in a vacuum.
6. Unpolarized or ordinary white light in a vacuum.
7. Plane polarized white light in glass. Ordinary refraction and dispersion.
8. Ordinary monochromatic light in calcite. Double refraction.

A trace of the motion of any ball can be taken at any time so that the motion of the parts of the medium as well as the distortion of the medium can be studied in detail.

STANFORD UNIVERSITY.

DIURNAL VARIATIONS IN TERRESTRIAL MAGNETISM AND IN THE ELECTRICAL POTENTIAL OF THE EARTH.

BY FERNANDO SANFORD.

IN an article entitled "The Electrostatic Charges of the Earth and Sun and their Relation to Terrestrial Magnetism" which was presented at the Cleveland meeting of the American Physical Society on November 27, 1920, were given some curves showing the diurnal variation of the earth's potential at Palo Alto, California, for the month of August, 1920, and a curve showing the mean diurnal variation of the *N* component of terrestrial magnetism at the Falmouth, England, magnetic observatory for the month of August for the

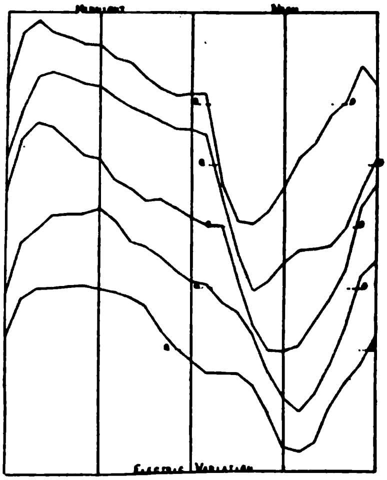

Fig. 1.

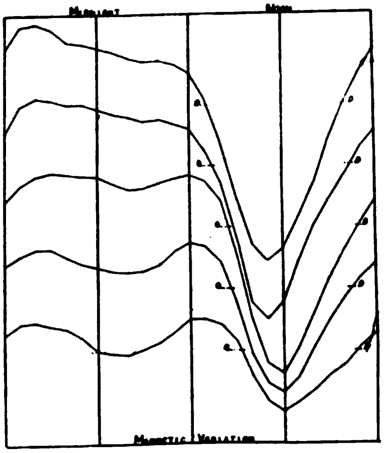

Fig. 2.

twelve years, 1891–1902. The measurements of the potential change at Palo Alto have been continued, and in Table I. are given the data expressed in millivolts of the observed change for the months August, September, October, November and December, 1920. Along with these, for purposes of comparison, are given Chree's values of the mean diurnal variation of the *N* magnetic component for the corresponding months for the twelve years above mentioned. The actual numbers given by Chree have all been divided by 5, thus making the magnetic force unit used $5 \cdot 10^{-6}$ C.G.S. unit, instead of $1 \cdot 10^{-6}$ C.G.S.

TABLE I.

Diurnal Variation in Millivolts in the Negative Charge of the Earth at Palo Alto, California, Compared with the Diurnal Variation in the N Component of Terrestrial Magnetism for the Corresponding Months at Falmouth, England, Expressed in $5 \cdot 10^{-6}$ C.G.S. Units.

The positive sign before the quantities in the table of electrical variations does not mean an increase of the positive potential of the earth, but an increase in the negative potential. It is used in this manner because an increase in the negative potential at a given place should cause an increase in the N magnetic component at that place.

Electric Variation.					Hr.	**Magnetic Variation.**				
Aug.	**Sept.**	**Oct.**	**Nov.**	**Dec.**		**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
+ 6.	+ 2.2	+11.8	− 1.2	+ 4.6	6 p.m.	+18.4	+11.4	+ 8.8	8.0	+ 5.6
+23.3	+17.3	+28.3	+14.8	+16.5	7 "	+26.0	+19.8	+13.8	+10.8	+ 9.0
+27.3	+28.7	+33.8	+19.3	+19.9	8 "	+17.2	+22.4	+16.6	+13.0	+ 9.6
+23.3	+30.2	+32.6	+23.8	+20.0	9 "	+25.4	+22.0	+17.8	+13.0	+ 8.2
+21.7	+28.7	+27.7	+24.0	+20.4	10 "	+21.2	+20.4	+17.4	+10.6	+ 7.4
+19.5	+26.7	+23.2	+24.0	+21.0	11 "	+20.6	+20.6	+17.0	+ 8.4	+ 4.6
+19.2	+25.7	+22.0	+25.8	+19.9	12	+19.2	+19.2	+16.6	+ 7.2	+ 0.4
+14.7	+22.2	+15.1	+22.0	+18.9	1 a.m.	+17.8	+17.6	+14.4	+ 5.8	− 0.8
+13.7	+19.2	+12.5	+15.3	+17.2	2 "	+15.8	+16.2	+12.6	+ 5.6	− 1.0
+ 8.7	+16.8	+ 8.5	+11.8	+14.2	3 "	+14.8	+14.8	+12.8	+ 6.0	+ 0.6
+ 5.0	+14.2	+ 9.0	+10.3	+ 6.6	4 "	+14.8	+15.6	+14.6	+ 8.0	+ 2.8
+ 3.0	+12.0	+ 6.2	+ 6.0	+ 0.8	5 "	+13.8	+14.0	+16.0	+12.4	+ 6.0
+ 3.5	+11.7	+ 3.0	+ 2.5	− 3.3	6 "	+11.0	+12.6	+17.0	+15.0	+10.2
+ 3.5	+10.0	+ 1.0	+ 1.5	− 7.3	7 "	+ 2.6	+ 5.2	+15.2	+14.0	+10.2
−26.0	−10.0	+ 0.6	− 4.4	− 7.4	8 "	−12.2	− 7.0	+ 8.6	+ 8.8	+ 8.4
−38.0	−29.3	−17.7	− 7.4	− 7.7	9 "	−30.4	−27.0	− 8.8	− 4.4	+ 2.8
−39.0	−41.0	−32.1	−14.7	−11.9	10 "	−45.4	−44.4	−30.0	−21.0	− 8.0
−34.8	−37.7	−40.3	−27.2	−20.5	11 "	−51.0	−49.8	−44.0	−30.6	−16.2
−27.3	−31.7	−40.8	−35.9	−31.9	12	−46.8	−44.2	−47.2	−33.2	−19.6
−17.5	−27.8	−39.0	−40.2	−33.2	1 p.m.	−36.8	−31.4	−39.2	−28.6	−16.6
−13.0	−27.4	−31.0	−34.4	−30.1	2 "	−26.2	−19.6	−27.4	−19.0	−12.6
− 3.7	−26.3	−23.8	−24.9	−19.3	3 "	−11.6	−10.8	−16.6	−10.4	− 7.8
+ 0.7	−20.2	−14.0	−12.5	−10.6	4 "	+ 1.2	− 3.0	− 7.4	− 3.0	− 4.2
+12.3	− 7.7	+ 6.6	+ 4.0	− 4.7	5 "	+10.8	+ 5.4	+ 1.4	+ 3.8	+ 1.6

In the table, an increase in the negative potential of the earth is marked + and a decrease −, in order to bring them into correspondence with the magnetic data, since an increase in negative potential should cause an increase in the *N* magnetic component.

The two sets of data are also expressed graphically in Fig. 1 and Fig. 2.

PALO ALTO, CALIFORNIA.

Sun Spots and Terrestrial Magnetic Storms.

By Fernando Sanford.

It has long been known that a close relation exists between sun spots and the irregular magnetic disturbances known as magnetic storms, but no satisfactory reason for this relation has been shown.

The work of Hale and others, at Mt. Wilson, has shown that sun spots are the visible manifestation of tremendous cyclonic disturbances upon the sun. It has also been shown that powerful magnetic fields are produced over large areas in the centers of these cyclonic vortices, which fields are apparently due to convection currents set up by the rotation of negative ions around the storm centers.

Stŏrmer's calculation of the electric and magnetic forces involved in a given sunspot show them to be great enough to produce strong induced currents at the distance of the earth.

It has frequently been shown that magnetic storms have the characteristics which would follow from vortex currents in the earth or in the upper atmosphere. It has been assumed that these currents are due to electrons expelled from the sun and forced into spiral paths by the earth's magnetic field.

The fact that both clockwise and counter-clockwise vortices are shown by magnetic storms in the same regions indicate that their rotational motion is not caused by the earth's magnetic field. Also, it is known that the sun has a magnetic field about 50 times as strong as the earth's field, hence no electrons can be expelled from the sun except in the direction of its magnetic axis, and such electrons would not reach the earth.

The diurnal variations in the earth's electrical potential which have been recorded at Palo Alto seem to show that the earth, as well as the sun, is highly electrified, and with a negative charge. This gives plausibility to the assumption that the other planets are similarly electrified, and furnishes a theoretical basis for the conclusion, hitherto based upon empirical data, that the periodicity of sun spots depends upon the relative positions of the planets. Reference is made to the evidence furnished by La Rue, Stewart and Loewy that the sun spot area varies with the angular distance between Mercury and Venus and to the fact that the mean sun spot period of a little more than 11 years is closely related to an integral number of revolutions of each of the nearer planets.

Thus: Mercury makes 45 revolutions in 11 years;
Venus makes 18 revolutions in 11.08 years;
Earth makes 11 revolutions in 11 years;
Mars makes 6 revolutions in 11.25 years;
Jupiter makes 1 revolution in 11.8 years.

Attention is also called to the fact that the diurnal variation of the *N* component of terrestrial magnetism has been shown by Chree to be about 70 per cent. greater in the four-year period including sunspot maximum than in the corresponding period at sunspot minimum. This phenomenon can not be due to the sun spots, themselves, since it is shown by the records of days of least magnetic disturbance. From the point of view of the present writer, such a variation could be caused by the sun only if the total electric charge of the sun varied periodically with the sunspots. Apparently, both the disturbances in the sun's electric field which gives rise to sun spots and the increase in the diurnal variation of the earth's magnetic field must be due to electrical induction by other charges in the solar system.

STANFORD UNIVERSITY.

NOTE ON THE FAHRENHEIT SCALE.

BY FLORIAN CAJORI.

QUOTATIONS were read from Fahrenheit's five papers in the Philosophical Transactions of London, for the year 1724, the three parts describing the thermometers Fahrenheit had used for the ten preceding years. Conclusions: (1) From the first quotation it follows that Fahrenheit used as fixed points the ice-water-salt or sal-ammoniac temperature and blood temperature; the interval was divided into 4 × 24 or 96 steps; (2) from the second quotation it follows that Fahrenheit (unlike other experimenters) used also a third point, the ice-water temperature, presumably for more perfect checking; (3) from all three quotations it follows that previous to 1724 the boiling point of water was not used by Fahrenheit as a fixed point in the graduation of his thermometers, that continuing his scale upward by equal steps the boiling point of water fell at 212°. On his scale the freezing point of water chanced to come at 32°. It is not known whether or not Fahrenheit changed his fixed points to the freezing and boiling points of water after the year 1724. A disregard of Fahrenheit's own statements has led to many false and contradictory statements, such as: (1) Before 1724 he used 32° and 212° as fixed points in his graduations; (2) the interval between ice-water-salt and ice-water was subdivided by continued bisection into 32° and the scale built up from these degrees; (3) he borrowed the number 96 of his subdivisions from Florentine thermometers; (4) he used as fixed points ice-water-salt and boiling mercury, subdividing the interval into 600 degrees; (5) he regarded his zero as an absolute zero (very probably untrue).

Corroboration of the conclusions of this paper is found in two contemporary writers, namely, C. Wolf in Acta Eruditorum for 1714 and C. Kirch in Miscellanea Berolinensia, T. VI, printed 1740. Both of them owned thermometers made by Fahrenheit.

NATURE OF IONIZATION IN THE POINT TO PLATE DISCHARGE.

BY EVALD ANDERSON.

IT has long been known that in the so-called point to plate, electrical unidirectional discharge the ions are principally of the same sign as the point. In 1911 Zeleny tried some experiments, the results of which indicated that at moderate currents, and gas pressures above half an atmosphere, very few if any ions of the opposite sign to the point, were present, while at lower pressures, these ions became more and more numerous. Field exploration experiments by H. T. Booth, in 1917 indicated the presence of ions of the opposite kind, but an investigation by Tolman and Carrer gave results which were not in accord with this latter conclusion. However, certain phenomena in connection with so-called electrical precipitation seem to show that at least at the higher voltages ions of the opposite sign are present.

Experiments were made for the purpose of throwing further light on this point. The method consisted in measuring the potential acquired by an insulated wire probe between the point and plate and determining how this potential varied with the potential of the point. The distance between the point and the plate was in all cases 25 mm. The probe was usually a fine wire bent in a small loop. It was carefully insulated from both the point and and plate. The potential was measured by a Braun type voltmeter reading to 5,000 volts, and by an attraction disk voltmeter, reading to 50,000 volts. Since both the potential of the point and the probe were measured on the same instruments the comparative values are correct, and independent of the absolute calibration of the instruments. The gas was air in all the experiments, and the pressure was either 10 or 76 cm. In the former case the air was dried over P_2O_5, but in the latter no drying agent was used.

For the low pressure experiments, a battery of ten 600 volts D.C. generators, connected in series, supplied current at a potential up to 6,000 volts, while for the other experiments a 50,000 volt D.C. generator was used.

At a pressure of 10 cm. the potential of the probe varied directly as that of the point up to the maximum potential tried, 4,800 volts. The arcing potential for this pressure and electrode setting was about 5,600 volts, for either positive or negative point. It is evident from the plotted results that the sign of the point does not markedly influence the character of the curve. The current passing, was, however, approximately three times as large when the point was negative.

The curves for atmospheric pressure are markedly different. In the first place, the arcing potential is approximately twice as high when the point is negative as when it is positive, although the potential at which ionization begins is nearly the same in both cases. Thus the curves for the point negative all show a break at about 30,000 volts, indicating that at this point the relative number of positive ions is suddenly increased. This could be explained by the assumption that at this potential the negative ions had sufficient energy to ionize the air at or in the surface of the plate. This explanation is supported by the phenomena noted when a thin plate of mica was placed on the plate so that its edge was directly under the point and probe. It has long been known that the presence of such a dielectric at the positive terminal causes ionization at this terminal. Such ionization is easily detectable by the characteristic light given off where positive ions are formed. In this case the break in the curve with the mica on the plate occurred at 24,000 volts, instead of at 30,000 volts, without the mica. The arcing voltage was also correspondingly lowered.

Curves showing the current, and the probe potential as functions of the potential of the point for one position of the probe, with and without mica on the plate indicate that at low potentials, the mica simply acts as an insulator, raising the potential of the probe and lowering the current. At higher potentials when ionization sets in at the edge of the mica, the potential of the probe is lowered, due to the positive ions, and the current is greatly increased. Sharp points, on the plate, as iron filings, have effects similar to an insulator, in that they lower both the arcing potential and the potential of the probe.

With the point positive none of the above effects are noticed. The data taken were not sufficient to definitely prove the shape of the curves in this case, due largely to the small range between the ionization and arcing potentials, but it was definitely proven that neither small points nor insulators at the plate affected the field in such cases. A test was then made on the effect on the arcing potentials of such materials, when the point was positive, and it was proven that not only this was entirely unaffected, but that also the arcing potential for a point and plate with the point positive was exactly the same as for two points the same distance apart. This shows that the phenomena at the positive terminal determined the maximum potential in such a system. It is probable, therefore, that the effect of dielectrics at the positive terminal is due to a modification of the plate into a point or edge, and local disturbance of the field due to the dielectric capacity and charges on its surface.

The behavior at 10 cm. pressure was quite different. Not only was the arcing voltage about the same for either sign, but a thin sheet of mica on the plate had no reproducible effect. It is probable, therefore, that the nature of the ions in a point to plate discharge is markedly different at low pressures, although it is not yet known whether the change is gradual or is abrupt at someone pressure.

These experiments have also shown that the potential acquired by a probe in a point to plate discharge probably is not simply and directly related to the concentration of ions at that point, when the probe is not there. Thus, the arcing voltage between the point and plate in these tests was only 35 kv., with the probe 5 mm. from the plate, while with the probe absent, the arcing voltage was 45 kv. showing that the probe had a decidedly disturbing influence. The probe, in fact, acts as a separate ionization center in the field, which is shown by silent discharge, from the probe to the plate, or sparks to or from the probe at the higher voltages.

UNIVERSITY OF CALIFORNIA.

An Explanation of X-Ray Diffraction Patterns from Rolled Metals

By C. T. Dozier.

When X-rays are passed through a thin sheet of rolled metal a diffraction pattern of two-fold symmetry is formed on a photographic plate set a few centimeters back of the sheet of metal. The diffraction pattern is weaker parallel to the direction of rolling than it is in the direction perpendicular to this. It is shown by photographs that this effect can be duplicated by parallel laying of elongated isometric crystals. The laterally diffracted rays are doubtless less absorbed than those emerging longitudinally. The effect in rolled metals is explained by assuming that the rolling causes the longer dimensions of embedded crystals to be forced into parallel alignment and the X-ray diffraction pattern thus affords a means of determining the direction of rolling in a metal specimen.

University of California.

The Concentration of Monochromatic X-Rays by Crystal Reflection.

By Elmer Dershem and C. T. Dozier.

It has previously been shown, Proc. Am. Phys. Soc., Phys. Rev., Vol. XI. p. 244, that a crystal surface if deformed into the shape of a logarithmic spiral of revolution will bring X-rays of a certain wave-length to a focus provided the source of X-rays is at the origin of the spiral.

Such a reflecting surface has now been formed by the use of small crystals of halite. The intensity of the monochromatic X-rays obtained in this way is 10 per cent. of the intensity of the entire radiation of the X-ray tube at the same distance, this intensity being measured with an air-filled ionization chamber.

University of California.

Minutes of the New York Meeting, February 26, 1921.

The 108th meeting of the American Physical Society was held in Room 305 Schermerhorn Hall, Columbia University, New York City, on Saturday, February 26, 1921. There was a single session beginning at ten o'clock a.m. with an attendance of about 225. President Theodore Lyman presided.

There was a meeting of the Council held at 2:30 o'clock p.m. on February 26, 1921, in the office of Dean Pegram, of Columbia University. The following elections to membership were made: *Elected to ordinary membership*: Frederick Barry, Richard T. Cox, T. M. Dahm, Marion Eppley, Roy Y. Ferner, Twao Fukushima, Warren K. Green, L. O. Heath, Usaku Kakinuma, Paul M. Mueller, Alexander McLean Nicolson, Simon Sonkin, W. Ewart Williams; *transferred from membership to fellowship*: Alfred H. Bucherer and Frank W. Ham.

At the annual meeting of the Society held in Chicago on December 30, 1920, amendments in the wording of the Constitution and By-Laws were adopted, the sole purpose and effect of which is to change the designations of the two classes of membership formerly named "regular members" and "associate members" to "fellows" and "members," respectively. All persons now designated regular members become fellows without further action, and all associate members become members. There is no change in the conditions for future election to either class of membership nor for transfer. The grade of honorary fellow replaces that of honorary member.

The following program of eighteen papers was presented, three papers being ready by title only:

Reflection of X-Rays from Crystals. W. M. STEMPEL.

The Reflection of X-Rays from Calcite. BERGEN DAVIS and W. M. STEMPEL.

Soft X-Rays of Characteristic Type. E. H. KURTH, introduced by K. T. Compton.

On the Absorption of X-Rays by Chromium, Manganese and Iron. WILLIAM DUANE and HUGO FRICKE.

The Piezo-Electric Resonator. W. G. CADY.

The Binaural Location of Pure Tones. R. V. L. HARTLEY and THORNTON C. FRY.

Note on the Characteristics of the New Singing Tube. CHAS. T. KNIPP. (Read by title.)

Broken Tone From Reed Instruments. JOHN B. TAYLOR.

Electron-Tube Drive for Tuning Forks. E. A. ECKHARDT, J. C. KARCHER and M. KEISER.

A High-Speed Oscillograph Camera. E. A. ECKHARDT.

A Method of Measuring Surface Tension of Liquids. HARRY CLARK.

The Spectral Structure of the Luminescence Excited by the Hydrogen Flame. HORACE L. HOWES.

Photoelectric Phenomena in Coated-Filament Audion Bulbs. ERNEST MERRITT.

Polarization Capacity and Polarization Resistance as Dependent upon Frequency. ERNEST MERRITT.

The Motion of Electrons Between Coaxial Cylinders in a Uniform Magnetic Field. ALBERT W. HULL.

An Electrical Doublet Theory of the Nature of the Molecular Forces of Chemical and Physical Interaction. R. D. KLEEMAN.

The Copernican Atom Radiating Energy.—A Physical Interpretation of Planck's Quantum Rule. ALBERT C. CREHORE. (Read by title.)

A Copernican Atomic Model Based on Electromagnetic Theory. ALBERT C. CREHORE. (Read by title.)

Reflection of X-Rays from Crystals.

By W. M. Stempel.

THE reflection of X-rays from crystals was treated theoretically by C. G. Darwin in 1914 on the assumption that the electrons were clustered so closely about the nucleus of the atom that their distances from the nucleus were small compared to the distance between the planes of atoms in the crystals.

Three years later (1917) A. H. Compton attempted to take into account, theoretically, the effect of assuming that the distances of the electrons from the nucleus was comparable with the distance of the reflecting planes, and arrived at the conclusion, after attempting to derive a general distribution function, that electrons arranged as in the Bohr atom or at least in rings was the only arrangement that would satisfy experimental facts. Both the above authors had to resort to plane waves or their equivalent to get their final workable equations.

The present writer starts with the rings of electrons and plane waves, and in addition carries out the integration in a somewhat different order as follows: A plane is considered in the first or surface atomic layer. This plane is chosen at random and the succeeding planes are taken equal distances apart from this, say D plus E, where D is the distance between the atomic planes of the crystal. The reflection from this series of planes is obtained. The initial plane is then changed and the process repeated. This integration is carried out for all possible positions of the initial plane. The result of this work is as follows:

The intensity of the beam of monochromatic X-rays reflected from a perfect crystal is

$$I_m = I_0 D \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1 + \cos^2 2\theta}{2} \frac{1}{E^2} \frac{1}{1 + \frac{\beta^2}{E^2}\delta^2} \cdot e^{-Bn^2}$$

$$\times \sum_s \left\{ N_s \left(1 + 2m - m(m+1)\frac{D \sin\theta}{2a_s}\right)\left(\frac{\sin \beta a_s}{\beta a_s}\right)^2 \right\},$$

where I_0 is the intensity of the incident beam,

e is the unit charge on the electron (in e^2/mc^2),

e in e^{-Bn^2} in base of Naperian log,

m_e is the mass of the electron,

m is an integer defined by $2a/mD > \sin\theta > 2a/(m+1)D$,

c is the velocity of light,

D is the grating space of the crystal used,

θ is the grazing angle,

E is the fractional part of the energy extracted from the incident beam per atomic layer that it passes through. Here the entire energy is being taken and includes as part both the pure absorption and that part re-radiated by the electron.

β is a constant and equal to $2\pi n/D$,

e^{-Bn^2} is Debye heat factor,

n is the order of the spectrum under consideration,
N is the number of electrons per c.c. belonging to one sized ring,
s refers to the particular electron orbit under consideration, and the summation is understood to be extended to each of the different orbits present no matter to what atom they may belong, the radius of the orbit is all that counts so long as the atom is in the reflecting plane, if not a correction must be made on account of this displacement.

It is to be observed that the intensity of the reflected beam given in equation one is in general not the maximum. Reflection can take place from a system of mathematical planes only when their distance apart is so adjusted as to satisfy the equation, $n\lambda = 2(D + \delta) \sin \theta$. The reflected intensity given by equation (1) is thus the intensity to be expected for any arbitrary wave-length and any arbitrary grazing angle. The maximum value will be reached only when the proper distance between reflecting planes becomes equal to the distance between the atomic planes of the crystal, or $\delta = 0$. The values of the intensity fall off very rapidly on either side of $\delta = 0$.

When a heterogeneous beam is used a small range of wave-lengths will be reflected. That wave-length for which $\delta = 0$ will be most intense, but on either side energy will still be reflected, the intensity falling off rapidly. The range of waves which would have any appreciable value is however very small and (I_0) for such a range is easily considered constant. To get the total energy reflected in the case of a heterogeneous beam we may integrate the equation (1) with respect to δ and get

$$I_{HT} = I_0 D \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1 + \cos^2 2\theta}{2} \frac{\pi}{E\beta} e^{-Bn^2}$$
$$\times \sum_s \left\{ N_s \left(1 + 2m - m[m + 1] \frac{D \sin \theta}{2a_s}\right) \left(\frac{\sin \beta a_s}{\beta a_s}\right)^2 \right\}.$$

In all these equations it is to be remembered that the first bracket under the summation sign is never less than one as will be noted on inspecting the condition which determines the value of (m).

If the beam reflected from the one crystal is allowed to fall upon another, placed with its atomic planes parallel with the first, the condition for reflection is correct for every element of the beam and thus serves an ideal method of studying reflection phenomena.

If for two parallel crystals, monochromatic light is allowed to fall upon the first crystal, the intensity of the beam coming from the second is

$$I_{m2} = I_0 D^2 \left(\frac{e^2}{m_e c^2}\right)^4 \left\{\frac{1 + \cos^2 2\theta}{2}\right\}^x \frac{1}{E^4} \frac{1}{1 + \frac{\beta^2}{E^2} \delta_1^2} \frac{1}{1 + \frac{\beta^2}{E^2} (\delta_1 - \delta_2)^2} e^{-2Bn^2}$$
$$\times \sum_s \left\{ N_s^2 1 + 2m - m[m + 1] \frac{D \sin \theta}{2a_s}\right)^2 \left(\frac{\sin \beta a_s}{\beta a_s}\right)^4 \right\},$$

where (x), the exponent of the polarization factor, will lie somewhere between

1 and 2, probably not far from 1. Its value will depend upon how completely the beam is polarized by the first reflection. δ_1 refers to the first crystal and δ_2 to the second. We will think of δ_1 being required because the wave-length is not correct for the grazing angle, and δ_2 because the second crystal is not quite parallel with the first.

When heterogeneous X-rays fall upon the first crystal, the total energy reflected from the second for any value of δ_2 will be obtained by integrating this last equation with respect to δ_1. The following equation is obtained,

$$I_{H2} = I_0 D^2 \left(\frac{e^2}{m_e c^2}\right)^4 \left\{\frac{1+\cos^2 2\theta}{2}\right\}^x \cdot \frac{f(\delta_2)}{\beta E^3} \cdot e^{-2Bn^2}$$

$$\times \sum_s \left\{ N_s^2 \left(1 + 2m - m[m+1]\frac{D\sin\theta}{2a_s}\right)^2 \left(\frac{\sin\beta a_s}{\beta a_s}\right)^4 \right\},$$

and the values of the $f(\delta_2)$ are given below for various values of δ_2 where $A = E/\beta$.

δ_2	0	.2A	.4A	.6A	.8A	1.0A	1.4A	1.8A
$f(\delta_2)$	1.571	1.555	1.510	1.441	1.354	1.257	1.054	.868

δ_2	2.0A	2.4A	2.8A	3.2A	3.6A	4.0A	5.0A	8.0A
$f(\delta)_2$	.785	.644	.531	.443	.371	.306	.218	.090

To get the fractional part of the beam that is reflected at the second crystal we have only to divide the last equation by the second and we get

$$D\left(\frac{e^2}{m_e c^2}\right)^2 \left\{\frac{1+\cos^2 2\theta}{2}\right\}^{x-1} \frac{f(\delta_2)}{\pi E^2} e^{-Bn^2}$$

$$\times \sum_s \left\{ N_s \left(1 + 2m - m[m+1]\frac{D\sin\theta}{2a_s}\right) \left(\frac{\sin\beta a_s}{\beta a_s}\right)^2 \right\}$$

The fractional part of the beam reflected may be easily obtained for all the other cases.

The theory has the advantage of disclosing the following facts which are new to X-ray reflection theory:

The reflected energy does not approach infinity for either large electron concentration or small grazing angles.

The theory definitely discloses the variation of the intensity of the reflected beam for conditions deviating from those for perfect reflection, which variation measures the sharpness of interference patterns. This sharpness of interference patterns is independent of electron distribution, a fact shown some years ago by Debye.

The theory discloses many other interesting facts too complex to point out here. One may be stated because of its extreme importance, and that is that for polished or irregular crystals the intensities of successive orders vary inversely as the square of the order, a fact shown experimentally to be true by the Braggs (they no doubt worked with polished crystals).

The possibilities of the theory together with some interesting experimental conformation will appear later when the details of this work are published.

POLARIZATION CAPACITY AND POLARIZATION RESISTANCE AS DEPENDENT UPON FREQUENCY.

BY ERNEST MERRITT.

THE success of the experiments described by Gen. Squier, in which radio frequency signals were sent along bare wires immersed in water, makes it reasonably certain that the electrolytic capacity of such wires is much less at high frequencies than for a slowly changing charging potential, and preliminary measurements at the Bureau of Standards have confirmed this conclusion. I have recently made determinations of the electrolytic capacity of platinum in sulphuric acid through a wide range of frequencies and find that in this case also the capacity depends upon the frequency. Platinum electrodes in sulphuric acid were chosen for test in order to make possible a quantitative comparison of the results with those predicted by theory. The electrodes were of wire 0.41 mm. in diameter and 18 mm. long. As measured by ordinary charge and discharge the charging potential being 0.1 volt or less, the capacity of the cell was approximately 5 microfarads.

In determining the capacity at different frequencies an alternating current of the frequency desired was developed by an oscillating audion, and a loosely coupled secondary circuit was tuned to resonance by its adjustable condenser. When the electrolytic cell was placed in series with the variable condenser it was necessary to make a small change in the setting in order to restore resonance; from this change and the original capacity the electrolytic capacity could be computed. When the cell was not used its place in the circuit was taken by a "dummy" having the same inductance and resistance. For measurements in the audible range a standard mica condenser was used. Frequencies were determined by a wave meter in the case of radio frequencies and in the case of lower frequencies by comparison with tuning forks. Current measurements were made with a crystal detector in an untuned circuit loosely coupled to the secondary.

The results obtained are in the main in agreement with the simpler form of the Warburg diffusion theory. For example, the capacity of the cell (two electrodes in series) was found to be 2.37 mf. for $n = 1,810$ and 0.12 mf. at $n = 1,000,000$. If the capacity were proportional to $1/\sqrt{n}$ throughout this whole range as predicted by the Warburg theory, the value for $n = 1,000,000$ should be 0.10 mf. In the range $n = 4,000,000$ to $n = 500,000$ the capacity appears however to be proportional to $1/n$ instead of $1/\sqrt{n}$. There is some reason to expect such a change in the law at high frequencies, since the theory is based upon the assumption that the ions are so numerous that equilibrium between the ionic concentration in the liquid and that in the electrode can be established instantly and without appreciable change in the concentration in the liquid. At sufficiently high frequencies this assumption must cease to be justified.

Cornell University.

Photoelectric Phenomena in Coated Filament Audion Bulbs.

By Ernest Merritt.

In audion bulbs having oxide-coated filaments a film often forms on the plate, sometimes appearing as an irregular discoloration of the polished surface, but often scarcely visible. It has been pointed out by Case that these films show photoelectric activity. In repeating the experiment described by Case I find that the activity, although smaller than that of a sodium or potassium cell of the Elster and Geitel type, is sufficient to permit of convenient measurement by means of a moderately sensitive galvanometer. The active rays appear to be at the violet end of the spectrum and in the near ultraviolet. The current produced by light from an open arc is reduced by about 20 per cent. by a piece of glass 5 mm. thick. Owing to the high vacuum the voltage current curve is remarkably flat; complete saturation is reached at about 10 volts and increase in voltage to 120 volts produces no further change in current.

Since the active films are produced by the evaporation of the oxide coating on the filament it seemed reasonable to expect that the filament would also show photoelectric activity I find this to be true in the case of all the tubes tested. With the plate and grid connected and maintained at a positive potential the current obtained by illuminating the filament is much smaller than when the plate is made negative with reference to grid and filament. In one case, for example, it was found that $i = 5 \times 10^{-9}$ for filament negative and $i = 4 \times 10^{-8}$ for plate negative. But the difference is not as great as the difference in area would lead one to expect.

The current obtained when the filament is illuminated increases greatly when the temperature of the filament is raised. To study this temperature change the effect of illumination was measured for different values of the filament current, the thermionic current being first measured and then the increase due to illumination. The photoelectric current, *i.e.*, the increase in current due to illumination, was found to increase steadily with the temperature, although less rapidly than the thermionic current. At a dull red heat the photoelectric current was in one case found to be 1,400 times as great as that from the same filament when cold. The photoelectric current measured was in this case 7 microamperes. With the same illumination and applied E.M.F. an Elster and Geitel potassium hydride cell, set at the angle of incidence showing maximum effect, gave a current of 0.34 microamperes. This result is all the more remarkable when it is remembered that the area illuminated in the case of the potassium cell must have been at least a hundred times greater than the area of the filament. Increased photoelectric activity at high temperatures has also been observed by Case with barium cells. While the results may prove to be due to secondary causes there is every indication that we have to deal with a real increase in photoelectric activity.

When polarized light was used there was no indication of any difference in the magnitude of the effect for different positions of the plane of polarization, either with the filament or plate.

Coated filament tubes, probably because of the active film referred to above, have been found to show readily the phenomena that result from secondary emission from the plate. With the grid maintained at a positive potential of 120 volts with respect to the filament the curve of plate potential-plate current is the typical dynatron characteristic described by Hull. In one of the tubes used in these experiments (a Western Electric "V" tube) the plate current reaches a positive maximum at about 70 volts, falls again to zero at 94 v., reverses and reaches a negative maximum at 97 v., passes through zero a second time at 107 v. and then continues on the positive side of the axis. The emission of secondary electrons from the plate is especially strong in the range between the positive and the negative maximum. In order to determine whether the secondary emission was influenced by light the plate voltage was set at the point corresponding to the first zero, *i.e.*, on the "negative resistance" part of the characteristic, and the plate was then illuminated. The effects observed were at first erratic, the deflection observed being sometimes positive and sometimes negative, depending upon the tube used, and occasionally changing from positive to negative with the same tube. This erratic behavior was found to be due to the fact that the effect of illuminating the plate is opposite to that of illuminating the filament, the former being negative, the latter positive. The two effects could be separated by using a narrow beam of light.

The effect of illuminating the plate is in the direction corresponding to increased electron emission from the plate. In some cases however the effect appears to be greater than would result from the simple addition of the photoelectric effect to the secondary emission.

When the filament alone is illuminated the deflection is positive, indicating a diminished secondary emission from the plate. The effect is probably due to the increased space charge near the filament, which reduces the velocity of the electrons reaching the plate. Experiments intended to test this explanation are not yet completed.

Cornell University.

The Reflection of X-Rays from Calcite.

By Bergen Davis and W. M. Stempel.

THE reflection of X-rays of various degrees of homogeneity from calcite was measured by means of a double X-ray spectrometer. The experiments covered a wave-length range from .3 to .8 Å. The radiation from a Coolidge tube under accurate control after passing through the slits of the spectrometer fell on the first crystal designated by A. A portion of the general radiation ($\Delta\lambda$) reflected from crystal A fell on a second crystal designated by B. Crystal B was placed so that its reflecting planes were parallel to those of A. A very great reflectivity was observed at this position of parallelism. The reflectivity was measured by comparing the ionization produced by the bean ($\Delta\lambda$) of X-rays before and after reflection from Crystal B.

The reflection obtained from B depended on the degree of homogeneity of the beam reflected from A. The reflectivity of B thus depended on the character of crystal A. The crystals were consequently investigated in combinations or pairs. The reflectivity depended also on the similarity of the crystals and on the perfection of the crystals and on the state of polish of their surfaces.

The pairs of crystals investigated are designated as follows:

($A_1 - B_1$): A_1 was a specimen of Montant calcite, not quite clear. Defects were visible in the interior. B_1 was a clear specimen of Iceland spar. The surfaces of both crystals were polished. ($A_2 - B_2$): A clear specimen of Iceland spar was split along a cleavage plane. The two surfaces were polished. The crystals were so mounted that the reflecting surfaces were the surfaces that had been contiguous before splitting. ($A_3 - B_3$): A clear specimen of Iceland spar was split along a cleavage plane. The surfaces were not polished. The crystals were so mounted that the reflection took place from the planes that were contiguous before cleavage. Maximum reflection was obtained when the several pairs of crystals were mounted with the reflecting planes parallel. Some reflection was obtained when crystal B was turned slightly out of parallelism with A. Interesting curves of energy distribution were obtained by rocking crystal B through small angles each side the position of maximum reflection. These curves differed in width. The dissimilar pair ($A_1 - B_1$) gave much wider rocking curves than the others. The curves were quite narrow for the more perfect pair ($A_3 - B_3$). The width at half maximum in this case was about 16″ of arc. A table is given of the per cent. of the energy reflected from crystal B for each pair. The results indicate that the reflection of homogeneous radiation from a perfect crystal of calcite (were such possible) would be greater than 50 per cent.

Angle.		Percent Reflection.		
		$A_1 - B_1$.	$A_2 - B_2$.	$A_3 - B_3$.
3	.317	25.8	37.3	
3–30	.369	24.6		
4	.422	23.4	33.6	44.4
5	.527	21.5	30.5	
5–30	.580	20.8		43.1
6	.632	19.8	27.9	
6–30	.686			42.8
7	.737	16.7	26.	
7-30	.790			42.8
8	.842	17.3	24.6	

Phoenix Physical Laboratory,
Columbia University.

SOFT X-RAYS OF CHARACTERISTIC TYPE.

BY E. H. KURTH.

THE following investigation was undertaken in the hope that, if special care were exercised to secure a high vacuum, troublesome ion effects experienced by earlier investigators, making the use of a window imperative or the interpretation of results uncertain, would disappear.

The apparatus used was of the type generally employed, the radiation being studied through its photo-electric action upon a metal plate. The target and filament were sealed into the small end of a large conical glass tube and a platinum detecting disk was supported at the other end. In the body of the cone proper a series of plates were arranged diverging from the target to the disk, and they were connected alternately together.

In order to secure as high a vacuum as practicable, the apparatus was joined by means of large bore tubing to a condensation pump. An appendix containing a small quantity of charcoal was also attached to the tube. Electric heaters were employed to bake out the apparatus for several hours at about 400 degrees before making a run. Finally at the end of this period the vapor traps and charcoal were immersed in liquid air.

The detecting disk was connected to an electrometer with a sensitivity of about 1,700 millimeters per volt.

The intensity of the measured effect was found to be essentially independent of potentials up to 800 volts which were applied to the diverging plates to prevent the passage of ions from one end of the tube to the other. Variations in the strength of a magnetic field applied perpendicularly to these plates likewise had no effect. However a potential of 135 volts on the plates and a weak magnetic field were applied during all the runs.

Three elements have thus far been tested, aluminium, iron and carbon. The radiation was found to be entirely characteristic of the particular element used, and was in every sense reproducible.

With aluminium very definite breaks were found in the intensity curve at 38 and 120 volts, indicating three types of radiation corresponding to 326 Å.U. and 103 Å.U., respectively. These values may be uncertain within possibly 5 per cent. It is likely that there is additional radiation shorter than 133, but, if so, it is not strong enough to be detected when superimposed on the effects of the longer wave-lengths.

It seems possible that 103 may be the L series, and 326 may be the M series.

The radiation curves of iron show very definite breaks at 62.8 Å.U., 48.4 Å.U. and 16.3 Å.U. The two long wave-lengths seem to represent the extremes of the M series for iron while the 16.3 break probably corresponds with the L series lines of the element. The radiation effect set in at approximately 25 volts and there was some indication of a break in the curve at 50 volts corresponding to 247 Å.U.

With the element carbon a break at 43.6 Å.U. was very pronounced. This agrees exactly with the extrapolated value for the K series within the limits of error of the extrapolation. No certain indication of any further breaks which might belong to a longer wave-length series have yet been observed. The radiation set in initially at approximately 20 volts.

This work is at present being extended to cover some of the other elements including beryllium.

Palmer Physical Laboratory,
Princeton, New Jersey.

On the Absorption of X-Rays by Chromium, Manganese and Iron.

By William Duane and Hugo Fricke.

In the experiments reported in this paper we have measured the K critical absorption wave-lengths for chromium, manganese and iron. The object of the research has been not only to obtain accurate values for these particular absorption wave-lengths, but also to collect evidence bearing on the following general questions:

(*a*) Has the nature of the chemical compound containing a chemical element any influence on the critical absorption wave-lengths characteristic of that element?

(*b*) Does a critical absorption wave-length represent a single sharp boundary separating wave-lengths of X-rays that are more absorbed from those that are less absorbed, or does it consist of several such boundary lines lying close together?

Evidence has been obtained by means of photographic spectrometers indicating that black phosphorus has a K critical absorption differing slightly from that of yellow phosphorus.[1] Further, on general theoretical grounds, we would expect energy changes due to transfers of electrons in atoms to depend to some extent upon the forces binding the atoms together; and, since the frequencies of vibration of X-rays are proportional (theoretically) to the energy changes, the X-ray wave-lengths should vary, therefore, with the nature of the chemical bonds.

In measuring critical absorption of light elements by means of photographic spectrometers, photographs have been obtained[2] that show certain irregularities between the light and the dark areas. These irregularities were interpreted as indicating a certain "structure" in the absorption limit. Attention was called to the fact that they could be explained by supposing that the critical absorption corresponds to the transfer of an electron from an inner orbit to any one of a number of possible orbits outside of the periphery of the atom, each one of these outside orbits representing an absorption line.

In our measurements we have used an ionization spectrometer with a calcite reflecting crystal. The general arrangement of the apparatus and the mode of procedure have been explained in several publications from this laboratory.[1] We have made only one change in the instruments. The X-rays now emerge from the X-ray tube through a very thin mica window fastened at the end of a long side tube, which projects out from the X-ray tube very near to the crystal. This reduces the absorption of the X-rays by the walls of the tube and by the air, and greatly increases the intensity of the X-rays in the long wave-length region of the spectrum. The rays leave the target making a very small angle with its surface, and the width of the slit is such as to define a beam of rays that has angular breadths ranging from 5′ to 9′ of arc in different experiments. These breadths are of the same order of magnitude as those used in the photographic work.

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[1] J. Bergensen, Comptes Rendus, Oct. 4, 1920, p. 624.

[2] Hugo Fricke, Phys. Rev., Sept., 1920, p. 202; and G. Hertz, Zeits. für Physik, Vol. 3, 1920, p. 19.

The absorbing screens are made by dipping a very thin sheet of paper into a solution containing the chemical element to be investigated, and then allowing it to dry. The deposit of salt obtained in this way is not very uniform. The screens are placed in the path of the X-rays before they strike the crystal. The potential on the tube is so chosen that we do not have the spectrum of second order.

The curves in the lantern slide shown represent the ionization current as a function of the angle that fixes the position of the crystal. A peak appears on one of these curves, which represents the longest L emission line (Ll) in the X-ray spectrum of the tungsten target. This line is quite faint and appears to be complex.

The sharp drops in the curves indicate the positions of the critical absorption. It is possible to estimate the values of the critical absorption wave-lengths to within considerably less than 1/20 of one per cent.

The curves represent the critical absorption of iron, manganese and chromium. The values of the wave-lengths are:

$$\begin{array}{cccc} & \text{Fe} & \text{Mn} & \text{Cr} \\ \lambda \times 10^8\ \text{cm.} = & 1.7377 = & 1.8893 = & 2.0623. \end{array}$$

We used two salts of iron, Fe_2O_3 and $FeSO_4$ ($7H_2O$), and could detect no difference between their critical absorption wave-lengths. This result indicates that the energy changes corresponding to the critical absorption must be sensibly the same for the iron atoms in the two states of chemical combination respectively.

All of the curves have sharp drops corresponding to the K critical absorption wave-lengths, but none of them show more than one such drop, nor do they indicate any irregularity in the neighborhood. In other words, there appears to be no evidence for a "structure of the absorption limit."

The failure of the ionization method to indicate an absorption structure can not be attributed to a lack of resolving power, for, as stated above, the breadth of the absorption drop is about the same as in the photographic measurements.

Irregularities in the thickness of the absorbing screen would tend to obliterate evidence as to the structure of the limit. Our screens, however, were about as uniform as those used in some of the photographic experiments. They were usually just thick enough to cut down the intensity on the short wave-length side of the drop to about one half that on the long wave-length side.

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[1] PHYS. REV., Dec., 1917, p. 624; ibid., Dec., 1919, p. 516; ibid., Dec., 1920, p. 526.

THE PIEZO-ELECTRIC RESONATOR.

By W. G. Cady.

A PLATE or rod suitably prepared from a piezo-electric crystal, and provided with metallic coatings, can be brought into a state of vigorous resonant longitudinal vibration when the coatings are connected to a source of alternating E.M.F. of the right frequency. Under these conditions, the plate reacts upon the electric circuit in a remarkable manner.

The vibrations are a consequence of the so-called "converse" piezo-electric effect, *i.e.*, the deformation resulting from an electric stress; while the periodically strained condition of the plate, through the action of the "direct" effect, sets up a periodic component of polarization which causes the reaction referred to.

On the theoretical side, the phase and magnitude of the counter-polarization in the plate are derived, as well as expressions for the current flowing to the plate and for the total current in the oscillatory circuit. The plate is assumed to be in parallel with the circuit capacity. It is shown that, owing to the piezo-electric polarization, and to the absorption of energy in the plate, the apparent electrostatic capacity and resistance of the plate are not constant, but depend upon the frequency somewhat as does the motional impedance of a telephone receiver. Over a certain range in frequency the capacity may even become negative.

A graphical method is developed for presenting the results of the theory. By an application of this method, it is possible, after making a series of purely electrical observations, to deduce the coefficient of viscosity of the material, even though the absolute value of the piezo-electric constant is not known.

Methods are described for mounting a small plate of piezo-electric crystal upon a rod of any solid elastic substance in such a manner as to excite longitudinal vibrations in the entire rod. The high-frequency viscosity of the rod can then be found, subject, however, to more or less error due to losses in the cement which attaches the crystal plate to the rod.

Experiments on longitudinal vibrations in rods or plates of steel, quartz, and Rochelle salt are described.

The possibility is discussed of using the piezo-electric resonator for a standard of high frequency; for excluding from a circuit oscillations of a given frequency; and as a coupling device to transfer small amounts of power from one circuit to another at a particular frequency.

Wesleyan University, Middletown, Conn.,
February 3, 1921.

THE BINAURAL LOCATION OF PURE TONES.

By R. V. L. Hartley and Thornton G. Fry.

THE theoretical explanation of experiments on the apparent location of sounds has been very largely confined to the relatively simple case of pure tones where the phenomena are almost entirely binaural. The theories advanced have been quite successful in explaining the results of that type of experiment in which the intensities at the two ears are kept equal and the direction of the image is observed for various phase differences. For those experiments in which the phases are kept equal and the intensity ratio is varied no satisfactory explanation has so far been offered.

The present paper presents a theory which when applied to the latter type of experiment explains why the observer has difficulty in assigning a definite direction to the image and why different observers get widely different results. The results given by Stewart[1] for this type of experiment are shown to be in entire agreement with the present theory. When the theory is applied to experiments with equal intensities and varying phase difference it indicates why the results of different observers are in substantial agreement with one another and why the theories advanced have explained them successfully even though certain factors have been neglected.

Experiments are suggested in which the phase difference and intensity ratio are adjusted simultaneously in such a manner as more nearly to approach the conditions of every day experience. To facilitate such experiments computed curves connecting the polar coördinates of a source with the phase difference and intensity ratio of the resulting stimulus are given for sounds of representative frequencies. Emphasis is placed upon the importance of recording the apparent distance as well as the direction of the sound image.

Research Laboratories of the American Telephone & Telegraph Co.
and the Western Electric Company, Inc.,
February 10, 1921.

NOTE ON THE CHARACTERISTICS OF THE NEW SINGING TUBE.

By Charles T. Knipp.

THE temperature difference that is necessary to cause the new singing tube to emit a tone, when the portion B (Fig. 1) is kept at room temperature while the tip A is heated, was observed to be about 400° Centigrade. If, however, B is cooled to the temperature of liquid air the temperature difference

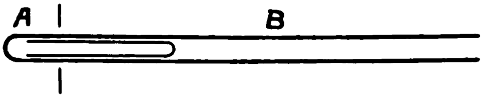

Fig. 1.
New singing tube.

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[1] Physical Review, May, 1920, p. 425.

necessary is greatly reduced, being about 200° Centigrade.[1] The pitch is also considerably lowered.

It was deemed desirable to make quantitative measurements of these temperature differences and also of the corresponding pitch of the tone emitted. To this end *B* was held successively at different temperatures, ranging from that of liquid air to values considerably above room temperature, while *A* was heated electrically in each case to a temperature where a tone was emitted continuously. The two portions *A* and *B* were each housed within a separate copper tube of about 1 cm. wall thickness and heavily insulated with asbestos. The temperatures were accurately measured by means of thermo-junctions,—three being attached to *A* and three to *B*. The temperature control of each section was good and could be held fairly constant at will.

Table I.

No.	A Average Temp. in Degrees C.	B Average Temp. in Degrees C.	Total Temp. Difference in Degrees C.	Temp. of *B* in Absolute Measure.	Vibrations per Second.
1.........	1	−181	182	92	213
2.........	204	− 88	292	185	300
3.........	355	− 16	371	257	378
4.........	448	+ 26	422	299	425
5.........	524	+ 57	464	330	450

The results from the only run thus far made are contained in Table I. In observation No. 1 the part *B* was placed within a glass jacket heavily wrapped with aspestos and cooled directly to − 181° C. by means of liquid air. The temperature of *A* was allowed to fall until the tone emitted was just main-

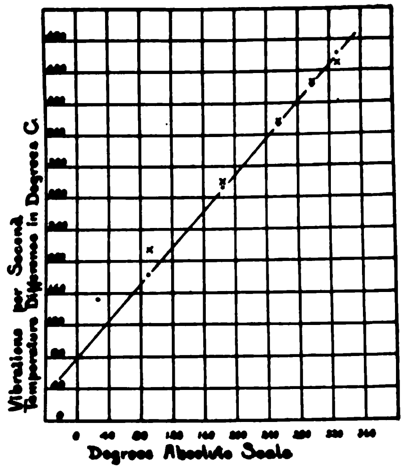

Fig. 2.

Characteristics of the new singing tube.

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[1] Phys. Rev., N.S., Vol. XV., p. 336.

tained. This by repeated trials was found to be at 1° C. Thus the temperature difference when B was cooled to $- 181°$ C., for this particular tube, was found to be 182° C. In observations 2 and 3 the part B was placed in a special copper tube designed by the author[1] some years back for the determination of intermediate temperatures. Observation 4 was for B at room temperature (note that it was now necessary to heat A to 448° C.), while in No. 5 the part B was warmed up to 57° C., and the tip heated electrically to 527° C. before the tube responded.

The absolute temperatures of the part B are given in the second last column, while the corresponding vibrations per second are listed in the last column. These data are represented graphically in Fig. 2, in which the total temperature differences as ordinates are plotted against absolute temperatures. The relation is strictly linear except possibly for the last reading at 330° absolute temperature. By extending the straight line to the left we are able to determine the temperature difference that should maintain the tone when B is cooled to absolute zero. For this particular tube the graph shows this temperature difference to be about 80° C.

The same figure also shows the corresponding vibration frequencies (indicated by crosses) plotted to the same scale against absolute temperatures. This relation also seems to be linear except for the point taken at 91° absolute. The pitch was determined by means of a tone variator.

Lastly the vibration frequencies and temperature differences in degrees centigrade, as shown by the graphs, are nearly equal numerically. This, however, should be considered as a coincidence.

Observations with tubes of different pitches and extending over wider temperature ranges are under way.

UNIVERSITY OF ILLINOIS,
February 10, 1921.

BROKEN TONE FROM REED INSTRUMENT.

BY JOHN B. TAYLOR.

IN musical instruments of the wood wind group, the pitch of the tone is generally considered as determined by the distance from the mouthpiece to the nearest of the side openings or vents which are uncovered. The opening or closing of vents below the first has some influence in modifying the pitch and quality.

If the vent hole nearest the mouthpiece is small, and located in a proper proportional relation to the total length of the air column, a harmonic tone is determined, in which case there may be considerable variation in the size and location of the harmonic vent without appreciably affecting the tone.

If the harmonic vent occupies a position intermediate to those positions proper for two harmonics, other factors may determine which one of the two harmonics will be heard—but, generally, whichever one starts to sound will continue so long as there is no change in the manner of blowing.

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[1] PHYS. REV., Vol. XV., p. 125.

The bassoon differs from the other wood wind instruments in a large degree because the theoretical locations of the ventages are much more widely separated than the spread of the finger tips. This difficulty was met in the early days before the development of key mechanisms by leaving an unusually thick wall on the wooden tube and boring the long ventages of small size at an angle. As a result, the upper and lower portions of the instrument are more "closely coupled" than is the case with the other wood wind instruments such as flute, oboe and clarinet, and the uncovering of a single vent does not approximate an open-end condition of the air column.

While experimenting with unusual cross fingerings, several combinations have been found producing tones quite foreign to the normal characteristic voice of the instrument. The term "broken tone" has been selected as best describing these unusual sounds.

One of these broken tones has been selected for illustration and wave form records have been made. The particular tone is that produced by sounding the low F (approximately 87 cycles). If while this F is sounding, the third finger of the left hand is raised, uncovering the vent which is ordinarily considered as that through which the tone d (approximately 145 cycles) speaks, the complex tone results.

A skilled observer judged the complex tone to be composed of B^b (116 cycles), f (174 cycles) and a differential resultant tone of BB^b (58 cycles).

Such tones would result if the third finger left hand vent (called d) is regarded as being near enough to the octave position for the low F (87 cycles), tending to produce the octave f (174 cycles.) Also, if the d vent is sufficiently long and restricted to produce B^b at 116 cycles, when the air column is regarded as ending at the outside of the third finger d vent, the second member of the complex tone may be explained.

The broken or somewhat rattling character of the tone was judged to indicate that the mode of vibration alternated rapidly between that producing B^b as a fundamental, and that giving f as a second harmonic. The rapid alternation of the two produces the low pitched differential tone BB^b.

The photographic records of the sound wave are considered to be in agreement with this suggested explanation. The complex or broken tone is not easy to decipher because the normal single tones of the bassoon are rich in harmonics and of complicated shape.

Schenectady, N. Y.,
February 11, 1921.

Electron Tube Drive for Tuning Forks.

By E. A. Eckhardt, J. C. Karcher and M. Keiser.

Eccles and Jordan and Abraham and Bloch have recently described methods for maintaining the vibrations of tuning forks by the use of electron tube circuits. Regenerative circuits are used, the coupling between the grid and plate circuits being provided by the tuning fork. There is no appreciable electrical or magnetic coupling.

It was found by the authors that this method of driving a tuning fork does not yield large amplitudes unless the circuit is carefully designed. If the circuit is properly adjusted for one electron tube it will frequently work less well with another of the same type and sometimes not at all.

A more flexible driving arrangement was devised by the authors which permits the driving of a given fork with any tube of a given type, provision having been made for the slight readjustments which are necessary when the tubes are changed. The adjustments are such that even the transition from one type of tube to another can be made in a few minutes.

In our driving arrangement the tuning fork does not provide the coupling between the two branches of the regenerative circuits. When the circuits are closed the fork driving magnet is energized and the fork is displaced from its equilibrium position. This sets up a small initial vibration of the fork which induces in the plate circuit a small E.M.F. of the fork frequency and hence an alternating component of the plate current. The plate circuit is coupled loosely to the grid circuit. If the mutual inductance between the grid and plate circuits is of the proper sign the alternating current of the fork frequency will be regeneratively amplified and the periodic driving forces on the fork will increase giving rise to larger fork amplitudes. The limit is set primarily by the operating characteristics of the electron tube.

A transformer may be inserted in the plate circuit and the arrangement may then be used as a generator of feeble alternating currents. By proper design of the circuits a very satisfactory wave-form can be obtained. The tuning fork acts as an automatic frequency control of high precision.

We believe that the apparatus is likely to be quite useful for the precise measurement of electrical quantities which are a function of the frequency.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
February 11, 1921.

A HIGH-SPEED OSCILLOGRAPH CAMERA.

BY E. A. ECKHARDT.

A HIGH-SPEED oscillograph camera was developed in our Sound Laboratory which has the advantage of being available for use in connection with any type of oscillograph. It has been used with G. E. oscillographs, an Einthoven galvanometer, and to record various purely mechanical vibrations.

An aluminum drum 5 feet in circumference and 4 inches wide is arranged to have a film of similar dimensions stretched over its convex surface. This drum is mounted in a light tight housing, which is something over 8 inches wide, to which access is obtained by means of a sliding door. The shaft protrudes from the housing on both sides. One end of it carries the driving pulley which is belted to a motor, the other carries a phosphor bronze worm which engages an idling cast iron wheel. In contact with the flat face of this wheel are the pole faces of an electromagnet which is energized only when an exposure is being made. During the exposure this electromagnet grips the cast iron wheel and prevents its rotation. Consequently the worm feeds along the wheel and the drum executes a spiral motion, the pitch of which is determined by that of the worm and the wheel. With a spiral of ½ inch per revolution nearly eight full revolutions are obtained on the film. This gives a film record 40 feet long. Several worm and wheel combinations can be mounted interchangeably on the drum, which permits the obtaining of longer and narrower records.

The shutter consists of a thin steel blade which covers the shutter opening. A plunger attached to the blade is pulled into a controlling solenoid against a spring when exposure is to be made. When the shutter solenoid is deënergized the shutter is closed by the action of this spring.

The shutter and spiral initiating solenoids are connected in series. They are energized simultaneously therefore and the shutter opens and the drum begins to spiral at the same moment. This operation can be controlled by a relay and the performance may therefore be subjected to remote control.

When a record is to be made the drum is brought to the desired speed. By closing the control key the shutter is opened and the drum begins its spiral motion. A contact is whipped open by the driving pulley when the drum has spiralled the desired distance. The shutter closes and the gripping magnet releases the cast-iron wheel which again begins to idle. A spring then restores the drum to its initial position in the housing.

Records have been obtained in which the 40 feet of film covered a time period of less than one second. With suitable time index marks and a steady rotational speed of the drum, time intervals may be read to one millionth of a second.

Bureau of Standards,
Washington, D.C.,
February 11, 1921.

A Method of Measuring Surface Tension of Liquids.

By Harry Clark.

Many good methods of measuring surface tension require either such elaborate mathematical treatment or such expensive apparatus that they cannot be used in elementary laboratory courses, and none of the other methods are quite satisfactory. The method here described requires little apparatus, is simple mathematically, and gives fairly accurate results.

The apparatus consists of a glass tube, perhaps 2 cm. in diameter and 10 cm. long, one end of which is provided with a metal plug held in place by a bit of rubber tubing. A fine hole is drilled axially through the plug, which is turned down so that it terminates externally in a small tube about 3 mm. long and of diameter appropriate to the liquid used—about 0.75 mm. for water.

If liquid is poured into the glass tube, held vertically in a clamp, a drop will form on the end of the metal tube, the horizontal diameter of the drop being equal to the external diameter of the tube. As more water is added, the drop increases in size vertically until it becomes hemispherical (to an excellent degree of approximation) beyond which it becomes unstable and falls off. The surface tension is given by

$$S = rdg\left[\frac{h}{2} + \frac{r}{3}\right],$$

where h is the maximum height of the liquid column above the end of the metal tube; d, the density of the liquid; r, the radius of the metal tube; and g, the acceleration of gravity.

The liquid is introduced slowly and continuously through a funnel, the tube of which is drawn down very fine and lies against the inner surface of the large glass tube. Heights are measured on a paper scale behind the tube, or better still, on graduations etched on opposite sides of the tube. The diameter of the metal tube is measured with a micrometer gauge. No cleaning is required beyond wiping the external edge of the metal tube with a cloth. The error in the results is proportional to that of the measurements, which need not exceed 2 per cent.

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH.

THE SPECTRAL STRUCTURE OF THE LUMINESCENCE EXCITED BY THE HYDROGEN FLAME.

BY HORACE L. HOWES.

THE discovery of the luminescence of many substances when partially bathed in the hydrogen flame but at temperatures below red heat has recently been made by Prof. E. L. Nichols. The present paper represents only one of several tests which have recently been developed to prove that this new type of luminescence exhibits properties similar to that excited at room temperatures by the iron spark, the cathode rays, etc. To this end calcium oxide under hydrogen flame excitation was found to yield the same series of bands as calcite under iron spark excitation and three samples of the Lenard and Klatt phosphorescent sulphides under the two distinct modes of excitation yielded similar series but the distribution of intensity was very different. A new Hilger spectro-photometer was employed with the specimen mounted on a rotating disk of brass gauze and the analysis of the spectra was rather difficult because of very extensive overlapping of the bands.

NEW HAMPSHIRE College,
February 9, 1921.

THE MOTION OF ELECTRONS BETWEEN COAXIAL CYLINDERS IN A UNIFORM MAGNETIC FIELD.

BY ALBERT W. HULL.

THE thermionic current between a straight filament and a concentric cylindrical anode is unaffected by a uniform magnetic field parallel to the filament so long as this field is smaller than a critical value, but falls abruptly to zero when the field is increased beyond this value. The same is true if the outer cylinder is cathode and the inner anode, except that the fall is less abrupt.

The relation between these quantities is very simple in practical cases.

The equations of motion of the electrons, using cylindrical coördinates r, θ, and z, are:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = F\frac{e}{m} - H\frac{e}{m}r\frac{d\theta}{dt}, \tag{1}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = H\frac{e}{m}\frac{dr}{dt}, \tag{2}$$

$$\frac{d^2z}{dt^2} = Z\frac{e}{m}, \tag{3}$$

where F and Z are the electric intensities in the radial and axial directions respectively, and H the magnetic field, which is uniform and parallel to the filament. F is a function of r only.

The first integrals of these equations are readily obtained, giving the velocity components dr/dt and $r(d\theta/dt)$. The critical magnetic field, at which the electrons are just able to reach the cylinder, is found either by placing the radial velocity component equal to zero; or by equating the square of the tangential velocity component, multiplied by half the mass, to the total work done on the electron by the radial electric force, *i.e.*,

$$\frac{dr}{dt} = 0,$$

or

$$\frac{1}{2}m\left(r\frac{d\theta}{dt}\right)^2 = eV. \tag{4}$$

This gives

$$V = H^2\frac{e}{8m}\left(R^2 - 2r_0^2 + \frac{r_0^4}{R^2}\right) + H\frac{r_0v_0}{2}\left(1 - \frac{r_0^2}{R^2}\right) - \frac{m}{2e}\left(u_0^2 + v_0^2\left[1 - \frac{r_0^2}{R^2}\right]\right),$$

where R and r_0 are the radii of outer and inner cylinders respectively, V the potential difference between them, u_0 and v_0 are the radial and tangential components, respectively, of the initial velocities of the electrons, and H the magnetic field that is just sufficient to prevent electrons reaching the anode.

If the electrons start from a filament of radius r_0 small compared with R, and the initial energies $\frac{1}{2}(m/e)u_0^2$ and $\frac{1}{2}(m/e)v_0^2$ are small compared with V, this becomes:

$$H = \sqrt{8\frac{m}{e}}\,\frac{V^{1/2}}{R}. \tag{5}$$

This is the case of a straight filament cathode in the axis of a cylindrical anode.

If the outer cylinder is cathode and the inner anode, eq. (4) becomes:

$$H = \frac{r_0}{R^2}\sqrt{\frac{8m}{e}V} + 2\frac{mv_0}{eR}. \tag{6}$$

This applies to a helical filament with anode wire in its axis. It applies also to the motion of positive ions produced by electrons from a straight filament in the axis of a cylindrical anode. In this case R is the distance from the axis at which the ions are produced, and e/m the ratio of charge to mass of the ion.

If the radii of both cylinders are large compared with their distance d apart Eq. (4) becomes:

$$V = H^2\frac{e}{2m}d^2 + Hv_0d - \frac{u_c^2}{2\frac{e}{m}}. \tag{7}$$

This is the case of plane parallel plates with electric field normal and magnetic field parallel to the planes.

The approximate paths of the electrons can be calculated for the case of the straight filament and external concentric anode, on the assumption that the space charge distribution is the same with magnetic field as without. This assumption is justified by the experimental fact that the positive ionization, as measured by a central negatively charged electrode (the filament is a small helix in this case, with collecting electrode in its axes) is unchanged by a magnetic field, even when the current to the anode is cut down to less than 1 per cent. of its maximum value by the field. The equation of the path on this assumption is:

$$r = R\left[\sin\frac{2}{3}\theta\right]^{3/2}. \tag{8}$$

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.,
February 10, 1920.

AN ELECTRICAL DOUBLET THEORY OF THE NATURE OF THE MOLECULAR FORCES OF CHEMICAL AND PHYSICAL INTERACTION.

BY R. D. KLEEMAN.

THE forces of interaction between molecules which give rise to the internal heat of evaporation of liquids, their surface tension, etc., are explained by means of the electric forces associated with each atom on account of the electrons and positive electrical charges it contains. Whatever the arrangement of electrical charges their effect at a point some distance from the atom could be represented by an electrical doublet. Now we may arrange a number of doublets in space so that they attract each other in all directions, by placing them at the points of intersection of three sets of parallel equidistant planes which are at right angles to each other, so that opposite charges face each other in a row of doublets, and the order of the charges is different in two adjacent rows. When the doublets are in motion it would follow from the principle that *the potential energy of a system tends to a minimum* that the doublets would undergo motion of translation and rotation in such a manner that on the average attraction between the doublets would result, and that it would be as large as possible. Thus though any direction of the axis of a doublet in space might be equally probable, attraction would result through an association of the directions of the various doublets. The attraction between two molecules in a liquid would accordingly vary inversely as the fourth power of their distance of separation, and vary directly as the product of the moments of the molecular doublets, multiplied by a factor which would depend on the nature of the interaction of the molecules, and hence depend on the temperature of the liquid, the distance of separation of the molecules, etc.

It can be shown that it follows from the stopping power of the α particle that the moment of the doublet of an atom is proportional to the square root of its atomic weight, and that the moment of a molecule is equal to the sum of the moments of its atoms. This is confirmed by the results of previous investigations by the writer on the law of attraction giving rise to the internal heat of evaporation of liquids, etc., according to which the attraction an atom exerts is proportional to the square root of its atomic weight.

Union College,
February, 1921.

A Copernican Atomic Model Based on Electromagnetic Theory.

By Albert C. Crehore.

The atomic model applied to the hydrogen atom is represented in meridian section in Fig. 1. The nucleus, C, is relatively enlarged, it being impractical to represent nucleus and electron to the same scale. The effective nucleus consists of two charges, namely the nucleus, C, with positive charge $2e$ and one negative electron, A or B, of charge e which are inseparable, thus making an effective nuclear charge of plus e. Thus from the neutral atom only one electron is detachable. Both electrons, A and B, in the normal state of the atom at the zero of temperature are in contact with the positive charge, C, one at each pole. All three charges rotate about a common axis, PQ, but the electron has no tendency to describe an orbit revolving around the nucleus on the Saha theory. The rotating nucleus, so far as great distances are concerned as I have shown elsewhere,[1] may be replaced by an equivalent closed circular charged rotating ring.

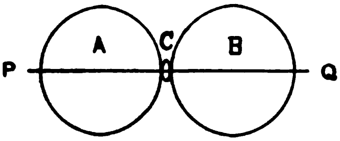

Fig. 1.

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[1] Abstract, Phys. Rev., Feb., 1921, p. 252.

The precise state of motion of rotation of the nucleus has not been rigorously solved on the basis of Saha's fundamental equations, but a state of rotation of the nuclear charge, which is supposed uniformly distributed throughout the volume as in the "solid" Lorentz electron, is not incompatible with the theory. It is not necessary to suppose that the angular velocity of an element of charge is constant for all radii; it may increase with the radius. It follows from the theory that two coaxial rings revolving in the same direction exert an accelerating effect upon each other when the distance between centers is several diameters of the rings. This problem is managable because the series employed remains convergent. The convergence disappears, however, as the rings are brought together to within a fraction of the radius, and an exact solution without resorting to these series must apparently be found before the mutual effect of two elementary rings within the body of the nucleus becomes known.

Assuming that a state of rotation meets the theoretical requirements for equilibrium, the spherical charge at rest is unstable, and becomes a pure fiction. This consideration makes possible an approximation to the shape of the positive charge in advance of an exact solution. The potential energy of a uniformly distributed spherical charge, E, at rest is[1]

$$W = 3E^2/5ak. \tag{1}$$

The mass[2] of the charge is

$$m = \frac{4}{5ak}\left(\frac{E}{c}\right)^2. \tag{2}$$

Eliminating the quantity $E^2/5ak$ between (1) and (2) gives the potential energy of the charged sphere at rest as

$$W = \frac{3}{4}\,mc^2. \tag{3}$$

Einstein and others have adopted mc^2 rather than $mc^2/2$ as the total energy associated with a mass m. In view of (3) the figure $mc^2/2$ cannot represent the total energy, and allowing for an equilibrium rotation the $3mc^2/4$ should be increased by the energy due to the rotation. Why is not the $mc^2/4$ the energy due to the rotation? For, together with (3) the total energy becomes mc^2. Let it be assumed that this is the case, and also that the shape of the sphere is distorted into an oblate spheroid when in equilibrium rotation. The pressure per unit area of surface of the sphere at rest is given by Schott[3]

$$P = 3E^2/8\pi a^4k.$$

Surface pressure and a tension along the surface are mathematically equivalent and convertible by the formula

$$T = P/\frac{1}{r}\left(+\frac{1}{r^1}\right) = \frac{a}{2}P,$$

T being the tension and r and r' the two radii of curvature of the surface, which merge into a when the surface is spherical. Let it now be supposed that the sphere is distorted by the rotation into an oblate spheroid, both the volume of it and the tension remaining constant. The work required is that to stretch the surface over a larger area, say A. The work is, therefore, AT, and this is to be equated to $mc^2/4$, giving

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[1] Schott, Electromagnetic Radiation, Eq. (381).

[2] Loc. cit., Eqs. (379), (380).

[3] Loc. cit., Art. 257.

$$AT = A\frac{a}{2}P = 3AE^2/16\pi a^3k = mc^2/4. \qquad (4)$$

Whence, by (2) and (4)

$$A = \frac{16}{15}\pi a^2.$$

The area of a sphere, being four great circles, must be stretched according to this result 16/15 of a great circle. If the shape is an oblate spheroid, this fact determines the eccentricity and the major and minor axes. The result is the eccentricity

$$e = 0.945,$$

and the ratio of the axes

$$a/b = 3.058.$$

Numerically, setting $E = 2e = 2 \times 4.762\ 10^{-10}$ E.S.U., and $m = 1.659 \times 10^{-24}$ grams, (2) gives the radius of the sphere of the nucleus at rest as

$$r = 4.86 \times 10^{-16} \text{ cm.}$$

The equatorial radius of the spheroid is, therefore, 7.22×10^{-16} cm., and the polar radius 2.20×10^{-16} cm.

In a former communication[1] the attraction between two such atoms as in Fig. 1 was given when the rotating nucleus is replaced by an equivalent ring of radius a_H, the frequency of rotation being $2K$, twice the Rydberg constant. When the atoms are given all orientations the force comes out an attraction equal to the known gravitational attraction, provided the ring has a radius

$$a = 6.4 \times 10^{-16} \text{ cm.}$$

This ring falls within the body of the rotating nucleus, being less than the equatorial radius 7.22×10^{-16} above. It is not possible to say how to calculate a ring equivalent to the spheroid above until the angular velocities of the elements at different radii are known or obtained from the Saha theory.

The ring equivalent to the negative electron, also a rotating body, is much larger in diameter, and has a linear speed about 1/30.4 part of the ring of the nucleus, and on this account has a small almost negligible gravitational effect.

The natural free period of the nucleus, when suddenly deformed in shape, as by a collision with one of it's own electrons, probably corresponds to the greatest frequency of any known body, as it is also the body of smallest known dimensions. Its natural frequency may be given by setting the whole energy content equal to $h\nu$, giving

$$h\nu = mc^2 \text{ or } \nu = 2.27 \times 10^{23} \text{ frequency.}$$

[1] Loc. cit.

This corresponds to a wave-length of

$$\lambda = c/\nu = 13.2 \times 10^{-13} \text{ cm.}$$

This appears to be the order of magnitude of the wave-length to be expected. The vibration implies a periodical interchange between the rotational energy and the potential energy due to the changing shape of the nucleus.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
February, 1921.

THE COPERNICAN ATOM RADIATING ENERGY. A PHYSICAL INTERPRETATION OF PLANCK'S QUANTUM RULE.

BY ALBERT C. CREHORE.

LET Fig. 1 of the preceding abstract represent an atom of hydrogen gas subjected to bombardment by a stream of electrons. To study the effect of a passing electron, begin by neglecting the rotation of the parts of the atom. The mechanical force upon the atom due to the moving electron may be divided into two parts, (1) the electrostatic part, and (2) the part arising from the velocity. The electrostatic force upon one electron in the atom due to the passing electron, barring an actual collision which is not necessary to account for the radiation, can never be as great as the electrostatic force exerted by the rest of the atom upon its own electron, because of the comparatively great distance to the passing electron. Hence, the electrostatic part (1) of the mechanical force alone can never pull an electron away from the atom.

The part (2) due to the velocity of the passing electron is very much less than the electrostatic force, unless the electron moves with a velocity very close to that of light. The experimental velocity observed to produce radiation is of the order 2×10^8 cm./sec. (11 volts) in hydrogen, a speed less than one hundredth that of light. Hence, no such low speed electron will ever succeed in separating an electron from the atom according to theory so long as the rotations of the parts of the atom are neglected. The chief effect of these forces (1) and (2) is to increase the pressure of one electron against the nucleus and diminish that of the other, thus rendering one more readily detachable than the other.

The key to the behavior of this atom is the rotation of the nucleus coupled with its elongated[1] shape. The passing electron exerts a couple on the nucleus tending to turn the direction of its axis of rotation. This effect is due to the term $(\mathbf{q}_1 \cdot \mathbf{q}_2)\mathbf{R}$ in the Saha ponderomotive force equation. Referring to Fig. 2, let *PQR* be the path of the electron passing the atom *H*. At the nearest point *Q*, where the effect is a maximum, this term gives a force shown by the arrow, *A*, directed toward *Q*, the vector **R** joining the two points. This being the force upon an element in the upper side, the force upon a corresponding element of the rotating nucleus diametrically opposite is reversed in direction and represented by the arrow, *B*, directed away from *Q*, the two forces forming a couple tending to turn the axis of rotation.

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[1] See abstract, "A Copernican Atomic Model Based on Electromagnetic Theory."

When the effect of this couple is small, the nucleus may turn a little and roll smoothly without losing contact with the electrons at any time. Then, as the passing electron leaves, this motion rights itself thus restoring all of the energy received back to the passing electron again, which departs with as much energy as it had on arrival. It is considered that the energy of the passing electron must be sufficient to cause the nucleus to roll over at a rate sufficient to make it leave contact with and bump against or collide with one of its own electrons before energy may be said to be received by the atom.

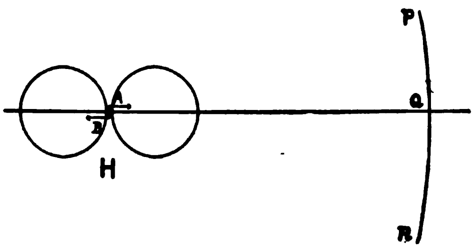

Fig. 2.

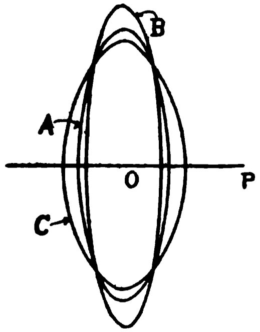

Fig. 3.

This process may be likened to the rolling of an elliptical wheel along a horizontal table. When the velocity of the wheel exceeds a certain critical value, the wheel leaves contact with the table and bumps along each half revolution. This analogy also points to the existence of a critical velocity.

These collisions set up the high frequency vibrations characteristic of the nucleus,[1] and the energy thus absorbed is not returnable to the passing electron as it was before, being transformed in character. It is postulated in advance of a rigid proof that this high frequency energy is absorbed by the nearest charges and is effectually imprisoned within the atom, very little, if any, of it being radiated into space as high frequency waves. The only way open to the atom under these circumstances to get rid of this energy is to eject an electron. How this takes place is made clear by electromagnetic theory.

Picture the nucleus thus set into high frequency vibration something like Fig. 3. The axis of rotation, *OP*, is supposed to remain fixed while the meridian section normally represented by the curve, *A*, becomes elongated into *B* and foreshortened into *C* as extreme distortions during a complete cycle. The motion is probably not as simple as this, but any complexity in the vibration does not affect the argument, for the best that can be expected is to obtain an approximation to the effect that such a vibration may produce. The area of the surface undergoes a periodical variation, and there must occur a corresponding periodical exchange of rotational energy with the potential energy due to the shape. But, the whole energy content of the nucleus takes part in this exchange, and the available energy is, therefore, large.

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[1] Loc. cit.

To approximate the force upon an electron in the immediate vicinity due to this vibration, divide the nucleus into two halves by the equatorial plane, and replace one half by a single equivalent point charge on the axis, say at the center of volume. This point will then oscillate along the axis *OP* during a complete cycle, since the center of volume moves along this line. The whole nucleus may thus be replaced by two point charges vibrating along the axis, first from and then toward each other, let us say harmonically. The average mechanical force that two such vibrating point charges exert upon an electron moving in its vicinity, together with the force exerted by the second electron inseparably associated with the nucleus, may represent the whole average force that the remainder of the atom exerts upon an electron ejected from itself. This force derived from the Saha theory is

$$F = \frac{e^2}{kr^2(1 - \beta_1^2)^{1/2}}[1 - 2\beta_2^2(1 - \tfrac{1}{2}\sin^2\alpha)]. \tag{1}$$

Let E_1, Fig. 4, represent the position of the ejected electron experiencing this force, and E_2 the position of the nucleus from which E_1 has become detached.

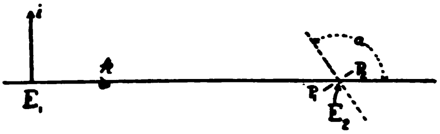

Fig. 4.

The broken line, P_1P_2, represents the direction of the axis, *OP* of Fig. 3, and the two point charges equivalent to the nucleus are supposed to move back and forth along these lines, P_1 and P_2. When the axis is directed toward the electron E_1, $\sin\alpha = 1$, and (1) reduces to

$$F = \frac{e^2}{kr^2}\,\frac{1 - \beta_2^2}{(1 - \beta_1^2)^{1/2}}, \quad \text{polar force.} \tag{2}$$

When E_1 is in the equatorial plane of E_2, $\sin\alpha = 0$, and (1) becomes

$$F = \frac{e^2}{kr^2}\,\frac{1 - 2\beta_2^2}{(1 - \beta_1^2)^{1/2}}, \quad \text{equatorial force.} \tag{3}$$

In these equations $\beta_1 c$ is the velocity of the electron E_1, and $\beta_2 c$ the maximum velocity of the two harmonically vibrating points substituted for the nucleus. When the expression is positive the electron E_1 is attracted toward the nucleus. The whole force acts along a radial line, there being no component perpendicular to the line joining the nucleus to the electron.

Equation (2) states that the force is always an attraction when the nucleus points its pole toward the electron E_1, for it may be assumed that β_2 cannot exceed unity. Equation (3) states that the force may be a repulsion provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Moreover, this repulsion may be very great indeed if the speed of the electron, $\beta_1 c$, approaches the velocity of light, because of the factor

$(1 - \beta_1^2)^{1/2}$ in the denominator. Although the mass increases with the speed, so also does the force in a similar fashion, and it should not be surprising if velocities approaching that of light are obtained. In fact, the force due to such a vibrating nucleus more than counterbalances the immense electrostatic force normally holding the electron in contact with the nucleus, and will in some cases succeed in ejecting an electron from the atom.

It also appears from (1) that there exists a radius at an angle, α, along which the force is zero, provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Equating (1) to zero gives this angle

$$\sin^2 \alpha = 2 - 1/\beta_2^2. \tag{4}$$

If the nucleus could turn its axis to this critical direction, the force upon the electron would be zero, and it would consequently move away with a uniform velocity. There would then occur no loss of energy by radiation, because no energy is radiated by uniform motion in a straight line. Under these circumstances the electron would carry away from the atom the whole of its own kinetic energy, and by this means the atom would lose far more energy than it normally does.

Regarding the nucleus and its electron, namely the effective nucleus, as a single physical system, like all physical systems left to themselves the entropy tends toward a maximum. In this instance the system strives to get rid of energy at a maximum rate, and would succeed in increasing the rate if the axis of the nucleus could turn immediately to the critical angle expressed by (4). The mutual forces between it and the electron govern the motion, and it must be expected that the pole of the nucleus will oscillate about this critical angle of zero force, being damped because of the loss of energy by radiation into space from the moving electron. The acceleration of the electron at the distance, x, should, therefore, be expressed by an equation of the type

$$\frac{d^2x}{dt^2} = k_1 \nu e^{-\nu t} \cos \nu t, \tag{5}$$

which multiplied by the mass is the force upon the electron. For, this force oscillates harmonically about a zero value, changing from repulsion to attraction, and decreasing in amplitude as the energy of the high frequency vibration is spent in the secondary process of radiation from the moving electron. The frequency $\nu/2\pi$ expressed by (5) refers to the oscillation of the pole of the nucleus. The whole motion is determined by this equation. A first integral with respect to time gives the velocity.

$$\frac{dx}{dt} = \tfrac{1}{2}k_1 e^{-\nu t}(\sin \nu t - \cos \nu t) + k_2. \tag{6}$$

Now the total energy radiated by an electron moving with this velocity may be calculated from the Liénard expression for the rate of radiation, which for straight line motion is

$$\dot{R} = \frac{2e^2}{3kc^3} \frac{\left(\frac{d^2x}{dt^2}\right)^2}{(1 - \beta^2)^3}. \tag{7}$$

The Saha expression for the rate of radiation is not yet known to the writer. Assuming that the electron goes out and back along a radius, the constant of integration becomes zero, because the velocity is zero when the time is infinite. The whole energy lost by radiation in one excursion out and back is then

$$E = \int_0^\infty \dot{R}dt = \frac{\pi e^2 k_1^2}{2kc^3}\left(1 + \frac{23}{90}\frac{k_1^2}{c^2} + \cdots\right)\nu', \tag{8}$$

where ν' is the frequency corresponding to the angular velocity ν in (5), and k_1 is constant by hypothesis. Consequently the whole coefficient of the frequency is a constant, and (8) exactly expresses Planck's quantum rule, radiated energy is equal to a constant times the frequency.

The value of Planck's constant, h, cannot be determined from this, but an estimate of the initial velocity of the electron may be obtained required to make (8) equal to $h\nu'$. The series in the parenthesis may have any value from 1 to infinity according to the value of k_1, and by (6) $k_1/2$ represents the initial velocity. Denoting the sum of this series by S, its value must be $S = 138.2$ to give the proper value of h, this value being

$$h = 2\pi e^2 S/ck. \tag{9}$$

This sum shows that the initial velocity must be very close indeed to that of light.

To obtain an estimate of the relative energies, the radiated energy and the kinetic energy of the electron at some point, consider the maximum velocity possessed by the electron, for example, on its return trip to the nucleus. This velocity is 3.23×10^9 cm./sec., not far from one tenth the velocity of light. The kinetic energy is approximately $47. \times 10^{-10}$ ergs. The maximum energy ever radiated from a hydrogen atom is $h\nu = hK = 0.2154 \times 10^{-10}$ ergs, and the kinetic energy just computed is about 220 times this maximum radiated energy.

The energy lost by the whole atomic system, however, is no greater than the transformed energy received from the passing electron, since the kinetic energy of its own electron is all returned again to the atom. Thus it becomes manifest that the internal energy of the nucleus must have been drawn upon temporarily to provide this large amount of kinetic energy for one of its own electrons. The available energy in the nucleus is $mc^2 = 1.49 \times 10^{-3}$ ergs. This conception of the process involved in radiation illustrates the trigger action, which becomes possible because the energy received from the passing electron is entirely transformed in character, the atom itself impressing upon all subsequent events its characteristic properties.

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THE

PHYSICAL REVIEW.

GRAPHICAL DETERMINATION OF HEXAGONAL AND TETRAGONAL CRYSTAL STRUCTURES FROM X-RAY DATA.

By Albert W. Hull and Wheeler P. Davey.

Synopsis.

Graphic Method for Interpreting X-Ray Patterns of Powders.—For each type of lattice the logarithms of the theoretical spacings of the different planes are plotted as functions of the axial ratio. Six such plots are reproduced, three for the hexagonal and three for the tetragonal system. By plotting the logarithms of the observed spacings on the edge of a strip of paper, this may be fitted by trial to the theoretical plot; the axial ratio and type of lattice being thus found in a few minutes.

Crystal Structure of Zn, Cd, In.—As examples three complete analyses are given. ***Zinc*** is shown to be a hexagonal close packed assemblage of prolate spheroids. The axial ratio is 1.860 and the side of the unit triangle 2.760 Å. ***Cadmium*** shows a structure like that of Zn, with axial ratio 1.89 and elementary triangle 2.980 Å. ***Indium*** shows a structure nearly like Al (cubic close-packed), viz., a tetragonal close packed arrangement of prolate spheroids, with axial ratio 1.06 and a unit square of 4.58 Å.

Introduction.

SUBSTANCES available for X-ray crystal analysis by the Bragg method fall into three classes:

A. Simple substances, *i.e.*, elements and simple compounds, regarding which no reliable crystallographic data exists;

B. Complex substances, *i.e.*, compounds containing three or more different kinds of atoms, with reliable crystallographic data;

C. Complex substances with no crystallographic data.

Substances of Class *B* are obtainable in large crystals, and hence are easy to investigate, and much useful information may be obtained. Their complete analysis, however, requires a better knowledge of the laws of X-ray scattering than we possess at present.

This knowledge may be most easily and reliably obtained from the study of Class *A* substances. There are a great many of these, and their investigation in powder form is very easy, by the method previously described.[1]

In the X-ray powder photographs the patterns that represent cubic arrangements can be recognized at a glance, and their complete analysis requires only a few minutes. Anisometric crystals are almost as simple if correct crystallographic data are available. Without such data one must proceed either by systematic mathematical analysis,[2] or by a series of guesses. Either method is, in general, very laborious. The process of guessing becomes very easy, however, by the use of plots.

DESCRIPTION OF PLOTS.

The plots (Figs 6–11) show the spacings of all possible planes (within the range of the plot) as a function of axial ratio.[3]

The scale of abscissas is logarithmic, so that if the planer spacings are plotted on the same logarithmic scale (shown on the bottom of each plot) they may be compared directly with the observed values without regard to the absolute length of the unit axes.

Three plots are given for each system. The first represents a single lattice of right triangular prisms and tetragonal prisms respectively; the second, two intermeshed lattices; and the third, three intermeshed lattices for the hexagonal and four for the tetragonal system.

The single lattice is an aggregate of points whose cartesian coördinates are m, n, pc, where m, n, and p represent all possible integers, c is the axial ratio, and the unit of length is the side of the unit triangle or square respectively.

The two intermeshed lattices of triangular prisms have coördinates

$$m,\ n,\ pc,$$
$$m + 1/3;\ n + 2/3;\ (p + 1/2)c.$$

This is the arrangement which, when the axial ratio is 1.633, gives the closest possible packing of equal spheres. It is therefore designated as "hexagonal close packing," though it is obviously not the closest packed arrangement of spheres for all axial ratios.

The two intermeshed lattices of tegragonal prisms have coördinates

$$m,\ n,\ pc,$$
$$m + 1/2;\ n + 1/2;\ (p + 1/2)c$$

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[1] A. W. Hull, PHYS. REV., 10, 661, 1917.

[2] C. Runge, Phys. Z., 18, 509, 1917.
Johnsen u. Toeplitz, Phys. Z., 19, 47, 1918.

[3] The term *axial ratio* is used in its strict sense, viz., the ratio of the fundamental translation distances of the point lattice along the vertical and lateral axes respectively.

For the hexagonal and tetragonal systems this is equal to the ratio of altitude to side of the unit triangular prism and tetragonal prism respectively.

and constitute a single lattice of body centered tetragonal prisms.

The three intermeshed triangular lattices have coördinates

$$m,\ n,\ pc,$$
$$m + 1/3;\ n + 2/3,\ (p + 1/3)c,$$
$$m + 2/3,\ n + 1/3,\ (p + 2/3)c.$$

This gives complete rhombohedral symmetry, and is equivalent to a single lattice of rhombohedra of which the edge is $\sqrt{1/3 + c^2/9}$ times the edge of the unit triangle, and the angles between edges *i.e.*, the face angles of the rhombhedra, are each equal to $2\csc^{-1}2\sqrt{1/3 + c^2/9}$.

The four intermeshed tetragonal prisms have indices

$$m,\ n,\ pc,$$
$$m + 1/2,\ n + 1/2,\ pc,$$
$$m + 1/2,\ n,\ (p + 1/2)c,$$
$$m,\ n + 1/2,\ (p + 1/2)c,$$

and constitute a single lattice of face-centered tegragonal prisms.

Use of Plots. Examples.

As illustrations of the use of the plots the complete analysis of zinc, cadmium and indium will be described.

Zinc.

Zinc has atomic weight 65.37, density 7.1, and crystallizes, according to Groth (measurements of Williams & Burton) holohedral hexagonal, with axial ratio 1.3564.

Powder photographs of pure zinc prepared in different ways gave identical patterns (using molybdenum monochromatic X-rays) with the spacings shown in Table I.

Table I.

Zinc

Intensity of Line (Estimated).	Planar Spacing in Ångstroms.	Intensity of Line.	Planar Spacing.
10	2.473	2	1.088
4	2.315	2	1.044
20	2.080	1	.947
4	1.684	2	.908
5	1.339	4	.857
5	1.333	2	.824
1	1.235	2	.770
4	1.173	1	.749
2	1.152	6	.734
4	1.121	3	.667
		2	.654

In order to compare these observed spacings with the theoretical ones of the plots, a strip of paper is placed beneath the scale of abscissas of the plot (position 1, Fig. 1) and the values of planar spacings from Table I. laid off along its edge. This strip with its pattern of lines is then moved over the plot, with its edge always parallel to the axis of abscissas, until a position is found where its pattern exactly coincides with that of the plot. "Position 2," Fig. 1, shows the best fit that can be obtained at the accepted axial ratio 1.356, and "position 3," Fig. 2, the correct position, at axial ratio 1.860. The agreement is exact. The plot shows that the second spacing, viz: 2.315 Å. belongs to the form $1 0 \bar{1} 0$, hence the side of the unit triangle, which is $2/\sqrt{3}$ times the $1 0 \bar{1} 0$ spacing, is 2.670 Å. The number of atoms per unit triangular prism is

$$n = \frac{\sqrt{3}}{4}\,\frac{a^3 c\rho}{M} = 1.00 \begin{cases} a & = \text{side of elementary triangle in cm.} \\ c & = \text{axial ratio,} \\ \rho & = \text{density,} \\ M & = \text{mass of 1 atom in grams.} \end{cases}$$

which is correct for two intermeshed triangular prism lattices. *The zinc lattice is therefore similar to that of magnesium (hexagonal close-packed) except that it is elongated 14 per cent. in the direction of the principal hexagonal axis.* The arrangement is that of *closest packing of prolate spheroids*, indicating that the zinc atom is polar and elongated, as suggested by Langmuir.[1]

Cadmium.

Filings of pure cast cadmium were photographed with Mo. monochromatic X-rays and gave the spacings in Table II.

TABLE II.

Cadmium.

Intensity of Line (Estimated).	Planar Spacings in Ångstroms.	Intensity of Line.	Planar Spacing.
20	2.817	1	1.225
10	2.585	2	1.165
60	2.336	2	1.060
12	1.900	2	1.025
10	1.515	2	1.023
8	1.490	(wide, 4 not resolved)	.958
1	1.400	1	.918
10	1.313	3	.861
6	1.257	2	.820

---

[1] Langmuir, Arrangement of Electrons in Atoms and Molecules. J. Amer. Chem. Soc., 41, 879, 1919.

The scale of abscissas on the plots is not quite long enough to enable all the values of planar spacings in Table II. to be laid off directly. They may be brought within the range of the plot, however, by dividing them all by 2; or the first two spacings may be laid off with the strip in position 1, and the rest in position 2, Fig. 3. The strip is then moved about on the plots until the correct position is found. This is shown at position 3, Fig. 3, at axial ratio 1.89. The best fit that can be obtained at the commonly accepted axial ratio 1.335 is shown in position 4, Fig. 4, and position 5, Fig. 4, shows the next most plausible guess, at axial ratio 1.633, which is hexagonal close packing. This fit is very bad, but close enough to tempt one to look for the correct axial ratio in the immediate neighborhood of 1.633. This would be a false lead, however, since the order of many of the lines, notably the first two, reverses in passing from axial ratio 1.633 to 1.89.

From the value of the $1\,0\,\bar{1}\,0$ spacing, as identified by the plot (position 3, Fig. 3), the side of the elementary triangle may be calculated, viz.,

$$a = \frac{\sqrt{3}}{2} d_{10\bar{1}0} = 2.980 \text{ Å}.$$

The number of atoms per unit triangular prism is

$$\frac{\sqrt{3}}{4} \frac{a^3 c\rho}{M} = 1.005,$$

which is sufficiently close to the correct value, 1,000, for two intermeshed triangular prism lattices in close packed arrangement.

The cadmium lattice is therefore a close-packed arrangement of elongated atoms, like that of zinc, the elongation being 16 per cent., as compared with 14 per cent. for zinc.

Indium.

A photograph of finely powdered indium, of unknown purity, gave the following lines (Table III.).

TABLE III.

Indium.

Intensity of Line (Estimated).	Planar Spacings in Ångstroms.
40	2.70
1	2.42
10	2.29
4	1.675
1	1.617
2	1.450
4	1.392
4	1.348
1	1.150
2	1.080

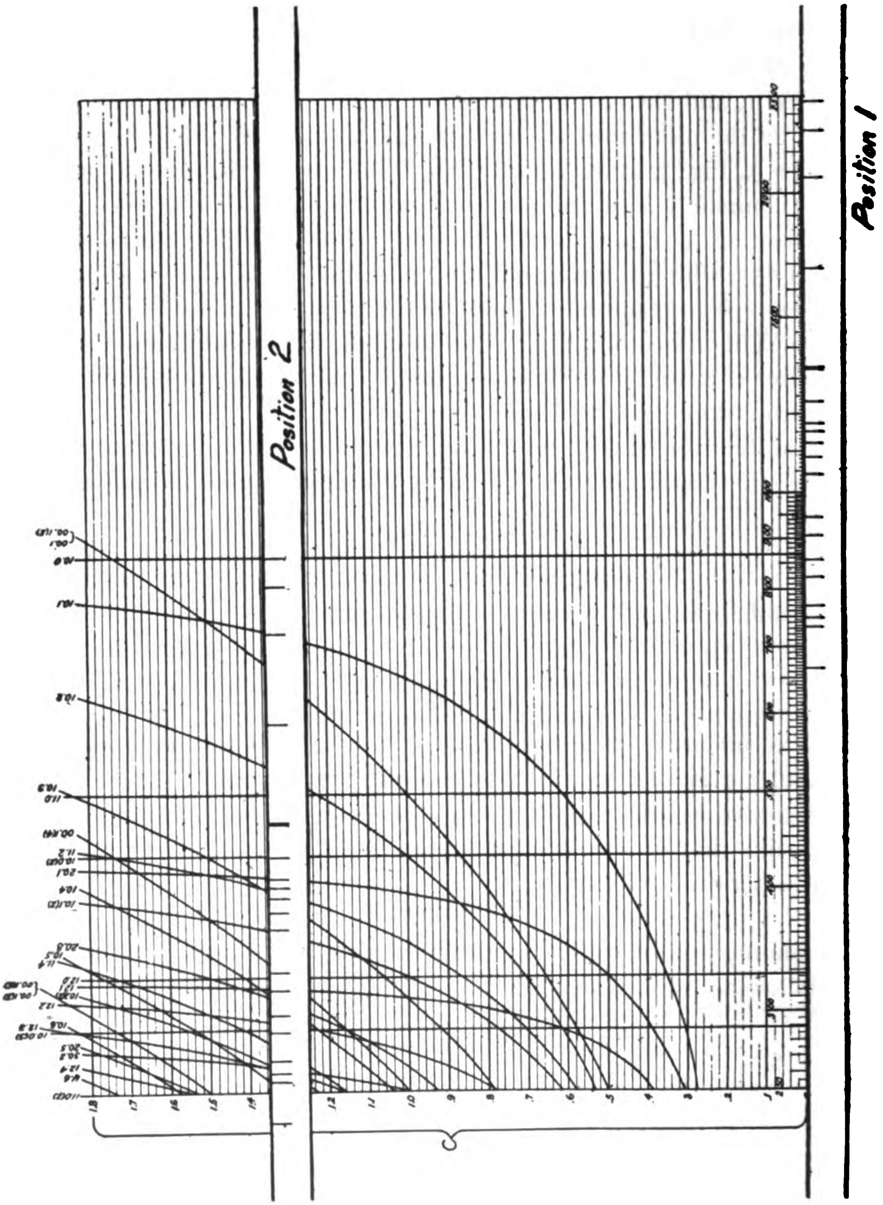

Fig. 1.

Zinc spacings compared with triangular close packing plot.

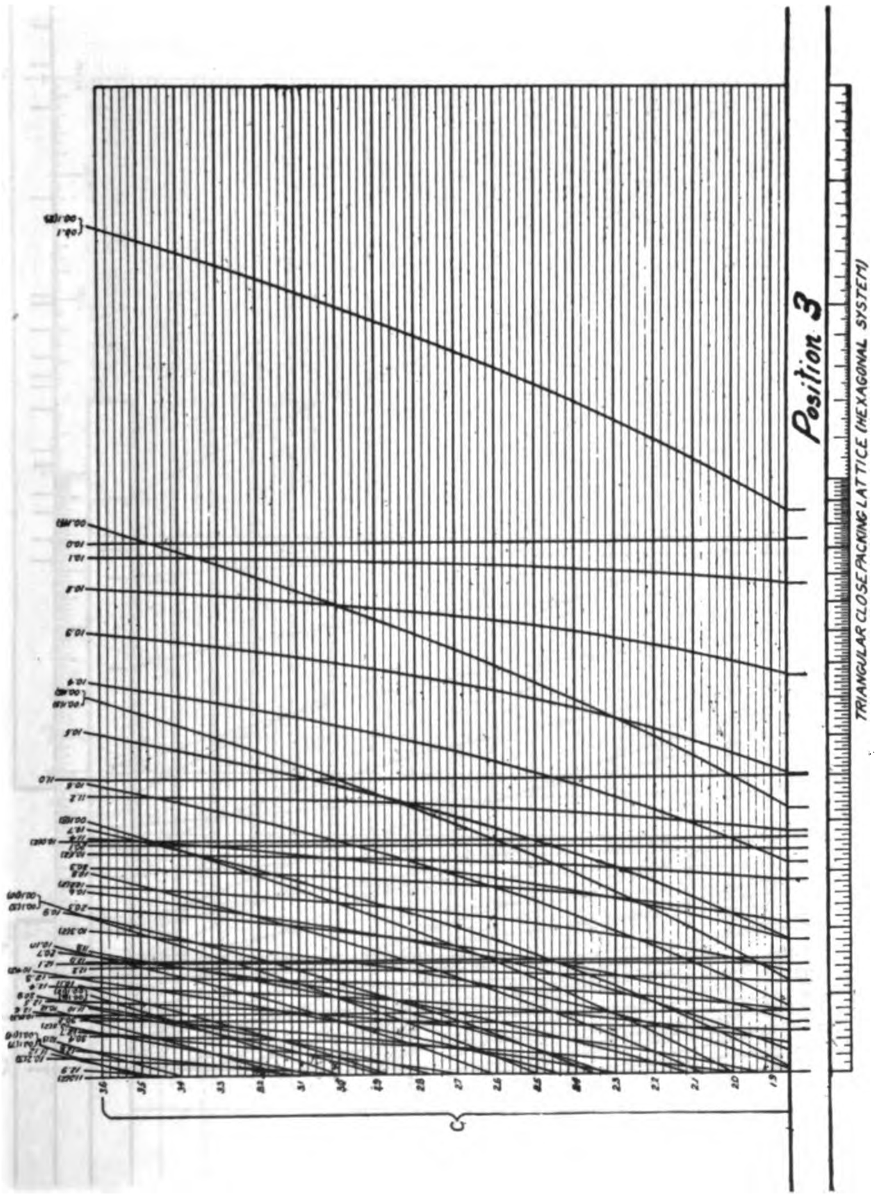

Fig. 2.

Zinc spacings compared with triangular close packing plot.

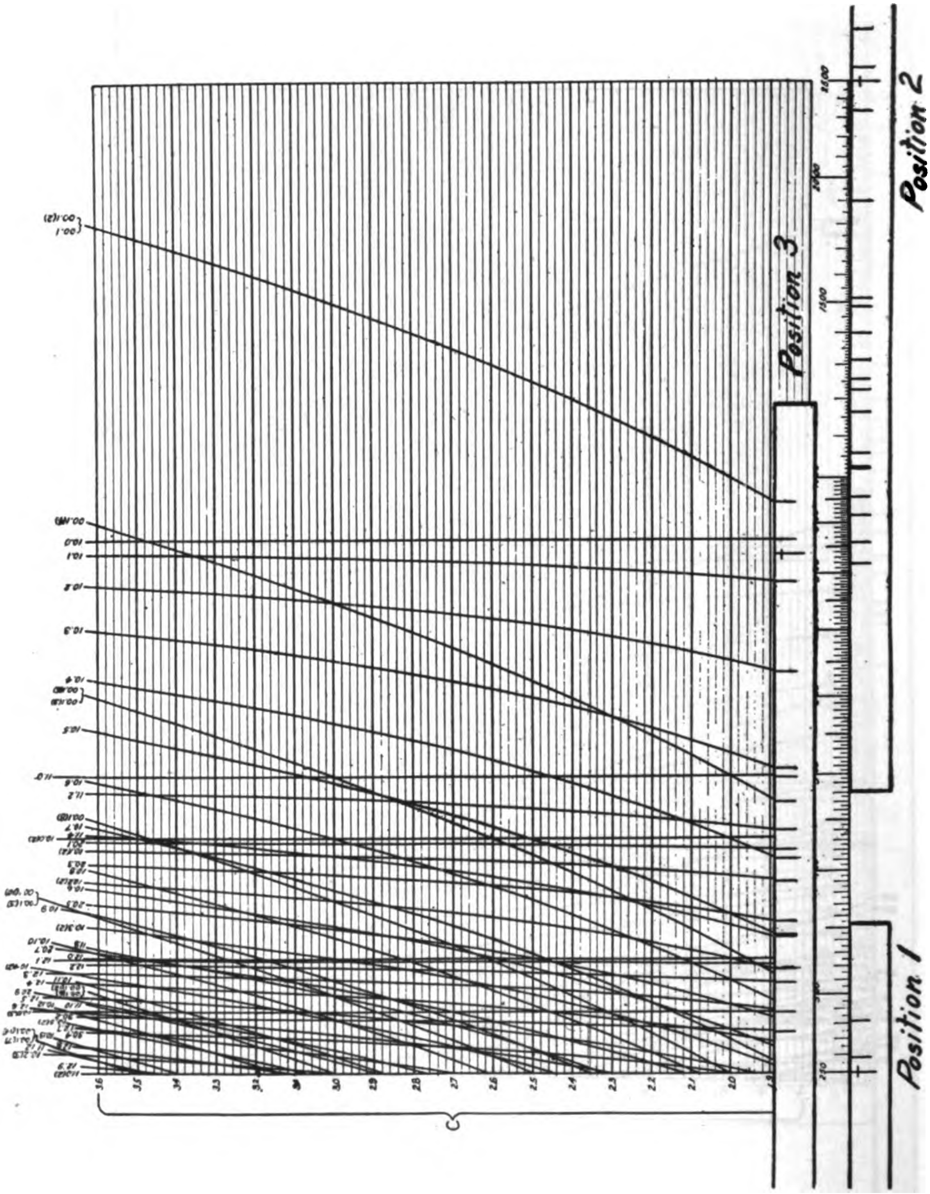

Fig. 3.

Cadmium spacings compared with triangnlar close packing plot.

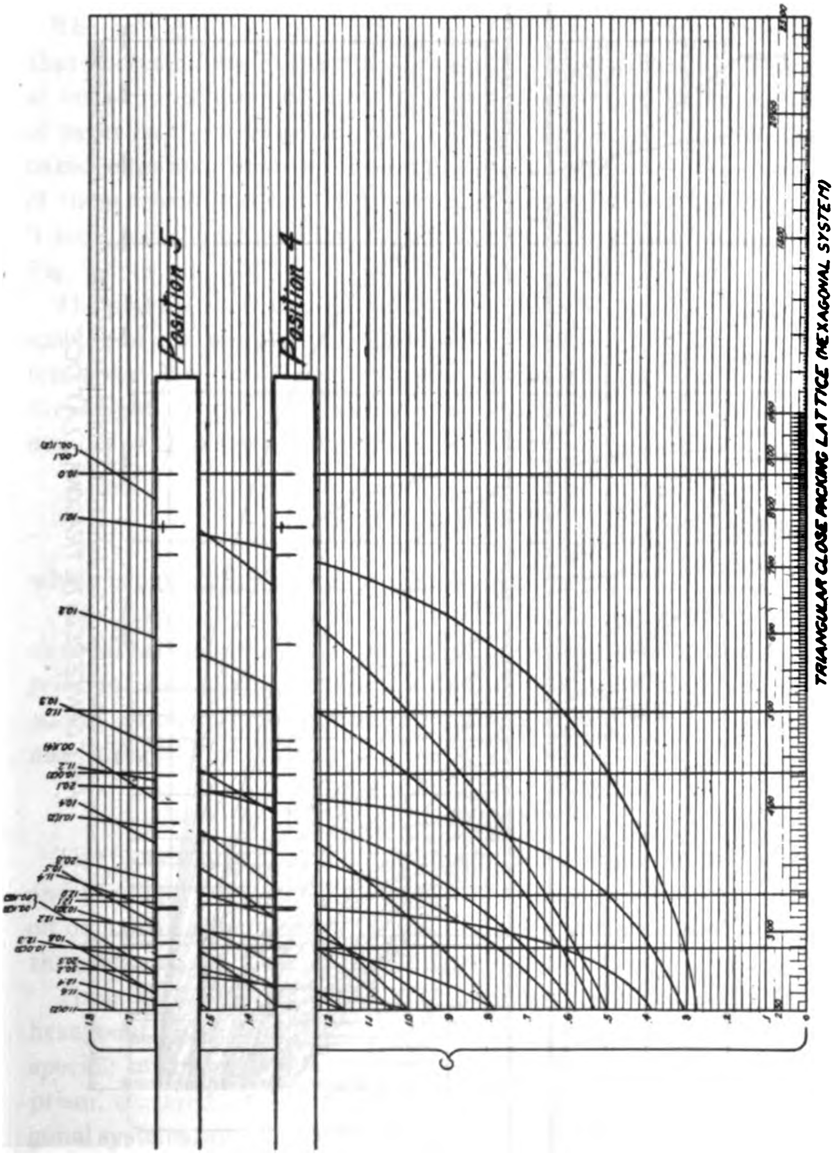

Fig. 4.

Cadmium spacings compared with triangular close packing plot.

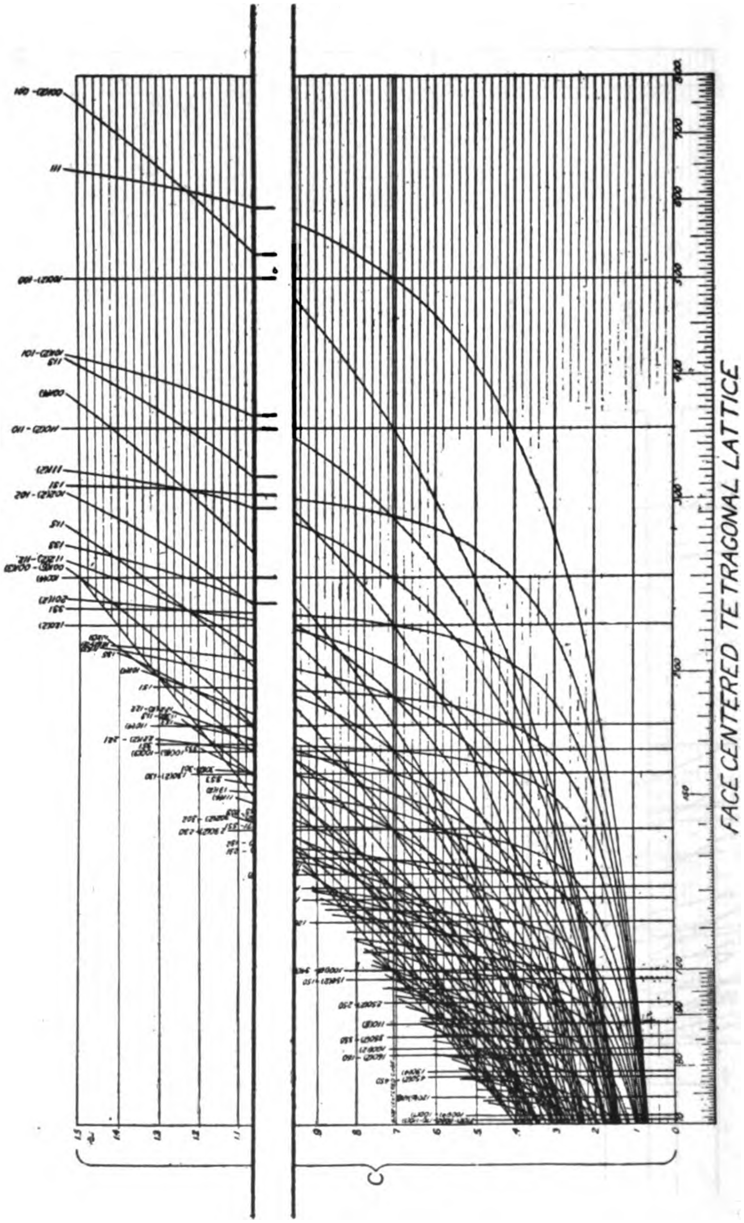

Fig. 5.

Indium spacings compared with face-centered tetragonal plot.

The spacings in Table III. are laid off on a strip of paper, and this strip moved about on the different plots until a position is found (Fig. 5) where the pattern exactly fits the plot.

The only data on indium given by Groth is an observation of Sachs that it crystallizes in regular octahedra. Inspection of the film showed at once that it was not cubic, and this was checked by trying the strip of paper on the tetragonal plots at axial ratio 1. It remained, therefore, to move the strip over the different plots until a position was found on one of them which matched the pattern of lines on the strip. In this case it took less than five minutes to find the correct position, as shown on Fig. 5, position 3.

The plot shows that the third spacing, 2.290Å., is half the fundamental spacing of the 100 planes (second order reflection from 100) of a simple tetragonal lattice. Hence the side of the unit square is $a = 4.58$Å., the height of the unit prism is $ca = 1.06 \times 4.58 = 4.86$Å., and the number of atoms per unit prism, taking the density of indium as 7.45 is

$$n = \frac{a^2c\rho}{M} = 4.00,$$

which is correct for a face-centered tetragonal lattice.

The lattice of indium is therefore similar to that of aluminum (cubic close-packed) except that it is elongated 6 per cent. in the direction of the principal axis. This lattice, like that of zinc and cadmium, is a close packed arrangement for oblate spheroids, indicating that the indium atom also is slightly elongated.

General Remarks.

The plots reproduced in Figs. 6–11 are all drawn to the same scale, and photographed without distortion, so that experimental values laid off on a strip of paper according to the scale at the bottom of any one of the plots may be used on all the plots.

These plots cover all possible arrangements of atoms in the cubic, hexagonal (including trigonal) and tetragonal systems. Only three specific arrangements have been given for each system, viz., simple prism, centered, and face-centered arrangements in the cubic and tetragonal systems, and simple prism, close packed, and rhombohedral arrangements in the hexagonal system. It is obvious, however, that these and all other possible arrangements are obtained from the simple tetragonal and triangular prism lattices respectively by simply omitting part of the lines. This will be more obvious, perhaps, if stated in the form of two fundamental principles:

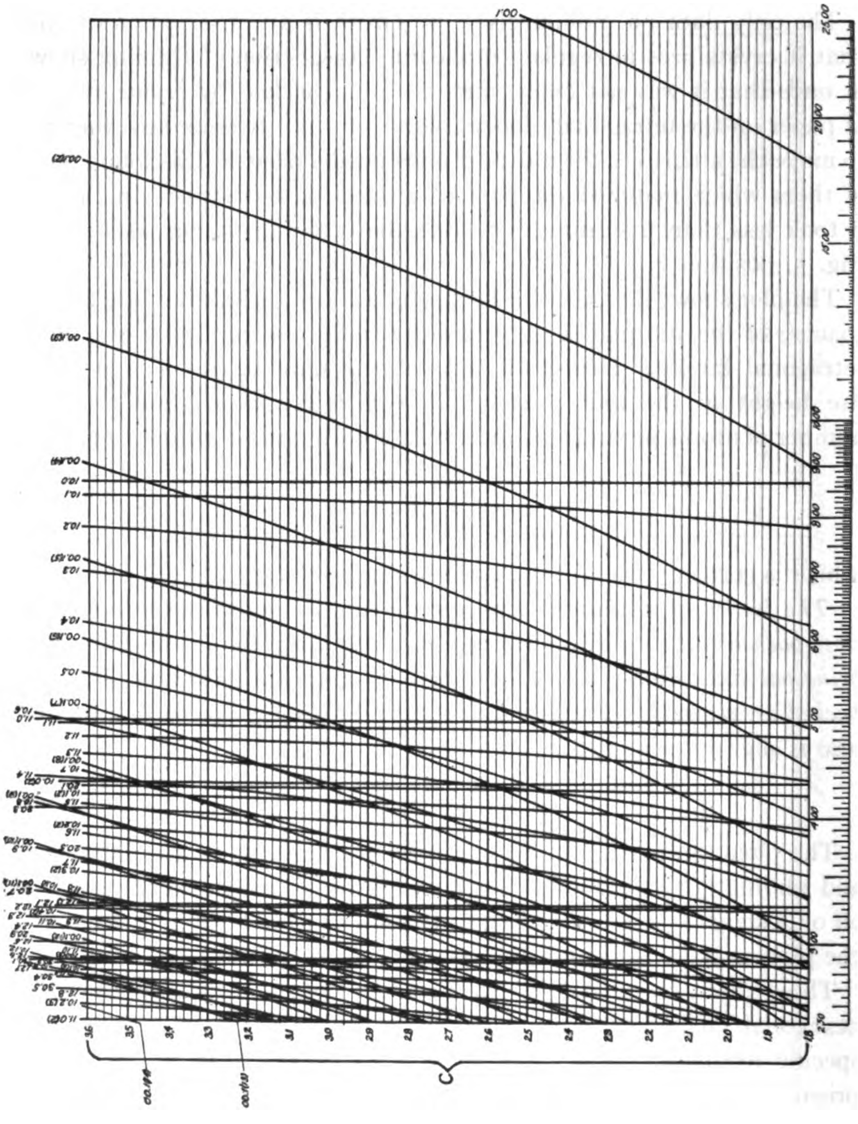

Fig. 6A.

Simple triangular lattice (hexagonal system).

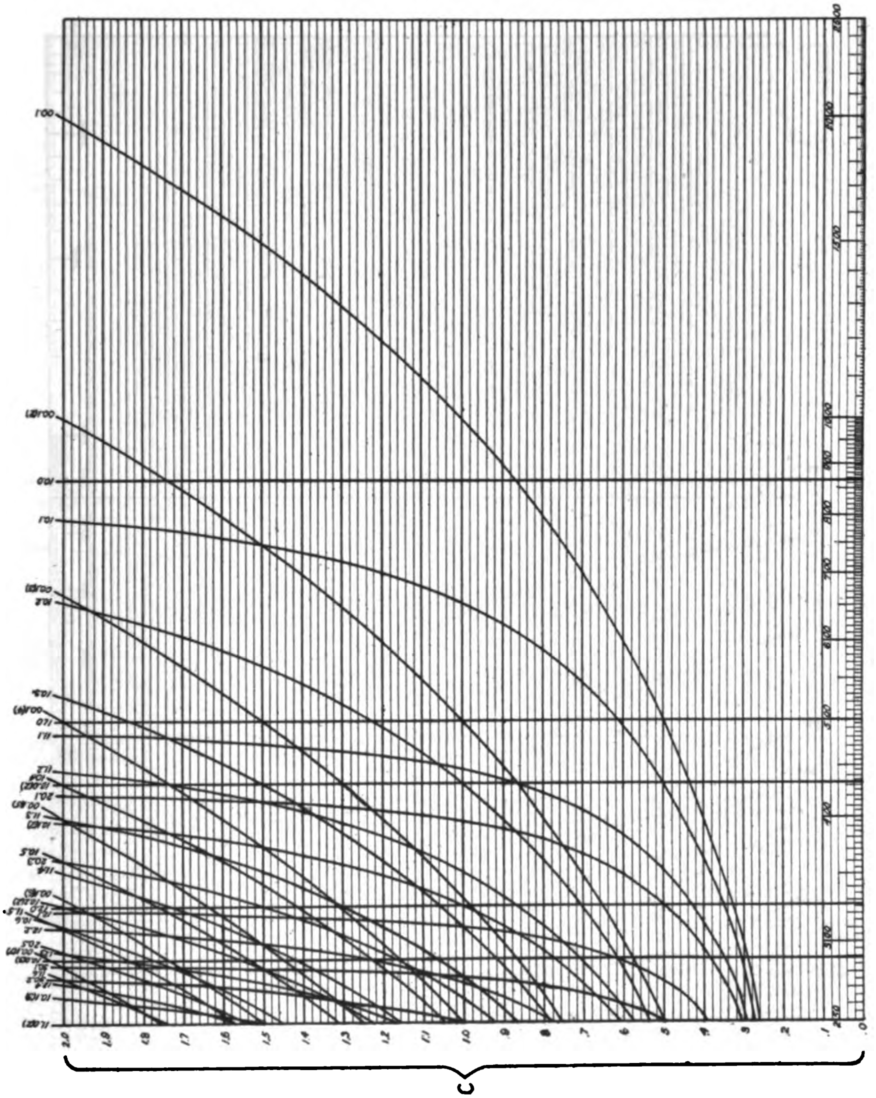

Fig. 6B.

Simple triangular lattice (hexagonal system).

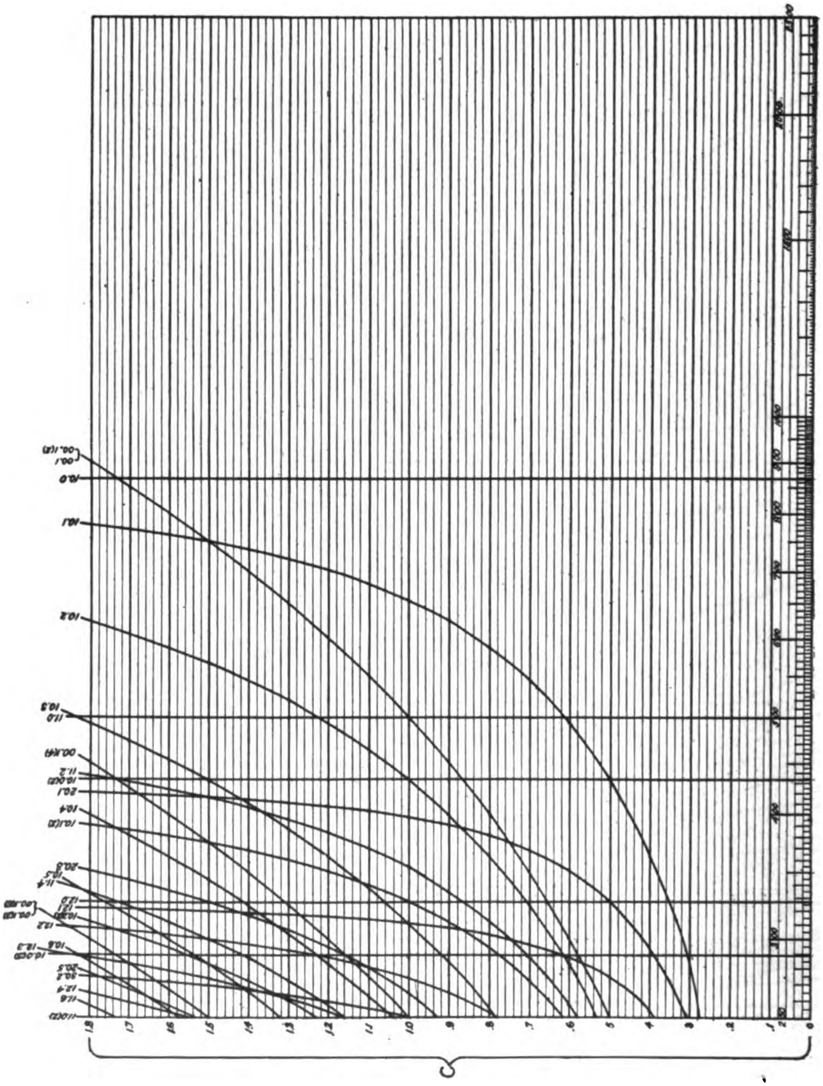

Fig. 7A.

Triangular close packing lattice (hexagonal system).

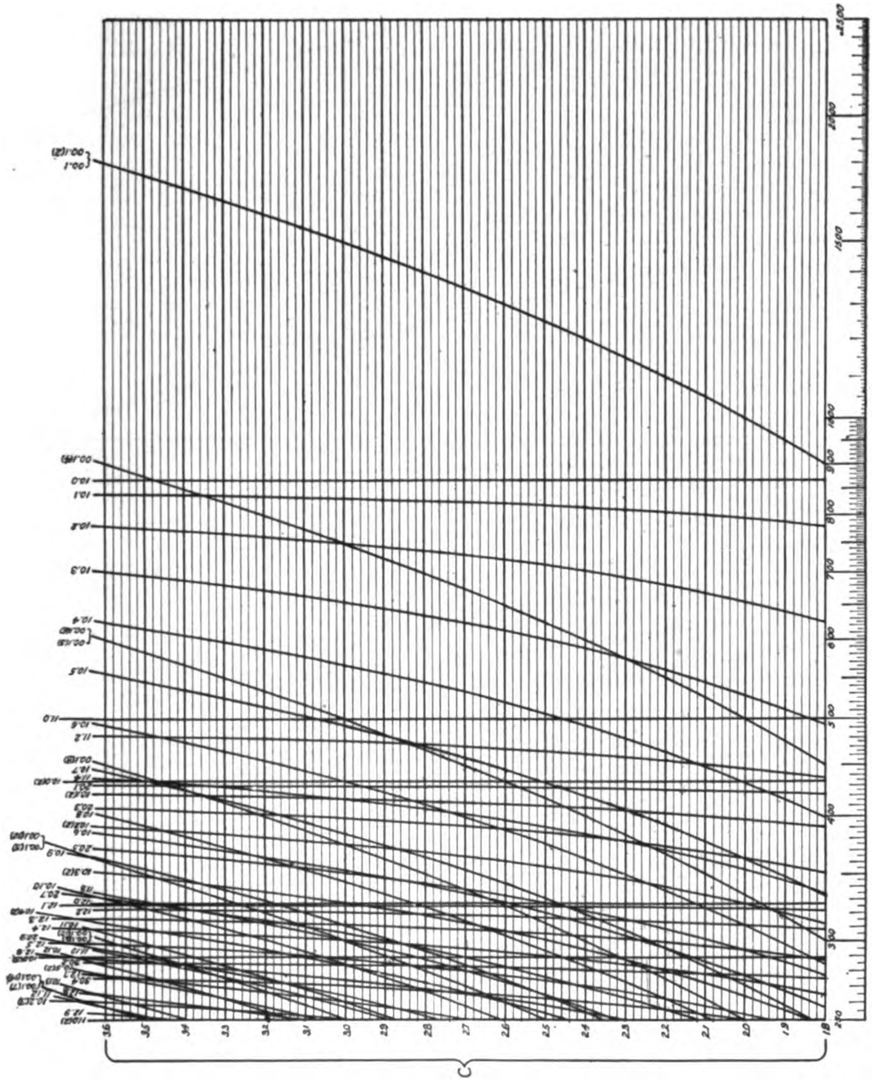

Fig. 7B.

Triangular close packing lattice (hexagonal system).

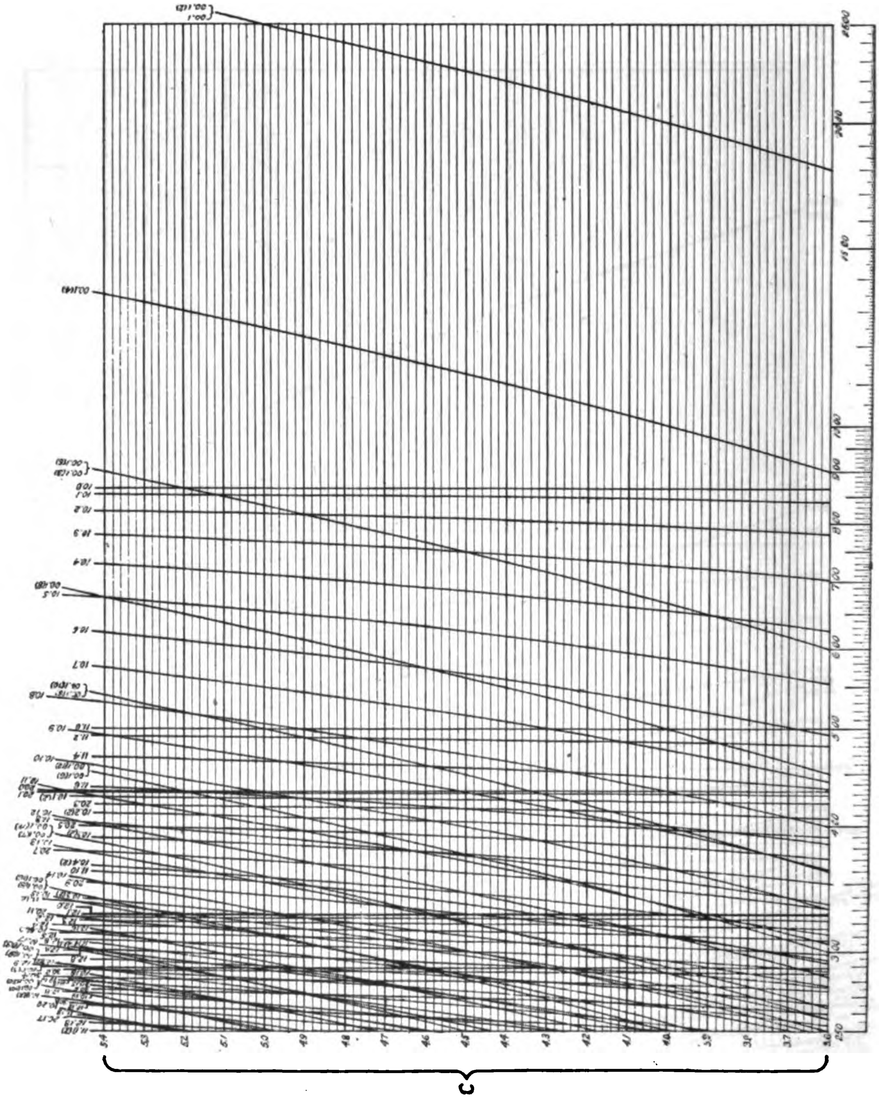

Fig. 7C.

Triangular close packing lattice (hexagonal system). [Scale 1 per cent. larger than the others.]

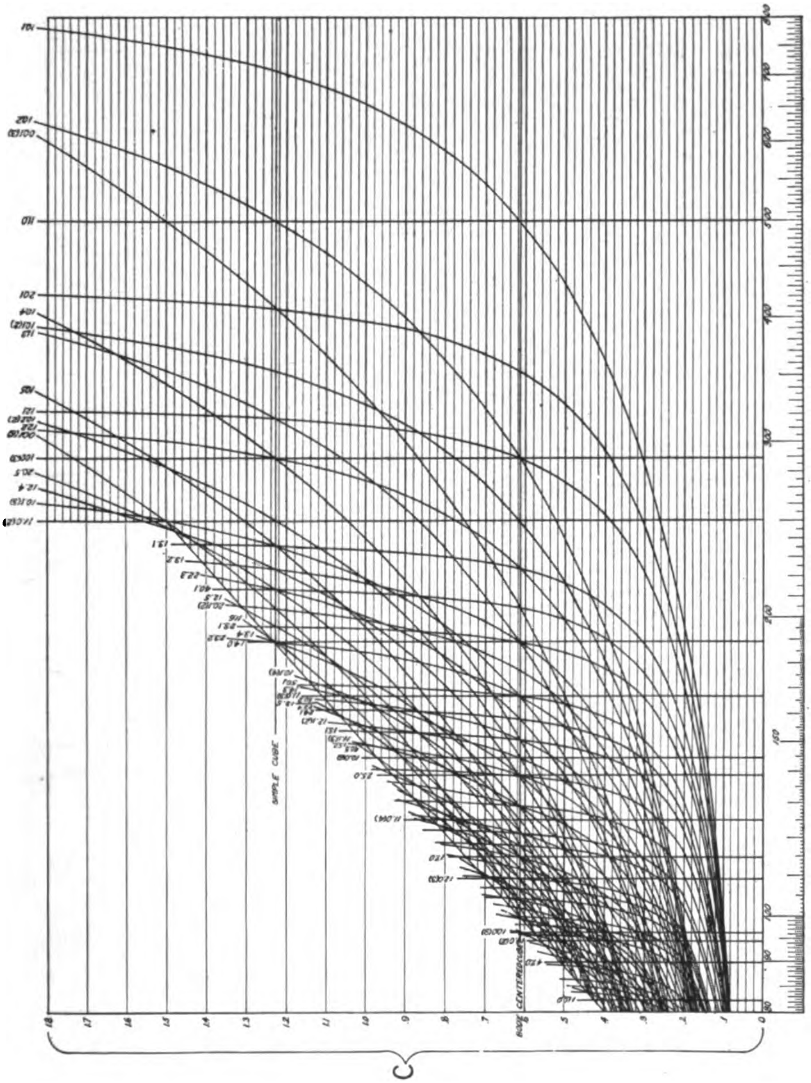

Fig. 8A.

Rhombohedral lattice (hexagonal system). [Scale 1 per cent. smaller than the others.]

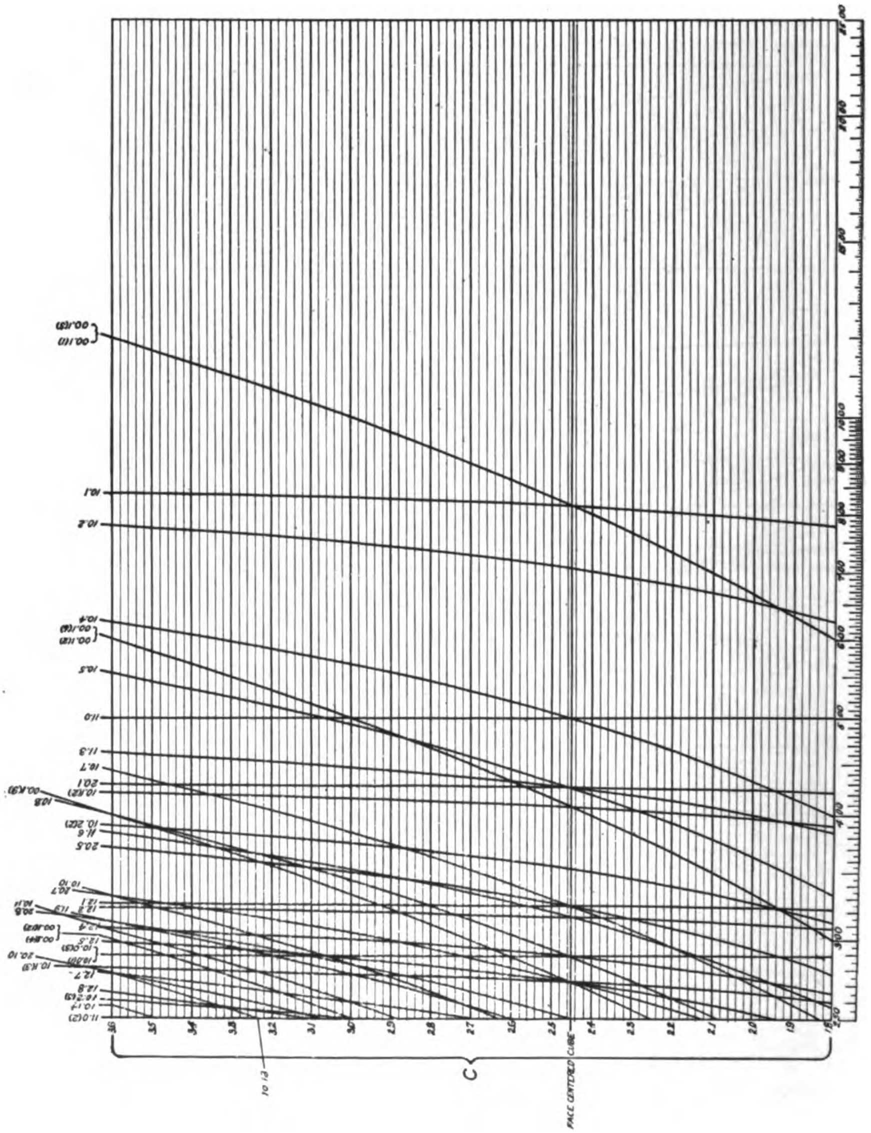

Fig. 8B.

Rhombohedral lattice (hexagonal system).

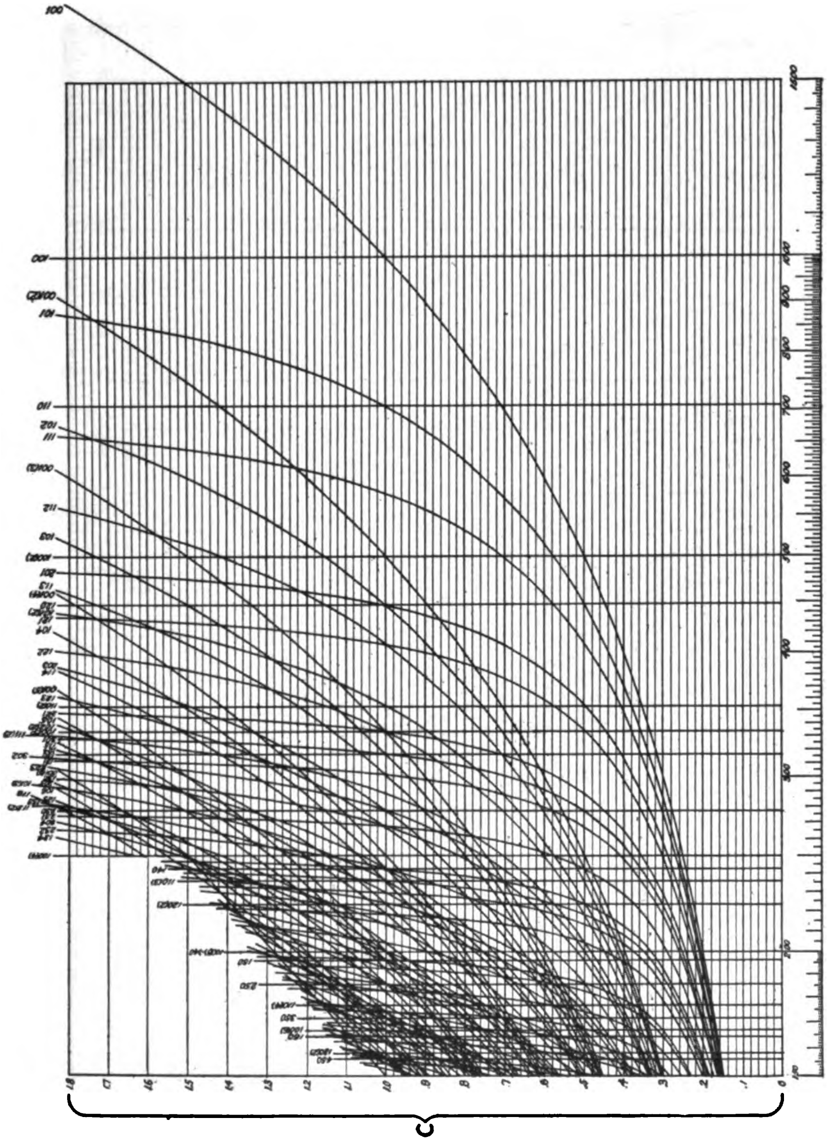

Fig. 9.

Simple tetragonal lattice. [Scale ½ per cent. smaller than the others.]

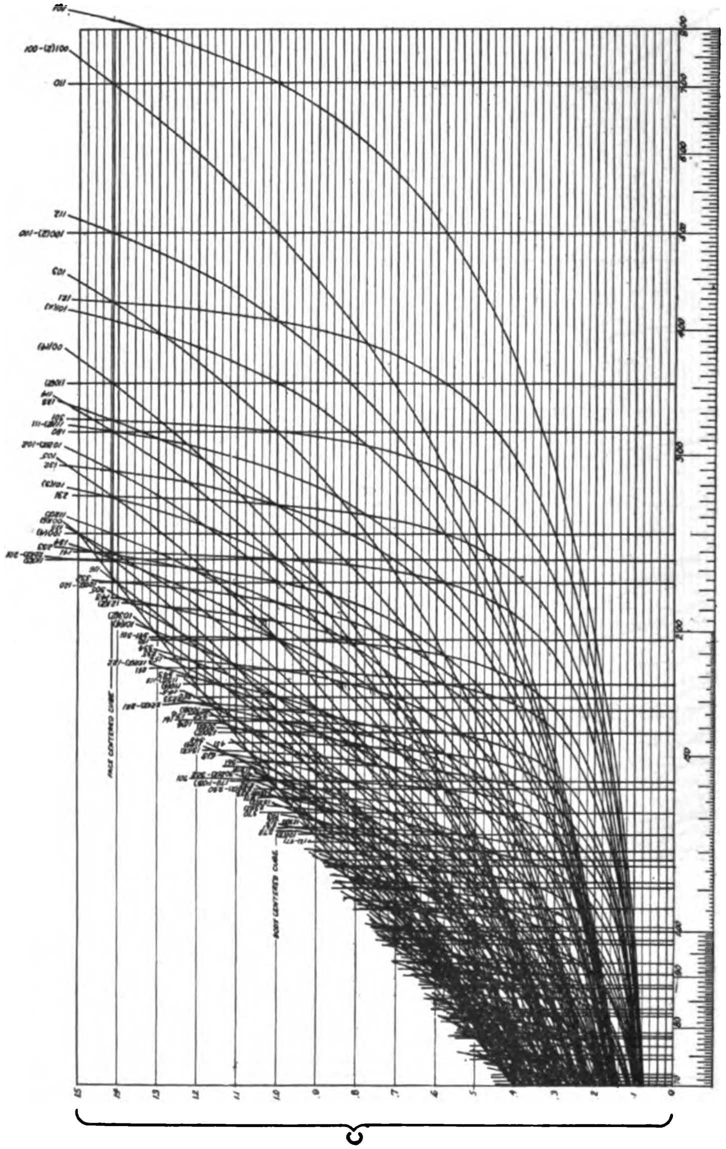

Fig. 10.

Body-centered tetragonal lattice.

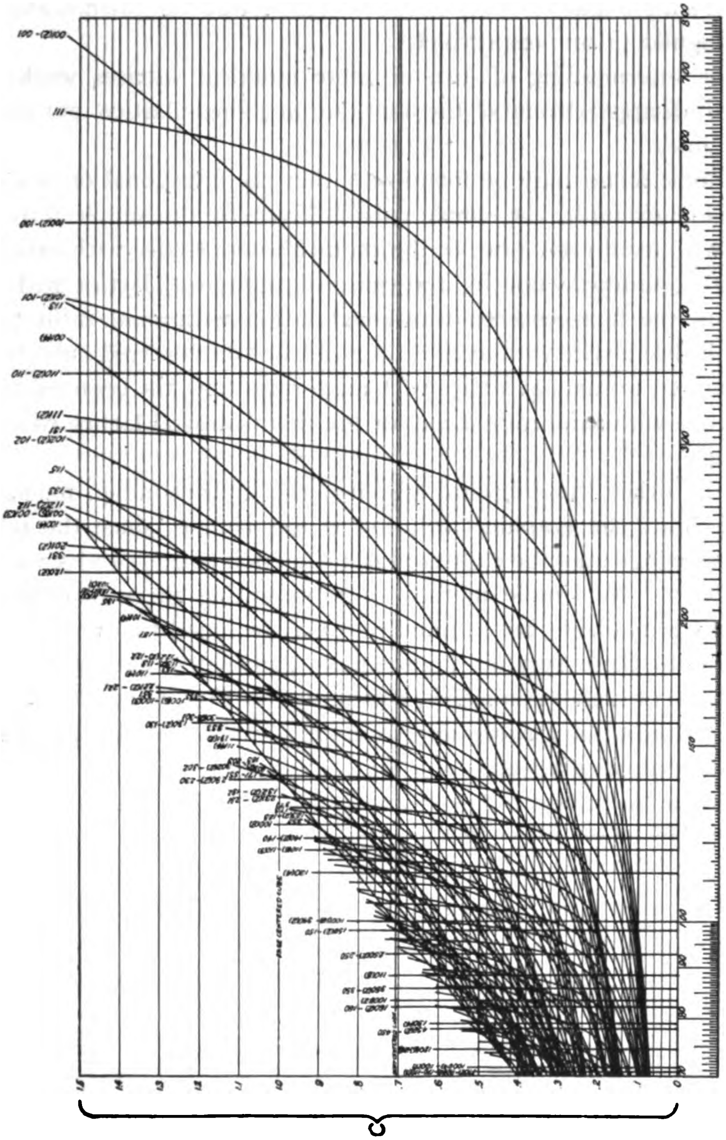

Fig. 11.

Face-centered tetragonal lattice.

1. All possible arrangements of atoms in the cubic, tetragonal, or hexagonal systems may be obtained by the combination of one or more identical simple space lattices of cubes, right tetragonal prisms, and right 60° triangular prisms respectively.

2. The intermeshing of two or more identical lattices weakens or causes to disappear some of the lines due to a single lattice, but can add no new lines.

The cubic forms may be found on either the tetragonal or hexagonal plots. For example, the simple cube is given by the simple tetragonal lattice with axial ratio 1.00, or the rhombohedral lattice with axial ratio 1.225; the centered cube by the centered tetragonal lattice with axial ratio 1.00, the face-centered tetragonal lattice with axial ratio .707, or rhombohedral lattice with axial ratio .612; the face-centered cube by the face-centered tetragonal lattice with axial ratio 1.00, the centered tetragonal lattice with axial ratio 1.414, or the rhombohedral lattice with axial ratio 2.45.

For the other three crystal systems, the orthorhombic, monoclinic, and tri-clinic, the graphical solution is less simple, since the relative crystal spacings must be expressed as a function of 2, 3, and 5 variables respectively. No simple method of representing these relations in a single plot is available, but in practice it may be found that many crystals belonging to these systems approximate sufficiently closely to one of the systems of higher symmetry, represented in Figs. 6–11, to enable the correct lattice to be guessed approximately from these plots, and found by a few trial calculations.

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X-RAY CRYSTAL ANALYSIS OF THIRTEEN COMMON METALS.[1]

By Albert W. Hull.

Synopsis.

Crystal Structure of Thirteen Metals Determined by X-ray Analysis.—The author's method of powder analysis was used. In the case of anisometric substances, the crystal system and axial ratio were found by means of plots by the method described in the preceding paper.[2] The lattices of ***chromium, molybdenum*** and ***tantalum*** were found to be body centered cubes with sides 2.895, 3.143 and 3.272 Å. respectively; ***cobalt alpha, nickel, rhodium, palladium, iridium*** and ***platinum*** have face centered cubic lattices with the sides of the cubes 3.554, 3.540, 3.820, 3.950, 3.805 and 3.930 Å. respectively; ***cobalt beta, zinc, cadmium*** and ***ruthenium*** have hexagonal lattices of the "close-packed" type with axial ratios 1.63, 1.86, 1.89 and 1.59 respectively and with the triangular sides 2.514, 2.670, 2.960 and 2.686 Å. respectively; and ***indium*** has a face-centered tetragonal lattice with axial ratio 1.06 and the side of elementary prism 4.58 Å. The structures found for cadmium, zinc and indium are close packed arrangements of solid prolate spheroids while that for ruthenium is a close packed arrangement of oblate spheroids.

Introduction.

The extension of the Bragg method of crystal analysis to powdered materials has been described elsewhere[3] and need be only briefly summarized here. The photographs were taken with the same apparatus and in the same manner as those previously described, and the interpretations were made in the same manner, except that the process of guessing the correct lattice was greatly simplified by the use of logarithmic plots of crystal spacings against axial ratio. These plots are fully described in the preceding paper.[4]

For convenience of reference, the arrangement of apparatus is reproduced in Fig. 1. Intense characteristic X-rays of wave-length 0.712 Å. were produced by a water-cooled molybdenum Coolidge tube, running at 30,000 volts constant potential and 30 milliamperes. A filter, consisting of a thin sheet of cellulose binding material containing .05 gram of ZrO_2 per cm.[2], absorbed almost completely all wave-lengths except the K-alpha doublet, giving a monochromatic source. The smooth face of the X-ray target made an angle of 10° with the line joining it to the two defining slits. These were each 2 mm. wide and 1 cm. high, and 9 and 13 cm. respectively from the focal spot. This gave an intense beam of approximately 1° angular width, with a halo, due to rays proceeding from the face and stem of the target, and secondary rays from other parts of the tube and lead box, of about 6°. A third slit, 6 mm. wide and 2 cm. high, was placed 4 cm. beyond the second defining slit; *i.e.*, 17 cm. from the focal spot. This slit was wide enough so that no primary rays which were able to get through the first two slits could strike it, and its function was to prevent secondary rays from the second slit from reaching the photographic film, except for an angular width of about 8° in the center.

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[1] The results of these analysis have been briefly announced elsewhere: Co, Ni, Cr, Mo, and Pt in Proc. A. I. E. E., 38, 1189, 1919; the rest in Science, 52, 227, Sept. 3, 1920.

[2] Graphical Determination of Hexagonal and Tetragonal Crystal Structures from X-Ray Data, by Albert W. Hull and Wheeler P. Davey. This number, pp. 549–570.

[3] Phys. Rev., 10, 661, 1917.

[4] Loc. cit.

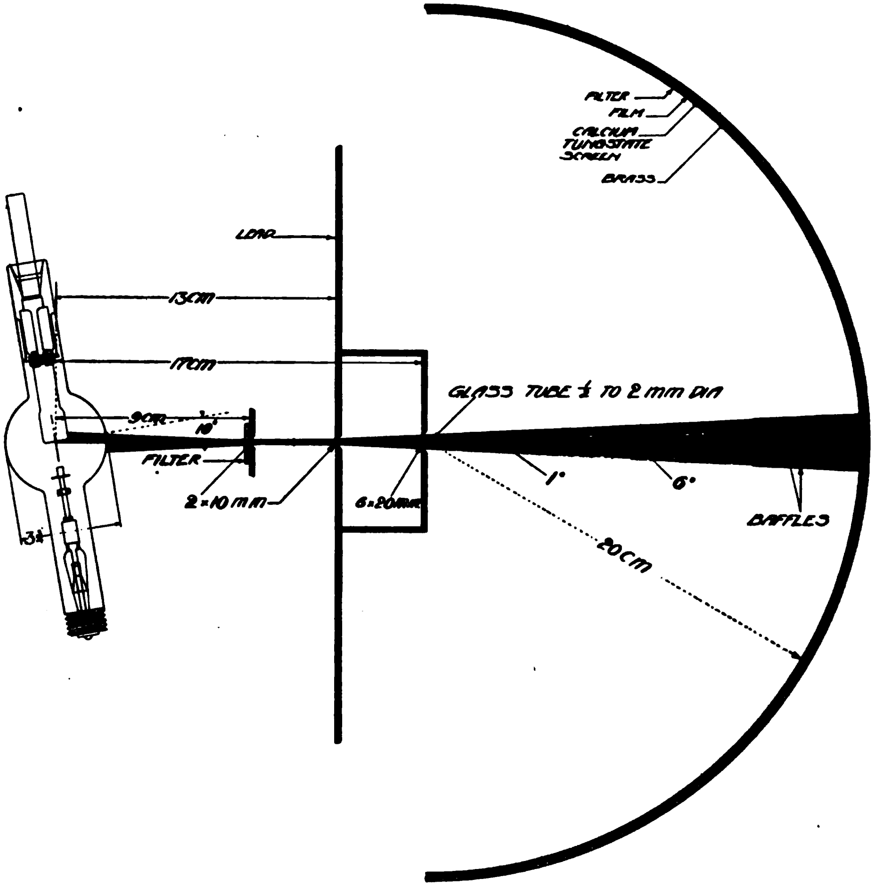

Fig. 1.

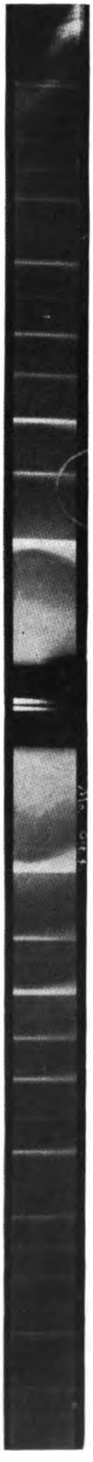

Fig. 2.

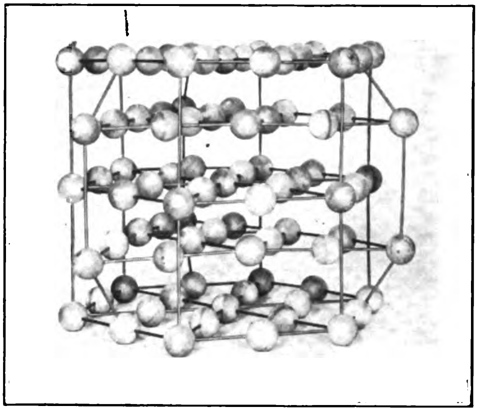

Fig. 3.

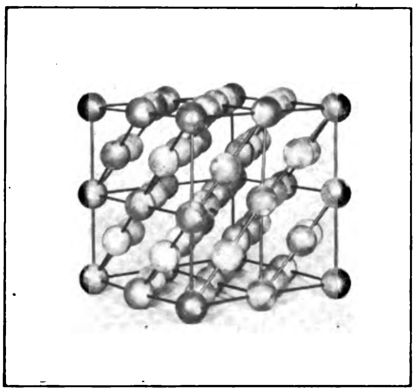

Fig. 4.

ALBERT W. HULL.

The powdered material to be examined was mounted in a thin-walled glass tube, of diameter ½ to 2 mm., according to density, directly in front of the third slit, and at the center of a semicircular brass frame of 20 cm. radius. Upon this frame was mounted a strip of standard Eastman portrait film 4 cm. wide and 60 cm. long, backed by a calcium tungstate intensifying screen. A second filter[1] sometimes of .0025 cm. iron, but generally and preferably a thin strip of cellulose containing .025 g. of ZrO_2 per cm.[2], was placed directly in front of the film to absorb fluorescent secondary rays from the powder. These rays, though they produce no lines, are often 100 times as intense as the scattered rays which produce the line patterns, and their fog completely obscures the lines. The exposure was usually over night, about 15 hours.

A typical film is shown in Fig. 2.

The films were measured on a standard Keuffel and Esser log log slide rule, with movable scale removed. The film was fastened with paper clips to the rule, with its center line at the zero of the scale of equal parts. The glass slider was then moved over the film, its index line placed successively over the centers of the lines on the film, viewed as a transparency, and the distances in metric inches read off on the scale of equal parts at the bottom. The lines on both sides of the center were read independently, so that any scratch or false line, as well as mistakes in measurement, could be detected by comparison, and the accuracy of reading could be estimated. The readings on the two sides always agreed within about ½ of 1 per cent.

These distances of the lines from the center of the film were reduced to crystal degrees by dividing by 2 (radius of film × $2\pi/360$), or 0.2777, and then to planar spacings in Ångstroms by taking the quotient $\lambda/(2 \sin \theta)$ according to Bragg's law.

In the following tables only the planar spacings, with the estimated intensities of the lines from which they are calculated, will be given. The scale of intensities is chosen so that intensity 1 represents the weakest line that can be identified with certainty. Lines with intensity ½ or even ¼ are considered reliable if found in identical positions on both sides of the film.

For comparison with the observed spacings, the calculated spacings are given; *i.e.*, those of a theoretical lattice of the type assumed, as calculated by the equations published in an earlier paper.[2]

The calculation of the theoretical intensities of the lines requires a knowledge of the effect of temperature, absorption, size of crystals, shape of atoms, and variation of ideal reflecting power with angle, in addition to the number of coöperating planes. The effect of all these factors except the last, is to produce a nearly uniform decrease of intensity with increasing angle. The large fluctuations in intensity of consecutive lines are due to the variation in the number of different sets of planes which coöperate to produce the respective lines. Hence a comparison of the number of coöperating planes with the observed intensities is a valuable check on the correctness of the identification of the lines. These numbers are given in the second column.

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[1] The first filter was between the X-ray tube and first slit.

[2] Phys. Rev., 10, 671, 677, 1917.

As a final check the densities calculated from the X-ray data, by dividing the mass of the atoms in an elementary cell by the volume of the cell, are compared with the best values obtained by standard methods, as given in Van Nostrand's Chemical Annual for 1918.

Table I.

Chromium.

A sample of very fine chromium powder, from the Metal and Thermit Company, gave the following spacings:

Intensity of Line.	Number of Coöperating Planes.	Planar Spacings.		Indices of Form.
		Observed.	Calculated.	
100	6	2.048	2.047	100
20	3	1.448	1.448	100 (2)
50	12	1.183	1.183	211
10	6	1.025	1.024	110 (2)
30	12	.915	.915	310
5	4	.837	.836	111 (2)
30	24	.774	.774	321
½	3	.723	.724	100 (4)
20	18	.682	.683	411 110 (3)
5	12	.648	.647	210
2	12	.617	.617	332
1	12	.591	.591	211 (2)
15	36	.567	.567	431 510

Type of Lattice		**Body-centered Cube.**
Side of elementary cube		2.895 Å.
Distance between nearest atoms		2.508 Å.
Density	from X-ray measurements	7.07
	from standard methods	6.92

The calculated spacings are those of a *body centered cubic lattice*, the side of the cube being 2.895 Å., and the distance between nearest atoms 2.508 Å.

Since there are two atoms per elementary cube, the ideal density is

$$\frac{\text{mass}}{\text{volume}} = \frac{2M}{d_0^3} = \frac{2 \times 52.0 \times 1.650}{(2.895)^3} = 7.07;$$

M = mass of 1 atom in grams; d_0 = side of elementary cube in cm.

The density given by Van Nostrand is 6.92.

Table II.

Molybdenum.

Very fine powder, by reduction of oxide in H_2. Excellent photograph.

Intensity of Line.	Number of Coöperating Planes	Spacing of Planes.		Indices of Forms.
		Observed.	Calculated.	
100	6	2.215	2.221	110
50	3	1.569	1.571	100 (2)
100	12	1.283	1.284	211
35	6	1.109	1.111	110 (2)
60	12	.993	.993	310
10	4	.907	.907	111 (2)
70	24	.839	.839	321
5	3	.784	.785	100 (4)
30	18	.739	.740	110 (3) 411
20	12	.702	.702	210 (2)
20	12	.669	.670	332
20	12	.641	.641	211 (2)
35	36	.616	.616	431 510
25	24	.574	.574	521
5	6	.554	.555	110 (4)
25	24	.538	.538	530 433
20	15	.523	.525	100 (3) 221

Type of Lattice..Body-centered Cube.

Side of elementary cube..3.143 Å.

Distance between nearest atoms..2.720 Å.

Density { from X-ray data..10.16

by standard methods..10.28

These spacings are identical, within experimental error, with those of tungsten. In order to detect the difference, if any, a special photograph was taken, giving tungsten and molybdenum on the same film, one above the other. This was accomplished by filling the lower half of a small glass tube with Mo and the upper half with W, and placing it symmetrically in the X-ray beam so that both halves received equal radiation. A brass septum prevented overlapping of the scattered radiation from the two samples.

This photograph showed the first 6 lines of W exactly above those of Mo. The seventh W line was noticeably displaced from that of Mo, and the fifteenth lines were displaced by the difference between the components of the clearly resolved α doublet (cf. Fig. 1), about 2 mm. The Mo lines were the farther from the center, on both sides, thus eliminating any error due to imperfect alignment of the samples.

This gives tungsten a spacing ¼ per cent. *greater* than that of Mo, viz., 3.150.

TABLE III.

Tantalum.

Filaments, taken from lamp, crushed. Very faint photograph.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
10	6	2.315	2.315	110
3	3	1.636	1.636	100 (2)
15	12	1.335	1.335	211
4	6	1.157	1.158	110 (2)
3	12	1.033	1.033	310
2	4	.946	.944	111 (2)
8	24	.872	.874	321
½	3	.818	.819	100 (4)
3	18	.773	.773	110 (3) 411
½	12	.732	.732	210 (2)
½	12	.697	.697	332
½	12	.667	.667	211 (2)
1	36	.643	.642	431 510

Type of Lattice	**Body-centered Cube.**
Side of elementary cube	3.272 Å.
Distance between nearest atoms	2.833 Å.
Density from X-ray data	17.09
Density by standard methods	14.49[1]

COBALT.

The powder photographs of cobalt show two crystalline forms, one face-centered cubic and the other hexagonal close packed, both stable at room temperature.

The relation of these two forms to each other, and especially to heat treatment and mechanical working, has not yet been studied.

Tests were made upon samples prepared in three different ways.

1. Two carefully analyzed samples of very pure cobalt were kindly

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[1] This is the value given by Van Nostrand. The most recent determination cited by Landolt & Börnstein (v. Bolton, Zs. Electroch., 11, 45, '05) gives 16.64, in better agreement with the X-ray data.

furnished for this investigation by Dr. H. T. Kalmus. Filings from these samples gave identical patterns of lines, with the spacings given in Table IV. They show *a perfect hexagonal close-packed lattice* with no trace of a cubic form. The same filings, after annealing in H_2 at 600° for 6 hours, showed a mixture of cubic and hexagonal close packing in about equal proportions. It was not found possible by annealing at this temperature to cause the crystallization to go over entirely from hexagonal to cubic form.

Table IV.

Cobalt (1).[1]

Filings from cast cobalt (Kalmus), 99.7 per cent. pure.

Number of Coöperating Planes, *N*.	Phase Factor, *F*.	*NF*.	Intensity of Line (Estimated).	Spacing of Planes.		Indices of Form.
				Observed.	Calculated.	
3	¼	.75	30	2.182	2.176	$10\bar{1}0$
1	1	1.	25	2.055	2.051	0001 (2)
6	¾	4.5	50	1.920	1.920	$10\bar{1}1$
6	¼	1.5	2	1.490	1.493	$10\bar{1}2$
3	1	3.	20	1.250	1.257	$11\bar{2}0$
6	¾	4.5	5	1.145	1.158	$10\bar{1}3$
6	¼	1.5	...	....	1.089	$10\bar{1}0$ (2)
6	1	6.	15	1.064	1.072	$11\bar{2}2$
6	¾	4.5	5	1.045	1.051	$20\bar{2}1$
1	1	1.	½	1.013	1.026	0001 (4)
6	¼	1.5	½	.953	.961	$10\bar{1}1$ (2)
6	¼	1.5	...	...	.928	$10\bar{1}4$
6	¾	4.5	2	.839	.853	$20\bar{2}3$
6	¼	1.5	...	...	.823	$21\bar{3}0$
12	¾	9.	2	.797	.807	$21\bar{3}1$
6	1	6.	2	.788	.794	$11\bar{2}4$
6	¾	4.5	1	.752	.768	$10\bar{1}5$
12	¼	3.	(wide)		.765	$21\bar{3}2$
6	¼	1.5	¼	.738	.747	$10\bar{1}2$ (2)
3	1	3.	¼	.717	.727	$10\bar{1}0$ (3)

Type of Lattice	**Hexagonal Close-packed.**
Axial ratio	1.633
Side of elementary triangular prism	2.514 Å.
Distance between nearest atoms	2.514 Å.
Density from X-ray data	8.66
Density by standard methods	8.718

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[1] This film was taken in a casette of only 10 cm. radius, with the Co filings spread on a thin film nearly 3 mm. wide, and there is a systematic error in the readings, amounting to about 1 per cent. for the largest spacings. The accuracy is sufficient for the purpose, which is to show the perfect hexagonal character of the lattice. The correctness of the value chosen for the fundamental spacing was checked by other films.

2. A sample of fine powder, prepared by reduction of cobalt oxide in H_2 at about 600°, gave the spacings shown in Table V. These correspond to *a perfect cubic close-packed lattice.* A very faint trace of the hexagonal form is present, as shown by the first and third lines, 2.162 Å. and 1.922 Å. respectively.

TABLE V.

Cobalt (2).

Fine cobalt powder, from oxide reduced in H_2 at 600°. Lines broken and irregular, due to large crystals, making the estimation of intensities difficult, but not interfering with the measurement of the spacings.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
1	..	2.162	....	...
12	4	2.035	2.051	111
¼	..	1.922	....	...
1	3	1.775	1.777	100 (2)
10	6	1.257	1.257	110 (2)
12	12	1.071	1.072	311
1	4	1.024	1.025	111 (2)
¼	3	.888	.888	100 (4)
½	12	.814	.815	311
5	12	.795	.794	210 (2)
1	12	.726	.725	211 (2)
2	16	.686	.684	511 111 (3)
1	6	.630	.628	110 (4)
1	24	.603	.601	531
1	15	.595	.594	100 (6) 221 (2)

Type of Lattice	**Face-centered Cubic.**
Side of elementary cube	3.554
Distance between nearest atoms	2.514
Density, from X-ray data	8.66
Density, by standard methods	8.718

3. A third sample, prepared by dissolving a portion of sample 1, pure Kalmus cobalt, in H_2SO_4 and electrolyzing, showed *a mixture of about equal amounts of the two forms.* The spacings are tabulated in Table VI., together with the theoretical spacings of both the cubic and hexagonal lattices. It is seen that practically *all the spacings of both lattices are present.*

In Table IV. two extra columns have been added, to take account of the fact that in the hexagonal close-packed lattice the number of planes which coöperate to produce a line is not a correct index of the intensity to be expected, even for consecutive lines, since in some sets of planes the planes are not equally spaced, and hence their reflections are not all in phase. The number, N, of coöperating sets of planes must be multiplied by the "phase factor," F, of each set of planes, which is the ratio of the square of the sum of the amplitudes of reflection from all the planes in the set *added as vectors*, to the square of their algebraic sum. The product NF will be seen to represent very well the fluctuations in intensity of consecutive lines, thus confirming their identification.

TABLE VI.

Cobalt (3).

Fine powder, obtained by rapid electrolysis of sulphate. Lines rather diffuse.

Intensity of Line.	Spacing of Planes.		
	Observed.	Theoretical Face-centered Cubic.	Theoretical Hexagonal Close-packed.
5	2.158	....	2.176
20	2.037	2.051	2.051
20	1.915	....	1.920
3	1.773	1.777	....
½	1.484	....	1.493
10	1.250	1.257	1.257
2	1.148	....	1.158
...	....	....	1.089
12	1.066	1.072	1.072
3	1.045	....	1.051
2	1.017	1.025	1.025
1	.952	...	.961
...	...	...	.928
...	...	.888	...
¼	.846	...	.853
...	...	...	.823
...	...	.815	...
½	.802	...	.807
1	.787	.794	.794
...	...	...	.768
...	...	...	.765
...	...	...	.747
¼	.723	.725	.727
¼	.701	...	.706
¼	.682	.684	.684

NICKEL.

In a previous paper[1] some preliminary measurements on nickel were reported, which seemed to indicate two crystal forms, one face-centered cubic, the other body-centered cubic. The photographs were very poor, and the results were given as questionable.

These experiments have been repeated with great care. Many photographs have been taken of nickel powder prepared in different ways, including electrolysis, reduction of oxide in hydrogen, filings from cast nickel, and the same annealed in hydrogen at different temperatures. *All gave identical patterns of lines, showing face centered cubic structure.* The samples used in the preliminary tests have been lost so that these tests cannot be checked; but sufficient data has been taken to show that the centered cubic form, if it exists, does not appear under ordinary conditions of preparation.

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[1] Phys. Rev., 10, 692, 1917.

The following measurements, obtained with fine-nickel powder prepared by rapid electrolysis of nickel ammonium sulphate, are typical:

TABLE VII.

Nickel.

Fine powder, by electrolysis of nickel ammonium sulphate.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
50	4	2.038	2.042	111
25	3	1.766	1.770	100 (2)
20	6	1.252	1.253	110 (2)
30	12	1.067	1.065	311
5	4	1.022	1.022	111 (2)
1	3	.884	.885	100 (4)
10	12	.812	.812	331
8	12	.791	.791	210 (2)
5	12	.723	.722	211 (2)
5	16	.681	.681	111 (3) 511
½	6	.625	.625	110 (4)
4	24	.598	.598	531
2	15	.590	.591	100 (6) 221

Type of Lattice	**Face-centered Cubic.**
Side of elementary cube	3.540 Å.
Distance between nearest atoms	2.505 Å.
Density from X-ray data	8.72
Density by standard methods	8.6–8.93

ZINC.

The planar spacings obtained from measurements of the powder diffraction pattern do not fit an hexagonal lattice with the axial ratio 1.356 calculated from crystallographic measurements.[1] The determination of the true axial ratio, 1.860, by means of graphs representing planar spacings as a function of axial ratio, is described in the preceding

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[1] See Groth, Chemische Krystallographie, Vol. 1.

Table VIII.

Rhodium.

Fine powder, from electrolysis of pure rhodium chloride.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
50	4	2.200	2.203	111
10	3	1.908	1.910	100 (2)
10	6	1.350	1.353	110 (2)
15	12	1.150	1.150	311
2	4	1.100	1.103	111 (2)
...	3	...	.955	100 (4)
2	12	.878	.875	331
1.5	12	.853	.853	210 (2)
1.0	12	.781	.779	211 (2)
½	16	.736	.731	111 (3) 511
...	6	...	.675	110 (4)
½	24	.647	.646	531

Type of Lattice..**Face-centered Cubic.**
Side of elementary cube... 3.820 Å.
Distance between nearest atoms.................................. 2.700
Density from X-ray data..12.18
Density by standard method....................................12.1

Table IX.

Palladium.

Powder very fine, lines diffuse and faint.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
10	4	2.274	2.280	111
4	3	1.966	1.975	100 (2)
10	6	1.398	1.399	110 (2)
15	12	1.192	1.189	311
...	4	...	1.140	111 (2)
...	3	...	.987	100 (4)
3	12	.909	.906	331
1	12	.886	.882	210 (2)
1	12	.807	.806	211 (2)
¾	16	.759	.760	111 (3) 511
...	6	...	.697	110 (2)
½	24	.668	.668	531
¼	15	.656	.660	100 (6) 221 (2)

Type of Lattice..**Face-centered Cubic.**
Side of elementary cube... 3.950
Distance between nearest atoms.................................. 2.795 Å.
Density from X-ray data..11.40
Density by standard methods..................................11.4 to 11.9

TABLE X.

Iridium.

Powder very fine, lines diffuse and faint.

Intensity of Line.	Number of Coöperating Plants.	Spacing of Planes.		Indices of Forms.
		Observed.	Calculated.	
4	4	2.185	2.193	111
2	3	1.898	1.902	100 (2)
4	6	1.346	1.347	110 (2)
4	12	1.150	1.145	311
1	4	1.092	1.096	111 (2)
..	3	...	.952	100 (4)
2	12	.872	.872	331
½	12	.847	.850	210 (2)
½	12	.776	.776	211 (2)
½	16	.732	.732	111 (3) 511
..	6	...	.672	100 (6)
2	24	.642	.643	531

Type of Lattice..**Face-centered Cubic**

Side of elementary cube.. 3.805 Å.

Distance between nearest atoms.. 2.690 Å.

Density from X-ray data..23.15

Density by standard methods..22.42

TABLE XI.

Platinum.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
10	4	2.265	2.266	111
4	3	1.958	1.964	100 (2)
10	6	1.387	1.390	110 (2)
15	12	1.183	1.183	311
2	4	1.137	1.134	111 (2)
1	3	.983	.983	100 (4)
5	12	.902	.901	331
5	12	.878	.878	210 (2)
4	12	.800	.802	211 (2)
2	16	.758	.756	111 (3) 511
..	6	...	.694	110 (4)
2	24	.663	.664	531
1	15	.658	.657	100 (6) 221 (2)

Type of Lattice..**Face-centered Cubic.**

Side of elementary cube.. 3.930 Å.

Distance between nearest atoms.. 2.780 Å.

Density from X-ray data..21.23

Density by standard methods..19.96

paper in this number, and only the summary of results need be given here.

The zinc lattice is found to be a combination of two simple right triangular prism lattices, each of axial ratio 1.860, and side of elementary prism 2.670 Å., with the atoms of one lattice at the centers of the prisms of the other lattice. The cartesian coördinates of the positions of the atoms are therefore

$$m,\ n,\ pc,$$

$$m+\tfrac{1}{3},\ n+\tfrac{2}{3},\ (p+\tfrac{1}{2})c,$$

Table XII.

Zinc.

Fine powder, from oxide reduced in H_2 at 600° C

Intensity of Line.	Number of Coöperating Planes × Phase Factor.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
3	1.0	2.472	2.466	0001 (2)
1	0.75	2.293	2.295	$10\bar{1}0$
10	4.5	2.077	2.080	$10\bar{1}1$
2	1.5	1.684	1.678	$10\bar{1}2$
10	4.5	1.339	1.337	$10\bar{1}3$
10	3.0	1.332	1.326	$11\bar{2}0$
½	1.0	1.235	1.232	0001 (4)
7	6.0	1.172	1.166	$11\bar{2}2$
½	0.75	1.152	1.148	$10\bar{1}0$ (2)
4	4.5	1.121	1.119	$20\bar{2}1$
½	1.5	1.088	1.087	$10\bar{1}4$
1	1.5	1.044	1.041	$10\bar{1}1$ (2)
2	4.5	.947	.948	$20\bar{2}3$
(doublet, not 2	4.5	.910	.913	$10\bar{1}5$
resolved) 2	6.0		.910	$11\bar{2}4$
...	1.5	...	.874	$21\bar{3}0$
3	9.0	.859	.861	$21\bar{3}1$
...	1.5	...	.845	$10\bar{1}2$ (2)
1	1.0	.827	.828	0001 (6)
	3.0		.825	$21\bar{3}2$
...	1.5	...	.778	$10\bar{1}6$
2	9.0	.773	.773	$21\bar{3}3$
	3.0		.771	$10\bar{1}0$ (3)
1	4.5	.756	.754	$20\bar{2}5$
2	6	.737	.736	$30\bar{3}2$
½	3	.717	.715	$21\bar{3}4$
½	6	.703	.703	$11\bar{2}6$

Type of Lattice		**Hexagonal Close-packed**
Axial ratio		1.860
Side of elementary triangular prism		2.670 Å.
Distance between nearest atoms	in basal plane	2.670 Å.
	in pyramid planes	2.920 Å.
Density	from X-ray data	7.04
	by standard methods	7.142

where *m*, *n*, and *p* have all possible integral values, *c* is the axial ratio 1.860, and the unit is the side of the elementary triangular prism, 2.670 Å.

The lattice is shown in Fig. 3. *It is one of the two closest packed arrangements of prolate spheroids whose axes are in the ratio* 1.140 : 1.

Each atom has six nearest neighbors in regular hexagonal arrangement about it (in the basal plane), at distances of 2.670 Å. (between centers), and six others almost as near, viz., 2.92 Å., three in the plane below, and three in exactly similar positions in the plane above.

The density is

$$\rho = \frac{\text{mass of 1 atom}}{\text{volume of triangular prism}}$$

$$= \frac{65.37 \times 1.650}{\frac{1}{4}\sqrt{3} \times (2.670)^3 \times 1.860} = 7.04.$$

CADMIUM.

The interpretation of the data obtained from powder photographs of cadmium, with the help of graphs of planar spacings against axial ratio, has been described in the preceding paper by A. W. Hull and W. P. Davey. The lattice is found to be hexagonal, of the close-packed type, with axial ratio 1.89, instead of the value 1.335 found in the literature.

A new and slightly better photograph gave the data in Table XIII.

The lattice is exactly like that of zinc, except for axial ratio, viz., *a close-packed arrangement of prolate spheroids whose axes are in the ratio* 1.158 : 1. The cartesian coördinates of the positions of the atoms are:

$$m,\ n,\ pc,$$

$$m + \tfrac{1}{3},\ n + \tfrac{2}{3},\ (p + \tfrac{1}{2})c,$$

where *m*, *n*, and *p* represent all possible integral numbers, *c* is the axial ratio, 1.890, and the unit is the side of the elementary triangular prism, 2.960. The lattice is therefore a combination of two simple lattices of right triangular prisms, each of side 2.960 Å. and height 1.89 × 2.960 = 5.60 Å., with the atoms of one lattice at the centers of the prisms of the other, and vice versa. Each atom is surrounded by six others, in the basal plane, in a regular hexagon about it, at distances of 2.960 Å., and by six others, almost as near, viz., 3.28 Å., three in the plane below and three in exactly similar arrangement in the plane above.

$$\text{The density} = \frac{\text{mass of one atom}}{\text{volume of triangular prism}} = 8.74.$$

Table XIII.

Cadmium.

Filings from pure cast cadmium. A good photograph.

Intensity of Line.	Number of Coöperating Planes × Phase Factor.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
10	1	2.790	2.798	0001 (2)
5	.75	2.565	2.564	$10\bar{1}0$
30	4.5	2.325	2.329	$10\bar{1}1$
6	1.5	1.893	1.892	$10\bar{1}2$
5	4.5	1.508	1.510	$10\bar{1}3$
4	3.0	1.476	1.480	$11\bar{2}0$
½	1.0	1.397	1.399	0001 (4)
5	6.0	1.306	1.311	$11\bar{2}2$
...	.75	1.287	1.283	$10\bar{1}0$ (2)
3	4.5	1.250	1.250	$20\bar{2}1$
½	1.5	1.224	1.230	$10\bar{1}4$
1	1.5	1.166	1.165	$10\bar{1}1$ (2)
1	4.5	1.055	1.057	$20\bar{2}3$
1	4.5	1.024	1.027	$10\bar{1}5$
1	6.0	1.014	1.017	$11\bar{2}4$
...	1.5	...	.969	$21\bar{3}0$
2	9.0	.953	.955	$21\bar{3}1$
...	1.5	...	.946	$10\bar{1}2$ (2)
...	1.0	...	.933	0001 (6)
½	3.0	.914	.915	$21\bar{3}2$
...	1.5	...	.878	$10\bar{1}6$
1.5	9.0	.860	.860	$21\bar{3}3$
...	3.0	...	.855	$10\bar{1}0$ (3)
½	4.5	.843	.844	$20\bar{2}5$
½	6.0	.817	.818	$30\bar{3}2$
...	3.0	...	.790	$21\bar{3}4$
...	6.0	...	.787	$11\bar{2}6$
½	4.5	.762	.765	$10\bar{1}7$
...	1.5	...	.756	$10\bar{1}3$ (2)
...	3.0	...	.740	$11\bar{2}0$ (2)
1	9.0	.732	.732	$21\bar{3}5$
½	6.0	.716	.715	$11\bar{2}1$ (2)
...	1.5	...	.710	$31\bar{4}0$
½	9.0	.706	.705	$31\bar{4}1$

Type of Lattice .... **Hexagonal Close-packed.**
Axial ratio 1.89
Side of elementary triangular prism .... 2.960 Å.
Distance between nearest atoms { in basal plane .... 2.960 Å.
Distance between nearest atoms { in pyramid plane .... 3.280 Å.
Density { from X-ray data 8.74
Density { by standard methods 8.642

RUTHENIUM.

There is no crystallographic data regarding ruthenium. The procedure was therefore as follows: A powder photograph was taken in the usual manner, the lines measured and the experimental crystal spacings calculated. These spacings were then laid off on a strip of paper, as described in the preceding article, and this strip moved over the different plots until a position was found on one of them where the experimental lines exactly matched the theoretical lines of the plot.

TABLE XIV.

Ruthenium.

Intensity of Line.	**No. of Coöperating Planes × Phase Factor.**	**Spacing of Planes.**		**Indices of Form.**
		Observed.	**Calculated.**	
3	0.75	2.315	2.325	$10\bar{1}0$
¼	1.0	2.160	2.134	0001 (2)
10	4.5	2.040	2.040	$10\bar{1}1$
2	1.5	1.572	1.575	$10\bar{1}2$
5	3.0	1.343	1.343	$11\bar{2}0$
5	4.5	1.212	1.214	$10\bar{1}3$
...	0.75	....	1.163	$10\bar{1}0$ (2)
5	6.0	1.139	1.136	$11\bar{2}2$
4	4.5	1.123	1.122	$20\bar{2}1$
...	1.0	....	1.067	0001 (4)
½	1.5	1.024	1.020	$10\bar{1}1$ (2)
1	1.5	.968	.971	$10\bar{1}4$
2	4.5	.902	.901	$20\bar{2}3$
...	1.5	...	.879	$21\bar{3}0$
3	9.0	.864	.861	$21\bar{3}1$
2	6.0	.838	.836	$11\bar{2}4$
¼	3.0	.816	.813	$21\bar{3}2$
½	4.5	.803	.802	$10\bar{1}5$
...	1.5	...	.788	$10\bar{1}2$ (2)
¼	3.0	.779	.776	$10\bar{1}0$ (3)
1	9.0	.750	.748	$21\bar{3}3$
1	6.0	.730	.728	$30\bar{3}2$

Type of Lattice......**Hexagonal Close-packed.**
Axial ratio......1.59
Side of elementary triangular prism......2.686 Å.
Distance between nearest atoms { in basal plane......2.686 Å.
Distance between nearest atoms { in pyramid planes......2.640 Å.
Density { from X-ray data......12.56
Density { by standard methods......12.26

The experimental values were found to agree exactly with an hexagonal close packed type of lattice, of axial ratio 1.59.—The indices of the crystal forms corresponding to each of the experimental spacings can be read off on the plot (cf. Fig. 3, p. 556, this number) and any one of them, *e.g.*, the $11\bar{2}0$

spacing, can be used to calculate the side of the elementary triangular prism. The density is then calculated as the ratio of the mass in the unit triangular prism, viz., one atom, to its volume. The side of the unit prism is thus found to be 2.686 Å., and the ideal density of ruthenium 12.56.

This lattice is a close packed arrangement of oblate spheroids whose axes are in the ratio 0.973 : 1. It is composed, like that of Zn and Cd, of two intermeshed lattices of right triangular prisms, each of side 2.686 Å. and height 1.59 × 2.686 = 4.272 Å., the atoms of the first lattice being in the centers of the prisms of the second and vice versa.

Each atom has as nearest neighbor the six atoms of the unit prism of which it is the center, at distances of 2.640 Å., and six others almost as near, in a regular hexagon about it (in the basal plane) at distances of 2.686 Å.

INDIUM.

The interpretation of the X-ray diffraction pattern of indium powder, with the help of graphs of axial ratio against crystal spacings, has been described in the foregoing article, and need only be summarized here.

Indium was found to consist of *a face-centered tetragonal lattice of atoms, with axial ratio* 1.06.

TABLE XV.

Indium.

Intensity of Line.	Number of Coöperating Planes.	Spacing of Planes.		Indices of Form.
		Observed.	Calculated.	
20	4	2.70	2.693	111
½	1	2.42	2.428	001 (2)
5	2	2.29	2.292	100 (2)
2	4	1.675	1.665	101 (2)
½	2	1.617	1.620	110 (2)
1	4	1.450	1.449	113
2	8	1.392	1.390	311
2	4	1.348	1.347	111 (2)
...	1	....	1.215	001 (4)
¼	2	1.150	1.146	100 (4)
1	8	1.080	1.082	133

Type of Lattice		**Face-Centered Tetragonal.**
Axial ratio		1.06
Side of elementary tetragonal prism		4.58 Å.
Height of elementary tetragonal prism		4.86 Å.
Distance between nearest atoms	in basal plane	3.24 Å.
	in pyramid planes	3.33 Å.
Density	from X-ray data	7.42
	by standard methods	7.12

The coördinates of the atoms are

$$m,\ n,\ pc,$$
$$m+\tfrac{1}{2},\ n+\tfrac{1}{2},\ pc,$$
$$m+\tfrac{1}{2},\ n,\ (p+\tfrac{1}{2})c,$$
$$m,\ n+\tfrac{1}{2},\ (p+\tfrac{1}{2})c,$$

where m, n, and p have all possible integral values, c is the axial ratio, 1.06, and the unit of length is the side of the elementary face-centered square which is the base of the unit prism. The lattice obviously consists of four intermeshed lattices of right tetragonal prisms, each of side 4.58 Å. and height 4.86 Å. It is shown in Fig. 4.

Each atom has four nearest neighbors, viz., the four corner atoms of the 001 face at whose center it is, at distances of 3.24 Å., and eight others almost as near, four in the centers of the lateral faces of the cube above and four similarly below, at distances of 3.33 Å.

This lattice is a close packed arrangement for prolate spheroids with axes in the ratio 1.06 : 1. It is an alternative arrangement to the hexagonal type exemplified by Zn and Cd, corresponding to the cubic and hexagonal arrangements, respectively, of close-packed spheres.

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THE BALMER SERIES OF HYDROGEN, AND THE QUANTUM THEORY OF LINE SPECTRA.

By Raymond T. Birge.

Synopsis.

Balmer Series of Hydrogen.—(1) *A rigid comparison of experimental with theoretical results* is the main purpose of the paper. It is concluded that Merton's measurements of the separation, half-width and relative intensity of the H_α and H_β doublets check with values derived from the recent Bohr-Kramers developments of the quantum theory, if a general field of 100 volts per cm. is assumed. After the fine structure has been analyzed and the "circle" line computed, determining the position of the components from the Sommerfeld theory and the relative intensities from the Bohr-Kramers theory, the frequencies of all lines are fitted into the Bohr relativity formula. (2) *The Rydberg constant* for H is then calculated to be 109,677.7 ± 0.2. (3) *A discussion of observations in mixtures of hydrogen and helium* made by Merton and Nicholson at relatively high pressures concludes the paper.

Rydberg spectral series constant for a nucleus of infinite mass is computed to be 109,736,9 ± 0.2.

Introduction.

The purpose of this paper is to make as accurate a comparison as possible between the best experimental results for the Balmer series of hydrogen, and the theoretical results to be expected according to the latest developments of the quantum theory of line spectra. The experimental data are as follows:

1. Grating measurements by Paschen[1] of H_α, H_β, H_γ, and H_δ, (all unresolved) giving 6562.797, 4861.326, 4340.465, and 4101.735 I.A. respectively. The experimental error is estimated at ±0.004 Å. in each case.

2. Grating measurements by Paschen[1] of H_α (resolved) giving 6562.853 and 6562.732 I.A. for the components.

3. Interferometer measurements by Meissner[1] of H_α, giving a doublet, 6562.849 and 6562.725 I.A., of intensity 7 and 5 respectively, and of H_β (unresolved) giving 4861.329 I.A.

4. Grating measurements by Curtis[2] of the first six lines of the series (all unresolved) giving 6562.793, 4861.326, 4340.467, 4101.738, 3970.075, and 3889.051 I.A. respectively. The probable errors range from 0.0017 Å. to 0.0006 Å. (see Table II., column 3). The correct reduction to vacuum

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[1] Ann. d. Phys. (4), 50, 901, 1916.

[2] Proc. Roy. Soc., A, 90, 605, 1914.

and a discussion of various possible series formulae was given by Curtis[1] in 1919.

5. A study by Merton[2] of the fine structure of H_α and H_β, under various experimental conditions. These results supplement and in part correct previous results by Merton and Nicholson.[3] The later results are to the effect that the most distinct doubling of both H_α and H_β is observed at liquid air temperature, in a mixture of helium and hydrogen at a few millimeters pressure. Under these conditions H_α is a doublet of 0.1446 Å. separation, the components having a half-width of 0.028 Å. and an intensity ratio of 2.18 to 1 (the red component being the stronger). H_β is a doublet of 0.093 Å. separation, and 0.045 Å. half-width of components. The intensity ratio was not measured. Further details of this work are given later in the present paper.

6. Data by Merton and Nicholson[4] and by Wood[5] as to the appearance of higher members of the Balmer series in mixtures of hydrogen and helium, at relatively high pressures, and in long vacuum tubes respectively.

There are also data on the Stark effect for the Balmer series, but this subject will not be discussed, since the agreement between theory and experiment, in the case of strong electric fields, has already been shown by Kramers[6] to be perfect. The results for weak fields will however be treated, in connection with the fine structure of the lines.

The quantum theory of line spectra, as applied to the Balmer series, may be summarized as follows:

In 1914–1915 Bohr[7] applied Rutherford's nuclear theory of the atom and general quantum theory to the case particularly of a single electron of charge e and mass m revolving about a nucleus of charge Ne and mass M. On the hypothesis of circular orbits, and the atomicity of angular momentum, the Balmer formula

$$\nu = \nu_1 = N_0 N^2 \left(\frac{1}{n''^2} - \frac{1}{n'^2} \right) \qquad (1)$$

was theoretically deduced. In this expression the angular momentum in the initial state of the electron is given by $n'h/2\pi$, in the final state by $n''h/2\pi$. For the Balmer series $n'' = 2$, $n' = 3, 4, 5$ etc. $N = 1$ for all Hydrogen series and $N = 2$ for Helium enhanced series. N_0 is the so-called Rydberg constant, and is *not* a constant, depending for its value on the mass of the nucleus. For Hydrogen it is designated N_0 and is given theoretically by $N_\infty \times M_H/(M_H + m)$, where N_∞ is the constant for a nucleus of infinite mass and given by $2\pi^2e^4m/ch^3$, in terms of frequency number. The spectroscopically observed value of N_0 and the theoretical value deduced from this expression check within the limits of experimental error.[1] Because however of the relatively large errors in "h" and "e," and the small error in N_0 (regardless of the particular formula used for the Balmer series), N_0 must be considered as an undetermined constant, in testing various series formulæ.

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[1] Proc. Roy. Soc., A, 96, 147, 1919.

[2] Proc. Roy. Soc., A, 97, 307, 1920.

[3] Phil. Trans., A, 217, 237, 1917.

[4] Proc. Roy. Soc., A, 96, 112, 1919.

[5] Proc. Roy. Soc., A, 97, 455, 1920.

[6] D. Kgl. Danske Vid. Selsk. Skr. natur og math. Afd., 8, R. III., 3, 1919.

[7] Phil. Mag., 26, 1, 476, and 857, 1913; 27, 507, 1914; 29, 332, 1915; 30, 394, 1915.

In 1915 Bohr[2] applied relativity mechanics to his atomic model, and thus obtained, in place of (1), the more complex expression

$$\nu = \nu_1 + \nu_2 = \nu_1 + 1/4N_0\alpha^2N^4\left(\frac{1}{n''^4} - \frac{1}{n'^4}\right), \tag{2}$$

where $\alpha = 2\pi e^2/hc$. Equation (2) is frequently written

$$\nu = N_0N^2\left(\frac{1}{n''^2} - \frac{1}{n'^2}\right)\left[1 + \frac{\alpha^2N^2}{4}\left(\frac{1}{n''^2} + \frac{1}{n'^2}\right)\right]. \tag{2'}$$

Equation (2) or (2′) indicates a shift of all wave-lengths to the violet as compared with the values given by equation (1). For the Balmer series this shift varies from 0.039 Å. for H_α to 0.010 Å. for the head of the series. Since the frequencies given by (2) correspond to an electron falling from one circle to another circle, the corresponding spectral lines will be called "circle lines."

In 1915–1916 Sommerfeld[3] extended the Bohr theory to the case of elliptic orbits. For a "hydrogen-like" atom, *i.e.*, a single electron revolving about a nucleus, there is both angular and radial momentum. The angular momentum is constant and is given by $n_2h/2\pi$. The radial momentum has an average value of $n_1h/2\pi$. The total momentum is given by $nh/2\pi$ where $n = n_1 + n_2$, and as before n' will be used to designate the initial state and n'' the final. The emitted frequency is then given by

$$\nu = \nu_1 + \nu_2 + \nu_3, \tag{3}$$

where

$$\nu_3 = N_0\alpha^2N^4\left[\frac{1}{n''^4}\left(\frac{n_1''}{n_2''}\right) - \frac{1}{n'^4}\left(\frac{n_1'}{n_2'}\right)\right]. \tag{4}$$

Equation (3) indicates the so-called "fine-structure" of spectral lines. Each line given by the Bohr theory (*i.e.*, each circle line) is split up into a number of components, obtained by combining the various possible states $n_1'' + n_2'' = n'' =$ constant, with the possible states $n_1' + n_2' = n$ = another constant. This is now known as the "combination-principle." Formulæ (2) and (3) are both approximate, terms in $(\alpha N)^4$ and higher powers having been neglected in comparison with $(\alpha N)^2$. It is only in the case of X-ray spectra that these higher powers need be considered. (See Sommerfeld, loc. cit., p. 55.)

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[1] Birge, Phys. Rev., 14, 363, 1919.

[2] Phil. Mag., 29, 332, 1915.

[3] Main article, Ann. d. Phys., 51, 1–125, 1916.

In discussing the relative intensity which the various fine structure components of a line might be expected to possess, Sommerfeld made certain arbitrary assumptions, as his theory gave no theoretical basis for the matter. Paschen (loc. cit.) tested these assumptions, using the "4686" helium enhanced series, in which the "fine structure" is sixteen times as broad as in the corresponding hydrogen series, due to the N^4 term in (4). Paschen concluded that the fine structure components agreed in position with Sommerfeld's theory, but agreed only partially in intensity, especially in the case of the spark discharge. Paschen found a striking difference in appearance between the spark and the constant current discharge, a difference not predicted by the theory.

A notable advance in the quantum theory of line spectra was made in 1918 by Bohr,[1] who worked out a logical basis for the intensity and state of polarization of the various fine structure lines. This theory he applied also to the Stark and to the Zeeman effect. Kramers (loc. cit.), working under Bohr, made a detailed application of Bohr's ideas to the Stark effect, in hydrogen and in helium, and to the fine structure of the lines, in these two gases. The agreement between theory and experiment, in the case of the Stark effect, is very remarkable. The position of the Stark effect components had already been predicted by Epstein[2] and Schwarzschild.[3] The advance made by Bohr and Kramers consisted in the correct prediction of the intensity and state of polarization of each component.

Kramers' work on the fine structure of the lines indicated that the intensity of the various components varied enormously with the strength of the electric field, especially in the case of the hydrogen series. Electric fields such as are found in the ordinary vacuum tube are evidently primarily responsible for the appearance of the lines, and especially for the change of appearance with varying physical conditions, as observed by Merton. The chief purpose of the present paper is to apply Kramers' theoretical results to the Balmer series, since Kramers himself made no detailed comparison except in one case to be mentioned presently.

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[1] D. Kgl. Danske Vid. Selsk. Skr. natur og math. Afd., 8, R. IV., 1 and 2, 1918.

[2] Ann. d. Phys., 50, 489, 1916; 51, 168, 1916.

[3] Ber. Akad. Berlin, p. 548, 1916.

The articles by Sommerfeld, (1916) Bohr (1918), and Kramers (1919), are all very long and involved, especially the Kramers article. Consequently it is not possible to give a logical summary of the assumptions and conclusions, without making the present paper unduly long. Such material as is needed will be used without explanation, except by reference to the page number of the original article.

THE FINE STRUCTURE OF THE BALMER SERIES, ACCORDING TO SOMMERFELD.

As shown by equation (4), each spectral line consists of a number of components. In this equation $\alpha^2 = 5.308 \times 10^{-5}$ (using 6.555×10^{-27} for "h"). Sommerfeld (p. 68) used 5.30×10^{-5} for α^2. The equation, as applied to the Balmer series, may therefore be written (in terms of frequency-number)

$$\nu_s = 5.822 \left[\frac{1}{n''^4} \left(\frac{n_1''}{n_2''} \right) - \frac{1}{n'^4} \left(\frac{n_1'}{n_2'} \right) \right], \tag{5}$$

where $n_1'' + n_2'' = n'' = 2$, and $n_1' + n_2' = n' = 3, 4, 5$, etc., and ν_s gives the distance of the various components from the circle line. Any spectral line may be indicated by $(n_1'n_2', n_1''n_2'')$. The H_α circle line is therefore (03, 02), the H_β circle line (04, 02). The separation $(n_1'n_2', 11) - (n_1'n_2', 02)$ is known as the fundamental hydrogen separation. It is given by $5.822/2^4 = 0.3639$, or 0.3640 approximately. For this very important constant Paschen (p. 910) derived the experimental value of 0.3645 ± 0.0045, and used this value in all his calculations. The author has, however, used the theoretical value 0.3640. The H_α line thus consists of two sets of identical triplets (see Table I., and Fig. 1 *a*), separated by 0.3640ν (= 0.1568 Å.). The H_β line consists of two identical quadruplets, separated by 0.3640ν (=0.0858 Å.), etc. The Balmer series therefore consists of a doublet of constant $\Delta\nu$, the components of each member being themselves complex, but (as shown by equation 4) the total width of each component growing rapidly narrower, as n' increases.

A convenient method (used by Paschen) for labeling the various fine structure components is indicated in Table I., where the components of H_α, beginning at the red, are given in order, with the successive separations, in terms of Ångstroms. (See also Fig. 1 *a*.)

States in which $n_2 = 0$ are excluded, for this indicates no angular momentum, and the electron would therefore collide with the nucleus. The number of possible states is therefore only $n' \times n''$. Sommerfeld's first assumption as to intensity is that it is proportional to $(n_2'/n')(n_2''/n'')$, thus giving the maximum intensity (unity) to the circle line I_a. The

TABLE I.

1	2				3	4	5
Name.	n_1'	n_2'	n_1''	n_2''	Separation in Å.	Intensity Paschen 1.	Intensity Paschen 2.
I_c	2	1	0	2		2/6	small
I_b	1	2	0	2	0.0464	4/6	4/6
I_a	0	3	0	2	0.0155	6/6	6/6
II_c	2	1	1	1	0.0949	1/6	1/6
II_b	1	2	1	1	0.0464	2/6	2/6
II_a	0	3	1	1	0.0155	3/6	zero

intensities for the H_α components are given in the 4th column of Table I. Sommerfeld then makes the second assumption that, in an electron transition, the radial momentum never increases. This excludes the II_a line of each member of the Balmer series. He also makes the third assumption that an increase of angular momentum rarely if ever occurs. This indicates a very small intensity for the least refrangible member of each Balmer series line. The results, applied to H_α, are given numerically in Table I., column 5, and graphically in Fig. 1 *b*. They indicate a

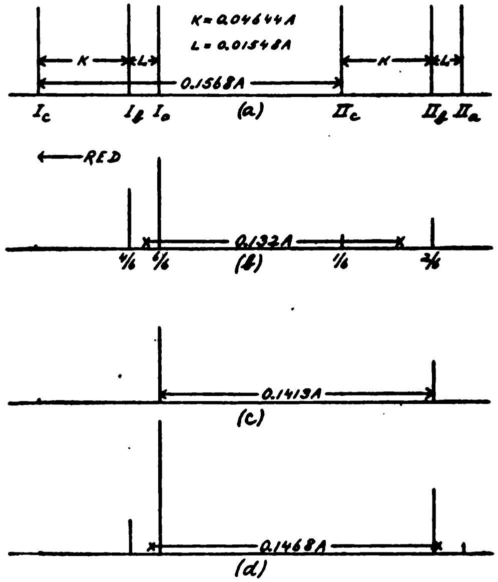

Fig. 1.

(*a*) H_α fine structure,—location only.
(*b*) Intensity relations according to Sommerfeld's theory.
(*c*) Intensity relations, with no electric field, according to Bohr-Kramers' theory.
(*d*) Intensity relations, with a field of 100 volts/cm., according to Bohr-Kramers' theory.
The crosses represent the centers of gravity of the two components of H_α.

doublet (each component of which is itself double), having a separation of 0.132 Å., and an intensity ratio of 3.3 to 1. Paschen, however, found that in the "4686" line the component III_a, corresponding to an increase of two units of radial momentum, appeared in the spark discharge with relatively great intensity. II_a, corresponding to an increase of one unit, was also very strong. Neither appeared in the constant current discharge. On the other hand, lines corresponding to an increase of angular momentum appeared faintly in both types of discharge. In fact, Sommerfeld made his third intensity assumption to agree with this observed fact.

SPECTRAL FORMULÆ FOR THE BALMER SERIES.

In measuring the wave-lengths of the Balmer series, as has been done by Curtis and others, what is actually observed is the apparent center of gravity of the complex line.[1] In order to interpret the results it is necessary to make assumptions in regard to the intensity, as well as the position, of the fine structure components. With such assumptions made, it is possible to compute, from the observed center of gravity of the line, the true wave-length of the circle component I_a. It is to this component alone that formula (2) or (2′) applies. This is the procedure followed by Paschen (loc. cit.). Curtis however, in his 1919 article, applied (2′) directly to the original measurements. He also took α^2 as an undetermined constant, obtaining by a Least Squares solution a value of 15.48×10^{-5}, as compared to the theoretical value 5.308×10^{-5}. The author has found that an almost equally good agreement between the observed and computed wave-lengths can be obtained by using the theoretical value of α^2. There is however a definite divergence between theory and experiment which appears more clearly by handling the data in a different way, as indicated below.

Curtis uses other two constant and even three constant formulæ. Since the deviations from equation (1) are very small, it is to be expected that the addition of one or two additional constants, in almost any functional form, will result in a fairly good agreement, such as Curtis found. But formula (2), as applied to the Sommerfeld theory, has only one undetermined constant (N_0). Even this would be a determined constant, if we had accurate enough values of "h" and "e." α^2 is also a function of these quantities, but considerable changes in the assumed value of α^2 make but slight alterations in the computed frequency. Equation (2) is then practically a one constant formula.

The deviations of the observed lines from equation (1) are best exhibited by computing a value of N_0 ($= N_\infty$) from each observed wave-length. The results,[1] taken directly from Curtis' 1919 paper appear in Table II.,

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[1] Curtis says that he intended to set on the actual center of the line. The matter is discussed in a later section of this paper.

TABLE II.

1, n'.	2, λ (Air).	3, Error.	4, N_∞ (1).	5, ΔN (1).	6, $\Delta\lambda$ (1).	7, ΔN (2).	8, $\Delta\lambda$ (2).
3.....	6562.793	0.0017	109,679.155	+0.402	−0.0251	+0.278	−0.0166
4.....	4861.326	0.0010	78.896	+0.143	−0.0063	+0.057	−0.0025
5.....	4340.467	0.0006	78.776	+0.023	−0.0009	+0.003	−0.0001
6.....	4101.738	0.0013	78.748	−0.005	+0.0002	−0.009	+0.0003
7.....	3970.075	0.0016	78.738	−0.015	+0.0005	−0.007	+0.0003
8.....	3889.051	0.0011	78.750	−0.003	+0.0001	+0.013	−0.0005

column 4. As this column shows, the values of N_∞ become sensibly constant, beginning with $n' = 5$ (H_γ). Accordingly the average value of N_∞, using the last four values only, has been computed, and then the deviations from this average value (which is 109, 678.753). The results, in terms of frequency, are given in column 5, and in terms of Ångstroms, in column 6.

In a similar manner, formula (2′) has been applied to the Curtis data, using the theoretical value of α^2. (This is the author's own work.) Again it appears that N_∞ is sensibly constant after the first two wave-lengths. Its average value, from the last four, is 109,678.351. In column 7 are given the deviations from this average, in frequency units, and in column 8, the deviations in Ångstroms. A comparison of columns 8 and 6 indicates a reduction of error to 66 per cent. for H_α, 40 per cent. for H_β, 10 per cent. for H_γ, and no sensible change for the others. The discrepancy for H_α is still however ten times the experimental error. It is thus quite clear that the application of relativity mechanics to the Balmer formula results in a definite reduction of the discrepancy between observed and computed values, but does not in any way obliterate the discrepancy. The explanation must therefore be sought in the fine structure of the lines.

To make a rigid test between theory and experiment, it is necessary to make definite assumptions as to the position and intensity of every fine structure component. It is then possible to treat each Balmer series line as a doublet, each member of which is complex. Knowing the structure of the red component of the doublet, it is possible to compute the frequency of the circle line I_a (always the most refrangible member of the red component) if the position of the center of gravity of the red component is known. This center of gravity, in turn, can be computed from the observed center of gravity of the entire line, knowing the separation of the centers of gravity of the two components of the doublet, and their intensity ratio. But these latter data depend on the exact assumptions made as to the intensity of the fine structure lines.

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[1] In the reduction to vacuum, Curtis uses the new figures of the Bureau of Standards. These will be used exclusively in the present article. Paschen uses older data, which are not so accurate, and therefore his vacuum values are not quite correct.

Table II., column 3, gives Curtis' estimate of the probable error of each line, in Ångstroms.

Paschen applied this general process to the analysis of his own measurements, using ostensibly the Sommerfeld theory as to the position and intensity of the fine structure components. He however made a number of simplifying assumptions, as well as one inadvertent error. The strict application of the Sommerfeld theory (using all three assumptions as to intensity) gives for the intensity ratio of the doublets, 3.3 to 1 (H_α), 3 to 1 (H_β), 2.8 to 1 (H_γ), 2.67 to 1 (H_δ), 2.57 to 1 (H_ϵ), etc., the limiting value being 2 to 1. The corresponding doublet separations are, 0.3130ν (= 0.1320 Å), 0.3489ν (= 0.0822 Å), 0.3587ν (= 0.0676 Å.), 0.3639ν (= 0.0613 Å.), 0.3640ν (= 0.0574 Å.), respectively. Paschen however used a constant intensity ratio of 2 to 1 for all lines, and a constant doublet separation of 0.3645ν. The distance of the circle line from the center of gravity of the red component is also variable, and has been used as such by the author. Paschen assumed the distance always equal to $1/4(I_a - I_b)$. This is incorrect even for the H_α line, and is due to Paschen's accidental assumption of an intensity ratio of 3 to 1 for I_a/I_b, instead of the correct 3 to 2 ratio. The error thus introduced is 0.0023 Å. for H_α. For the other lines it is practically negligible. The correct H_α structure, on the Sommerfeld theory, is shown in Fig. 1 *b*.

As has already been noted, Sommerfeld's assumptions as to intensity do not agree with experimental results, in the case of the helium lines. The discrepancies have now been explained by Kramers, who shows that the relative intensity of all fine structure components depends upon the exact strength of the electric field in the vacuum tube. This matter will presently be treated in detail. Another serious conflict between the Sommerfeld theory and observed facts is in the matter of the half-breadth of the red and violet components. Merton observed 0.028 Å. for H_α, and 0.045 Å. for H_β. A careful computation of the Doppler effect gave 0.023 Å. and 0.017 Å. for these widths. But on the Sommerfeld theory, the red component of H_α should be about 0.015 Å. wide, and the violet component 0.046 Å. wide, regardless of *any* Doppler effect. Conversely, for H_β, the Sommerfeld theory gives a width of only 0.005 to 0.014 Å., much less than the observed, with the full Doppler effect counted in.

Using all available data and the new figures for reduction to vacuum, the author has computed the position of the circle line, from the observed center of gravity of the entire line. This has been done according to the strict Sommerfeld theory, and also on the assumption of a constant 2 to 1 intensity ratio for the doublet. In view however of the discrepancy of this theory with observations, in the case of H_α and H_β, it does not seem advisable to present the results in detail. The values of N_∞ obtained from the various lines are as consistent as those of Paschen.[1] Using a 2 to 1 intensity ratio, but the theoretical value of the doublet separation, an average value for N_∞ of 109,677.826 is obtained. Using the theoretical intensity ratios given previously, the value is 109,677.944. The Curtis and Paschen wave-lengths, treated separately, yield the same average by the first method, but slightly different averages by the second. Neither of these averages includes H_α, since this line requires individual treatment. It will be noted that the values of N_∞ are somewhat higher than that of Paschen.[2]

THE BOHR-KRAMERS THEORY OF INTENSITY.

Bohr's new "Selective" principle (Auswahlprinzip) for obtaining at least a rough estimate of the intensity of the fine structure components is based on the idea that the actual orbit of the electron, in the initial and in the final state, can be considered as the sum of a number of harmonic vibrations. The intensity of an emitted line of any given frequency is then connected with the amplitudes of the harmonic vibrations of corresponding frequency in the initial and in the final state of the electron. The type of harmonic vibration (linear, circular, or elliptic) indicates the polarization (linear, circular, or elliptic) of the emitted line.

As applied by Kramers to the case of a strong electric field, the principle yields results in remarkable agreement with experiment. The intensity, as well as the state of polarization, of every component of the Stark effect, in the hydrogen and helium (enhanced) series is theoretically accounted for. The same principle, applied to the fine structure of these series, with no electric field, indicates (Kramers, p. 355) that only those transitions are possible for which the angular momentum *increases* or *decreases* by $h/2\pi$, *i.e.*, $n_2' - n_2'' = \pm 1$. In the case of the Balmer series this indicates the presence of only *three* fine structure components for each member, contrasted to $n'(n'' - n' + 1)$ components on the Sommerfeld theory. They are I_a, I_c and II_b for H_α, I_b, I_d and II_c for H_β, I_c, I_e and II_d for H_γ, etc.

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[1] P. 935, loc. cit. Paschen obtained a weighted mean of 109,677.691, and gives the probable error as ±0.06, although the probable error, by the ordinary Least Squares formula is only ±0.0145. The author, in all computations in this paper, has used only an unweighted average.

[2] All the computations of the present article had been completed, and the article partly written, when there appeared a brief paper by Herbert Bell (Phil. Mag., 40, 489, 1920) on the "Helium-Hydrogen Series Constants." The only point at which this paper overlaps the present work is in regard to the results just given. Bell uses both Paschen and Curtis wave-lengths, and the new reduction to vacuum. He however follows exactly the method of Paschen, assuming a 2 to 1 intensity ratio, and a constant doublet separation of 0.3650. Like Paschen, he uses the erroneous intensity ratio 3 to 1, for I_a/I_b. Bell also makes a computation with an assumed 3 to 1 intensity ratio for the doublet components. His results naturally differ slightly from those just presented due to the different method of making the fine structure corrections.

The intensity of each component is given by an expression (Kramers p. 356) involving Bessel functions. The results for H_α, on an arbitrary scale, are 5.00 for I_a, 1.36 for II_b, 0.04 for I_c. The true intensity of I_a is then taken as about one half of 5.00, because I_a is a circle line, and in the case of the Stark effect it was found that the theory gave too high values for the intensity, in the case of circle lines (due presumably to the various approximations introduced in the application). This conclusion was deduced by checking the results with the known fact that in the Stark effect, the polarized components into which an unpolarized spectral line is split will, when taken together, show no characteristic polarization in any direction. The results thus show that there may be errors up to possibly 100 per cent. in Kramers' computed intensities, but these are relatively small compared to the discrepancies in Sommerfeld's work, which were sometimes in excess of 1000 per cent.

The Kramers results for H_α are given in Fig. 1 *c*. They indicate a doublet separation of 0.1413 Å., and an intensity ratio of about 2 to 1. For H_β the results give a doublet separation ($I_b - II_c$) of 0.0822 Å., and an intensity ratio of about 1.25 to 1. The half-width of the H_β components, observed by Merton, is still less accounted for than on the Sommerfeld theory, although the H_α separation and intensity ratio checks very closely with Merton's experimental results.

Kramers (p. 359–378) then considers the effect of a weak electric field on the fine structure lines. His previous formulæ for the Stark effect hold only for strong fields. A rigid theory holding in all cases would be extremely complex, and has not yet been worked out. The results for a weak field indicate (as expected) that each fine structure component is broken up into a number of Stark effect components. Since the electric field is assumed very weak, the separation of these electric components may be neglected, and the intensity of any fine structure component may be judged by the sum of the intensities of all the Stark effect components composing it.

The calculations then show the extremely important result that various fine structure components previously missing may now appear, some of them with considerable intensity. In fact (Kramers, p. 364), while previously there appeared only those transitions for which the angular momentum changed by one unit, we may now have the additional transitions for which the angular momentum remains unchanged, or changes by two units. If the intensities of the new components are small, compared to the original components, the intensity of the old components will be practically uninfluenced by the perturbing electric field. These new components, in the case of the helium lines, explain with one questionable exception the experimental observations of Paschen (see Kramers, pp. 372–376).

The equations for obtaining the intensity of the new components are given on page 367 of Kramers. They are very complex, and contain, it may be noted, several typographical errors. These errors may however be readily identified by comparing the expressions with those on pages 318–320, from which they were derived. Kramers computes numerical results for H_α and for Helium 4686 and 3203 only. The author has checked most of Kramers' calculations. The only serious error found is in the case of the II_a component of both H_α and Helium 4686, for which Kramers computes practically zero intensity, while the correct results are 1.588 and 0.115 respectively.[1] These intensities are for an electric field of 300 volts per cm. The intensity of the line varies as the square of the field intensity, and inversely as the fifth power of N. It also depends, in a more complex manner, on n' and n'', increasing very rapidly with increase in either or both.

The author believes that the last two sentences furnish the clue to the interpretation of all of Merton's results, in mixtures of hydrogen and helium.[2] Merton himself remarks (p. 310) "the only cause of broadening to be suspected would be . . . due to the influence of the electric field of neighboring charged particles on radiating atoms, operating in accordance with the Stark effect. It is true that this effect is greater for hydrogen than for other elements, but the effect in the case of helium is comparable in magnitude and the excellent agreement between the experimentally measured widths of helium lines and the widths calculated on the basis of the Doppler effect in the experiments of Buisson and Fabry would seem to leave little room for broadening of this type."

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[1] For H_α, R'^2 is 1.141 and 0.447 for the two components of II_a, *i.e.*, (03,11). For He 4686 the R'^2 of the five components of II_a, *i.e.*, (04,12), is 0.025, 0.039, 0.0149, 0.0189, and 0.0173, respectively. The Kramers results for R''^2 are correct. The author's corrected intensity for II_a of 4686 accounts better for Paschen's observed line, in the spark discharge, at 4685.687.

[2] The Balmer series, with the fine structure predicted by theory, seems to appear only in mixtures of hydrogen and helium. No attempt is made in this paper to explain the various effects noted by Merton in pure hydrogen, and in hydrogen containing a trace of impurity. The cause of these effects is undoubtedly to be looked for in the *irregular, local* fields of force of the surrounding atoms, as contrasted to the *general* electric field discussed here. It is only the helium atom which has practically no electron "affinity," and so no appreciable local field.

Using Kramers' results, this conclusion is not justified. The author has found that by assuming a uniform field of 100 volts per cm., such as has experimentally been found to exist in the hydrogen positive column, under a few mm. pressure, the effect for helium and for hydrogen is very different, and is quite capable of explaining the experimental results. It is not possible to give with authority the exact quantitative figures. In the case of H_β such a field produces a Stark effect splitting which is comparable in magnitude with the fine structure separation. For this line one therefore is effectively dealing with a "strong" electric field, to which the simpler Epstein-Schwarzschild formula applies. Moreover, the computed intensity of some of the new fine structure components is many times as great as the intensity of the strongest of the original components, whereas the formulæ used are intended only for the case where these new components are comparatively weak. When they are strong the original components themselves will have changed in magnitude by an amount comparable with the intensity of the new components, but Kramers has derived no formula for this increase. The actual relations are therefore only very approximate.

The results are as follows. For a field of 100 volts per cm., the total Stark effect width for He 4686 is 0.0085 Å. But the strong components are all near the center (Kramers, Plate III.) so that the observed broadening might appear much less. For He 3203, the breadth is 0.0092 Å. This should be observed, since the strong components are on the outside. The regular fine structure components of 4686 are all at intervals of 0.05 Å. or more (Paschen, p. 916), and the observed breadths 0.03 Å. or more. The electric fields in Paschen's work may therefore be much stronger than 100 volts per cm.

For H_α, a field of 100 volts per cm. gives a total possible Stark effect width of 0.0224 Å., but the outer components are much weaker than the central (Kramers, Plate I.) so that the effective half-width is probably less than half as great. Merton computes 0.023 Å. for the Doppler effect and observes 0.028 for the half-width. This is in reasonable agreement with theory, especially as the broadening due to the electric field and that due to the Doppler effect may not be simply additive. The only fine structure components that are really effective are still I_a and II_b so there is no effective broadening due to the fine structure. The exact results are shown in Fig. 1 *d*, where, according to Kramers' suggestion, the intensity of the new components is added to the neighboring old components. The figures then become (with corrections to

Kramers' computations previously noted) $I_a = 2.237$, $I_b = 0.577$, $II_a = 0.196$, $II_b = 1.096$. The intensity ratio for the doublet is $(2.237 + 0.577)/(0.196 + 1.096) = 2.18$ to 1, or exactly equal to Merton's observed value. The center of gravity of the red component is 0.0032 Å. to the red of I_a, while that of the violet component is 0.0023 Å. to the violet of II_b. The separation of the doublet is 0.1468 Å., compared to Merton's observed value of 0.1446 Å.

For H_β with a field of 100 volts per cm., the total Stark effect width is 0.0304 Å., and the strongest components are on the outside (Kramers, Plate I.) so that this value should represent the observed broadening. The Doppler effect is 0.017 Å., and if this is simply additive, the total width is 0.0474 Å., as compared to Merton's observed value of 0.045 Å. It is probable that 0.002 to 0.004 Å. should be added for the fine structure, making a total expected half-width of as much as 0.051 Å. Again however this assumes the Stark effect broadening and the Doppler effect broadening as simply additive.

The fine structure broadening of 0.002 to 0.004 Å. is the result of the author's computations, Kramers having done no work on this line. The original components, with their relative intensities are I_b (0.104), II_c (0.085), and I_d (0.002). The *new* components, due to an electric field of 100 volts per cm., with their intensities, are I_a (11.85), II_b (8.29), and I_c (1.72). We have thus, as previously noted, new components much stronger than the old, with no means of accurately computing the intensity change in the old. Assuming it to be the same as in the new components, we have the red member of the doublet composed of two almost equally strong lines I_a and I_b, with a separation of 0.0018 Å., together with the comparatively weak I_c line, while the violet member is composed of two strong lines II_b and II_c, separated 0.0036 Å. The intensity ratio is roughly $11.85/8.29 = 1.43$ to 1, while the separation[1] is about 0.0840, compared to Merton's 0.093 Å. (Merton himself thinks that this value may be too large.) Merton did not measure the intensity ratio for H_β but a mere visual inspection of the published photographs (p. 316) indicates that the ratio is much less for H_β than for H_a.

For H_γ the Stark effect broadening is 0.0435 Å., with again the strong components on the outside. As the total doublet separation is only about 0.068 Å., it is not to be expected that this line can be resolved, under any of the experimental conditions realized by Merton. Many of the changes of appearance observed by Merton, with varying chemical and electrical conditions, can thus doubtless be explained by changing electric fields, interatomic if not general. That the electric field is competent to explain all observed half-widths, for resolved helium and hydrogen lines, seems evident in the light of the above figures.

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[1] This assumes the center of gravity of the violet component halfway between II_b and II_c, and of the red component at I_b (due to the influence of I_c.)

THE VALUE OF THE RYDBERG CONSTANT N_ω.

We are now ready to return to the original problem of testing equation (3). The test consists of computing a theoretical value for the circle line and substituting this in equation (2). The values for N_ω ($= N_0$ of equation 2) should be constant, and may be compared to the values given in Table II.

As previously mentioned, the Sommerfeld theory of intensities yields a constant N_ω, provided H_α is excluded. The Bohr-Kramers theory is equally satisfactory. The results are given in Table III.

TABLE III.

1.	2, N_ω.	3, $r(\nu)$.	4, $r(A)$.	5, N_ω.	6, $r(\nu)$
H_β Paschen	7.815	−0.006	+0.0003	7.665	+0.048
" Meissner	7.747	−0.074	+0.0033	7.597	−0.020
" Curtis	7.815	−0.006	+0.0003	7.665	+0.048
H_γ Paschen	7.851	+0.030	−0.0012	7.660	+0.043
" Curtis	7.800	−0.021	+0.0008	7.609	−0.008
H_δ Paschen	7.887	+0.066	−0.0025	7.642	+0.025
" Curtis	7.807	−0.014	+0.0005	7.561	−0.056
H_ϵ Curtis	7.821	±0.	±0.	7.571	−0.046
H_ζ Curtis	7.848	+0.027	−0.0010	7.587	−0.030
Average	7.821	0.027	0.0011	7.617	0.036

Column 2 gives the value of N_ω (the integral part 109,677 being omitted) using a constant intensity ratio of 2 to 1 for the doublet, but the theoretical separation.[1] In order to obtain the circle line, the center of gravity of the red component, in the case of H_β, is assumed at I_b, as explained in the previous note, *i.e.*, 0.0018 Å to the red of the circle line I_a. For H_γ the corresponding correction is about 0.0007 Å. For the other lines it is negligible.

On these assumptions, the average value of N_ω is 109,677.821, and the average residual, in frequency, is 0.027, as shown in column 3, or 0.0011 Å, as shown in column 4.

The same process has been repeated, using the true intensity ratio given by the Bohr-Kramers theory, *i.e.*, 2.18 to 1 for H_α and 1.43 to

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[1] This separation is 0.0840 Å. for H_β, and 0.0674 for H_γ, with the electric field used. For the other lines the constant separation of 0.3640 ν is used, as the error thus introduced is negligible.

1 for H_β. The results for the other lines have not been determined as the computations are very laborious; but it is evident that the ratio is rapidly declining, and will reach unity in the limit. I have accordingly assumed ratios as follows, 1.2 to 1 for H_γ, 1.1 to 1 for H_δ, 1.05 to 1 for H_ϵ, and 1 to 1 for H_ζ. The results are given in column 5 of Table III. (where again the integral part 109,677 is omitted), and the frequency residuals in column 6.[1]

The final problem is to determine how the experimental results for H_α check with these values of N_ω. To do this it seems simpler to use the average N_ω already obtained to compute a theoretical value for the wave-length (in air) of the circle line of H_α. Using 109,677.821 for N_ω, we obtain 6562.8413 Å. Using 109,677.617, we obtain 6562.8535 Å. As previously deduced, using the Bohr-Kramers theory and 100 volts per cm., the center of gravity of the red component should be 0.0032 Å. to the red of the circle line. This gives for its theoretical position 6562.8445 Å. or 6562.8567 Å. The average is 6562.8506. For the center of gravity of the red component Paschen observed 6562.853, while Meissner observed 6562.849. The average is 6562.851 which agrees exactly with the theoretical average. In other words, various theories as to the intensity of the electric field and the resulting intensity of the components of the lines lead to somewhat different expected results for the position of the red component of H_α, but the observed results also vary, and the various experimental and theoretical results all lie in the same region, and have the same average.

The second method of checking H_α is from the observed position of the center of gravity of the entire line. This position is 6562.797, according to Paschen, and 6562.793, according to Curtis. If 6562.8445 is the true position of the center of gravity of the red component, and if Merton's experimental value of 0.1446 Å. is assumed for the doublet separation (the theoretical value being about 0.1468 Å.), then Paschen's observation requires an intensity ratio of 2.04 to 1, while Curtis' requires 1.81 to 1. If 6562.8567 is the true value of the center of gravity, then Paschen's observation requires an intensity ratio of 1.42 to 1, while Curtis' requires 1.27 to 1. Meissner observed 1.4 to 1 ratio, while Merton observed 2.2 to 1.[1] With such discordant experimental results, it is hard to decide what ratio should be used, and it is evident that the value of the ratio depends greatly upon experimental conditions. A measurement of the center of gravity of the entire line is thus valueless, unless the intensity ratio, for the *same* conditions, is known. Moreover, the slightest change in the ratio makes a perceptible difference in the final comparison of theory and experiment. The only valid method at present available seems then that of using the directly observed center of gravity of the red component, and this, as shown, leads to a fairly satisfactory agreement of theory and experiment.

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[1] Curtis mentions that, in measuring the position of a line, he attempted to set on the actual center of the line, and not on its center of gravity. Paschen however says he attempted to set on the center of gravity. Bell calls attention to a slight systematic drift of Paschen's wave-lengths, as compared to Curtis'. There *is* such a drift, but it is doubtless due to the different reference standards used, and not to the method of setting. Thus if Curtis *really* set on the center of the line, his readings should be corrected (to a first approximation) by assuming a 1 to 1 intensity ratio of doublet components, instead of that actually used. This change in method would shift the H_β circle line by 0.0092 Å, from its predicted position on a 1.43 to 1 intensity ratio, or by 0.0158 Å., on a 2 to 1 ratio. Yet the Curtis and Paschen measurements agree for this line, (to 0.001 Å.), and differ by 0.004 Å. or less for all the others. It thus does not seem possible that Curtis and Paschen used essentially different methods of setting. If it *is* assumed that Curtis set on the actual center of the line, then, using only his values of wave-length, the average N_ω is 109,677.524 and its constancy is about the same as before.

The various values for N_∞ are summarized as follows,

Sommerfeld theory, 2 to 1 ratio	109,677.826
Sommerfeld theory, theoretical ratio	109,677.944
Bohr-Kramers theory, 2 to 1 ratio	109,677.821
Bohr-Kramers theory, theoretical ratio	109,677.617

These may be compared to previous results,

Paschen (Sommerfeld theory)	109,677.691
Curtis (empirical formulæ)	109,677.58 to 109,678.76

The author recommends for N_∞ the value **109,677.7**, which is probably correct to 0.2.

The best value of e/m is 1.773×10^7,[2] and this, combined with 4.774×10^{-10} for e, and 96,470 for the Faraday,[1] gives 1.6636×10^{-24} grams for the mass of the hydrogen atom, and 1853 to 1, as the ratio of its mass to that of the electron. We have therefore

$$N_\infty = \frac{2\pi^2 me^4}{ch^3} = 109{,}677.7\left(1 + \frac{1}{1853}\right) = \mathbf{109{,}736.9} \pm 0.2.$$

This may be compared to Paschen's determination (p. 936, loc. cit,) of 109, 737.18, using 1843.7 for the above ratio of M_H to m.

It has thus been shown that the quantum theory of line spectra, using the "Selective principle" of Bohr, leads, in the case of the Balmer series, to consistent values of N_∞, as well as to a satisfactory explanation of the observed doublet separation, the observed intensity ratio of the components, and the observed half-width of the components.

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[1] On the assumption that Curtis set on the center of each line, thus giving 109,677.525 for N_∞, we obtain 6562.8622 Å. for the center of gravity of the red component. For this Curtis' H_α value demands an intensity ratio of 1.09 to 1, as compared to a theoretical 1 to 1 ratio on this assumption. We again have consistent results, but a slightly lower value of N_∞.

[2] Birge, Phys. Rev., 14, 361, 1919.

THE BALMER SERIES IN MIXTURES OF H AND He AT HIGH PRESSURES.

An entirely different type of phenomena is that illustrated by the appearance of the Balmer series, down to $n' = 14$, in mixtures of H and He at 41 mm. pressure. Merton and Nicholson consider this important evidence against the entire Bohr theory of the hydrogen atom, but in view of the many exact quantitative agreements of this theory, little doubt can remain as to its general validity. It is therefore necessary to "explain" these observations. Assuming a *constant* temperature of 15° C., and a *constant* separation of the atoms, it appears, as stated by Merton and Nicholson, that the diameter of the ring (2.08×10^{-6} cm.) for $n' = 14$, is twice the distance apart of the atoms (1.0×10^{-6} cm.)

Qualitatively, all phenomena agree with the Bohr atom. Wood (loc. cit.) finds that, aside from the disturbing effects due to the secondary spectrum, the successive lines of the Balmer series appear as the pressure is lowered. Merton and Nicholson find that the higher members have relatively less intensity at high pressures. Wood finds that H_{20} has only 1/600,000 the intensity of H_α, in a long vacuum tube at 0.4 mm. pressure, while in the nebulæ the intensity decrement is much smaller. The Merton and Nicholson figures for the intensity decrement lead to 1/4,000,000 for this ratio.

. It thus follows that, considering *only* those atoms which, at any instant, are radiating monochromatic energy, it is merely necessary that an extremely small fraction of this number need to be several times as far from all neighboring atoms as the average distance. With a Maxwellian distribution of velocities and distances, this seems easily possible. The question as to what fraction of the total number of atoms is at any instant emitting monochromatic radiation does not enter here. Except in the case of the "first resonance" line, this fraction also is believed to be very small. Furthermore, it is well known that the helium atom has practically no effect, either attractive or repulsive, on near-by electrons. This explains simultaneously why the predicted fine structure of the Balmer series appears *only* in mixtures of H and He, why the 12th member of the series can appear at all in such mixtures at high pressure, and why the higher members are extraordinarily sharp in such mixtures. Merton and Nicholson themselves remark the absence of broadening of the higher members, indicating an absence of any interatomic electric field, in spite of the proximity of the helium atoms.

There still remains the question of the temperature of the radiating source. Because of the work of Buisson and Fabry[1] on the width of the helium lines, this temperature is now usually considered roughly that of the walls of the tube, and Merton uses this assumption, in his calculation of the Doppler effect for H_α and H_β. In view of the narrowing of these lines, at liquid-air temperature, it would appear that the assumption is certainly approximately correct for hydrogen, as well as for helium.

It seems to the author, therefore, that the well-known neutral effect of helium on neighboring electrons, together with the Maxwellian distribution of velocities, separations, etc., is quite sufficient to account for the experimental results thus far obtained. The fact that a small trace of impurity such as oxygen (see Wood, loc. cit.) seems necessary to the best production of the Balmer series would indicate that a strong field of force (due to the oxygen atom) is present under these conditions in the immediate neighborhood of the emitting atom. Yet this produces only a slight blurring of the emitted line (*i.e.*, frequency shifts of possibly one part in 50,000). All the more then can helium atoms be very close to the emitting center, without appreciably disturbing the emission.

University of California,
December, 1920.

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[1] Jour. de Phys., 2, Ser. 5, pp. 442–464, 1912.

AN EXPERIMENTAL STUDY OF THE REFLECTION OF X-RAYS FROM CALCITE.

By Bergen Davis and W. M. Stempel.

Synopsis.

Per cent., of monochromatic X-rays reflected from calcite, for wave length ranging from .3Å. to .8Å. was studied by means of a *double X-ray spectrometer*. A narrow beam of general radiation fell at an angle upon a crystal designated as A and the reflected beam was allowed to fall upon a second crystal designated B. The intensity of the beam from B was then compared, by means of the ionization produced in an ionization chamber with the direct beam from A. *Per cent., reflection was found surpisingly large* when the surfaces of the two crystals were strictly parallel. The per cent., reflection will depend upon the character of the beam reflected and therefore upon crystal A.

Significant energy distribution curves were found when the crystal B was rocked through a small angle on either side of parallelism. *A very sensitive micrometer motion* was provided for rotating crystal B about both a horizontal and also about a vertical axis, necessitated by the extremely narrow energy distribution curves obtained.

Three pairs of crystals were investigated. Pair (A_1–B_1) where A was a rather imperfect specimen of *Montana calcite* and B_1 was a clear specimen of Iceland spar. The surface of both crystals were polished. Pair ($A_2 - B_2$). Here a clear specimen of *Iceland spar* was split. The two split surfaces were polished and the crystals mounted so that reflection was from the surfaces that had been contiguous before splitting. Pair ($A_3 - B_3$) was another pair obtained by splitting a clear specimen of Iceland spar, but leaving surfaces unpolished. Mounted same as ($A_2 - B_2$).

The rocking curves vary greatly in width, depending upon the nature of the reflecting surface; 16″ of arc for width of half maximum when crystal is a good sample and the surface is the natural split surface (not polished). When crystals are not perfect samples and surface polished as much as 57″ of arc was obtained. For any one pair of crystals the width of the half maximum of these rocking curves is a *linear function of the wave length* with a *definite finite intercept* depending upon the condition of the surface. The more perfect the crystal the less does this width depend upon wave-length.

Per cent., reflection is a function of wave length, but is less so the more perfect the crystal. For the pair ($A_3 - B_3$), it was nearly independent of wave length. It would seem that the *reflection from a really perfect crystal would be independent of wave-length and would probably be greater than fifty per cent.*

A KNOWLEDGE of the proportion of the incident X-rays of any frequency that are reflected from the cleavage surface of a crystal is of great importance in determining the distribution of energy in the continuous X-ray spectrum and also the partition of energy among the many lines of the characteristic spectra. It is also of prime importance in the development of an adequate explanation of crystal reflection itself.

Measurements of the relative reflection of X-rays in the several orders have been made by Professor W. H. Bragg and W. L. Bragg, as well as the relative reflectivity of various crystals for the same radiation. The percentage reflection or the coefficient has also been measured by Professor Bragg, using a single crystal. Such determinations must contain an error due to the presence of the general radiation in addition to the line radiation in the original beam before reflection. A. H. Compton[1] has made a direct determination of the coefficient of reflection of rock salt and calcite for one particular wave-length. Compton allowed the radiation reflected from a crystal placed on one spectrometer to fall on a second crystal placed on another spectrometer suitably placed near the first. The apparatus was arranged so as to give an integral area of the curves as the crystals were rocked through a small angle. The small value of the reflection obtained by Compton would indicate that the two crystals were not placed with the crystal faces parallel, as we have done, but were placed nearly (180°–2θ) to this position so that only a small portion of the ray from first crystal could be reflected from the second crystal in any one position. (This point is not mentioned in his description of the experiment.)

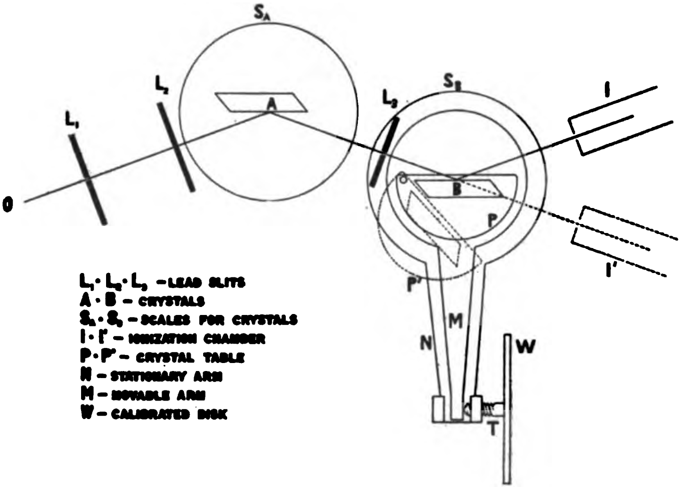

Fig. 1.

For the purpose of this study of crystal reflection a special double X-ray spectrometer was constructed. A plan of this instrument is shown in Fig. 1. The arrangement of the parts is sufficiently shown in the figure as not to need a detailed description. The heterogeneous beam passing through the slits L_1L_2 falls on Crystal A. The position of this crystal is read on the vernier scale S_A in the usual manner. The pencil of rays reflected from A is nearly homogeneous, the range of wave-lengths ($\Delta\lambda$) included depending on the degree of divergence of the original beam. The beam reflected from A falls on crystal B and suffers a second reflection. The intensity of this reflected beam is measured by the ionization current in the ionization chamber I as shown. The crystal B is mounted on a movable plate P which can be thrown back as indicated by the dotted lines in the figure. The beam from crystal A can then pass directly into the ionization chamber placed in the position I'. The intensities of the two beams before and after reflection are thus compared by the ionization produced in the chamber in the two positions I and I'. A stop is provided so that the crystal B will return exactly to the same position each time after displacement.

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[1] Proc. Am. Phys. Soc., Phys. Rev., July, 1917.

The source of X-rays was a standard Coolidge tube with a tungsten target. This tube was energized by a high tension transformer supplied by a 500-cycle generator. The alternating potential was rectified by a kenetron set and the electricity stored in a large plate-glass condenser immersed in oil. The capacity of this condenser was such that the voltage applied to the tube was practically constant, the momentary fluctuations being of the order of one half of one per cent. of the total voltage (50,000) when a current of 2 milliamperes was flowing through the tube. The speed of the 500-cycle generator was also controlled to a constancy of one half of one per cent. The voltage applied to the tube and the current flowing could be controlled and measured to this degree of accuracy. The voltage was measured by a specially designed repulsion electrometer that had a range from 5,000 to 100,000 volts.

The scale S_B, the mounting for the crystal and the ionization chamber I were all mounted on an arm (not shown in the figure) and could be rotated about center of scale S_A as axis and the angle read by a vernier on scale S_A in the usual manner. The angle of crystal B was read by a vernier on scale S_B. The position of the ionization chamber was read by another vernier on this scale. The arm M was attached to the table carrying crystal B. This arm extended out from the axis as shown and could be moved through a small angle when so desired by the finely threaded screw T. The disc W attached to the screw was graduated on its edge. The parts were so proportioned that one scale division on the periphery of disc W represented a rotation of crystal B through 2 seconds of arc. In this way it was possible to give crystal B an angular displacement as small as one fifth second of arc and read the same with considerable accuracy.

The slightly divergent beam passing through the slits L_1, L_2 after reflection from crystal *A* falls on the reflecting face of crystal *B*. A third slit L_3 was introduced in the path of this beam as shown in the figure. This narrow slit was necessary to keep the beam sufficiently narrow that it would not spread over an area broader than the face of crystal *B* at small glancing angles.

Maximum reflection was obtained at a certain setting of crystal *B*, presumably this is the position in which the reflecting planes of *B* are parallel to those of *A*. The position of parallelism referred to in this paper is the position at which maximum reflection is obtained. When the reflecting planes of the two crystals are parallel, the ray *ee′* from crystal *A* for instance will strike crystal *B* in such manner that angle *b* will equal angle *a*. Another ray striking crystal *A* at *f* will meet *B* at *f′* and the angle *c* will equal angle *d*. The same is true for every ray reflected from *A* between the points *e* and *f*. *Every ray reflected from A will meet B at the proper angle for maximum reflection for the same setting of B.* The above statement is only strictly true if the planes of *A* and *B* are perfect and the reflection follows strictly the law $\lambda = 2d \sin \theta$. An imperfection of any kind in crystal *A* would make the reflected ray slightly non-homogeneous, that is along any path as *ee′* for instance the radiation would not only consist of a beam of wave-length given by $\lambda = 2d \sin \theta$, but would contain other radiation of slightly different wave-length.

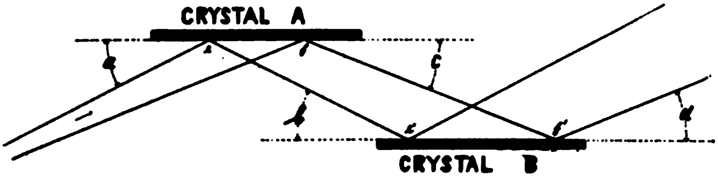

Fig. 2.

Since every pencil of rays reflected from *A* to *B* falls on *B* at the proper angle for best reflection (when the planes are parallel) we shall consider the coefficient of reflection as determined in this investigation to be the ratio of the intensity of the beam after reflection from *B* to its intensity before such reflection. These two intensities are measured by means of the ionization chamber in the two positions *I* and *I′*.

It is at once evident from Fig. 2 that a ray *ee′* of given wave-length reflected from *A* and a ray *ff′* of another wave-length so reflected cannot both meet the crystal *B* at proper angle for reflection if the two crystals differ at all in structure or in crystal spacing. If the crystals *A* and *B* do differ in structure or spacing a small rotation of crystal *B* will bring the crystal to the proper position for reflection of the rays falling on different portions of its surface.

In addition to the lever arm *M* and slow screw device *T* for rotating *B* through small angles about a vertical axis, a similar device (not shown in the figure) was constructed for rotating crystal *B* about a horizontal axis. In this way the reflecting planes of *B* could be brought accurately parallel to those of crystal *A*.

The first pair of crystals investigated was A_1 a Montana calcite, and B_1 a very old and clear specimen of Iceland spar. The reflection was from the (100) planes of each crystal. The surfaces were polished on a plane parallel plate in the usual manner of obtaining an optical polish. The crystals being carefully mounted and crystal A_1 set for reflection of a portion ($\Delta\lambda$) of the general radiation of a desired wave-length, it was found on rotating crystal B_1 through small angles each side the maximum, that a very regular curve of intensity was obtained. A typical curve for these two crystals is shown in Fig. 3 and is given to indicate how accurately such curves can be obtained. The observed

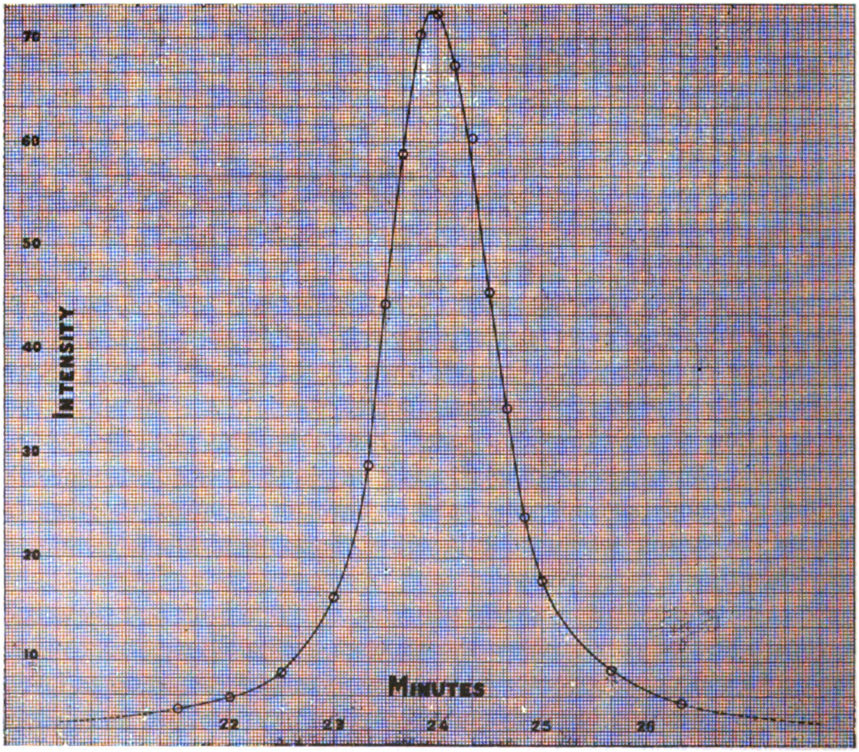

Fig. 3.

points lie on a very smooth regular curve. It should be pointed out that the principal irregularities in X-ray spectrometer observations do not arise from irregularities in the radiation but are due to the fact that close settings and readings of small angular changes can not be made with verniers. The smoothness of the curves here presented is due to the accuracy of measuring small angular changes by the slow screw device.

The curve (Fig. 3) is seen to have considerable width. The width of curve referred to throughout this paper is the width at half-maximum indicated by the length of the lines a *aa'*, *bb'*, *cc'* in Figs. 4, 5, 6 and 7.

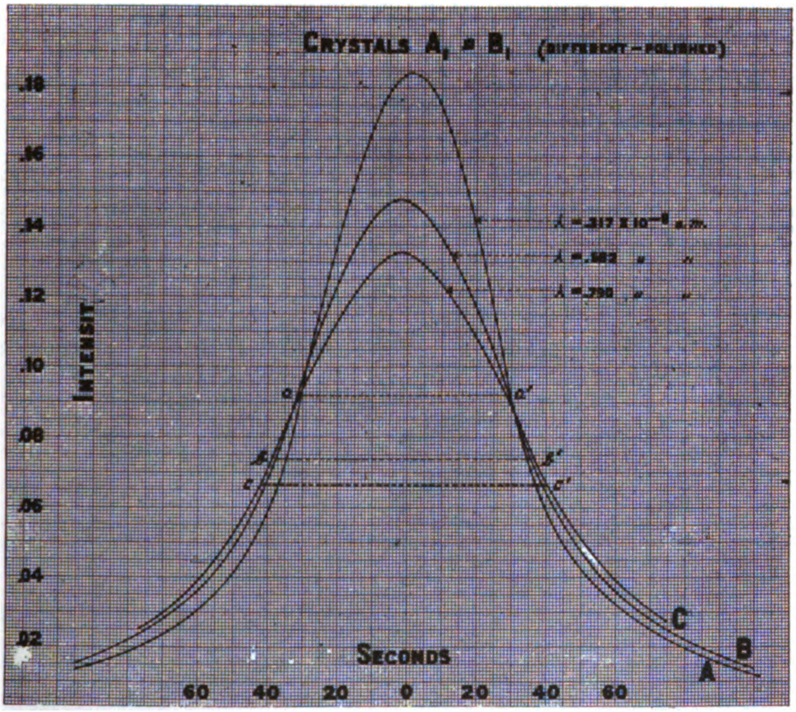

Fig. 4.

After observation of this effect due to *rocking the crystal* through small angles a systematic investigation was undertaken to ascertain its cause.

The width of curve at half-maximum was found to be independent of:

(*a*) Width of the slits L_1L_2.

(*b*) Voltage applied to X-ray tube.

(*c*) Current through X-ray tube.

It was found to depend on:

(*a*) Angle of reflection (wave-length).

(*b*) The nature of the crystals.

(*c*) The state of polish of the surfaces and the similarity of crystals.

Three combinations of crystals were investigated designated as follows:

Crystals $A_1 - B_1$. Crystal A_1 was a Montana calcite. Slight flows were visible in the interior. Surface polished. Crystal B_1. An old clear specimen of Iceland spar. Surface polished.

Crystals $A_2 - B_2$. A clear specimen of Iceland spar was split in two portions. The surfaces polished. The two reflecting surfaces as mounted (Fig. 2) were the surface planes that had been in contact before splitting.

Crystals $A_3 - B_3$. A clear specimen of Iceland spar was split into portions. The freshly split surfaces were *not polished.* The two reflecting surfaces as mounted were the contiguous planes before splitting.

These combinations of crystals showed marked differences in reflectivity and in width of *rocking curves.*

TABLE I.

Crystals A_1 and B_1.

Grazing Angle.	Wave-length.	% Reflection.	Width of Half Maximum.	Width of Half Max. × % Reflec.	Width of Half Maximum Minus Intercept = Q.	Sin θ.	$\frac{\text{Sin }\theta}{Q}$.
3° 00′	$.317 \times 10^{-8}$	25.8	58.0	1,495	11.7	.0523	
					(58.0 − 46.3)		.0045
3 30	$.369 \times 10^{-8}$	24.6	67.8	1,670	21.5	.0610	.0028
4 00	$.422 \times 10^{-8}$	23.3					
		22.2					
		24.8					
		23.4					
5 00	$.527 \times 10^{-8}$	21.7					
		21.3					
		21.5					
5 30	$.580 \times 10^{-8}$	20.8	72.0	1,510	26.4	.0958	.0037
			73.5				
			72.5				
			72.7				
6 00	$.632 \times 10^{-8}$	19.5					
		19.1					
		20.7					
		19.8					
7 00	$.737 \times 10^{-8}$	18.3					
		19.0					
		18.7					
7 30	$.790 \times 10^{-8}$	18	78.2				
			79.6				
			78.8	1,475	32.5	.1305	.0040
8 00	$.842 \times 10^{-8}$	17.3	83.4	1,445	37.1	.1392	.0038

The curves obtained by slightly rocking crystal B_1 through small angles each side of the maximum are shown in Fig. 4. Observations were made at three different wave-lengths. The coefficients of reflection obtained at a number of glancing angles are given in Tables I and IV.

The fourth column of Table I. gives the widths at half-maxima (aa', bb' and cc') expressed in seconds of arc. The fifth column gives the product of width at half-maximum and the per cent. reflection (proportional to maximum ordinate). This product is probably proportional to the areas of the curves. The table shows these areas to be nearly constant. The widths at half-maxima are plotted in Fig. 8. The straight line drawn through the points intercepts the ordinate axis at points designated by g_1. The width at half-maxima minus the intercept g_1 is given in the sixth column of the table (Q).

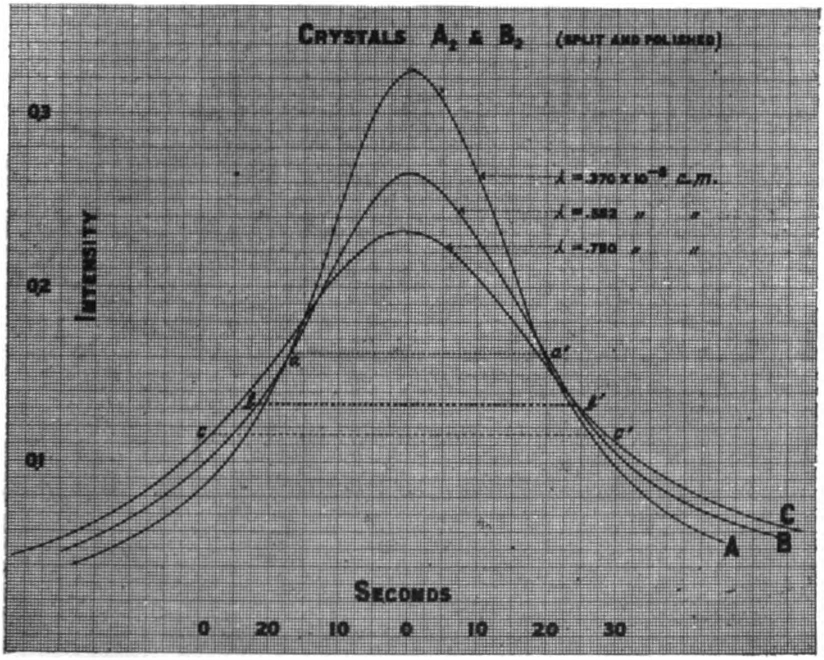

Fig. 5.

The fact that the points in Fig. 8 lie approximately on a straight line suggests that the curve width is made up of two parts, one part a constant g_1 and another part proportional to the sine of the glancing angle. This is indicated by the constancy of the ratio $\sin\theta/Q$ shown in the last column of the Table, where Q represents the widths at half-maxima minus the intercept g_1.

Crystals A_2 and B_2. A clear specimen of Iceland spar was split into two parts along a cleavage plane. The two split surfaces were polished on a plate using fine rouge and water as the abrasive. The crystals were mounted on the spectrometer in such position that the reflection took place at the two surfaces that had been continguous before cleavage. The coefficient of reflection was considerably larger than that obtained with the two dissimilar crystals $A_1 - B_1$. The width of rocking curves on the other hand was much less. The widths at half-maxima, the products of widths at half-maxima and per cent. reflection are given in Table II. The product of width at half-maxima and per cent. reflection is not quite constant but progressively increases with glancing angle as

TABLE II.

Crystals A_2 and B_2.

Grazing Angle.	Wave-length.	% Reflection.	Width of Half Maximum.	Width of Half Max. × % Reflec.	Width of Half Maximum Minus Intercept = Q.	Sin θ.	$\frac{\text{Sin }\theta}{Q}$.
3° 00′	$.317\times10^{-8}$	37.3					
3 30	$.369\times10^{-8}$	(curve) 35.4	34.3	1,245	18.7	.0610	.00326
			36.3		(35.2–16.5)		
			35.0				
			35.2				
4 00	$.422\times10^{-8}$	34.3					
		32.9					
		33.6					
4 30	$.474\times10^{-8}$	(curve) 32.9	41.5	1,371	25.2	.0785	.00312
			41.9				
			41.7				
5 00	$.527\times10^{-8}$	30.5					
5 30	$.580\times10^{-8}$	(curve) 29.2	45.6	1,343	29.5	.0958	.00325
			46.5				
			46.0				
6 00	$.632\times10^{-8}$	27.9					
6 30	$.684\times10^{-8}$	(curve) 26.9	54.6	1,417	36.2	.1132	.00313
			50.9				
			52.7				
7 00	$.737\times10^{-8}$	26.0					
7 30	$.790\times10^{-8}$	(curve) 25.3	57.0	1,452	40.9	.1305	.00319
			57.8				
			57.4				
8 00	$.842\times10^{-8}$	24.6					

shown in the fifth column of the table. The widths at half-maxima plotted against glancing angle lie sensibly on a straight line (Fig. 8). The widths at half-maxima minus the intercept g_2 is given in the sixth column of the table (Q). The ratio sin θ/Q is a constant. The width of rocking curve is composed of two parts. One part is a constant g_2, and the other part varies as the sine of the glancing angle.

Crystals $A_3 - B_3$. A clear crystal of Iceland spar was split into two parts along a cleavage plane. The crystals were so mounted that the molecular planes that were contiguous before splitting became the two reflecting surfaces of the crystals. The surfaces were not polished, and care was taken not to touch them in any way so that the surface molecular layers should remain in as perfect a state as possible.

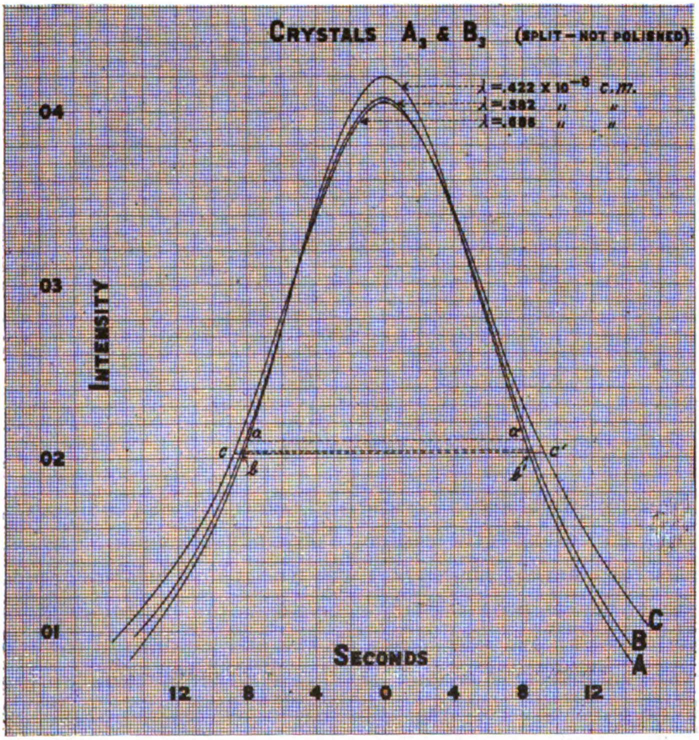

Fig. 6.

The reflection was very much greater than that obtained with the other crystals described. The results are shown in Table III., where the several columns have the significance previously described. The curves obtained by *rocking* crystal B_3 are shown in Fig. 6. The product of widths at half-maxima and per cent. reflection is nearly constant, increasing slightly with increasing wave-length. The intercept on the width at half-maxima ordinate at zero angle is shown at g_3 in Fig. 8. The ratio sin θ/Q is a constant. The curve width may be regarded as comcomposed of two parts, one part is a constant g_3, the other part varies as the sine of the glancing angle.

TABLE III.

Crystals A_3 and B_3.

Grazing Angle.	Wave-length.	% Reflection.		Width of Half Maximum.		Width of Half Max. × % Reflec.	Width of Half Maximum Minus Intercept = Q.	Sin θ.	$\frac{\text{Sin } \theta}{Q}$.
4 00′	$.422 \times 10^{-8}$	44.4		16.1		717	3.6		
		44.4		16.2			(16.2–12.6)	.0698	.0199
			44.4		16.2				
5 30	$.580 \times 10^{-8}$	42.9		17.3		752	4.9	.0958	.0195
		43.3		17.6					
			43.1		17.5				
6 30	$.686 \times 10^{-8}$	42.4		18.1		787	5.8	.1132	.0195
		43.2		18.6					
			42.8		18.4				
7 30	$.790 \times 10^{-8}$	42.4							
		43.3							
			42.8						

The effect of similarity of crystal and treatment of surface on the width of rocking curves and their areas is clearly shown in Fig. 7. These curves are for glancing angles of 3°–30′ and 4°. They are given to scale. The areas of curves progressively decrease as the crystal surfaces become more perfect. This is also shown by the fifth column of the tables.

For convenience of reference the per cent. reflection for the several crystals at various wave-lengths is assembled in Table IV. and shown graphically in Fig. 9.

The last two columns of the table give the calculated variation of reflection with wave-length by an equation derived by C. G. Darwin[1] for imperfect crystals. All of the constants in this equation including the Debye heat term are collected into a single constant A, and the equation written in the form

$$R = A \frac{\lambda^3(1 + \cos^2 2\theta)}{\mu \sin \theta \cos \theta}.$$

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[1] Phil. Mag., Feb. and April, 1914.

For purposes of comparison the constant A is taken to be such as to give the observed reflection at a particular angle (6°). The reflection at other angles is then calculated for the absorption $\mu \sim \lambda^{2.4}$. For this value of μ the equation agrees very closely with the observed results for the two pairs of polished crystals. This is particularly true for crystals $A_2 - B_2$.

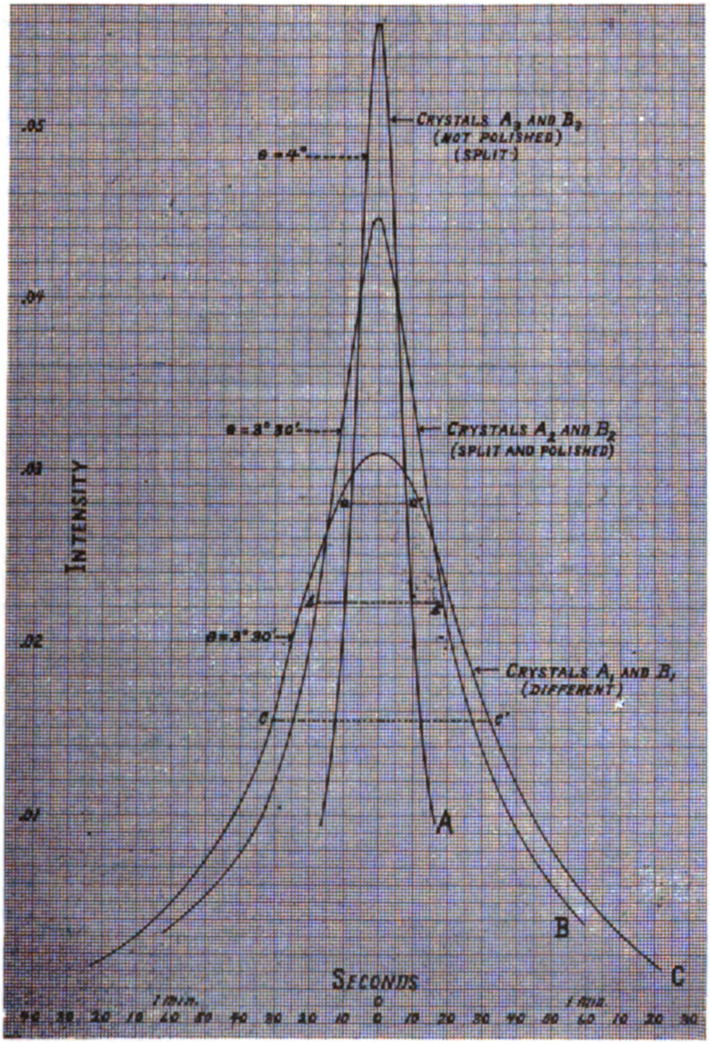

Fig. 7.

It should be pointed out that the per cent. reflection or coefficient of reflection here determined is considered to be that of crystal B in every

TABLE IV.

Angle.	λ.	Per Cent. Reflection.				
		Observed.			Computed (Darwin).	
		A_1—B_1.	A_2—B_2.	A_3—B_3.	A_1—B_1.	A_2—B_2.
3	.317	25.8	37.3		26.5	37.2
3-30	.369	24.6				
4	.422	23.4	33.6	44.4	23.5	33.1
5	.527	21.5	30.5		21.6	30.1
5-30	.580	20.8		43.1		
6	.632	19.8	27.9		19.8	27.9
6-30	.686			42.8		
7	.737	18.7	26		18.6	25.9
7-30	.790			42.8		
8	.842	17.3	24.6		17.5	24.6

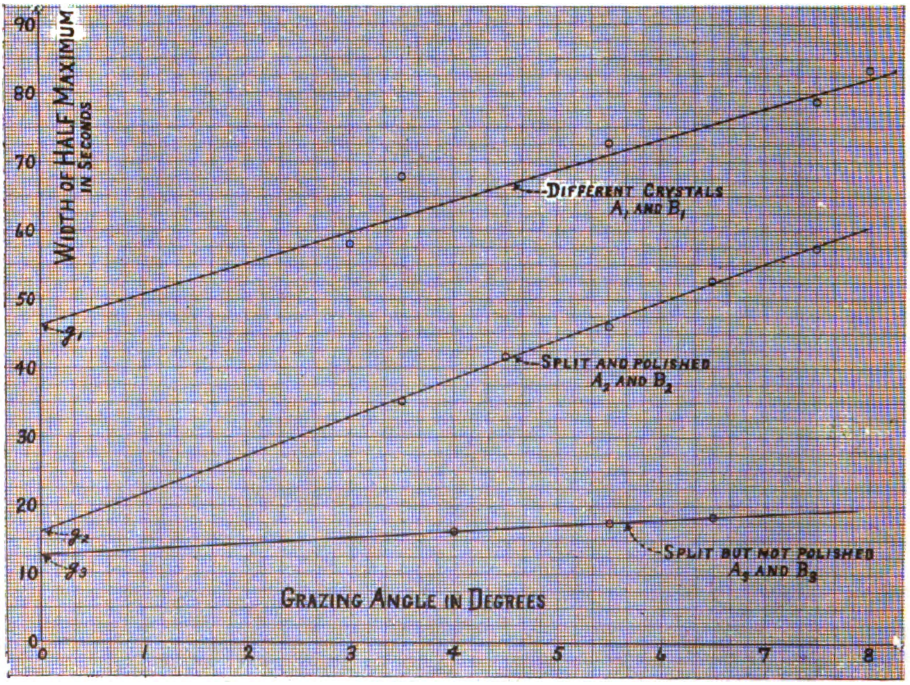

Fig. 8.

case. The reflection obtained from *B* however depends not only on its perfection but also on the perfection and surface of crystal *A*. This is indicated by the performance of the combination $A_1 - B_1$ which differed in character from that of the other set of crystals. Crystal B_1 was a perfect specimen of Iceland spar, as perfect in appearance as crystal B_2.

Crystal A_1 on the other hand, a specimen of Montana calcite showed imperfections that were visible in its interior. The irregular results and low value of the reflection obtained with these two crystals can be fairly attributed to the imperfection of A_1. It will be specially noticed by Table I and Fig. 8 that the width at half-maximum in particular is too large at glancing angle 3° + 30′. The other points in Fig. 8 lie reasonably on a straight line. In addition to the visible evidence, the imperfections of crystal A_1 are indicated by curves of the continuous and line spectrum obtained by B. A. Wooten using this crystal (Phys. Rev., Jan., 1918—Fig. 1). The minute false lines there observed are undoubtedly due to imperfections in the crystal. Crystal B_1 on the other hand appears to be fairly perfect as indicated by smooth curves of the continuous and line spectrum obtained by C. T. Ulrey using this crystal (Phys. Rev., May, 1918).

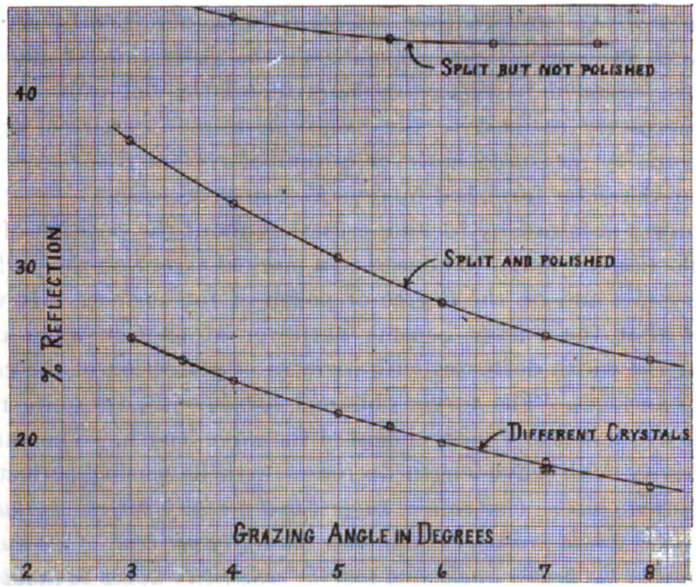

Fig. 9.

What shall be defined as the coefficient of reflection is a matter for discussion. Using single crystal reflection Professor Bragg has taken the ratio of curve areas obtained by rocking the crystal and moving the slit of the ionization chamber across the beam. This of course can only be done in case of the line spectrum and even then is subject to the error of the general radiation which is present to a considerable extent.

A. H. Compton (*l.c.*) has applied the method of integration of curve areas in case of reflection from two crystals. The coefficient of reflection is expressed in terms of angular measure (per degree). It is not clear how such a quantity which is really the ratio of intensities before and after reflection can contain angular measure.

We have taken as the coefficient the ratios of the intensities of the beam of X-rays before and after reflection from crystal B at position of maximum reflection. This is the coefficient for crystal B. The reflection from crystal B depends on the degree of homogeneity of the radiation falling on it. It is the function of crystal A to sort out from the general radiation a nearly homogeneous radiation for examination by crystal B. The degree of homogeneity depends on the perfection of crystal A. Since this crystal determines the character of the radiation falling on B it is necessary to describe the results in terms of pairs of crystals. The reflectivity measured however is that of crystal B for rays of the particular character furnished by crystal A in every case.

The curves obtained by rocking crystal B about the position of maximum reflection are found to depend to a marked extent on the perfection of the crystals and on the degree of polish of the surfaces. It will be observed that the curves become narrower at shorter wave-lengths. Also that the width decreases with increase of perfection of the crystals and their surfaces. The curves obtained for $A_3 - B_3$ are quite narrow and their width is nearly independent of the wave-length. The variation of reflectivity with wave-length decreases with increasing perfection of crystal surface also. *Our results suggest that a really perfect crystal (were such possible) would probably reflect more than fifty per cent. of the radiation falling on it at the proper angle, and that the reflectivity would be independent of wave-length.*

The attention of the reader is directed to a short paper on reflection of radiation from an infinite series of equally spaced planes appearing in this number of the PHYSICAL REVIEW. The results obtained are of considerable interest in view of the determination of reflectivity just described.

The width at half-maximum of 16″ obtained for crystals $A_3 - B_3$ is many times too great to be attributed to the Fresnel line width that should be expected with radiation of such short wave-length and a grating of such a large number of elements. The width of 16″ of arc for a reflectivity of about 45 per cent. would indicate that even in the case of a perfect crystal the curve width would still be of the order of a few seconds of arc. This width might be due to very minute differences in spacing in the case of the two crystals, but since contiguous planes were used for reflecting surfaces in case of $A_3 - B_3$ it is doubtful if the crystals varied sufficiently in grating space in such a small distance as to produce so great an effect.

Another more reasonable explanation of curve width (for perfect crystals) is that it is due in some way to the width of each reflecting plane. That is, it may be due to the distribution of the electrons about the nucleus of the atoms of the crystal. One of us (Stempel) has recently developed a theory of crystal reflection involving the electron distribution that accounts with some success for these curves.

Since the width of the rocking curve would depend directly on a possible difference of crystal spacing in the two crystals one should obtain a great effect by heating one of the crystals slightly. Also by heating both crystals (*A* and *B*), keeping them at the same temperature it should be possible to measure the Debye heat effect with great accuracy by this arrangement of a double spectrometer.

Phoenix Physical Laboratory,
Columbia University,
January, 1921.

REFLECTION OF RADIATION FROM AN INFINITE SERIES OF EQUALLY SPACED PLANES.

BY K. W. LAMSON.

SYNOPSIS.

Reflection of Radiation from an Infinite Series of Equally Spaced Planes.—If the fractions of the incident radiation upon any plane which are absorbed, reflected and transmitted by that plane are constant and denoted by a, r and t respectively, and if all possible internal reflections are taken account of, the *coefficient of reflection* is shown to equal $(1/2r)\ \{1 - t^2 - r^2 - \surd\ [1 - 2t^2 - r^2 + (t^2 - r^2)^2]\}$. When both a and r are very small, as in the case of the *reflection of X-rays from a crystal surface*, the expression becomes $[1 + a/r - \surd\ (a^2/r^2 + 2a/r)]$; thus the proportion reflected depends only on the ratio of absorption to reflection per plane.

THE surprisingly large value of the reflection of X-rays from calcite (about 45 per cent., obtained by Bergen Davis and W. M. Stempel, an account of which appears in this number of the PHYSICAL REVIEW, suggests the problem of investigating the reflection that might be expected from a system of equally spaced reflecting planes.

No special assumptions are made as to the physical nature of the radiation or of the reflecting medium other than that it consists of an indefinite number of equally spaced planes. As the radiation passes through each plane a minute fraction r is reflected and a minute fraction a is absorbed. The amount transmitted is $t = 1 - r - a$. The fraction reflected per plane is quite small being of the order perhaps of 10^{-5} to 10^{-6}.

In the case of a perfect crystal the absorption per plane is probably much less than the reflection[1] for the particular wave-length for which the reflection is a maximum for any given angle. The percentage of transmission, reflection and absorption is assumed to be the same for all planes.

The final emergent (front reflecting face) ray from a system of planes is made up of parts, each of which has been reflected an *odd* number of times and transmitted through an *even* number of planes.

Assume the intensity of the ray incident at the first plane to be unity. Then the intensity of a ray after $2j - 1$ reflections and $2i$ transmissions is

$$I = t^{2i} r^{2j-1}. \tag{1}$$

Let C_{ij} be the number of ways in which it is possible for a ray to be transmitted $2i$ times and reflected $2j - 1$ times. Then the total amount reflected is

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[1] C. G. Darwin, Phil. Mag., April, 1914.

$$R = \sum_{ij=1}^{\infty} C_{ij} t^{2i} r^{2j-1}. \tag{2}$$

The values of C_{ij} were computed directly for all values of i and j up to 5 and were seen to be given by the following formula.

$$C_{ij} = \frac{(i+j-2)!\,(i+j-1)!}{(i-1)!\,i!\,(j-1)!\,j!}$$
$$= \frac{i(i+1)^2(i+2)^2\cdots(i+j-2)^2(i+j-1)}{(j-1)!\,j!}. \tag{3}$$

This formula has been checked by the independent computation of C_{ij} for $1 = 8$ and $j = 7$.

For the case when $j = 1$, $C_{i_1} = 1$, the contribution of a single reflection from all planes is

$$\sum_{i=1}^{\infty} (C_{i1} t^{2i}) r = \frac{r}{1 - t^2} = \frac{1}{2}\left(\frac{1}{1 + a/r}\right). \tag{4}$$

The sum appearing in the formula for R Equation (2) has been found by T. H. Gronwall to be

$$\frac{1}{2r}\left[1 - t^2 - r^2 \sqrt{1 - 2t^2 - 2r^2 + (t^2 - r^2)^2}\right]. \tag{5}$$

For the values of a, r, t which we are considering, this is equal to

$$R = \frac{a}{r} + 1 - \sqrt{\frac{a^2}{r^2} + \frac{2a}{r}}. \tag{6}$$

Thus the total reflection depends only on the ratio of absorption to reflection at each plane. As r is so small it might be expected that the terms in r^3, r^5, etc., might be neglected, that is, that the contributions of repeated internal reflections might be neglected. This however, is not the case. The number of possible paths increases so rapidly with the number of reflections that for probable values of a/r, the repeated reflections contribute a considerable portion of the total energy reflected.

The following table is computed from formula (6) for various values of the ratio a/r.

$\dfrac{\text{Absorption per Plane}}{\text{Reflection per Plane}}$.	0	.02	.05	.1	.2	.3	.4	.5	1	2	3	10
Per cent. reflection.......	100	82	73	64	54	47	42	38	27	17	13	5

Columbia University,
January, 1921.

A STUDY OF THE PHOTO-ACTIVE ELECTROLYTIC CELL, PLATINUM—RHODAMINE-B—PLATINUM.

BY CARLETON C. MURDOCK.

SYNOPSIS.

The Range of the Current-time Equations.—Goldmann and Thompson have each suggested equations to represent both the growth and the decay of the current given by a photo-active electrolytic cell having electrodes of platinum and an alcoholic solution of Rhodamine-B as electrolyte. The author points out that in each of these equations it is necessary to use different parameters after the first minute of the growth or decay than before and suggests that this may be due to the failure to correct the data for the inertia and damping of the galvanometer coil. *A method for correcting the galvanometer deflections* for these effects is discussed and correction equations developed. These equations involve constants of the galvonometer, the resistances of the circuit and the polarization capacity of the cell.

The polarization capacity of the cell used is measured by impressing a small potential difference on the cell and observing the instantaneous galvanometer deflections. The capacity as computed from the time constant of the corrected current-time curves gives 26 microfarads per square centimeter of each electrode. This is in good agreement with the results obtained by other observers for the capacity of platinum electrodes in aqueous solutions of inorganic substances.

The Dark Current.—Cells having as electrodes sputtered platinum films are found to give a current when not illuminated if the electrodes are not alike. This current tends to flow (*a*) from the denser to the less dense electrode, (*b*) from the warmer to the cooler electrode. Its value is independent of the external resistance of the circuit within the range 0 to 1 megohm and decreases slowly as fatigue effects the photoelectric sensitiveness of the cell.

Tests of the Current-time Equations.—Data taken by subjecting the cell to exposures of 10 minutes or more (*a*) can not be repeated (*b*) are not when corrected in agreement with either of the above types of equation. Data taken by exposures of 1 minute (*a*) may be repeated very exactly, (*b*) are not, when corrected, in agreement with either type of equation unless the parameters are allowed to change and then only in the case of Goldmann's equation for the growth and Thompson's equation for the decay. In both these cases the change in the parameters occurs at about 8 seconds.

The Hyperbolic Decay Equation.—It is shown that the decay of the current following an exposure of 1 minute or less is well represented throughout the range of experiment by the equation

$$y = y_0 - \frac{t}{Mt + L},$$

an equilateral hyperbola.

The effect of the length of exposure on the decay is investigated for exposures of 15, 30, and 60 seconds making use of this equation. It is found that the initial rate of decay is independent of the length of exposure. This is shown to be not consistent with Goldmann's equation.

THE fact that electrolytic cells containing alcoholic solutions of certain organic dyes have photo-electric properties, was first shown by the work of Nichols and Merritt[1] in 1904. That the effects are electromotive in character rather than resistive, was not understood, however, until the work of Hodge[2] in 1908. He showed that the effects are produced by the action of the light on a very thin layer of the solution at the boundary surface between the illuminated electrode and the electrolyte. The same results were independently obtained by Goldmann[3] who made a systematic study of cells of this type. He found that the current produced in the external part of the circuit is proportional to the light intensity and to the area of the illuminated electrode and that it is practically independent of the external resistance for the range from 3,000 to 100,000 ohms.

When the surface between one of the electrodes and the electrolyte is illuminated, the electrode becomes positively charged and if it is connected to the other, a current will flow from the illuminated to the unilluminated electrode. This current is not constant but increases rapidly at first and then more and more slowly until it reaches a maximum after which it slowly decreases. When the exciting light is cut off the current decreases in a similar manner and very gradually approaches its initial value.

Two types of equations have thus far been proposed to express the law of the variation with time of this photoelectric current. Both apply to the case where the duration of the exposure is so short that the maximum current is not reached. The first of these was proposed by Goldmann. He assumed that the effect is due to an ionization of the solute molecule by the incident light in a manner similar to the Lenard-Hallwachs effect and that there results a charging of the electrolytic capacity of the illuminated electrode. On the basis of this he derived equations (1) and (2) for the growth and decay respectively.

$$i = q(1 - \epsilon^{-mt}), \tag{1}$$

$$i = i_0 \epsilon^{-mt}. \tag{2}$$

In these equations, i represents the instantaneous value of the current at time, t; q, the charge liberated per second by the light; i_0, the initial value of the decay current; and m, $1/CR$, where C is the electrolytic capacity and R the external resistance.

In order to represent his experimental data, he found it necessary to modify his equations by the addition of constant terms.

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[1] Nichols and Merritt, Phys. Rev., Vol. 19, p. 415.

[2] Hodge, Phys. Rev., Vol. 26, p. 540 and Vol. 28, p. 25.

[3] Goldmann, Ann. d. Phys., Vol. 27, p. 449, 1908.

$$i = A_1 + B_1(1 - \epsilon^{-m_1t}), \tag{3}$$

$$i = A_2 + B_2\epsilon^{-m_2t}. \tag{4}$$

Equations (3) and (4) show these modified forms. A_1, A_2, B_1, B_2, m_1, and m_2 are empirical constants and the values of m_1 and m_2 are not equal but differ by about 40 per cent. The values of i computed from equation (3) and (4) are in good agreement with Goldmann's observed values except when t is less than one minute, in which case there is a marked discrepancy.

The other type of equation was proposed by Thompson[1] and was suggested by the fact that most of the dyes exhibiting this property are fluorescent and by the similarity between the curve of the decay of the photoelectric current and that of the decay of phosphorescence. If the reciprocal square root of the intensity of phosphorescent light is plotted against time either a straight line or a series of intersecting straight lines will result. Thompson plotted the reciprocal square root of the photoelectric current against time and obtained with both the growth and decay data, points that lay upon two straight lines. In both cases these lines intersected at a point whose abscissa was approximately one minute.

At present the evidence to show that either of these laws has real physical significance seems inconclusive, the first because of the added terms and the failure for the first part of the curve, the second, because of the absence of any theoretical derivation. These two attempts to express the law of the variation of the current have one striking feature in common. In both the equation which represents the data over most of the range fails to do so during the first minute. This may be due as both Goldmann and Thompson suggest to the fact that there are two effects present of different rates of growth and decay, it may be caused by a systematic error in the method of measurement or it may indicate that the proposed laws are only approximately correct. In the first part of the curves the slope and curvature are large and consequently the damping and inertia of the suspended system of the galvanometer must affect the form of the experimentally determined curves to a considerable extent. Failure to correct the curves for these effects would involve errors that would be of appreciable size during approximately the first minute and this may account for the apparent change in the laws during that interval. It seemed desirable, therefore, to study the current-time curves in this interval making correction for the failure of the glavanometer deflection to follow the rapidly changing current. This correction can be computed from the differential equation of motion of the galvanometer.

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[1] Thompson, PHYS. REV., Ser. 2, Vol. 5, p. 43.

For the case of a circuit consisting of pure resistances this equation is

$$(\alpha\ddot{x} + \beta\dot{x} + x)j = u, \tag{5}$$

in which x represents the scale reading of the deflection and u, the current to be measured at time, t, and j the current constant of the instrument for the scale used. α is also a constant of the instrument but β is a linear function of the reciprocal resistance of the circuit. If we let $\beta = \kappa + \lambda/\rho$ where ρ is this resistance; α, κ and λ are constants whose values are easily determined from observations on the period and logarithmic decrement of the instrument as functions of ρ. Let

$$u = jy, \tag{6}$$

then y is the value of x corrected for the errors introduced by the galvanometer and the desired correction equation is

$$y = x + \alpha\ddot{x} + \beta\dot{x}. \tag{7}$$

Given x as a function of t, $\dot{x}$ and $\ddot{x}$ may be obtained graphically and the correction terms of (7) computed.[1]

In the experiments described below I used a galvanometer having the following constants: $\alpha = 5.3$, $\beta = .46 + (61 \times 10^3)/\rho$ and $j = 3.9 \times 10^{-10}$ (where x is measured in centimeters at 282 centimeters distance; t, in seconds; i, in amperes and ρ in ohms). The condition for critical damping of the galvanometer is that $\beta^2 = 4\alpha = 21.2$ giving $\beta = 4.6$ and $\rho = 14{,}700$ ohms. In practise values of ρ larger than this were used, thus decreasing the value of the correction coefficient, β. The values ranged from 17,000 to 21,000 ohms.

This method of correction was carefully tested and found to be capable of an accuracy considerably greater than demanded by the experiments to be performed. In order to be able to change the effective sensibility of the galvanometer and the resistance of the circuit and at the same time keep the value of ρ within the above range, the galvanometer was shunted as shown in Fig. 1. Because of the inductance of the galvanometer, variable currents will not divide between the galvanometer and the shunt branches strictly in accordance with the law of shunts for steady currents. It will be allowable, however, to assume this law provided $l\dot{u} < pgu$ in which l is the inductance and g the resistance of the galvanometer and p the fractional error allowed. This may be written

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[1] This method is in effect the same as that used by Einthoven (Amst. Acad. Proc., 8, p. 210, 1906) in analyzing the curves obtained with his string galvanometer. The details of the measurements are necessarily somewhat different due to the difference in the characteristics of the instruments. The author has failed to find any account of the method as applied to a D'Arsonval galvanometer.

$$\frac{y}{\dot{y}} = \frac{u}{\dot{u}} > \frac{1}{p} \cdot \frac{l}{g}$$

$y/\dot{y}$ may be readily evaluated from the correction curves and the condition is that it shall be greater than the inductive time constant of the galvanometer, l/g, divided by the allowable fractional error. When this condition holds the introduction of a network between the galvanometer and the external part of the circuit does not change the effective values of α, κ and λ but the effective value of j becomes $n \cdot j$ where n is the shunt ratio for the network.

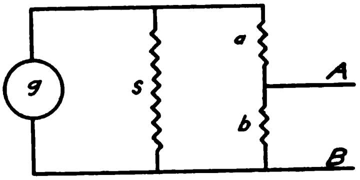

Fig. 1.

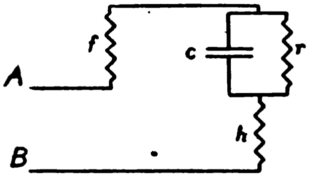

Fig. 2.

In case there is an electrolytic cell in the external branch, AB, the resistance external to the galvanometer cannot be regarded as a pure resistance as the polarization of the cell by the currents generated in the moving coil reacts to affect the motion of the coil. For the purpose of studying its polarization a cell may be considered as equivalent to a capacity, c, shunted by a resistance, r, and the two in series with a resistance, h, equal to the resistance of the cell as measured on an alternating current bridge. This network is shown in Fig. 2. f is a resistance box which was in series with the cell. The values of r and c which would make such a network strictly equivalent to the cell are not constants but are functions of the polarization. The measurements of r for the cells used in the experiments here described indicate that it is nearly constant within the range of potentials used. No data has been taken with these cells on which conclusive evidence concerning the variation of c with polarization can be based. However it has been shown that the variation of c with polarization in cells containing aqueous solutions is small at small potential differences.[1] It seems safe to assume r and c as constants for the purpose of calculating the correction to the galvanometer deflection due to polarization provided the correction so calculated is small.

Suppose the network shown in Fig. 2 to be connected to a galvanometer through another network for controlling the sensibility and damping of the galvanometer. Let n be the shunt ratio of this network; R, the effective resistance of this network and the galvanometer; and ρ the damping resistance. Let $\zeta = h + f + R$ and $\omega = r\zeta/(r + \zeta)$. Then the equation of motion is

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[1] Blondlot, Journ. d. phys., 10, p. 333, 1881.
Sheldon, Leitch and Shaw. Phys. Rev., 2, p. 401, 1895.

$$\left[\alpha\ddot{x} + \beta\dot{x} + \frac{\lambda\omega}{n^2\zeta^2}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}[\ddot{x}]_s ds + x\right] nj = u \tag{8}$$

in which α, β, λ and j are the same as in equation (5) and t_0 is the value of t when $\ddot{x} = \dot{x} = 0$. (For the proof of this equation see the appendix.

The correction equation now becomes

$$y = x + \alpha\ddot{x} + \beta\dot{x} + \frac{\lambda\omega}{n^2\zeta^2}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}[\ddot{x}]_s ds, \tag{9}$$

which differs from equation (7) only in addition of the last term. For the purposes of computation it has been found convenient to modify the form of this new correction term. Let $z = \epsilon^{(s-t)/\omega c}$ and $z_0 = \epsilon^{(s-t_0)/\omega c}$. Using z as the variable of integration instead of s, the term becomes

$$\frac{\lambda\omega^2 c}{n^2\zeta^2}\int_{z_0}^{1} \ddot{x}dz.$$

Now

$$\int_{z_0}^{1} \ddot{x}dz = \int_{0}^{1} \ddot{x}dz - \int_{0}^{z_0} \ddot{x}dz$$

and if we select the time t_0 as some time before the beginning of the experiment when the galvanometer coil was at rest for a finite time, we can write

$$\int_{0}^{z_0} \ddot{x}dz = 0$$

without error as far as the experiment is concerned. Thus we obtain

$$y = x + \alpha\ddot{x} + \beta\dot{x} + \gamma\int_{0}^{1} \ddot{x}dz, \tag{10}$$

where

$$\gamma = \frac{\lambda c}{n^2}\frac{\omega^2}{\zeta^2}.$$

$\int_{0}^{1} \ddot{x}dz$ is the mean value of $\ddot{x}$ weighted with respect to z from $z = 0$ to $z = 1$. When ωc is small the values of $\ddot{x}$ for the time just preceding t weigh heavily and the integral closely approximates the value of $\ddot{x}$ at time t. When this approximation is justified the correction becomes

$$y = x + \alpha'\ddot{x} + \beta\dot{x}, \tag{11}$$

where $\alpha' = \alpha + \gamma$. This form of equation may be used whenever

$\gamma\ddot{x}\omega c$ is negligible in comparison with y. Otherwise equation (10) must be used and the value of the integral evaluated graphically. In order to make the corrections indicated in equations (10) and (11), it is necessary to know, in addition to the galvanometer constants required for the use of equation (7), the values of r and c for the electrolytic cell.

The cells used by the earlier experimenters had opaque platinum electrodes pressed closely against the front wall of the glass box containing the electrolytic solution. In this type of cell a thin layer of the solution is traversed by the light before it is incident upon the surface of the electrode. This layer absorbs some of the light. There are two serious objections to this type of cell for quantitative work. The thickness of the absorbing layer cannot be measured with sufficient accuracy to enable one to estimate the effective intensity of the light. In such a cell there is always a part of the electrode surface upon which no light falls but which is in contact with the solution. Between this portion of the electrode surface and the illuminated portion there is an electromotive force which the electrode itself short-circuits. Goldmann used as electrodes semitransparent platinum films deposited by Kundt's method on the glass of the front wall of the cell. The light was transmitted through the electrode and was thus incident upon the sensitive layer without previous absorption in the solution. Thompson used, for the most part, cells of the opaque electrode type but he also used cells whose electrodes were semitransparent silver films. In connection with his results on the current-time curve, he does not state which type of cell was used.

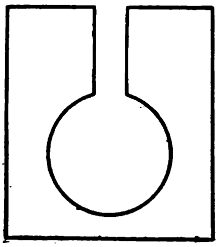

Fig. 3.

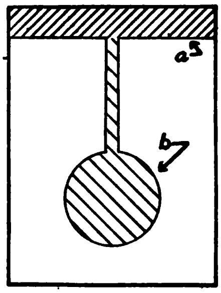

Fig. 4.

The cell used in the experiments described below consisted of a piece of plate glass, 4 x 5 cm. and 6 mm. thick, cut as shown in Fig. 3 and placed between two plates of optical glass upon which were semitransparent films of platinum as shown in Fig. 4. The part of the film marked, *a*, in the figure was thick enough to be opaque and was used to make contact between the semitransparent film, *b*, and the external part of the circuit. The films were deposited by cathodic sputtering and were then baked at a dull red temperature to make them more permanent. The diameter of the circular part of the film was 2 cm. and the cell was filled so that the meniscus was tangent to the circle at its upper edge. This made the effective electrode area 3.2 sq. cm. The cell as used in the experiments described below was filled with a 6 per cent. solution of rhodamine-B in absolute alcohol and had a resistance of about 15,000 ohms. A large part of this resistance was in the narrow strips of platinum film leading from the opaque film above to the circular part in contact with the solution.

The values of the quantities r and c were measured by placing the cell in series with the shunted galvanometer, a megohm box and a small constant electromotive force obtained from the "E. M. F." terminals of a low resistance potentiometer. If we imagine the cell replaced by the equivalent capacity and resistances, the circuit is represented by the network shown in Fig. 2, the shunted galvanometer and the electromotive force in series. The expression for the instantaneous value of the current through h is

$$i = \frac{E}{\zeta + r} + C\epsilon^{-(t/\omega c)}$$

in which C is a constant determined by the initial conditions. If E remains constant, the second term approaches zero and the current, a steady value, $i = E/(\zeta + r.)$ Since i, E, and ζ are known, the value of r can be computed. If the value of E is now changed the current will approach its new value by the exponential curve represented by the above equation and the value of ωc can be determined from the curve. Since both ζ and r are now known ω is known and the value of c may therefore be computed. This method of measuring the polarization capacity of an electrolytic cell, while not yielding results of as high accuracy as other methods, is particularly well adapted to the measurement of cells of high resistance. The large time constants of such cells make most of the methods usually employed, impractical.

Fig. 5 shows results obtained by this method with the cell described above. At time, $t = 0$, E was changed from 0 to .07 volts and at time, $t = 900$ sec., it was again made 0. The variation of the galvanometer deflection with time is shown by the dotted line curve, the small circles representing the actual observations. Equation (7) was used to correct these curves for the errors introduced by the galvanometer as the constants required for the use of the more exact equation were not known. The small crosses indicate the corrected observations wherever the corrections are larger than the radii of the small circles. The two full line curves are plots of the equations

$$y = 4.6 + 6.7\epsilon^{-.033t} \text{ and } y = 7\epsilon^{-.032t}.$$

The experiment illustrated in Fig. 5 gives as values of ωc, 30 and 31 seconds and, of r, 3.2 megohms. A considerable number of experiments of this type were tried. The resulting values of ωc varied considerably.

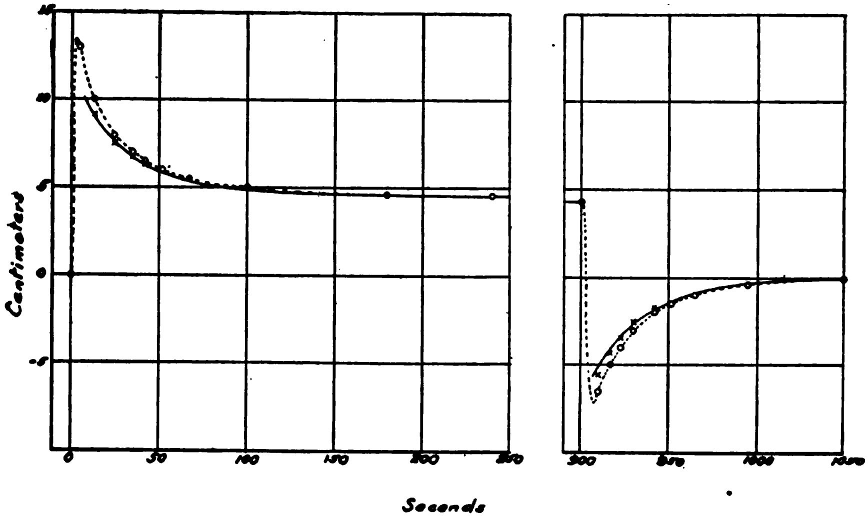

Fig. 5.

The mean vaue was 33 seconds. The values of the steady currents for the whole set of experiments were plotted against the corresponding impressed electromotive forces. The resulting line curved slightly but was practically straight throughout the range covered in the photoelectric experiments. Its slope indicated a value of r of 3.1 megohms. In these experiments $n = 13.4$, $f = 1$ megohm, $h = 15{,}000$ ohms and $R = 4{,}600$ ohms giving $\zeta = 1.02$ megohms, $\omega = .78$ megohms, $\gamma = .01 \text{ sec.}^2$ and $c = 42$ microfarads. The value of γ is negligible in comparison with that of α. Thus the additional correction which would have been involved had equation (10) been used instead of equation (7) is zero and no further correction is here needed.

The electrolytic capacity of cells with platinum electrodes has been measured by several observers using a variety of methods. The capacity is proportional to the area of the electrode and varies with the material of the electrode and somewhat with the potential but it is practically independent of the electrolyte. The value here obtained, 42 microfarads, is the capacity of two condensers in series. The capacity of each electrode is therefore 84 microfarads and this gives 84/3.2 = 26 microfarads per sq. cm. This compares well with the values that have been obtained by other experimenters who, however, worked with aqueous solutions. Some of the values obtained with platinum electrodes are: Varley,[1] 50; Kohlrausch,[1] 26; Blondlot,[1] 31; Hemholtz,[1] 39; Bouty,[2] 20; Scott,[3] 50. Goldmann obtained the value of 20 microfarads per sq. cm. for his cell which would give 40 microfarads per sq. cm. for each electrode. This was obtained from data taken on open circuit with an electrometer during illumination and the computation was based on his theory of the photoelectric action of the cell, the assumptions at the basis of which are given above.

Many electrodes were made and tested before the two finally used were selected. When a cell was set up and connected in a circuit with a galvanometer there was always a current even with the cell in the dark. This current would vary rapidly at first but when the polarization effects produced by the illumination incident to the operation of setting up the cell had disappeared it would assume a fairly steady value. This dark current was the greater, the more the difference in the density of the two films and when the two were at the same temperature, the denser electrode was always positive. Raising the temperature of either electrode made it more positive. The current was so sensitive to temperature variations that unless precautions were taken to keep the temperature of the cell uniform, the dark current varied in an erratic manner.

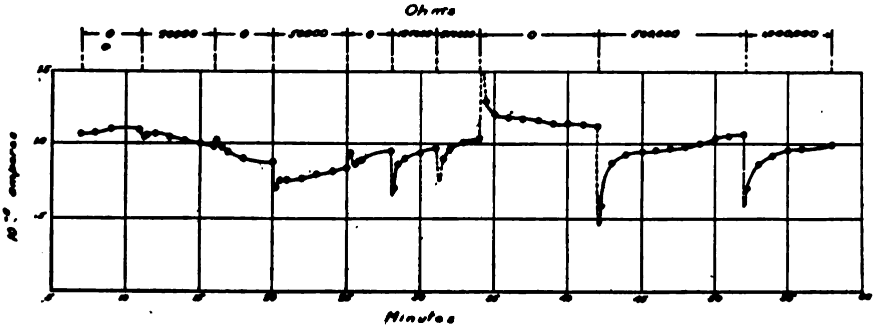

Fig. 6.

The dark current was found to be similar to the photoelectric current in that its value is independent of the resistance of the circuit within a large range of variation. This is shown in Fig. 6 in which the value of the dark current is plotted against time. At every change in the value of the resistance there is indicated a throw of the galvanometer such as one might expect as the result of the charging or discharging of the electrolytic capacity due to the change in the value of the potential drop in the external resistance. In addition to these throws the diagram shows a slow and apparently erratic drift in the value of the current. There seems to be no relation between this and the resistance changes. This drift is characteristic of that due to temperature variations. In this experiment the resistance of the galvanometer was about 1,000 ohms and the resistance of the cell as measured on an alternating current bridge, 15,000 ohms. We have therefore varied the resistance of the circuit through the range, 16,000 to 1,016,000 ohms without producing an effect upon the current which can be attributed to this variation.

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[1] Sheldon, Leitch and Shaw, loc. cit.

[2] Bouty, Jour. de phys. (3), Vol. 3, p. 498, 1894.

[3] Scott, Ann. der phys., Vol. 67, p. 402, 1899.

A dark current was also found by Thompson which, however, he was able to partially eliminate by the introduction of resistance into the circuit. This would seem to indicate that there was present in addition to the current of the type described above, one due to a chemico-electromotive force. In all previous work with semitransparent electrodes the films have been deposited simultaneously, side by side on the same piece of glass. Under these conditions the electrodes would ordinarily be so nearly alike in density and temperature that the dark current would be so small as to escape notice. This dark current should be made the subject of a thorough study. Its evident simularity to the photoelectric current makes it probable that the explanation of the latter will also explain the former. Further work on this subject is now under way. In this investigation the effort was rather to reduce the dark current to zero by producing two electrodes of equal densities.

The two films finally selected transmitted respectively 68 per cent. and 72 per cent. of the incident light. The transmission of each was nonselective throughout the visible spectrum. The cell made up with these two electrodes was filled with a 6 per cent. solution of rhodamine-*B* and placed in a glass box which was sealed to prevent the evaporation of the solution. This was then placed in a dark box arranged with a shuttered opening through which one of the electrodes could be illuminated at will. The dark current observed was the smallest that had been obtained with a cell of this type. Its value was observed from time to time for more than two weeks. Table I. gives data on the dark current during

TABLE I.

Day.	0	1	2	3	10	15
Dark current	3.0	2.5	2.0	1.5	1.3	.5
Sensitivity	1.	.95	.85		.7	.6

this period. The current varied continually because of temperature changes and the values are the estimated mean values for the day in question measured in 10^{-8} amperes.

During the same period a series of photoelectric experiments was performed under varying conditions as to length of exposure and resistance. Estimated values of the photoelectric sensitivity measured on an arbitrary scale are included in the table for comparison. The data shows a decrease in the value of the dark current with time. It is accompanied by a corresponding decrease in the photoelectric sensitivity. It seems probable that there is a relationship between the two fatigue effects.

Several of the photoelectric current curves of the series just described have been corrected for the errors introduced by the galvanometer and tested for agreement with the Thompson and Goldmann equations. In no case have satisfactory results been obtained. When the Thompson test was applied a curved line always resulted. In most cases, however, it was of such a form that if fewer observations had been plotted it might easily have been interpreted as two straight lines. In taking data for these curves a chronograph was used and a large number of points were recorded. Thompson's diagrams show but few observation points. It seems probable that the difference in the results obtained is due to the fact that his observations were too few and too far apart to indicate the steady character of the curvature.

The cell with which Goldmann obtained the current-time curves whose equations are given above had for its unilluminated electrode a copper plate in an aqueous solution of copper sulphate connected to the alcoholic rhodamine — B solution by a capillary tube. This electrode was designed to avoid the errors due to the polarization of the unilluminated electrode by the photoelectric current. Goldmann found that, when this electrode was used, the current was in general greater than that obtained at corresponding times with the ordinary type of electrode. However during approximately the first minute of an exposure the two results did not differ measurably. I have, as yet, made no attempt to correct for the error due to the polarization of the dark electrode by the photoelectric current. If Goldmann's method of eliminating this error is correct, it is negligible for exposures of less than one minute's duration and in as much as it is the beginning of the growth and decay curves which is the particular object of this study, a series of curves has been made in which the exposure is limited to one minute.

It having been found not feasible to eliminate the dark current with the type of cell in use, its value was made as constant as possible by surrounding the cell by an oil bath in a glass box. This steadied the temperature variations and no further trouble was experienced from variations of the dark current. The deflection due to the dark current was then taken as the zero from which the photoelectric current was measured.

In exposures of short duration the exact time of the opening and the closing of the shutter becomes important. In order to insure an accurate record of these times the shutter was so arranged that its openings and closings were automatically recorded on the same chronograph used to record the growth and decay curves.

Fig. 7 shows the results of a series of four experiments performed by this method. The cell was illuminated with an intensity of 5.2 foot candles for one minute. The symbols ○, ×, + and □ represent the data from the first, second, third and fourth experiment respectively. The time is taken as zero at the instant of the opening of the shutter in each experiment and the deflection due to the dark current has been substracted from the ordinates before plotting in each case. The striking feature of these observations is the accuracy with which the results are repeated. In all previous experiments with longer exposures it was found impossible to reproduce results. In every case the current would be less at the second trial.

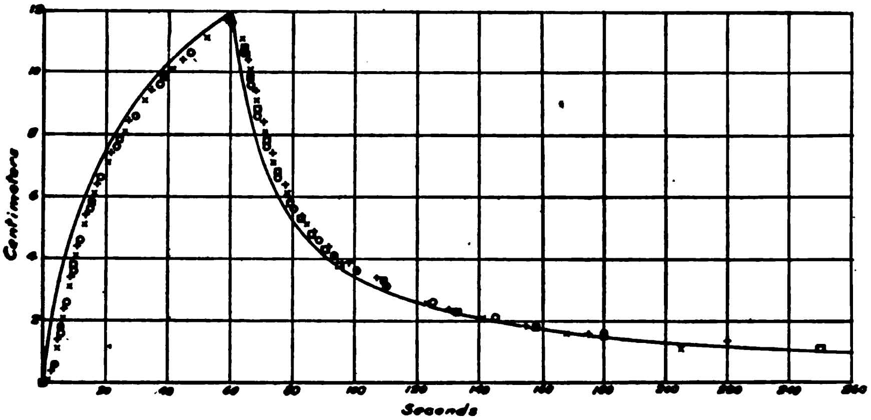

Fig. 7.

The curve shown in Fig. 7 is the composite of the four experiments corrected by equation (11). The following are the values of the constants involved.

a,	20,000,	h,	15,000,	n,	2.6,
b,	13,000,	ζ,	22,950,	ωc,	.96,
s,	50,000,	r,	3,100,000,	α,	5.3,
g,	480,	ω,	22,750,	β,	3.5,

R,	7,950,	c,	4.2×10^{-6},	γ,	.4,
f,	0,	l,	.015,	α',	5.7,
p,	20,400,	l/g,	.00003,	nj,	10^{-9}.

Fig. 8 shows the graphical plots from which the corrections were computed: (*a*) is $\dot{x}$; (*b*), $\ddot{x}$ and (*c*), $\int_0^1 \ddot{x}\,dz$. Plot (*c*) was only made for the growth curve as it so closely approximates (*b*) as to make it possible to use equation (11). The value of the test function, $\gamma\omega c\ddot{x}$ is nowhere greater than .02 which is less than the uncertainty in the value of y. The computation of the values of y for the third experiment of the series is shown in Table II.

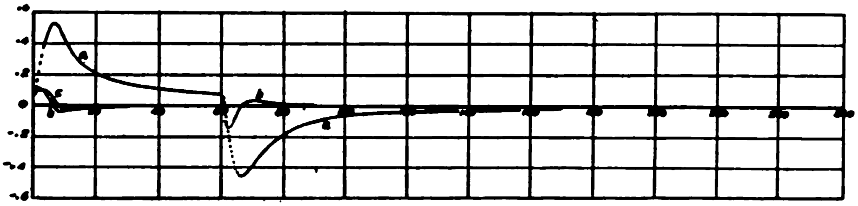

Fig. 8.

In this computation the result is nowhere more accurate than one per cent. In order therefore to justify the assumption of the truth of the law of shunts $y/\dot{y}$ must be greater than $100 \times .00003 = .003$ seconds. An examination of the curves of Fig. 7 shows that this is true except in the range from 0 to .003 seconds. As no data was taken in this range, the assumption is justified.

Fig. 9 shows a test of the data of Table II. for compliance with Thompson's type of equation. The result here shown is typical of those obtained in all of the experiments performed. In all, the growth data give a smoothly curving line, the point at $t = 60$ lies below the first straight

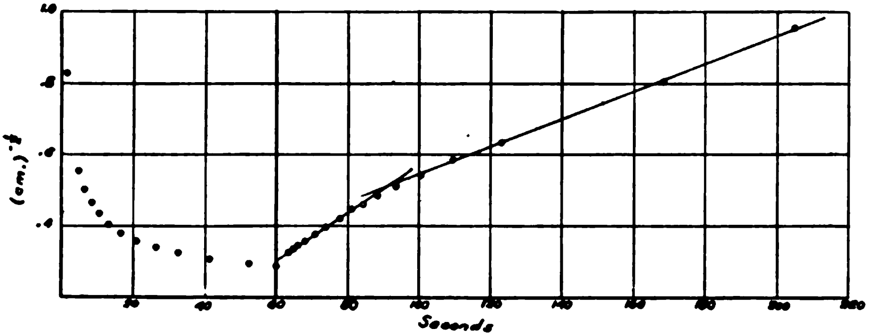

Fig. 9.

TABLE II.

t.	x.	$3.5\dot{x}$	$5.7\ddot{x}$	y.
0	0			0
1.4	.1	.6	.75	1.45
4.4	1.1	1.65	.5	3.25
6.3	2.1	1.85	0.	3.95
8.4	3.1	1.7	–.2	4.6
10.5	4.1	1.45	–.2	5.35
13.2	5.1	1.15	–.15	6.1
16.5	6.1	.9	–.1	6.9
20.9	7.1	.7	–.05	7.75
26.2	8.1	.55	–.05	8.6
32.7	9.1	.45		9.55
41.6	10.1	.4		10.5
52.4	11.1	.3		11.4
60.0	11.7	.3		12.0
63.9	11.1	–1.25	–.55	9.3
65.1	10.6	–1.55	–.15	8.9
66.1	10.1	–1.6	–.05	8.45
68.4	9.1	–1.45	.15	7.8
71.1	8.1	–1.2	.2	7.1
74.0	7.1	– .95	.15	6.3
78.1	6.1	– .75	.1	5.45
80.9	5.6	– .65	.05	5.0
84.2	5.1	– .45	.05	4.7
88.5	4.6	– .35	.05	4.3
93.6	4.1	– .25		3.85
100.4	3.6	– .2		3.4
109.9	3.1	– .15		2.95
123	2.6	– .1		2.5
141	2.1	– .1		2.0
168	1.6	– .05		1.55
205	1.1			1.1

line of the decay and the points near the intersection of the two straight lines form a curve tangent to both. It is evident that Thompson's law does not hold for short exposures in the case of the growth data. This result might have been predicted for the case of short exposures from the fact that $y^{-1/2}$ is necessarily infinity when t is zero. In the case of the decay data Fig. 9 probably gives as good verification of the law as that obtained by Thompson. The results, however, show conclusively that the knee in the plot is not to be associated with the errors due to the galvanometer but that it is characteristic of the law of decay. The fact that the point at $t = 60$ in every case falls below the first straight line might be interpreted as indicating the existence of another line through this point and intersecting the first between this point and the next one shown. Similar cases of three intersecting straight lines have been found in phosphorescence decay curves.

It has been found impossible to represent the data by an equation of the type of (1), Goldmann's theoretical equation for the current during illumination. Line "*a*" in Fig. 10 shows the best result that has been obtained in this attempt. Here the values of y from the growth curve have been plotted against $e^{-.048t}$. However an equation of the type of (3) may be found that will represent the data except during the first 10 seconds. This is the same sort of agreement that Goldmann found between his data and the equation. This is shown in Fig. 11 in which y is plotted against $e^{-.033t}$. In this case, however, it is doubtful if the agreement is truly significant. If similar plots are made using various larger values of m_1, there is in each case a range within which the points lie upon a straight line. Line "*b*" in Fig. 10 is an example of this and represents the data in the range from 5 to 25 seconds. The deviation of the points from this line for times less than 5 seconds is of the same type as that observed by Goldmann. The deviation for times greater than 25 seconds is in the wrong direction to be explained by the polarization of the dark electrode. As one takes smaller or larger values of m_1, this range of agreement comes later or earlier but the deviation outside of it remains of the same type as shown. In Fig. 11, m_1 has so small a value that the range of agreement extends to the end of the time of exposure. There is every reason to think that, if the exposure had been longer, deviations from the straight line would have been found for times later than 60 seconds. In fact the last point of the plot shows evidence of

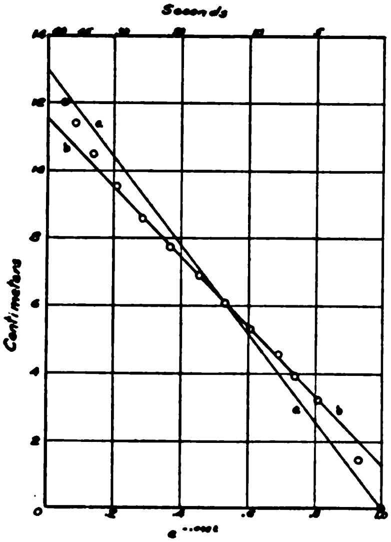

Fig. 10.

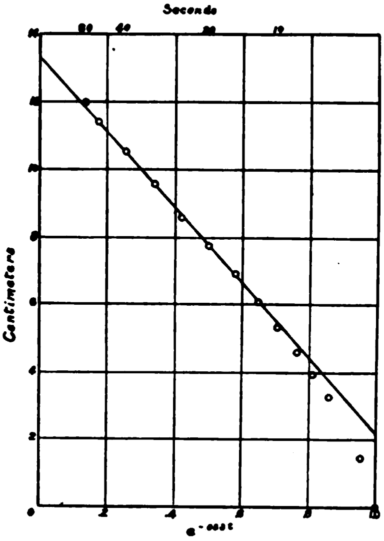

Fig. 11.

the beginning of such a deviation. For values of m_1 less than .033 the agreement is not satisfactory. Fig. 12 shows a test of the data for compliance with (2), Goldmann's theoretical equation for the current during decay. Here $\log_e y$ is plotted against t. The result should be a straight line if the equation holds. This is obviously not the case. Attempts to represent the decay data by an equation of the type of (4) have also been unsuccessful.

It seems fair to conclude that the change in the laws of growth and decay observed by both Goldmann and Thompson at about one minute are not due to the damping and inertia of the galvanometer coil. None of the laws, with the exception of Thompson's law of decay, accurately represents the data for short exposures. This cannot but cast some doubt upon Goldmann's theory of the mechanism of the photoelectric action as it would seem that if his ideas were correct equations (1) and (2) would more nearly represent the data for short exposures than for long. To make this result conclusive, however, experiments should be carried on over longer periods of exposure under conditions which would allow of correction being made for the polarization of the dark electrode.

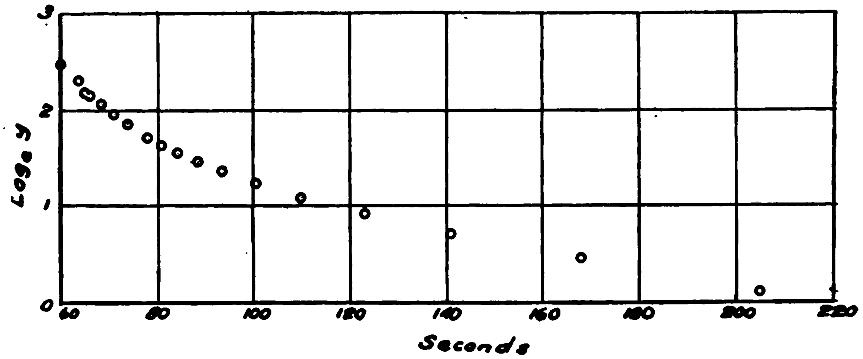

Fig. 12.

In order to study the effect of variations in the composition and concentration of the electrolyte and in the material and density of the electrodes upon the photoelectric properties of cells, it is desirable to have some equation which at least approximately represents the current-time curve. The numerical constants of such an equation, even though it be empirical, are very useful in making comparisons. Thompson's law of decay might be used for this purpose but it has the disadvantage that it involves at least four constants. I have found that an equation of the type,

$$y = y_0 - \frac{t}{Mt + L}, \qquad (12)$$

in which t is measured from the instant of closing the shutter and y_0 is the value of y at that instant, very closely represents the decay curves following exposures of not more than a minute's duration. This equation may be written in the form

$$\left(t - \frac{L}{M}\right)\left(y - y_0 + \frac{1}{M}\right) = \frac{L}{M^2}, \tag{13}$$

the equation of an equilateral hyperbola if y is plotted against t as in Fig. 7. It may be also written,

$$\frac{t}{y_0 - y} = Mt + L, \tag{14}$$

the equation of a straight line if $t/(y_0 - y)$, is plotted against t as in Fig. 13. In making this plot the data from Table II. was used. The values of the constants found to give the best results are $L = 1.28$, and $M = .0844$,

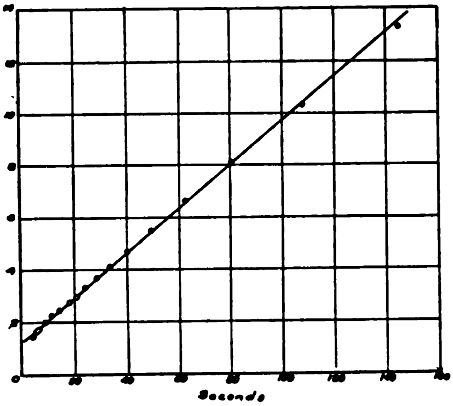

Fig. 13.

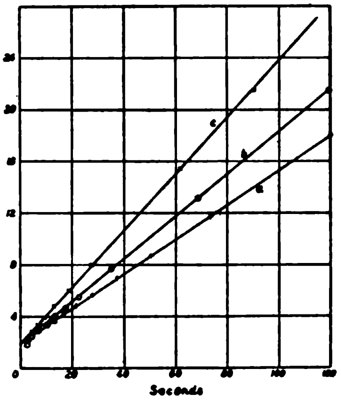

Fig. 14.

giving $1/M = 11.85$, $L/M = 15.2$, $L/M^2 = 180$ and

$$y = \frac{12.2 - t}{.0844t + 1.28}.$$

Table III. gives the observed and computed values of y and their differences.

There seems to be a possible systematic deviation from the hyperbolic law during the early part of the decay. This is indicated in the table by the positive differences in the second, third and fourth items and in Fig. 13 by a downward trend of the points near the left end of the line. Deviations of this type have been found in several experiments and always in the same sense. Frequently, however, there is no such indication.[1] The data in this region is less accurate than the rest and has

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[1] See Fig. 14, curve (C).

TABLE III.

y.	y_c.	Diff.	y.	y_c.	Diff.
12.0	12.0	0	4.7	4.7	0
9.3	9.6	.3	4.3	4.25	−.05
8.9	9.0	.1	3.85	3.85	0
8.45	8.6	.15	3.4	3.4	0
7.8	7.8	0	2.95	2.9	−.05
7.1	7.0	−.1	2.5	2.45	−.05
6.3	6.3	0	2.0	2.0	0
5.45	5.55	.1	1.55	1.6	.05
5.0	5.1	.1	1.1	1.25	.15

been given less weight in determining the values of the constants M and L. Moreover the method of plotting used in Fig. 13 is particularly sensitive to errors at the left end of the line.

Use has been made of the hyperbolic relation to study the effect of the length of exposure on the nature of the decay. In another series of experiments, performed with the same cell, decay curves were taken after exposure of 60, 30 and 15 seconds. The cell was only partly filled with the electrolytes as some of the solution had leaked out and the effective area of the electrodes was observed to be about 2/3 of that in the first series. The values of the constants involved in the correction equation were the same as before except that the change in the effective area of the electrodes modified the values of c and r and of the constants depending on them. Assuming that the new area was 2/3 of the old, $c = 28$ microfarads, $r = 4.9$ megohms, $\omega c = .64$ seconds and $\gamma = .3$ sec.^{2}. When the cell was illuminated with light of the same intensity as before a growth curve was obtained of which the corrected ordinates were .64 times those obtained in the previous experiments. This verifies the estimated value, 2/3, as the photoelectric current has been found to be proportional to the effective electrode area.[1]

Fig. 14 shows corrected data for the decay curves plotted in accordance with equation (14). The lines (a), (b), and (c) are the plots for the decays following exposures of 60, 30 and 15 seconds respectively. These lines are seen to have nearly the same value for their y-intercept, L. The significance of this may be shown by differentiating equation (12) obtaining for the rate of decay of the current

$$-\frac{dy}{dt} = \frac{L}{(Mt + L)^2}.$$

Now when $t = 0$, this becomes

$$-\frac{dy}{dt} = \frac{1}{L}.$$

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[1] Goldmann, loc. cit., p. 478.

Thus we see that the y-intercept L is the recriprocal of the initial rate of decay of the current and that the intersection of the three lines upon the y-axis signifies that the initial rate of decay of the current is not dependent upon the length of exposure. In order to test this conclusion the equations for all the decay curves of this series have been computed. The results are shown in Table IV. Column 1 gives the duration of the exposure in seconds, 2 and 3 give the values of L and M, and 4, the ordinate of the asymptote of the hyperbola.

TABLE IV.

1, T.	2, L.	3, M.	4, y'.
60.0	2.0	.139	.3
60.3	1.88	.137	.4
60.7	2.0	.134	.15
	Av. 1.96		
31.0	1.77	.166	0
29.5	1.85	.167	0
29.9	1.9	.169	.05
30.0	1.85	.170	.1
	Av. 1.84		
15.7	1.98	.222	0
15.0	1.94	.223	0
15.0	1.91	.230	.05
	Av. 1.94		

An examination of the values in column 2 confirms the conclusion of the constancy of L within its accuracy of measurement which is about 5 per cent. The values of y' in column 4 are either zero or small and positive. It has been frequently observed that the current does not in general decrease to zero but to a small positive value.[1] This value is seen to be practically zero for exposures up to 30 seconds duration but to certainly have a finite value for an exposure of 60 seconds. The possible deviation from the hyperbolic equation during the early part of the decay which was discussed above in connection with Fig. 13 and Table III. does not occur in any of the 15 second exposure data and is not marked in the 30 second exposure data. This suggests the possibility of this deviation being related to y'.

We may in fairness conclude that following a short exposure, the current decreases from an initial value, y_0, to a small value, y', at a variable rate, v, whose initial value, $v_0(=1/L)$, is independent of the duration of the exposure and that the curve of decay is fairly well represented by an equilateral hyperbola which is completely determined by these three constants y_0, v_0 and y'. Its equation in terms of them is

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[1] Goldmann, loc. cit., p. 512.

$$(y - y')\left[t - \frac{1}{v_0}(y_0 - y')\right] = \frac{(y_0 - y')^2}{v_0}.$$

A detailed study of v_0 and y' as functions of the physical and chemical properties of the electrodes and the electrolyte and of y' as a function of the length of exposure promises to give valuable information concerning the nature of the effect.

The invariance of v_0 with the length of exposure in a way explains our failure in the attempt to represent the data for the decay by an exponential equation. It is a well known characteristic of this type of equation that the first derivative of the dependent variably, y, is a linear function of y. Now if v_0 is invariant with respect to the time of exposure it will be invariant with respect to η_0 which is a function of that time. If an exponential equation is to meet this condition it must have exponents which are themselves functions of η_0. This is not the case in Goldmann's theory of the effect.

In a theoretical discussion of this effect there should be taken into account three facts not hitherto considered. The fact that the rate at at which the current starts to decay after a short exposure varies only slightly, if at all, with the length of exposure, is inconsistent with any theory in which the rate of decay depends upon the value of the current. The existence of a dark current, simillar in character to the photoelectric current and varying in value with the photoelectric sensitivity and with the nature and temperature of the electrodes, makes the theory that the current originates solely in the ionization of the rhodamine molecule by the incident light, insufficient. The discovery by Thompson[1] that the material of which the electrode is composed has a marked effect upon the photoelectric sensitivity confirms this view and further points to the probability that the electrodes play an active part which must be considered in explaining the effect.

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APPENDIX.

Proof of Equation (8).—Let i, i_r, and i_c represent the currents in h, r, and c respectively, due to the motion of the galvanometer coil. (See Fig. 2.) Then

$$i = i_c + i_r. \tag{15}$$

The potential difference between the ends of the resistance, r, is

$$v_1 - v_2 = ri_r \tag{16}$$

$$= \frac{1}{c}\int i_c dt. \tag{17}$$

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[1] Loc. cit.

Eliminating i_r between (15) and (16) and writing

$$e = r\,i_c, \tag{18}$$

$$v_1 - v_2 = ri - e. \tag{19}$$

This equation is of the form of the expression for the difference of potential between the ends of a conductor of resistance, r, carrying a current, i, and containing an electromotive force, e. We may therefore consider the branches c and r replaced by such a conductor for the purpose of studying effects outside of these branches. If we eliminate $v_1 - v_2$ and i_c between equations (17), (18), and (19) we obtain for the condition on the value of e

$$\frac{de}{dt} + \frac{e}{rc} = r\frac{di}{dt}. \tag{20}$$

Let E be the electromotive force induced in g by the motion of the coil. The current in r due to E equals the current which E would produce in g if it were situated in r which is $E/[n(r+\zeta)]$.[1] This current is superimposed upon the current in r due to e. Therefore

$$i = \frac{E}{n(r+\zeta)} + \frac{e}{r+\zeta}. \tag{21}$$

Eliminating i between (20) and (21) and writing ω for $r\zeta/(r+\zeta)$,

$$\frac{de}{dt} + \frac{e}{c\omega} = \frac{r}{n\zeta}\frac{dE}{dt} \tag{22}$$

or

$$e = \frac{r}{n\zeta}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}\left[\frac{dE}{dt}\right]_s ds, \tag{23}$$

where s is the variable of integration and t_0 is the value of t when $e = o$.

Now the total current in the galvanometer coil will be that resulting from the superposition of the current due to E, that due to e and that portion of the current to be measured which flows through the galvanometer.

$$i_g = \frac{E}{\rho} + \frac{e}{n(r+\zeta)} + \frac{u}{n}. \tag{24}$$

The electromotive force E equals $-N d\theta/dt$, where N is product of the area and number of turns of the coil and the magnetic induction of the field in which it is suspended. Substituting this value of E in (23) and (24) and eliminating e between the two equations,

$$i_g = \frac{u}{n} - \frac{N}{\rho}\dot{\theta} - \frac{N\omega}{n^2\zeta^2}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}\,\ddot{\theta}_s\,ds. \tag{25}$$

There act upon the galvanometer coil the following torques:—that due to the current, $-Ni_g$; that due to the elastic suspension, $-\tau\theta$; that due to friction, $-\mu(d\theta/dt)$. Equating the sum of these to the rate of change of the angular momentum, $I\ddot{\theta}$, and eliminating i_g by equation (25), we obtain

$$I\ddot{\theta} + \left(\mu + \frac{N^2}{\rho}\right)\dot{\theta} + \frac{N^2\omega}{n^2\zeta^2}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}\,\ddot{\theta}_s\,ds + \tau\theta = \frac{N}{n}u \tag{26}$$

Suppose θ to be measured on a scale at distance, m, from the mirror. Then $\theta = x/2m$. Substituting this value for θ and multiplying through by $2m/\tau$, we obtain

$$\alpha\ddot{x} + \left(\kappa + \frac{\lambda}{\rho}\right)\dot{x} + \frac{n^2\zeta^2}{\lambda\omega}\int_{t_0}^{t} \epsilon^{(s-t)/\omega c}\,[\ddot{x}]_s\,ds + x = \frac{u}{nj}, \tag{27}$$

in which α is written for I/τ; κ, for μ/τ; λ for N^2/τ and j, for $\tau/2mN$. Writing β for $\kappa + \lambda/\rho$ and multiplying by nj gives equation (8).

Now t_0 has been defined by the condition, $t = t_0$ when $e = 0$. This condition holds when $\ddot{x}$ and $\dot{x}$ are both zero for then $u = njx$, a constant, and the current in each resistance of the network of Fig. 2 is constant. But

$$e = ri_c = cr\frac{d}{dt}(v_1 - v_2) = c\frac{di}{dt}$$

and thus must be zero when i is constant.

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[1] Maxwell, "Electricity and Magnetism," Sec. 281.

NEW BOOKS.[1]

The Concept of Nature. By A. N. **Whitehead.** Cambridge, England: The University Press, 1920. Pp. ix + 202.

A sequel to the author's Principles of Natural Knowledge in which his method of extensive abstraction is further elaborated.

The dependence of the character of space on the character of time is used as a basis for the systematic explanation of both.

Memoria de la Facultad de Ciencias Fisicas, Matematicas Y Astronomicas. Correspondiente A 1918. No. 8. La Plata: Facultad de Ciencias Fisicas, Matematicas Y Astronomicas, 1920. Pp. 1 + 136.

The annual Record of the Scientific Activities of the University of La Plata for 1918 containing:

1. A list of titles of scientific work published by the faculty with rather full abstracts.

2. A statement of courses offered and various statistics concerning the university.

Tables Logarithmiques et Trigonometriques A Quatre Décimales et Tables a Trois Décimales A L'Usage des Physiciens et des Navigateurs. By H. **Rocques Desvallees.** Paris: Gauthier-Villars et Cie, 1920. Pp. xxiv + 72.

This set of tables contains the usual directions for use; logarithms and anti-logarithms; addition logarithms and subtraction logarithms; and values of the trigonometrical functions conveniently arranged.

Organizacao do Estudo da Fisica. By **Alvaro R. Machado.** Porto: Tipografia Sequeira, 1920. Pp. 1 + 146.

Notes and Observations upon the facilities for the study of physics in Spain, France, Switzerland, Belgium and England.

Course de Cinematique Theorique. By H. **Lacaze.** Paris: Gauthier-Villars et Cie, 1920. Pp. 1 + 138.

The five chapters of this brief treatise deal respectively with:

Vectors; Kinematics of a point; Movement of a Solid; The Composition of Accelerations; The Displacement of a Vector in a Plane.

Second Series *June, 1921* *Vol. XVII, No. 6*

THE

PHYSICAL REVIEW.

THE DISTRIBUTED CAPACITY OF INDUCTANCE COILS.

By G. Breit.

Synopsis.

Effective Capacity of a Coil Defined.—Experiments show that if a coil is connected in series with a condenser of capacity C, the frequency $(\omega/2\pi)$ with which this combination is in resonance is given by $L(C + C_0) = 1/\omega^2$, where L and C_0 are constants.

The constant C_0 is called the *effective capacity* of the coil, sometimes simply "the capacity" of the coil. A general formula, equation (6), is derived for its calculation.

Single-layer Solenoid. The formula is applied to the short single-layer solenoid, used when grounded in an elliptical shield and when insulated from the shield, and to the short single-layer solenoid used when grounded and insulated in free space. An explanation is given of the remarkable *constancy of C_0* as found by experiment in the case of short coils.

Experimental Verification.—An experimental verification is given by direct measurement of capacity and inductance. The current distribution in a coil has also been studied experimentally. The results have verified the theory.

Introduction.

This paper is intended to call the attention of physicists and mathematicians to some interesting aspects of the subject of distributed capacity of coils. The subject is of practical importance because inductance coils are used extensively in radio communication and because the distributed capacity, taken in connection with the value of inductance, determines the range of wave-lengths within which the inductance coil can be used to advantage. Furthermore, there is considerable mathematical interest connected with the calculation of the effective capacity caused by the capacity distributed along the wire of the coil.

The subject has been largely neglected by mathematical physicists. Lentz[1] and Drude[2] seem to be the only ones who have made a study of it.

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[1] W. Lentz, Ann. d. Phys., *34*, p. 923–974, 1912. W. Lentz, Ann. d. Phys., *43*, p. 749–797, 1914.

[2] P. Drude, Ann. d. Phys., *9*, p. 293, 1902.

However, there are errors in Drude's mathematics, while Lentz's results are not adapted to numerical calculation and involve too many assumptions to be used generally.' For example his treatment applies only to wires having circular section and further it combines the assumption of negligible curvature of a single turn with that of an infinitely long coil. This combination of assumptions may be justified for treatment of skin effect. Its applicability to capacity calculation is questionable.

In this paper an outline of the general method of calculating the effective capacity will be given and then illustrated by working out some special cases.

THE EFFECTIVE CAPACITY OF INDUCTANCE COILS.

Definition of Effective Capacity.—It is an experimental fact that the resonance frequency $\omega/2\pi$ of an inductance coil across whose terminals a condenser of capacity C is connected is given by

$$L(C + C_0) = \frac{1}{\omega^2} \tag{1}$$

where L, C_0 are constants for the coil in question. The international electrical units are used in this paper, where not otherwise specified. Formula (1) means that if an electromotive force $E_0 \cos \omega t$ acts in some part of the circuit and the capacity of the condenser is varied then the current is a maximum when (1) is satisfied. The constant C_0 is called the *Effective Capacity* of the coil. It is due to the capacities which are distributed along the wire of the coil.

The assumptions made in the following calculation are as follows:

1. The field of the condenser does not affect the field of the coil appreciably.
2. The resistance of the coil is negligible compared with its reactance, so that the wire may be treated as a perfect conductor.
3. The dimensions of the coil are so small in comparison with the wavelength used that the retarded values of the scalar and vector potential may be equated to their contemporaneous values.
4. The value of the E.M.F. induced in the whole coil by the current in a small section of the coil at a given point of the coil is proportional to the length of the section.
5. In the special cases considered the diameter of the coil is very much larger than the depth of the coil's winding or the axial length of the coil.

These assumptions will now be discussed in detail. Consider the case when the field of the condenser does not affect appreciably the electric intensity at the wires of the coil. It is essential to understand that this may not always be the case as is seen from the following considerations.

The wire of the coil is connected to the condenser whose capacity is C. If there is a difference of potential between the two condenser plates the charges on the plates give rise to an electric field whose line integral along any curve between the two plates is equal to the difference of potential between the plates. If the curve is taken along the wire it follows that the electric intensity along all of the wire cannot be negligible since its line integral is equal to a large difference of potential.

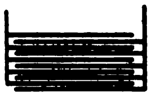

Fig. 1.
Interleaving effect of condenser plates.

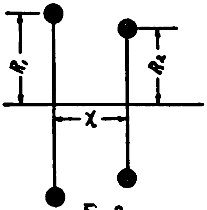

Fig. 2.
Diametral section of two turns of wire.

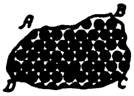

Fig. 3.
Cross section of multi-layer coil by a plane through the axis.

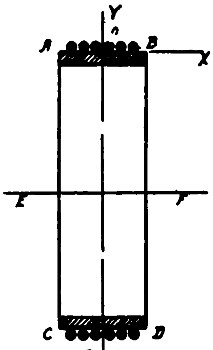

Fig. 4.
Diametral section of short single-layer solenoid.

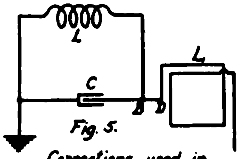

Fig. 5.
Connections used in measuring the capacity of a coil to ground.

Nevertheless it is legitimate to neglect this electric intensity under conditions which are satisfied in most practical cases. Consider the field which arises from the charges on the plates of an ordinary condenser whose plates are interleaved as shown on Fig. 1. The electric field is most intense between the plates. It also has a value at all points outside. But this value becomes small at a large distance. The principal contribution to the line integral is, therefore, confined to a small length along the leads connecting the coil to the condenser, and even if it should happen that there is an intense field due to the condenser at some part of the coil, it usually happens that this field is either compensated in some other part of the coil or else is along the wires of the coil.

However, there are cases when this assumption cannot be justified. No general treatment of these cases can be given unless the dimensions and position of the condenser are known. For this reason the present treatment considers the ideal case when the condenser is connected to the coil by long leads and in practical applications one must constantly bear in mind that this assumption has been made.

Let, then, the electric intensity caused by the condenser be negligible and let the electromotive force be applied between the condenser and the coil in one of the long leads. Then the electric intensity at any point on the surface of the coil is due solely to two causes:

(1) the currents in the coil,
(2) the charges on the wire of the coil.

If further the material out of which the coil is made should be a perfect conductor then the electric intensity must be normal to the surface of the coil at every point.

The question arises immediately as to whether the current is the same through every cross section of the wire. It is apparent at once that it is not. In fact the current in the coil gives rise to an electric intensity which has a component tangential to the wire at least at some points because if this were not the case the coil would not have any self-inductance, because the self-inductance is the line integral along the wire of the electric field due to the current per unit rate of change of current. But the total electric intensity must be normal to the surface of the wire and therefore has no tangential component. Consequently there must be charges on some parts of the coil which give rise to an electric intensity whose tangential component is equal and opposite to the tangential component due to the current. In the case considered the tangential component of the current varies periodically. Therefore, the charges also vary periodically. If x is an arbitrary parameter along the wire such, *e.g.*, as a length measured from an arbitrary fixed point along the wire, if i is the current, and if Qdx is the net charge in an infinitesimal segment dx, *i.e.*, if Q is the charge per unit length, then it is easily shown that

$$\frac{\partial i}{\partial x} = -\frac{\partial Q}{\partial t} \tag{2}$$

by the conservation of charge. As shown

$$\frac{\partial Q}{\partial t} \neq 0.$$

Therefore, $\partial i/\partial x \neq 0$. Thus the current is different at different points on the wire.

Suppose that the frequency dealt with is so low that this non-uniformity of current is small. Then as an approximation the component of the electric intensity may be computed as if a uniform current i were flowing through the coil. Let (1) and (2) be arbitrarily selected points on the surface of the wire of the coil. Let the self-inductance of the portion of the coil included between (1) and (2) be L_{12} and denote the mutual inductance of this section to the rest of the coil by M_{12}. Then the line integral of the electric intensity due to i and taken along the wire from (1) to (2), i being reckoned positive when the current is flowing from (1) to (2), is

$$e_{12} = (L_{12} + M_{12})\frac{\partial i}{\partial t}.$$

If the electrostatic potential is V_1 at (1) and V_2 at (2), the line integral of the electric intensity due to the charges and taken from (1) to (2) is

$$V_1 - V_2.$$

The total line integral is then

$$e_{12} + V_1 - V_2.$$

But the total electric intensity is normal to the wire and, therefore, the total line integral is zero. Hence

$$V_2 - V_1 = e_{12}, \tag{3}$$

i.e., the line integral of the field due to the current between any two points on the surface of the wire is equal to the difference of potential between the two points.

Attention must be called here to the fact that e_{12} is independent of the path on the surface of the wire along which the integration is effected because no matter how intricate this path may be the result of the integration must always give $V_2 - V_1$. This means that even though the electric field due to the currents cannot be derived from a single-valued potential at all points in space, the component tangential to the surface of a perfect conductor may be derived from a single-valued potential on the surface of that conductor.

The same may also be seen from the fact that the magnetic intensity has a constant normal component to the surface of a perfect conductor so that the flux of magnetic induction through any closed curve on the surface of the conductor is constant and the line integral of the electric intensity around any closed curve is zero.

Thus from a knowledge of i, $V_2 - V_1$ may be derived. If it is also known that some part of the coil is grounded then the point grounded is at zero potential and therefore V is known at all points of the coil. According to the assumption made as to the condenser the only charges which give rise to V are those on the wire of the coil and the objects in the neighborhood of the coil. Suppose that it is possible to solve the electrostatic problem of finding such a distribution of charges on the coil and its surroundings as to give rise to the specified value of V. This problem can be always solved and, moreover, the solution of it is unique because it is equivalent to finding V in a region of space within which V satisfies Laplace's equation and on the boundaries of which V is known. (Theoretically it is known that such a distribution and that only one such distribution exists.) Then Q in (2) is known and consequently i may be obtained from the formula

$$i = i_1 - \int_{x_1}^{x} \frac{\partial Q}{\partial t}\, dx,$$

where i_1 is the value of i at $x = x_1$.

Further, since the relation between the charge density $Q(x)$ and the "difference of potential" between the coil terminals is necessarily linear one can conveniently write

$$Q(x) = + \alpha(x) \frac{di_1}{dt}, \tag{4}$$

where $\alpha(x)$ is a function of x whose form depends on the shape and dimensions of the coil and the nature of the medium around the coil. The expression di/dt is written only as an approximation, which, however, is legitimate if $i - i_1$ is small because in the computation carried out only the terms depending on the first power of the frequency will be taken into account and $Q(x)$ itself depends on the first power and powers higher than the first. Since $i - i_1$ is zero for direct current, it is small for a sufficiently low frequency. Thus

$$i(x) = i_1 - \left(\frac{d^2 i}{dt^2}\right) \int_{x_1}^{x} \alpha(x) dx, \tag{5}$$

where x_1 is the value of x at one of the terminals of the coil.

The coil may be divided mentally into a number of small sections. Thus if the coil has 100 turns each of the turns may be looked at as one such small section. Within one of the sections the current is uniform. If the parameter assigned to one of them is x, the current in the section is $i(x)$. Then $i(x)$ is given by the preceding equation (5). If the section is small enough it is possible to find such a function $M(x)$ that $M(x)\Delta x$ is the sum of the self-inductance and the mutual inductance to the rest of the soil for a section included between x and $x + \Delta x$. The electromotive force induced between the coil terminals due to the current in the section is then

$$M(x) \frac{\partial i(x)}{\partial t} \Delta x.$$

The electromotive force due to all sections is

$$e_{12} = -\Sigma M(x)\frac{\partial i(x)}{\partial t}\Delta x,$$

where the summation is extended over all the sections. If Δx becomes infinitesimal this summation is

$$e_{12} = -\int_{x_1}^{x_2} M(x)\frac{\partial i(x)}{\partial t}dx$$

where x_1, x_2 are the values of x at the ends of the coil. Substituting the value of $i(x)$ given by (5) the electromotive force is:

$$e_{12} = -\frac{di_1}{dt}L + \frac{d^3 i_1}{dt^3}\int_{x_1}^{x_2} M(x)\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx,$$

where

$$L = \int_{x_1}^{x_2} M(x)dx$$

and is therefore the self-inductance of the coil. If the oscillations in the current i are simple harmonic, *i.e.*, if they are represented by

$$i = I_0\cos(\omega t - \theta),$$

then

$$\frac{d^3 i_1}{dt^3} = -\omega^2\frac{di_1}{dt},$$

so that the electromotive force induced in the whole coil is

$$e_{12} = -L\frac{di_1}{dt}\left[1 + \omega^2\int_{x_1}^{x_2}\frac{M(x)}{L}\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx\right].$$

The current i_1 is the current which flows through the condenser connected to the coil to constitute an oscillatory circuit. Let C be the capacity of the condenser and $-Q_c$ the charge on the plate connected to the ungrounded point of the coil given by $x = x_1$. Then

$$\frac{dQ_c}{dt} = i_1$$

and

$$\frac{di_1}{dt} = -\omega^2 Q_c.$$

Therefore, in terms of Q_c the electromotive force is

$$e_{12} = L\omega^2 Q_c\left[1 + \omega^2\int_{x_1}^{x_2}\frac{M(x)}{L}\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx\right].$$

But the difference between the potential of the plate connected to $x = x_2$

and the plate connected to $x = x_1$ is Q_c/C. Therefore,

$$\frac{Q_c}{C} = Q_c L\omega^2 \left[1 + \omega^2 \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \alpha(x)dx \right\} dx \right].$$

If

$$\omega^2 \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \alpha(x)dx \right\} dx$$

is small compared to 1, that is, if ω is sufficiently small, the above formula gives

$$\omega^2 = \frac{1}{LC}.$$

Using this value of ω^2 and substituting in the integral term of the above expression a second approximation is obtained in the form

$$\frac{1}{\omega^2} = L\left[C + \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx \right].$$

If this is to be identical with

$$\frac{1}{\omega^2} = L(C + C_0)$$

which is the experimental relation then

$$C_0 = \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx. \tag{6}$$

Although approximations were made in the formula it is general and correct as long as C_0 is a constant because the formula is exact at low frequencies. For convenience of reference the meaning of the symbols in this formula is restated.

If the charge in an element dx is $Q(x)dx$, then

$$\alpha(x) = \frac{Q(x)}{\left(\frac{di}{dt}\right)},$$

$M(x)dx$ is the mutual inductance between the section dx and the whole coil. L is the self-inductance. x is an arbitrary parameter. x_1, x_2 are the terminal values of x, x_1 being the value at the ungrounded condenser terminal.

The main steps in the derivation of this formula are summarized below:

(*a*) Since the wire was assumed to be a perfect conductor the E.M.F. induced between any two points is equal to the difference of potential between the points on the surface.

(*b*) This difference of potential calls for charges on the surface of the wire.

(*c*) The charges on the wire cause a non-uniform distribution of current in the wire.

(*d*) When the frequency is low the E.M.F. induced between any two points may be calculated as if the current were uniform. Hence the potential at all points of the wire is known and, therefore, the charge in any section is known.

(*e*) Knowing the distribution of charge the distribution of current is derived.

(*f*) From a knowledge of the connection between the current at the ungrounded condenser terminal and the current at any other point the self-induced E.M.F. is expressed in terms of the current at the condenser terminal.

(*g*) This E.M.F. is equated to the potential difference between the condenser plates expressed in terms of the charge and capacity. By comparison with the formula

$$L(C + C_0) = \frac{1}{\omega^2}$$

C_0 is derived.

(*h*) The theory given so far applies strictly only to very low frequencies. However, it will be shown later that C_0 is independent of the frequency for many cases. This is true experimentally in all important cases. Therefore, in the cases when C_0 does not vary with the frequency the formula (6) is general.

Applications to Problems Solved by Two-Dimensional Methods.

In this paper only problems solved by two-dimensional methods will be considered. A large class of problems reduce themselves to two-dimensional problems. This is the case with any circular coil whose diameter is large compared to the maximum distance between two of its turns.

Let R_1 be the radius of one of the turns. The charge density per unit length of wire is sensibly constant within one turn, if the number of turns is large. Let this charge density be ρ_1. The potential due to the turn considered at any point in space is now obtained by direct integration. By symmetry it is the same at all points of any circle coaxial with the first. Let this circle be of radius R_2, and let x denote the distance between the planes of the two circles. (See Fig. 2.) Then the potential in volts is

$$V = \frac{c^2}{10^9 K}\int_0^{2\pi} \frac{\rho_1 R_1 d\varphi}{\sqrt{(R_1\cos\varphi - R_2)^2 + R_1^2\sin^2\varphi + x^2}},$$

where $c = 2.9982 \times 10^{10}$ and is the ratio of the electromagnetic to the electrostatic unit of charge. K is the specific inductive capacity of the medium around the coil. Now

$$\int_0^{2\pi} \frac{d\varphi}{\sqrt{(R_1 \cos\varphi - R_2)^2 + R_1^2 \sin^2\varphi + x^2}}$$

$$= 4\int_0^{\pi/2} \frac{d\varphi}{\sqrt{x^2 + (R_1 + R_2)^2 - 4R_1R_2\cos^2\varphi}}$$

$$= \frac{4}{\sqrt{x^2 + (R_1 + R_2)^2}}\, F\left(\frac{2\sqrt{R_1R_2}}{\sqrt{x^2 + (R_1 + R_2)^2}}\right),$$

where $F(k)$ denotes complete elliptic integral of the first kind with modulus k.

If x is small compared to both R_1 and R_2

$$\frac{2\sqrt{R_1R_2}}{\sqrt{(R_1 + R_2)^2 + x^2}}$$

is nearly 1. Consequently the elliptic integral

$$F\left(\frac{2\sqrt{R_1R_2}}{\sqrt{(R_1 + R_2)^2 + x^2}}\right)$$

is approximately[1]

$$\log \frac{4}{\sqrt{1 - \frac{4R_1R_2}{(R_1 + R_2)^2 + x^2}}} = \log\left(\frac{8R}{d}\right),$$

where

$$d = \sqrt{(R_1 - R_2)^2 + x^2}$$

and where

$$\sqrt{(R_1 + R_2)^2 + x^2}$$

is replaced by $2R$, R now being used for R_1 as well as R_2. Thus

$$V = \frac{2 \times 10^{-9}c^2}{K}\rho_1 \log\left(\frac{8R}{d}\right) = \frac{17.978 \times 10^{11}}{K}\rho_1 \log\left(\frac{8R}{d}\right). \qquad (7)$$

This is the approximate expression for the potential caused by a circular ring at a small distance from it. It is essential to note here that the potential caused by the same ring at infinity vanishes to the first order.

Let then $ABCD$ (see Fig. 3) represent a section of the coil by a plane through its axis, the small circles being the sections of the wires of the coil. Each one of the wires is charged, the charge being distributed with a certain surface density, say σ. Let ds be an element of length along one of the circles. Then $\sigma\, ds$ is the corresponding linear density of charge for a ring of width ds and on the surface of the wire. If r is the distance from any point in the cross section to *ds*, then the potential at that point is obtained as

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[1] See Appendix

$$V = \Sigma \frac{10^{-9}c^2}{K}\int 2\sigma \log\left(\frac{8R}{r}\right)ds = \Sigma \frac{8.989 \times 10^{11}}{K}\int 2\sigma \log\left(\frac{8R}{r}\right)ds,$$

where the integration extends over the whole circle and the summation covers all the circles. *V* can be broken up into two parts by writing

$$V = -\Sigma \frac{8.989 \times 10^{11}}{K}\int 2\sigma \log(r)ds + (\log 8R)\Sigma \frac{8.989 \times 10^{11}}{K} 2\sigma ds.$$

The second of these two vanishes if

$$\Sigma \int \sigma ds = 0,$$

i.e., if the total charge on the wires of the coil remains constant. Such is the cause if the coil is not grounded. In this case of the ungrounded coil the first term is the only term and consequently the potential is the same as that which would exist if the wires of the coil were straight because a straight wire may be considered as the limiting case of a circular wire of infinite radius. *Thus the case of the ungrounded short coil is always reducible to a two-dimensional problem.*

If the short coil is grounded the only modification is that introduced by the second term. This contributes the same potential at all points of the cross section. Thus only a slight modification in the first solution is introduced by grounding. It must be carefully remembered here that in the two-dimensional case the logarithmic potential is infinite both at the filament causing it and at infinity. This makes it impossible to apply the two-dimensional treatment in general. If, however, the total charge is zero, the potential vanishes at infinity in the two-dimensional case and as was shown above is approximately the same as in the case of circular turns. It is only in this case, therefore, that the two-dimensional treatment applies.

There is an additional simplification in the case considered. This is introduced by the fact that *the E.M.F. induced in any one turn is practically the same as that in any other.*

In fact the expression for the mutual inductance between two coaxial circles whose radius *R* is large compared to their distance apart *d* is

$$M = 4\pi R\left[\log\left(\frac{8R}{d}\right) - 2\right].$$

Draw a system of rectangular axes (OX, OY) in the plane perpendicular to the wires of the coil. To each circle coaxial with the coil there corresponds a point (x, y), the point of intersection of the circle with the plane.

Suppose the current distribution is given by the fact that a current

$$\frac{1}{\mu}f(x, y)dxdy$$

crosses an area $dx\,dy$ at (x, y), μ being here the permeability of the medium. Then the flux through a circle passing through the point (x_0, y_0) is

$$N = 4\pi R\int\int\left\{\log\left(\frac{8R}{\epsilon^2\sqrt{(x-x_0)^2+(y-y_0)^2}}\right)\right\}f(x, y)dxdy,$$

where the double integral is taken over the entire area through which a current may pass, and ϵ is the natural base. Let a be the maximum possible value of

$$\sqrt{(x-x_0)^2+(y-y_0)^2}.$$

Then

$$\log\frac{a}{\sqrt{(x-x_0)^2+(y-y_0)^2}}$$

is always either zero or positive.
But

$$N = 4\pi R\int\int\left\{\log\left(\frac{8R}{a\epsilon^2}\right)\right\}f(x, y)dxdy$$
$$+ 4\pi R\int\int\left\{\log\frac{a}{\sqrt{(x-x_0)^2+(y-y_0)^2}}\right\}f(x, y)dxdy.$$

Therefore,

$$N > 4\pi R\left\{\log\left(\frac{8R}{a\epsilon^2}\right)\right\}\int\int f(x, y)dxdy. \tag{8}$$

Further,

$$\begin{aligned}\frac{\partial N}{\partial x_0} &= 4\pi R\int\int\frac{x_0-x}{(x_0-x)^2+(y_0-y)^2}dxdy,\\ \frac{\partial N}{\partial y_0} &= 4\pi R\int\int\frac{y_0-y}{(x_0-x)^2+(y_0-y)^2}dxdy,\end{aligned} \tag{9}$$

and by virtue of (8)

$$\frac{\partial(\log N)}{\partial x_0} < \frac{1}{\log\left(\frac{8R}{a\epsilon^2}\right)}\frac{\int\int\frac{(x_0-x)dxdy}{(x_0-x)^2+(y_0-y)^2}}{\int\int f(x, y)dxdy},$$

$$\frac{\partial(\log N)}{\partial y_0} < \frac{1}{\log\left(\frac{8R}{a\epsilon^2}\right)}\frac{\int\int\frac{(y_0-y)dxdy}{(x_0-x)^2+(y_0-y)^2}}{\int\int f(x, y)dxdy}.$$

Thus by making $\log(8R/a)$ large the maximum variation in $\log N$ can be made negligible. This fact is not by any means self-evident because the magnetic field at an infinitesimal distance from a current is infinite.

Therefore, in coils with sufficiently large diameter the magnetic flux through any one turn is the same as that through any other.

These results will now be applied to the case of a short closely wound solenoid.

(*a*) *Short Closely Wound Solenoid.*—Let *A B C D* (Fig. 4) be a diametral section of a circular cylinder, the axis of the cylinder being *E F*. The coil is imagined to be wound on the surface of the cylinder, each turn being circular, the turns being wound close together and the number of turns being large. The distance *A B* is taken to be negligible in comparison with the distance *A C*.

In the diametral plane points will be named by means of a cartesian system of axes OX, OY. The origin O of the system is placed at the middle of the line AB. OX is along $A\,B$, and OY is perpendicular to $A\,B$. The number of turns being large the current is uniform within a few turns. Thus the x coördinate of a turn may be chosen as the parameter previously denoted by x. The width of the coil is denoted by $2a$.

It was shown that the flux through one turn of the coil is the same as that through any other. *Therefore in* (6) *the quantity* $M(x)$ *is a constant.* Since

$$\int_{-a}^{+a} M(x)dx = L,$$

$$\frac{M(x)}{L} = \frac{1}{2a}.$$

Hence

$$C_0 = \frac{1}{2a}\int_{-a}^{+a}\left\{\int_{-a}^{x}\frac{\alpha(x)}{L}dx\right\}dx. \tag{10}$$

It now remains to compute the quantity $\alpha(x)$. This depends on the objects surrounding the coil. A simple condition is that in which all these objects are removed to an indefinite distance. However, practically this is not always realized. For this reason in the design of a standard inductance with a definite distributed capacity it seems advisable to surround the coil with a metallic shield. The mathematics in the special instance considered will be simplest if the shield is in the form of a solid of revolution obtained by constructing an ellipse whose foci are at A and B and whose plane is that of $A\,B\,C\,D$, and revolving the ellipse about the axis $E\,F$. The potential of the shield will be arbitrarily taken as zero. The middle of the coil (*i.e.*, the point O) is considered as connected to the shield by a short wire so that the potential of O is also zero.

Then by virtue of (3) the potential at x on $A\ B$ is

$$V = -\frac{di}{dt}L\frac{x}{2a}, \tag{11}$$

the positive direction of i being from negative to positive values of x.

In order to find $\alpha(x)$ it is necessary to find $Q(x)$. This is attained by finding the distribution of potential between $A\ B$ and the shield.

The problem is solved without difficulty by the use of elliptical coordinates defined by the transformation

$$x + jy = a \cosh (u + jv), \tag{12}$$

where

$$j = \sqrt{-1}$$

or its equivalent:

$$\begin{aligned} x &= a \cosh u \cos v, \\ y &= a \sinh u \sin v. \end{aligned} \tag{13}$$

It follows from (12) that the real or the imaginary part of any monogenic function of $u + jv$ is a solution of Laplace's equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

Again if u is kept constant then in virtue of (13)

$$\frac{x^2}{a^2 \cosh^2 u} + \frac{y^2}{a^2 \sinh^2 u} = 1,$$

so that the point (x, y) lies on an ellipse whose foci are $(\pm a, o)$. Let then u_0 be the value of u which is assigned to the ellipse of the shield. Then

$$V = -\frac{L}{2}\frac{di_1}{dt}\frac{(\epsilon^{-u} - \epsilon^{u-2u_0}) \cos v}{1 - \epsilon^{-2u_0}}, \tag{14}$$

satisfied Laplace's equation because $(\epsilon^{-u} - \epsilon^{u-2u_0}) \cos v$ is the real part of $\epsilon^{-(u+jv)} - \epsilon^{-2u_0}\epsilon^{u+jv}$. Also if $u = u_0$, then $V = 0$, and if $u = 0$ $V = -L(di_1/dt)(x/2a)$ so that (11) is satisfied. Finally the above expression for V, as well as its first derivatives are finite and continuous in the space between $u = 0$ and $u = u_0$ except when the segment $A\ B$ is crossed. Consequently this is the potential which exists in the space under the conditions of the problem.

The segment $A\ B$ is given, of course, by $u = 0$. If u is slightly greater than 0 the segment expands into an ellipse. The half of the ellipse corresponding to positive values of y is given by the values of v between 0 and π. The negative values of y are similarly given by values of v between π and 2π. Thus the line AB is the limiting state of such an ellipse for which $u = \delta$, δ being infinitesimal.

Differentiating (12) one obtains:

$$\frac{d(x + jy)}{d(u + jv)} = a \sinh (u + jv) = a[\sinh u \cos v + j \cosh u \sin v];$$

$$\therefore \left| \frac{d(x + jv)}{d(u + jv)} \right| = a \sqrt{\sinh^2 u \cos^2 v + \cosh^2 u \sin^2 v}. \qquad (15)$$

The surface density of charge on the ellipse $u = \delta$ is

$$- \frac{10^{-11}K}{35.956\pi} \left(\frac{\partial V}{\partial n} \right)_{u=\delta},$$

where $\partial/\partial n$ is the directional derivative as to the normal drawn out from $u \mp \delta$. But by (15)

$$\frac{\partial V}{\partial n} = \frac{1}{a \sqrt{\sinh^2 u \cos^2 v + \cosh^2 u \sin v}} \frac{\partial V}{\partial u}.$$

Therefore the surface density

$$\sigma_1 = - \frac{10^{-11}K \left(\dfrac{\partial V}{\partial n} \right)_{u=\delta}}{35.956\pi a \sqrt{\sinh^2 \delta \cos^2 v + \cosh^2 \delta \sin^2 v}}. \qquad (16)$$

Substituting $2\pi - v$ for v in (16) it is seen that the surface density at the reflection of the point given by v in $A\,B$ is also σ_1. The sum of the two surface densities is then

$$\sigma = 2\sigma_1 = - \frac{10^{-11}K}{17.978\pi a} \frac{\left(\dfrac{\partial V}{\partial n} \right)_{u=\delta}}{\sqrt{\sinh^2 \delta \cos^2 v + \cosh^2 \delta \sin^2 v}}.$$

In the limit when $\delta = 0$ it is found from (14) that

$$\sigma = - \frac{10^{-11}KLl \coth u_0 \cos v}{35.956\pi a \, |\sin v|} \frac{di_1}{dt}.$$

Let l be the length of one turn of the wire of the coil. Then the charge in the infinitesimal interval $(x, x + dx)$ is

$$- \frac{10^{-11}KLl}{35.956\pi a} \coth u_0 \frac{\cos v}{|\sin v|} \frac{di_1}{dt} dx.$$

Thus by the definition of $\alpha(x)$ given in (4)

$$\alpha(x) = - \frac{10^{-11}KLl}{35.956\pi a} \cosh u_0 \frac{\cos v}{|\sin v|}.$$

Substituting in (6) and remembering that A corresponds now to $x = -a$,

$$C_0 = \frac{10^{-11}K}{8.989} \int_{-a}^{+a} \frac{1}{2a} \left\{ \int_{-a}^{x} \frac{-Ll}{(4\pi a L)} \coth u_0 \frac{\cos v}{|\sin v|} dx \right\} dx.$$

The integration is effected without difficulty by the substitution

$$x = a \cos v.$$

The result is

$$C_0 = \frac{10^{-11}Kl}{16 \times 8.989} \coth u_0 = 0.06952 \times 10^{-12}Kl \coth u_0. \qquad (17)$$

Since the major semi-axis of the ellipse $u = u_0$ is $a \cosh u_0$, and the minor is $a \sinh u_0$, the result could be also written as

$$C_0 = 0.06952 \times 10^{-12}Kl \frac{\alpha}{\beta} \text{ farads}$$

$$= 0.06952Kl \frac{\alpha}{\beta} \mu\mu f,$$

where α = major semi-axis,
β = minor semi-axis.

In particular if the major and minor semi-axes are both made infinite the ratio α/β becomes unity. Thus the *effective capacity of a short single layer solenoid used with its middle grounded and undisturbed by surrounding objects is*

$$C_0 = 0.06952Kl\mu\mu f = \frac{Kl}{16} \text{ cgs electrostatic units of capacity.} \qquad (19)$$

It is remarkable that according to the formula (18) the only modification introduced by the shield is given by the factor α/β which even for comparatively narrow shields becomes approximately 1.

(*b*) *Short, Closely Wound Solenoid Grounded at Terminal When in Shield.*—Consider now the case when the terminal B is connected to the shield. The method of solution is almost identical with that of the preceding case. The only difference is that the potential at any point, x, is

$$-\frac{L(x-a)}{2a}\frac{di_1}{dt}$$

rather than

$$-\frac{Lx}{2a}\frac{di_1}{dt}.$$

It is clear that if on the solution given by (14) a solution for Laplace's equation which is 0 at $u = u_0$ and $(L/2)(di/dt)$ at $u = 0$ be superposed, the result will satisfy all the conditions of the problem. The expression

$$\frac{L}{2}\frac{di}{dt}\left(\frac{u_0 - u}{u_0}\right)$$

actually vanishes when $u = u_0$ and becomes $(L/2)(di/dt)$ when $u = 0$.

Therefore,

$$V = \frac{L}{2}\left[\frac{u_0 - u}{u_0} - \frac{\epsilon^{-u} - \epsilon^{u-2u_0}}{1 - \epsilon^{-2u_0}}\cos v\right]\frac{di}{dt} \tag{20}$$

is the required solution for the potential.

From (20)

$$\left(\frac{\partial V}{\partial u}\right)_{u=0} = \frac{L}{2}\left[-\frac{1}{u_0} + \coth u_0 \cos v\right]\frac{di}{dt},$$

so that

$$\alpha(x) = -\frac{10^{-11}KLl}{35.956\pi a}\left[-\frac{1}{u_0} + \coth u_0 \cos v\right]\frac{1}{|\sin v|} \tag{20a}$$

and

$$C_0 = \frac{10^{-11}L}{17.978a}\int_{-a}^{+a}\left\{\int_{-a}^{+x}\frac{l}{4\pi a}\left[\frac{1}{u_0} - \coth u_0 \cos v\right]\frac{dx}{|\sin v|}\right\}dx$$

$$= 0.06952 \times 10^{-12}Kl\left[\coth u_0 + \frac{2}{u_0}\right]\text{farads}$$

or if it is preferrable to express the result in terms of α, and β, as before, then

$$C_0 = 0.06952 \times 10^{-12}Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right]$$

$$= 0.06952Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right]\mu\mu f. \tag{22}$$

In the derivation of this formula the reader probably noticed that the current at A was chosen as i_1 while the terminal B was grounded. The reason for this choice is that when the capacity of a condenser is measured one set of plates is connected to ground permanently and the other is connected to and disconnected from the measuring apparatus. Thus the charge measured (either directly or used indirectly in theoretical derivations of formulas) is the charge supplied to the *ungrounded plate.* This is, therefore, the charge Q_c used in the derivation of formula (6).

Care should be taken not to use the formula (22) in cases when $\alpha/\beta - 1$ becomes very small because then the dimensions of the ellipse becomes large and the equation

$$\frac{\partial^2 V}{\partial z^2} = 0$$

is no longer true.

Only if α is negligible in comparison with R is this legitimate.

For this reason it is essential to work out the distributed capacity of a short solenoid for the case when it is grounded at one terminal and is kept at a considerable distance from surrounding objects.

Short Single Layer Solenoid Grounded in Free Space.—Consider now the same coil removed from surrounding objects and grounded at some point by a fine wire. Let B be grounded. It is evident that

$$V_1 = \frac{-L}{2}\frac{di}{dt}\epsilon^{-u}\cos v \tag{23}$$

is an expression for the potential which gives correct differences of potential for the case considered. However, at B, $V_1 = 0$. In fact its value at B is

$$-\frac{L}{2}\frac{di}{dt}.$$

It is thus necessary to find a solution of Laplace's equation which becomes

$$+\frac{L}{2}\frac{di_1}{dt}$$

at the coil and which vanishes at infinity and then to add this solution to V_1. Or if it is hard to find this solution it is sufficient to know what distribution of charges on the coil will give rise to such a solution. The answer is given by the expression (7). This expression is interpreted most easily by writing

$$d' = \frac{d}{8R},$$

for then

$$V = -10^{-9}c^2\frac{2p_1}{K}\log d'.$$

This shows that if $8R$ were the unit of length V would be the same as is taken in the two-dimensional case when the wires are straight. Consequently, the distribution of charges is the same as in the two-dimensional case. Now in this case the charge density varies as $1/|\sin v|$, *i.e.*, as $1/\sqrt{a^2 - x^2}$, as is seen from (20*a*). Consequently such a distribution gives a uniform potential over $A\ B$. The exact value of this potential may be ascertained by letting the charge density

$$\sigma = \frac{\rho}{\sqrt{a^2 - x^2}}. \tag{24}$$

The potential of the coil due to this charge is obtained most easily at the center of the winding of the coil and as has been shown above is the same at the center as at any other point of the coil. It is

$$V_2 = \frac{8.989 \times 10^{11}}{K}\int_{x=-a}^{x=+a}\frac{2\rho\log\left(\frac{8R}{|x|}\right)}{\sqrt{a^2 - x^2}}dx = \frac{8.989 \times 10^{11}}{K}2\pi\rho\log\left(\frac{16R}{a}\right).$$

But this must be $+ (L/2)(di_1/dt)$. Therefore,

$$\rho = + \frac{10^{-11}KL\dfrac{di_1}{dt}}{35.956\pi \log\left(\dfrac{16R}{a}\right)}$$

and the charge density corresponding to this value of ρ is

$$\therefore \sigma_2 = + \frac{10^{-11}K\dfrac{di_1}{dt}L}{35.956\pi \log\left(\dfrac{16R}{a}\right)\sqrt{a^2 - x^2}}$$

$$= + \frac{10^{-11}KL\dfrac{di_1}{dt}}{35.956\pi a \left\{\log\left(\dfrac{16R}{a}\right)\right\}|\sin v|}. \qquad (24a)$$

The charge density called for by V_1 (see (23)) is

$$\sigma_1 = - \frac{10^{-11}KL}{35.956\pi a}\frac{di_1}{dt}\frac{\cos v}{|\sin v|}.$$

Hence the total charge density

$$\sigma = \sigma_1 + \sigma_2 = \frac{10^{-11}KL}{35.956\pi a}\frac{di_1}{dt}\left[\frac{1}{|\sin v| \log\left(\dfrac{16R}{a}\right)} - \frac{\cos v}{|\sin v|}\right]. \qquad (25)$$

If as before l is the length of one turn of wire (*i.e.*, $l = 2\pi R$) then the charge in the infinitesimal interval $(x, x + dx)$ is

$$\sigma lxd = \frac{10^{-11}KLl}{35.956\pi a}\frac{di_1}{dt}\left[\frac{1}{|\sin v| \log\left(\dfrac{16R}{a}\right)} - \frac{\cos v}{|\sin v|}\right]$$

and

$$\alpha(x) = \frac{10^{-11}LlK}{35.956\pi a}\left[\frac{1}{|\sin v| \log\left(\dfrac{16R}{a}\right)} - \frac{\cos v}{|\sin v|}\right]. \qquad (26)$$

Therefore (see (6))

$$C_0 = \frac{10^{-11}K}{35.956}\int_{-a}^{+a}\frac{1}{2a}\left\{\int_{-a}^{x}\frac{Ll}{\pi a L}\left[\frac{1}{|\sin v| \log\left(\dfrac{16R}{a}\right)} - \frac{\cos v}{|\sin v|}\right]dx\right\}dx.$$

Performing the integrations:

$$\begin{aligned} C_0 &= \frac{10^{-11}Kl}{8.989}\left[\frac{1}{16}+\frac{1}{8\log\left(\frac{16R}{a}\right)}\right] \text{ farads} \\ &= 1.1124\left[\frac{1}{16}+\frac{1}{8\log\left(\frac{16R}{a}\right)}\right] Kl \text{ micromicrofarads.} \end{aligned} \tag{27}$$

The formulas (22) and (27) are both seen to be made up of two parts. The first part is the effective capacity of the coil when ungrounded. The second may, therefore, be called the effective capacity due to grounding.

In the following paragraph, it will be shown that *for the case of a coil used in free space the effective capacity due to grounding is ¼ of the electrostatic capacity of the coil to ground.*

Equation (24*a*) gives the surface density of charge which maintains the surface of the coil at a potential $\frac{L}{2}\frac{di_1}{dt}$. Therefore, the electrostatic capacity of the coil is

$$\int_{v=0}^{v=\pi}\frac{\sigma_2 la\sin v dv}{\left(\frac{L}{2}\right)\frac{di_1}{dt}} = \frac{10^{-11}K}{17.978\log\left(\frac{16R}{a}\right)}.$$

But in (27) the distributed capacity due to grounding is

$$\frac{1}{4}\,\frac{10^{-11}c^{-2}Kl}{17.978\log\left(\frac{16R}{a}\right)}.$$

Therefore, the distributed capacity due to grounding is one fourth of the electrostatic capacity to ground.

This must be true as long as the distribution of charge introduced by grounding is the same as in the case of a coil grounded in free space, *i.e.*, removed from surrounding objects and grounded by a fine wire. Since the only difference between (14) and (20) is given by the term

$$\frac{L}{2}\,\frac{u_0-u}{u_0}\,\frac{di_1}{dt}.$$

The distribution of charge introduced by the grounding of a coil in an elliptical shield is of the form $A/|\sin v|$, where A is a constant. Comparing this with (24) it is seen that *the distributed capacity introduced by grounding of the coil in an elliptical shield is one fourth of the electrostatic capacity of the coil to the shield.*

Further, even if the coil is not closely wound, the distributed capacity due to grounding must still be approximately equal to one fourth of the electrostatic capacity to ground as long as the winding is sufficiently close to make the average distribution of charge the same as that of a coil with close winding.

Neglecting minor corrections it is easy to measure the electrostatic capacity of a coil to ground in given surroundings. For this purpose a circuit *L*, *C*, Fig. 5, is tuned to the frequency of a generating set *A* and the reading of the shielded condenser *C* is noted. The shield of *C* is grounded and a wire *BD* extends permanently to the coil *L′* whose electrostatic capacity to ground is to be measured. By a slight motion of the end *D*, contact is made with *L′* and *C* is retuned. The difference between the capacity of *C* at the two settings gives the capacity of *L′* to ground. *L′* is of course not connected to ground directly during the measurement.

This has been verified experimentally. Thus for a coil for which the distributed capacity when ungrounded is 30 micromicrofarads the capacity when grounded came out 42 micromicrofarads while the measured electrostatic capacity to ground is 47 micromicrofarads. And it is seen that

$$30\ \mu\mu f + \frac{47\ \mu\mu f}{4} = 42\ \mu\mu f \text{ (approximately).}$$

The formulas derived are generally in satisfactory agreement with experiment if the capacity of leads is taken into account and the effect of condenser shields is made negligible. It is also of interest to note that formula (27) gives values for C_0 which are of the order of magnitude of the radius. This agrees with data accumulated at the Bureau of Standards.

Explanation of Constancy of C_0.

So far we have not explained why the number C_0 in (1) comes out a constant. In this section it will be shown that in the limit when the width of a coil becomes negligible compared to the diameter C_0 becomes a constant.

It was seen that the magnetic flux through all turns of the coil becomes the same under these conditions. Let then (L/N^2) be a constant such that the magnetic flux through a turn due to a current i in another turn is

$$\frac{L}{N^2}i,$$

N being the total number of turns. Then the flux through any one turn when i flows in all turns is

$$N\frac{Li}{N^2} = \frac{Li}{N}$$

and, therefore, the total flux through the coil is

$$N\frac{Li}{N} = Li$$

when the current is uniform. Thus L is the inductance of the coil.

Since the flux is the same through all turns, even if the current is not uniform the E.M.F. induced in a turn is

$$\frac{-L\left(\frac{di}{dt}\right)}{N},$$

where $\bar{i}$ is the average value of i, *i.e.*,

$$\bar{i} = \frac{i_1 + i_2 + \cdots + i_N}{N}, \tag{28}$$

$i_1, i_2 \cdots i_n$ being the currents in the 1st, 2d, 3d $\cdots$ Nth turns respectively. Thus the charge on the 1st, 2d, $\cdots$ Nth turns is

$$\frac{-f_1L}{N}\frac{d\bar{i}}{dt}, \qquad \frac{-f_2L}{N}\frac{d\bar{i}}{dt} \cdots \frac{-f_NL}{N}\frac{di}{dt},$$

where $f_1, f_2 \cdots f_N$ are independent of the frequency.

Let i_0 be the current entering the first turn. Then

$$i_2 - i_0 = \frac{f_1L}{N}\frac{d^2\bar{i}}{dt^2},$$

$$i_3 - i_1 = \frac{f_2L}{N}\frac{d^2\bar{i}}{dt^2},$$

etc.

Consequently the excess of the current through any turn over i_0 is proportional to $d^2\bar{i}/dt^2$ and the current can be written as

$$i_n = -\varphi_n\frac{d^2i}{dt^2} + i_0 = i_0 + \omega^2\varphi_n\bar{i}.$$

Here φ_n is independent of the frequency. According to (28)

$$\bar{i} = i_0 + \omega^2\bar{i}\bar{\varphi},$$

where

$$\bar{\varphi} = \frac{\varphi_1 + \varphi_2 + \cdots + \varphi_N}{N};$$

$$\therefore \bar{i} = \frac{i_0}{1 - \omega^2\bar{\varphi}}. \tag{28a}$$

The E.M.F. induced in the coil is then

$$\frac{-\frac{di_0}{dt}L}{(1 - \omega^2\bar{\varphi})}.$$

If Q is the charge on the condenser

$$\frac{dQ}{dt} = -\,i_0.$$

Hence

$$\frac{1}{LC} = \frac{\omega^2}{1 - \omega^2\overline{\varphi}}$$

or

$$\omega^2 = \frac{1}{L\left(C + \frac{\overline{\varphi}}{L}\right)}.$$

One can now write

$$C_0 = \frac{\overline{\varphi}}{L},$$

C_0 being independent of ω. Then

$$\omega^2 = \frac{1}{L(C + C_0)},$$

where C_0 is a constant, which proves that the relation (1) is exact as long as the flux is the same through all the turns.

It may be worth mentioning here that if a coil is represented as a line with distributed constants C_0 is not independent of ω.

It must be mentioned also that in any of the applications mentioned the effective capacity is unchanged if the cross section is turned through an angle with reference to the axis of the coil. Thus the formula for a short single layer solenoid applies also to a pancake coil of small depth.

Current Distribution.

It is possible to study the current distribution experimentally in coils by inserting non-inductive resistors at different points when the coil is tuned to resonance. It can be shown that if R_1, R_2 are the values obtained for the resistance by the resistance variation method[1] at two points (1) and (2) of the coil then

$$\frac{i_1}{i_2} = \sqrt{\frac{R_2}{R_1}},$$

where i_1, i_2 are the currents at (1) and (2).

By this method the curves of Figs. 6, 7, 8 were obtained. The curves of Figs. 7 and 8 show the dissymmetry introduced into the circuit by a galvanometer or by grounding. The coil experimented on in the case of the curves on Fig. 6 was a 4-foot coil wound with 14 turns of number 19 double cotton covered wire with 1 cm. spacing on a wooden frame. The frame was so designed that as little as possible of the wood was in the field of the coil. The dissymmetry was avoided by using a very small galvanometer (Weston thermogalvanometer) and making the readings

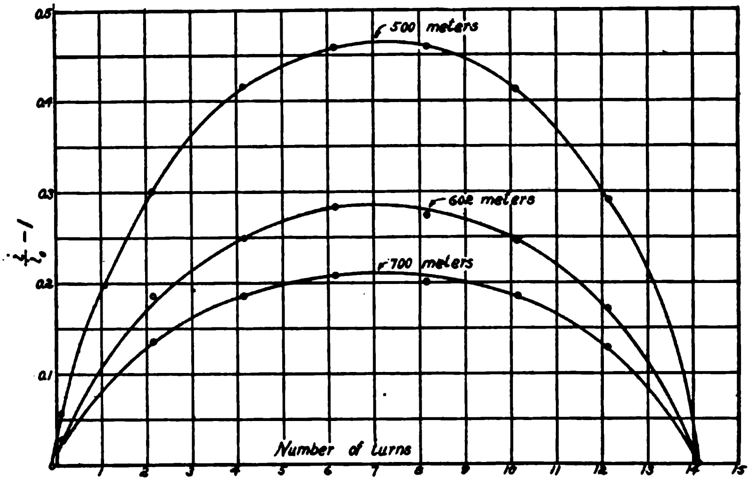

Fig. 6.
Current distribution in a coil determined experimentally at various wave-lengths.

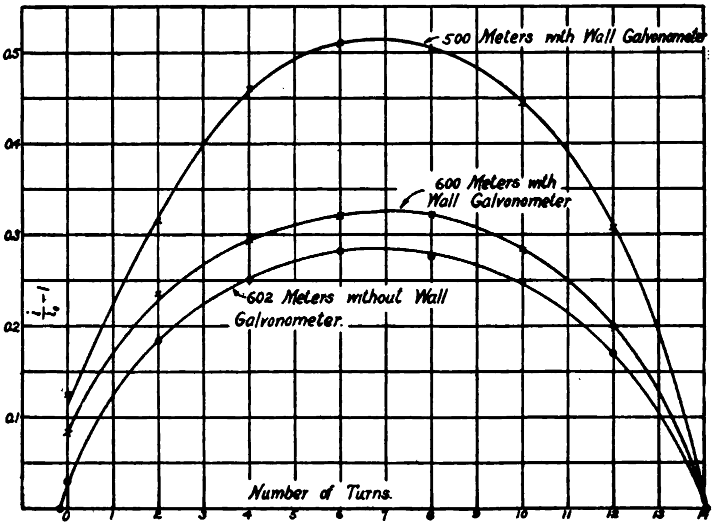

Fig. 7.
Current distribution in a coil determined experimentally at various wave-lengths with slight dissymmetry caused by wall galvanometer.

from a large distance with a telescope. In the case of Fig. 8, i_0 was used for the current at the shielded condenser terminal.

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[1] See Bureau of Standards Circular No. 74, p. 180.

It is interesting to see how this measurement (obtained during the course of some work at the Johns Hopkins University) verifies the theory. In the first place, since the width of the coil is small compared to the diameter $(1/i_0) - 1$ must be nearly the same for all frequencies. Taking the experimental values for $(1/i_0) - 1$ at 700, 602, and 500 meters the following table is obtained

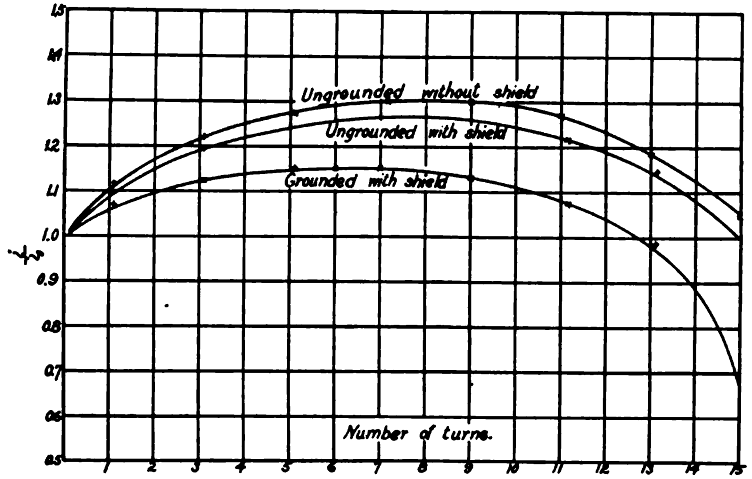

Fig. 8.

Effect of grounding and of condenser shield on current distribution.

$\left(\frac{i}{i_0}-1\right)_{700}$	$\left(\frac{i}{i_0}-1\right)_{602}$	$\left(\frac{i}{i_0}-1\right)_{500}$	$\dfrac{\left(\frac{i}{i_0}-1\right)_{602}}{\left(\frac{i}{i_0}-1\right)_{700}}$	$\dfrac{\left(\frac{i}{i_0}-1\right)_{500}}{\left(\frac{i}{i_0}-1\right)_{700}}$
0.135	0.186	0.297	1.38	2.20
0.186	0.247	0.414	1.33	2.22
0.208	0.284	0.459	1.36	2.20
0.207	0.273	0.461	1.36	2.29
0.184	0.247	0.413	1.34	2.24
0.127	0.174	0.291	1.37	2.29
	Mean:		1.36	2.24

Measurements by the resistance-variation method are difficult to make with an accuracy higher than 1 per cent. This would give an accuracy of 0.5 per cent. in i/i_0 and worse than 10 per cent in $(i/i_0) - 1$. The largest deviation in the table is 2.3 per cent. in the case of the comparison between 602 and 700 meters; and 2.5 per cent. in the comparison between 500 and 700 meters. The agreement must thus be regarded as satisfactory.

Further a knowledge of the natural period of the coil enables one to calculate the ratios tabulated. In fact from (28*a*)

$$i - i_0 = \frac{i_0\bar{\varphi}\omega^2}{1 - \bar{\varphi}\omega^2} = \frac{i_0}{\dfrac{1}{\bar{\varphi}\omega^2} - 1}. \tag{29}$$

But

$$C_0 = \frac{\bar{\varphi}}{L}, \qquad i.e., \qquad \bar{\varphi} = LC_0.$$

Hence

$$\bar{i} - i_0 = \frac{i_0}{\dfrac{\omega_0^2}{\omega^2} - 1}, \tag{30}$$

where

$$\omega_0^2 = \frac{1}{LC_0}$$

and is the value of ω which would correspond to the natural frequency of a pure inductance L connected in series with a capacity C_0. In terms of wave-length λ

$$\bar{i} - i_0 = \frac{i_0}{\dfrac{\lambda^2}{\lambda_0^2} - 1}, \tag{31}$$

where

$$\lambda_0 = 2\pi\sqrt{LC_0}.$$

For the coil in question the value of l in (18) is 470 cm. Hence using the formula (19)

$$C_0 = 32.6\ \mu\mu f \text{ when } K = 1.$$

The measured value of C_0 with leads was 38 $\mu\mu f$ and the measured value of L_0 was 547 μh. The capacity of the leads was calculated as 3.4 $\mu\mu f$. Thus the measured value without leads is 34.6 $\mu\mu f$. The lead correction being uncertain 33.6 $\mu\mu f$ can be taken as the value of C_0. The corresponding value of λ_0 is 255 meters. Then for

	700 meters	602 meters	500 meters
$\dfrac{1}{\dfrac{\lambda^2}{\lambda_0^2} - 1} =$	0.153	0.219	0.350

Here

$$\frac{0.350}{0.153} = 2.29,$$

$$\frac{0.219}{0.153} = 1.43.$$

According to the theory (Eq. 31) these must be the same as the measured quantities; viz.,

2.24 and 1.36.

It is seen that a difference exists between the measured and theoretical values. However, the difference is not large. It becomes less as the absolute value of the ratio increases and in the overall comparison between 500 and 700 meters it is 2.5 per cent. Although this is a serious discrepancy if we are concerned with current distribution, its effect on the constancy of the effective capacity of the coil is very small because 2.5 per cent. in 38 $\mu\mu f$ is barely 1 $\mu\mu f$ and is just within the limits of ordinary measurement.

SUMMARY.

1. The effective capacity of an inductance coil is defined as the constant C_0 in the equation

$$L(C + C_0) = \frac{1}{\omega^2},$$

which is known to be true experimentally.

2. A general formula is derived for the effective capacity. This is formula (6) of the paper.

$$C_0 = \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx$$

Here x is an arbitrary parameter along the wire

$$\alpha(x) = \frac{Q(x)}{\left(\dfrac{di_1}{dt}\right)},$$

where $Q(x)dx$ is the charge between x and $x + dx$.
i_1 is the current at the terminal,
L is the self-inductance,
$M(x)dx$ is the mutual inductance of the element dx to the rest of the coil.

3. The formula is illustrated in special cases. These are: the short single-layer solenoid, used when grounded in elliptical shield, insulated in elliptical shield, insulated and grounded in free space.

The formulas for the cases discussed are:

(I.) Short single-layer solenoid one turn of which has a perimeter l or pancake of small depth used in elliptical shield of major semi-axis α and minor semi-axis β and insulated from shield

$$C_0 = 0.06952Kl\frac{\alpha}{\beta}\mu\mu f = \frac{Kl}{16}\frac{\alpha}{\beta}\text{C.G.S. electrostatic units.}$$

(II.) Same coil connected to shield at terminal

$$C_0 = 0.06952Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right]\mu\mu f$$

$$= \frac{Kl}{16}\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right] \text{ C.G.S. electrostatic units.}$$

(III.) Same coil used outside shield and insulated

$$C_0 = 0.06952Kl\mu\mu f = \frac{Kl}{16}\text{ C.G.S. electrostatic units.}$$

(IV.) Same coil used outside shield and grounded

$$C_0 = 1.1124\left[\frac{1}{16} + \frac{1}{8\log\left(\dfrac{16R}{a}\right)}\right]Kl\mu\mu f$$

$$= \left[\frac{1}{16} + \frac{1}{8\log\dfrac{16R}{a}}\right] \text{ C.G.S. electrostatic units.}$$

R being the radius. All the formulas with the exception of the last apply not only to coils having circular turns but also to other shapes such as square, rectangular, etc.

4. An explanation is given of the remarkable constancy of C_0 as found by experiment in the case of short coils.

5. An experimental verification of the theory is given by direct measurement of capacity and by investigating the current distribution in a coil at various frequencies.

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APPENDIX.

The approximation used here is

$$F(k) = \log\frac{4}{\sqrt{1-k^2}}.$$

In order to see the truth of this expression use can be made of Maxwell's formula for the mutual inductance of two coaxial circles whose radii are both approximately a and whose shortest distance apart is b. This is in electromagnetic units in air.

$$M = 4\pi a\left[\log\left(\frac{8a}{b}\right) - 2\right].$$

The above expression is only approximately true. The expression will now be equated to Maxwell's exact formula in elliptic integrals in its limiting form and hence the value of $F(k)$ for small $k - 1$ will be derived. This exact formula for two concentric circles of radii a_1, a_2 is in the same units.

$$M = 4\pi\left[\frac{a_1^2 + a_2^2}{a_1 + a_2} F\left(\frac{2\sqrt{a_1a_2}}{a_1 + a_2}\right) - (a_1 + a_2)E\left(\frac{2\sqrt{a_1a_2}}{a_1 + a_2}\right)\right].$$

If a_1 is nearly a_2 it becomes approximately

$$M = 4\pi a\left[F\left(\frac{2\sqrt{a_1a_2}}{a_1 + a_2}\right) - 2E\left(\frac{2\sqrt{a_1a_2}}{a_1 + a_2}\right)\right].$$

Also, since

$$\frac{2\sqrt{a_1a_2}}{a_1 + a_2}$$

is nearly 1,

$$E\left(\frac{2\sqrt{a_1a_2}}{a_1 + a_2}\right)$$

is nearly 1. Identifying the two expressions for M the approximation for $F(K)$ follows immediately.

A different proof is found in Whittaker and Watson, Modern Analysis, pp. 514–515, Sections 22–737.

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MAGNETIC SUSCEPTIBILITY OF NICKEL AND COBALT CHLORIDE SOLUTIONS.

BY LAURA BRANT.

SYNOPSIS.

The Magnetic Susceptibilities and Magneton Numbers of Nickel and Cobalt in Aqueous Solutions of their Chloride Salts.—The susceptibilities of solutions have been found by a modified form of the "Kelvin-Wills" method. Data is given for nickel chloride solutions ranging in molar concentration from 0.001000 to 3.765 and for cobalt chloride solutions from 0.001018 to 2.0353, for field ranges of 3,000 to 14,000 gausses for the lower concentrations and 3,000 to 10,000 for the higher. All measurements were made at a temperature of 20° C.

The susceptibilities of the salts have been computed from the susceptibilities of the solutions by application of the Wiedemann law, and the susceptibilities of the metals obtained by extending the Wiedemann relation to the salts.

The susceptibilities of the solutions measured were found to be independent of the field strength throughout the ranges of field used, and the susceptibilities of the salts, both nickel and cobalt, were found to be independent of the concentrations of the solutions. The value found for the molecular susceptibility of nickel is 0.004423 and of cobalt 0.010362. These values give upon application of Weiss's equation 16.0 and 24.5 magnetons respectively for the nickel and cobalt atoms.

Susceptibility of Air.—An auxiliary result obtained in the experiments is that the volume susceptibility of air at 20° C. and average conditions as regards pressure and moisture extending over a number of months is 0.0288×10^{-6}.

FROM a consideration of the values of Pascal[1] and of Liebknecht and Wills[2] for the magnetic susceptibility of paramagnetic salts in solution Professor Weiss conjectured[3] that the molecular magnetic moment was an integral multiple of an elementary moment which he called the "magneton." The existence of this natural magnetic unit is fundamental in his magneton theory. Whatever may have been, or may be, the fate of this theory, a result of it was a quickened interest in the magnetic study of solutions, a study which had held the attention of experimenters since the classical magneto-chemical investigations of Wiedemann. Of the large literature which has followed, five publications are of special interest in connection with the subject of this paper. Two of these deal with the susceptibility and magneton number of nickel in its salts in aqueous solutions and three with the susceptibility and magneton number of cobalt in aqueous solutions of its salts.

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[1] Ann. Chim. Phys., 8ᵉ Ser., T. xvi, p. 531; 1909: T. xix, p. 5; 1910.

[2] Ann. d. Phys., Bd. I., p. 178; 1900.

[3] Jour. de Phys., 5ᵉ Ser. T. 1, p. 965, 1911.

The first of these papers is by the Spanish physicists Cabrera, Moles, and Guzman,[1] and gives the results of precision work upon the normal salts, nickel sulphate, nickel chloride, and nickel nitrate. Measurements were made upon solutions varying in concentration over a considerable range. In the case of nickel chloride this was from 0.02208 to 0.34630 grams of salt per gram of solution. The second paper gives the values obtained from the careful measurements of Weiss and Mlle. Bruins[2] upon solutions of the same normal salts. The concentrations used for the case of nickel chloride ranged from 22.690 to 0.623 per cent. of nickel chloride. The results for the susceptibility of the solutions in both of these researches were obtained by means of a modified form of Quincke's ascension method, after the manner of Piccard in his determination of the susceptibility of water; and the value for the metal content found by an application of Wiedemann's law. The conclusions from the two experiments were in good accord. Both led to a susceptibility coefficient for nickel which was independent of the concentration, and to an integral number, 16, of magnetons for the nickel atom.

Of the papers on cobalt, the first gives the results of measurements by Cabrera, and his co-workers, Jimeno, and Marquina[3] on solutions of cobalt chloride, cobalt sulphate, and cobalt nitrate, obtained, as in the case of the nickel solutions, by Piccard's modification of Quincke's classical method, and application of the Wiedemann law. The susceptibility of cobalt was found to vary with the concentration. Fractional numbers of magnetons between 24 and 25 resulted, but in such a manner that whole numbers were approached as limiting values. Thus there were found 24 magnetons for the cobalt atom from measurements on the more dilute solutions, and from measurements on the stronger concentrations, 25. The second paper dealing with cobalt is by Trümlper,[4] a student at Zürich, who had completed two independent series of measurements on solutions of the chloride, nitrate, and sulphate salts at the time the earlier publication appeared. The first of these series was made by means of a magnetic balance of the Curie-Cheneveau type and resulted in a magnetic coefficient for cobalt which varied with the concentration. Comparison later with the work of Cabrera, Jimeno, and Marquina showed that there was not a close quantitative agreement, but that Trümpler's values were, nevertheless, consistent with the interpretation given by the Spanish physicists and pointed to the same integral numbers, 24 and 25, of magnetons for the cobalt atom in limiting conditions in solution. For the second series of measurements Trümpler also adopted the Quincke-Piccard ascension method. These measurements gave a value of the susceptibility of cobalt which did not depend upon the concentration of the solution, and a fractional number, approximately 24.5, of magnetons for the cobalt atom. The third paper contains results for cobalt obtained by Quartaroli[1] from dilute solutions of the same three cobalt salts by means of his differential method, a nickel chloride solution having been used as standard. Under the condition that the magneton number of nickel is a constant, 16, the results for the chloride solutions gave for cobalt a molecular coefficient which increased when the concentration was increased, and a magneton number ranging from about 24.5 to 25.3 for the concentrations used.

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[1] Sci. Arch., p. 325, 1914.

[2] Amst. Akad. Proc., p. 246, Vol. 18.

[3, 4] See "Le magneton et les sels paramagnetiques dissous," by P. Weiss, Revue Gen. de L'El., p. 925, Dec. 15, 1917. These papers are known to the writer only through this review.

Experiments for determining the susceptibilities and magneton numbers of nickel and cobalt in aqueous solutions of their chloride salts by a method which differs from those mentioned above had been practically completed in this laboratory before Weiss's review (l.c.) came to the writer's attention. The experiments were entirely finished before Quartaroli's paper was known. To present the results obtained by the method here employed is the object of the present paper.[2]

METHOD.

Lord Kelvin[3] suggested, as a means of determining the susceptibility of diamagnetic and feebly magnetic solid substances, placing a suitably shaped portion of the substance, such, *e.g.*, as a cylinder or a rectangular slab, having known dimensions in a non-uniform magnetic field and measuring the mechanical force exerted by the field upon it. He observed that the field strength, which must be known, could be obtained conveniently through finding the force acting upon a chosen length of wire carrying a measured current across the magnetic lines of force. The susceptibility could then be computed from known relations. Professor Wills[4] determined the susceptibility of solids by a method in which these suggestions are embodied and pointed out that the process was applicable to measurements on liquids and gases. He later used it, adapted to liquids, in determining the susceptibility of water, and others have since made use of it in the study of other liquids. A particularly favorable feature of the method as developed by Professor Wills is that the determination of the field is made at the same time that the force upon the body is found.

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[1] Soc. Chim. Ital. Gazz., 48^1, p. 79, 1918.

[2] The measurements reported in this paper were completed during the summer of 1919.

[3] Brit. Assoc. Report, p. 745, 1890.

[4] PHYS. REV., *6*, p. 223, 1898.

The method of obtaining the susceptibility of the solutions in the present experiments is also based upon the suggestions of Lord Kelvin. The method follows a different procedure for obtaining mechanical force than was used in the earlier adaptation by Professor Wills, but borrows from that adaptation the details for the field determination. In the earlier application the mechanical force was found by means of a balance from one arm of which a rectangular slab of the body, and the current element, were suspended in the field: in these experiments the force has been determined from the currents required to carry the body to positions of known displacements when suspended as a pendulum in the field.

Wiedemann's law has been applied in computing the susceptibility of the metal from that of the solution, and Weiss's equation in computing the number of magnetons.

Apparatus.

The line drawing (Fig. 1) gives a general idea of the distribution and relative size of the parts of the apparatus making up the suspended system. A narrow glass platform, *G*, upon which a rectangular glass vessel, *V*, for containing the liquid, rested by means of a point, dot, slot device (not indicated in the figure), was suspended horizontally between the poles, *P*, of an electromagnet by four fine copper wires *w*, *w*, *w′*, *w′*, of equal length which sustained equally the weight of the platform and its load. The wires *w* and *w′* passed through the capillary bore of small glass tubes (2.2 cm. in length) attached to the platform, and through two similar but longer tubes (9.7 cm. long) attached to a brass bar supported above. The tubes were parallel and symmetrically placed at the ends of the platform and bar, respectively, at right angles to their length. The ends of the tubes, forming the points of suspension, were well rounded to reduce friction, and slipping of the wires was prevented by a touch of china cement in the capillary bore. The planes *w*, *w* and *w′*, *w′* were parallel and 20.4 cm.

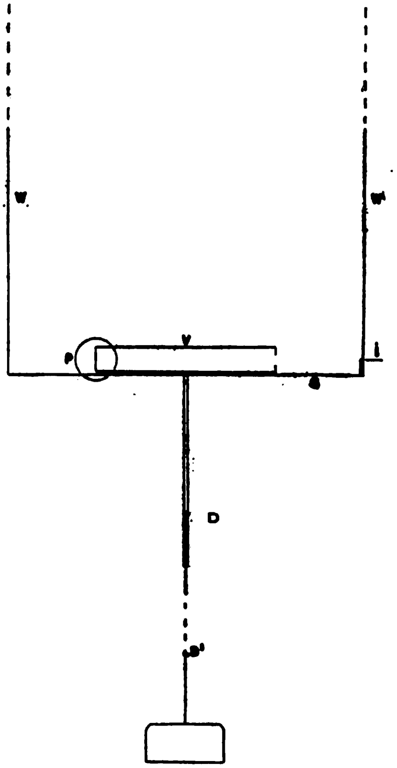

Fig. 1.

distant, and were cut by the horizontal planes containing the upper and lower suspension points to a length of 81.50 cm. The position with respect to the poles was such as to bring the length of the vessel, which was parallel to the long axis of the platform, perpendicular to the lines of force and one end (in the absence of a magnetic force), approximately in the position of the strongest part of the field. The other end was well out of the field.

The cell, *V*, measured about 10.4 × 1.4 × 1.0 cm. externally. Around the top, the end to be placed in the field, and the bottom was pasted a strip of tinfoil which served to conduct a current across the lines of force, the length of the foil across the field end of the cell forming an element of circuit used in the field determination. The ends of the tinfoil were in contact, at the outer end of the cell, with short lengths of thin copper sheet to which were soldered a few cm. of brass wire. Small slotted brass tips slipped over the ends of the wires connected the ends of the tinfoil to fine silk-covered strands of a long spiral of two-ply copper twist, which hung vertically from the brass support above, and formed part of a current circuit described further in a following paragraph. The width of the foil was equal to the internal width of the vessel. The field end of the vessel was cut from very thin glass, so that the tinfoil was separated from the inner surface by a small fraction of a mm. only. The side walls were about 1 mm. in thickness, and the outer end, which was bored to permit of filling and emptying the vessel, was about 3 mm. thick. When filled with liquid the cell was sealed by means of a thin coating of paraffin.

D and *D′* (of brass and copper respectively) are parts of an electromagnetic damper which was in position but not used when the readings recorded in this paper were taken.

A glass upright, at the outer end of the glass platform, supported a vertically ruled glass index, *i*, the plane of the rulings being parallel to the length of the platform. In the early part of the work the index was a square of glass lined with very light diamond traces about 0.5 mm. apart. Later this was replaced by the scale of a Bausch and Lomb stage micrometer ruled to 0.01001 mm.

The brass bar which supported the whole pendular system was pinned in a manner permitting rotation about a central axis perpendicular to its length to a brass plate the elevation and tilt of which was adjustable by means of screws; the plate was mounted upon two crossed, screw-controlled, guided brass slides, providing adjustment parallel and perpendicular to the field. A thermometer graduated at 0.2° C. hung from the guide above so that its bulb was just above the cell. The whole was borne by a brass arm extending from a heavy iron casting which was firmly bolted to the base of the magnet. The entire system·was enclosed in a well-lighted wooden box as a protection against draughts.

A second iron casting bolted to the base of the magnet and finished with brass fittings furnished a rigid support and the necessary means of adjustment for keeping a microscope at the level of the index, *i*, and in focus upon the scale. A stout brass rod bracing the two castings against each other effectively eliminated any disturbing motion of either due to the field. The microscope was provided with a filar micrometer having a screw head divided into one hundred parts. For the optical combination as adjusted throughout the experiment, one division of the head of the screw was equivalent to 0.0000588 cm.; one tenth of a division could easily be estimated. A Leitz Wetzlar object micrometer reading to 0.01 mm. was used for the calibration. The observations of the index were made by means of the microscope through a window of very thin clear mica in the box enclosing the suspension.

The electromagnet used was of the Du Bois half-ring type. The core was of about 10 cm. diameter and was fitted with right conical pole pieces having a base angle of approximately 30° and faces initially 1.5 cm. in diameter. These faces were later cut away giving new surfaces about 2.5 cm. across. The distance between the faces was about 1.3 cm. The coils had a capacity of 25 amperes. Current was supplied by storage batteries at about 110 volts. A reversing key provided a means of demagnetizing the magnet. The magnet was mounted on foot-screws which rested upon steel rails placed across the top of a pier built up from the cement floor of a sub-basement chamber.

For some time before measurements were to be made, the temperature of this chamber was kept at approximately 20° C. by means of an adjustable electric heater which supplemented the steam heat provided in the building; due to the subterranean location of the room the temperature seldom went above 20° C. even in the hottest days of summer.

The upper end of the copper spiral previously mentioned was led out of the wooden box at the top and was then connected through a commutator, in series with storage batteries and resistances adapted to sending a desired current through the foil. A Weston milliammeter provided with shunts and a Siemens-Halske milliammeter placed in the circuit were used in determining the strength of the current, the instrument read for any particular value being the one giving the most favorable range of scale. Both ammeters were carefully calibrated by means of a potentiometer and the two were checked against each other at frequent intervals.

SOLUTIONS.

The nickel and cobalt salts used were pure products obtained from Kahlbaum, and were found, upon analysis in the Chemical Laboratory of this University, to be free from cobalt and nickel respectively and from other ferromagnetic material. Conductivity water was used for the solutions. The nickel chloride was analyzed by Professor Fales of the department of chemistry at Columbia. The solutions of this salt were made up and analyzed by Professor Fales about two years before being used and had been carefully kept in non-soluble glass bottles. The densities were determined shortly before the susceptibility measurements were made. The cobalt salts were analyzed and new solutions made up for this work under the direction of Professor Beans, also of the department of chemistry at Columbia. In preparing the solutions a stock solution of the highest concentration was first made up. The lower concentrations were prepared from this by dilution. The concentrations of the several solutions were later determined by analysis. These determinations were for a temperature of 20 ± .005° C. The data giving the concentrations were supplied by the chemists. The water used in the magnetic measurements was from the same conductivity water (this having been preserved together with the solutions), from which the nickel solutions had been prepared.

The densities were obtained by means of a pycnometer of about 10 c.c. capacity. They were found for a temperature well within 20 ± .02° C.

THEORY OF METHOD.

The theory of the method may be expressed in terms of well-known relations. Any magnetic substance placed in a non-uniform magnetic field is acted upon by a mechanical force due to the field. Considering Maxwellian stresses in the media, it can be shown that the resultant force, F_l, acting upon a liquid slab contained in the vessel, V, is, for the position of the vessel given in these experiments, a force in the direction of the length of the vessel and of magnitude

$$F_l = \tfrac{1}{2}KA(H_1^2 - H_0^2), \tag{1}$$

where A is the area of cross-section of the slab at right angles to the direction of F_l, K the product of the permeability of the medium in which the system is suspended and the susceptibility of the liquid against the medium, and H_1 and H_0, with close approximation, the impressed magnetic forces at the ends of the slab in the stronger and weaker parts of the field respectively.

On account of the symmetry of the suspended system with reference to the central vertical plane parallel to the pole faces, the resultant force, F, due to the field effective in deflecting the system has also the direction of the length of the vessel. If F_t and F_e denote this force when the vessel is filled with liquid and when empty, respectively, the difference between these forces for a given field is a measure of the force due to the field upon the liquid:

$$F_l = F_t - F_e \tag{2}$$

for a given field.

Let f be defined as any force acting upon the system in the direction of the length of the vessel. (F_t and F_e are then particular values of f.) If d is the displacement of the system due to the force f in the direction of the force, elementary mechanics gives the following relation applicable for the case of small displacements and negligible stiffness of the suspension:

$$f = \frac{M'gd}{l}, \tag{3}$$

in which M' is the mass suspended, l the length of the suspension, and g the acceleration due to gravity.

If a current I amperes is sent through the tinfoil of length l' across the end of the vessel when the foil is placed in a field H_I there results, in accordance with electromagnetic theory, a force f given by

$$f_I = \frac{H_I l' I}{10}. \tag{4}$$

With a given excitation of field suppose a current I', and again a current I'', to flow through the tinfoil. Let δ' and δ'' denote respectively the resulting displacements of the system. The displacements being due to the concurrent action of the field upon the system and upon the respective currents, we may equate the sum of the magnetic and electro-magnetic forces to the mechanical force, and thus obtain from (3) and (4)

$$F \pm \frac{H'l'I'}{10} = \frac{M'g\delta'}{l}$$

and

$$F \pm \frac{H''l'I''}{10} = \frac{M'g\delta''}{l},$$

where H' and H'' are the fields at the positions of the tinfoil at distances δ' and δ'' respectively from the undeflected position.

If the currents I are so directed that F and the respective forces f_I are opposing forces only the negative signs of the last two equations need be retained. If δ' and δ'' are small it may be assumed that $H' = H'' = H$, where H is the field at the undeflected position of the tinfoil. Under these conditions the equations give

$$H = \frac{10M'g}{ll'} \frac{(\delta'' - \delta')}{(I' - I'')}$$

and

$$F = \frac{M'g}{l} \frac{(\delta''I' - \delta'I'')}{(I' - I'')}.$$

This method of determining H and F may be made all but a zero method since δ may be made small at will by suitably adjusting I.

When (as here) displacements are measured by means of a micrometer screw, it is convenient to write $\delta' = kn'$ and $\delta'' = kn''$ in which k is the value in cm. of one head division of the screw, and n' and n'' readings in head divisions. We have, then,

$$H = \frac{10M'gk}{ll'} \frac{(n'' - n')}{(I' - I'')} \tag{5}$$

and

$$F = \frac{M'gk}{l} \frac{(n''I' - n'I'')}{(I' - I'')}. \tag{6}$$

It is assumed (a) that the equalities $H' = H'' = H$ given above hold for the range of displacements used in these experiments. Assuming (b) that the field H_1 at the field end of the liquid slab is sensibly equal to the field at the position of the tinfoil, and (c) that H_0^2 is negligible in comparison with H_1^2, equations (1) and (2) give for the undeflected position of the system,

$$K = \frac{2(F_t - F_c)}{AH^2}. \tag{7}$$

Equations (5), (6), and (7) serve for computing K by the method of these experiments. C.G.S. units are used throughout.

If the atmosphere in which the system is suspended has a volume susceptibility κ_a, the volume susceptibility of the liquid, κ_l, is

$$\kappa_l = K + \kappa_a. \tag{8}$$

The specific (mass) susceptibility of the liquid, χ_l, is determined from this by the defining equation for χ_l, namely,

$$\chi_l = \frac{K_l + \kappa_a}{\rho_l}, \tag{9}$$

ρ_l being the density of the liquid.

Under the assumption that the additive relation expressed in Wiedemann's law[1] holds for the solutions used in these experiments, (an assumption made tentatively but allowed to stand in view of the constancy of the respective results obtained for χ_s),[1] we have

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[1] Ann. der Phys. und Chem., 126, 1, 1865.

$$m_l'\chi_l = m_s'\chi_s + m_w'x_w, \tag{10}$$

in which m' denotes mass (in grams), χ specific susceptibility, and the subscripts l, s, and w refer to the solution, salt, and water respectively. If m is molecular weight, M the concentration (molar) of the solution, and L the number of c.c. in a liter, equation (10) may be written

$$\chi_l = \frac{m_s M}{L\rho_l}\chi_s + \left(1 - \frac{m_s M}{L\rho_l}\right)\chi_w. \tag{11}$$

Equations (9) and (11) give

$$\frac{m_s M}{L}\chi_s + \left(\rho_l - \frac{m_s M}{L}\right)\chi_w - \kappa_a - K_l = 0, \tag{12}$$

from which χ_s may be computed if both χ_w and κ_a are known, or if K_w is measured, ρ_w being given, and either χ_w or κ_a is known.[2]

In these experiments K_w has been determined, ρ_w taken from tables, and Piccard's[3] value for χ_w found for a temperature of 20° C. used.

Postulating that susceptibility is an additive (atomic) property of the dissolved salts[4] for the solutions used, we have in conformity with the Wiedemann law,

$$m_s\chi_s = m_m\chi_m + m_j\chi_j, \tag{13}$$

subscripts m and j relating to the metal and acid radical of the salt respectively. From this equation the molecular susceptibility, $m_m\chi_m$, may be computed if $m_j\chi_j$ is known. Pascal's[5] value for $m_j\chi_j$ (corrected) has been used.

Weiss's equation from which the number of magnetons has been found is:

$$N = \frac{\sqrt{3RT\chi_m}}{1123.5}, \tag{14}$$

---

[1] Wiedemann, himself, pointed out, and others have since noted again and again, that results for the susceptibility of the solute obtained by the application of this law were dependent upon the chemical state of the solution. Among the more recent papers of interest with reference to this (and to the additive relation, equation 13), are:

A. E. Oxley, Camb. Phil. Soc. Proc., Vol. 16, p. 421, 1912; Vol. 17, p. 65, 1912.

A. Quartaroli, Soc. Chim. Ital. Gazz., 46^1, p. 371, 1916, and the review by P. Weiss (l.c.).

[2] The three equations of the form (12) corresponding to three solutions of the same salt having different concentrations (for one of which M may be zero) may be solved, if χ_s and χ_w are constants, for χ_s and also χ_w and κ_a in terms of accepted constants, m, L, (and ρ_w) and the quantities measured in these experiments: the data given in this paper have not been found with the precision desirable for use in the solution of these equations.

[3] C. R., 155, p. 1234, 1912.

[4] See note 2 above.

[5] C. R., 148, p. 413, 1909.

in which R is the gas constant, T the absolute temperature, and 1123.5 the moment of the gram magneton.

MEASUREMENTS.

Test experiments showed the assumptions (*a*), (*b*), and (*c*) of the preceding section to be permissible. The values of M', l, l', and A were found by usual methods. The quantities n and I were obtained as follows: with a given mass suspended, preliminary trial gave the approximate value of I required for a chosen current through the magnet in order that the resulting electromagnetic force, f_I, and the force, F, due to the field should balance. This current I was then passed through the tinfoil and the field excited; the current through the foil was then so adjusted as to bring the index alternately to within n' and n'' units of the undeflected position, and the corresponding readings of the ammeter I_r' and I_r'' recorded. The displacements n' and n'' were observed through the microscope and recorded in terms of "head divisions" of the micrometer screw. The readings I_r' and I_r'' multiplied by the ammeter constant, s, gave the currents I' and I'' respectively. An odd number of settings (sometimes three, sometimes more) were made so that the initial and final readings were for the same position. A set of five or more readings could easily be obtained in a few seconds; the averages of the values for I_r' and I_r'' were used. These readings were taken at a temperature which did not differ from 20° C., as shown by the thermometer above the cell, V, by more than ± 0.2° C.[1] The determination of l, l', and A, and of M', n', n'', I_r' and I_r'' for the system with the empty cell in position and with the cell filled with each of the several liquids, together with the microscope and ammeter constants, k and s, completed the measurements required (the densities and chemical data being obtained) for the determination of the values $m_m\chi_m$ and N.

DATA AND COMPUTED VALUES.

The following have been taken as constants in this work:

Accepted Values.	*Measured Quantities.*
$g = 980.2$ cms/sec^2,	$l = 81.50$ cms.,
$L = 1000$ c.c.,	$l' = 1.387$ cms.,
$R = 83.155 \times 10^6$ ergs/degree,	$A = 0.848$ cm^2.,

---

[1] A variation within a half degree or less in temperature had no noticeable effect upon the observations; but a temperature range of a few degrees resulted in marked differences in the values obtained. Since it was the aim in these experiments to determine the susceptibility at a constant temperature no quantitative work was done with the purpose of determining accurately the value of the temperature coefficient. Qualitative work however showed clearly that the coefficient was far greater for the glass or other parts of the suspended system han for the liquids used.

ρ_w = 0.99823 gms/c.c.,

k = 0.00005883 cm.

$\chi_w = -0.71927 \times 10^{-6}$,

$m_j\chi_j = -40 \times 10^{-6}$,

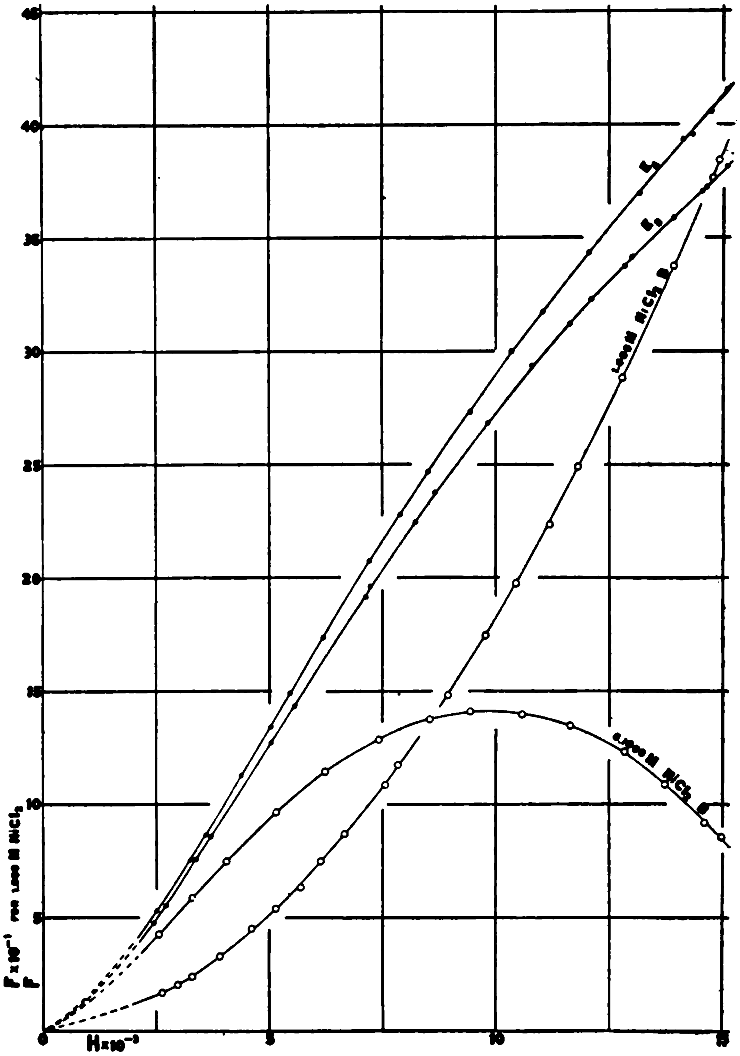

Fig. 2.

Figures 2–6 present graphically values obtained for F and H. Curves E are $F_c - H$ curves, others $F_t - H$ curves for the several liquids as indicated. Subscripts A, B, C, and D designate the particular $F_c - H$ curve, E, which obtained when the measurements giving the respective $F_t - H$ curves A, B, C, and D were taken. (In all cases points on the

$F_c - H$ curve were determined both before and after the observations for an $F_t - H$ curve were made.) The difference in the curves E are attributed to incidental changes in the apparatus.

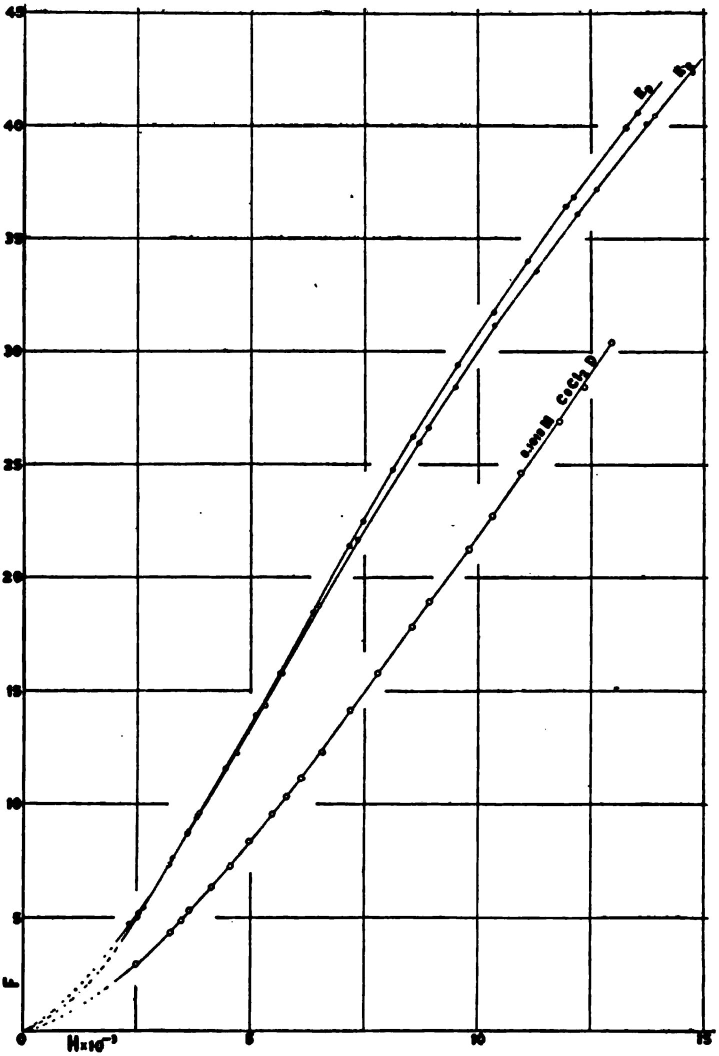

Fig. 3.

Tables I., II., and III. (curves E_A, H_2O A, and 0.001000 M $NiCl_2$ A) are illustrative of the 17 similar tables containing data from which, together with given constants, the $F - H$ curves were obtained.

Tables IV. and V. are representative of tables giving K for each of the liquids (including two for water). The values of H (at 1,000 gauss intervals within the range used) and of F used in computing K are averages of several determinations read from curves so plotted that the fourth significant figures (where given) of the F columns were estimated tenths of millimeters along the axis of ordinates, and that estimated tenths of millimeters along the axis of abscissas corresponded to 5 units in the H columns. The several tables are summarized in Table VI.

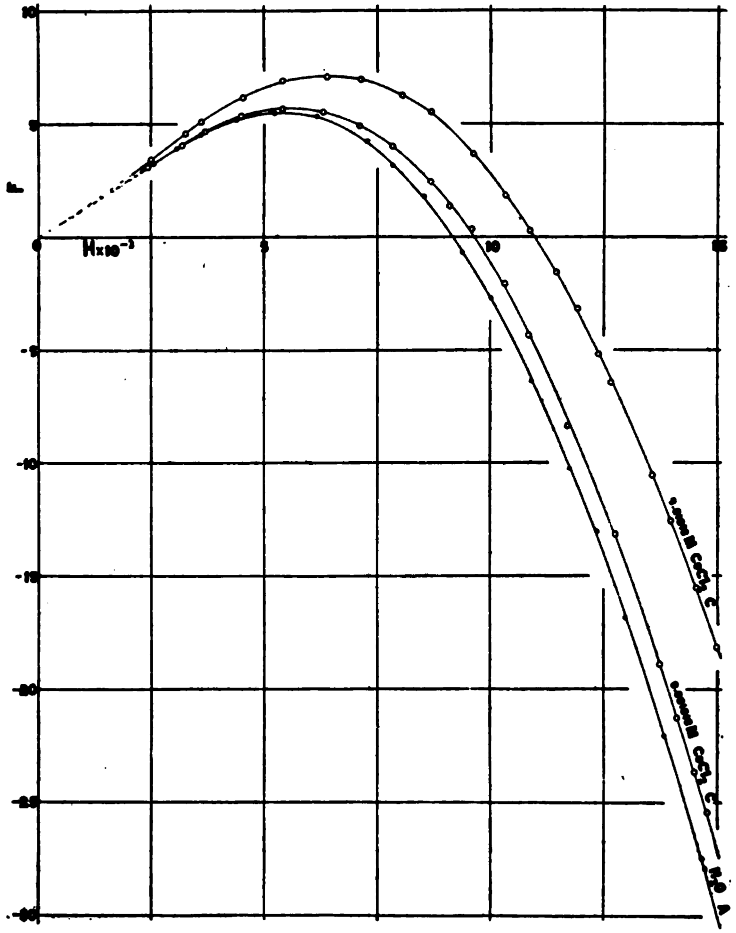

Fig. 4.

The densities ρ_t recorded in Tables VII. and VIII. are average values written, as in the case of the values of K, (Table VI.), with two deviation figures. Columns 5 and 6 of Tables VII. and VIII. give the values found by these experiments for the molecular susceptibility and magneton number of nickel and cobalt respectively. The deviations in the respective values are shown in columns 7.

If K_l, K_w, ρ_l, χ_w, and M are known to within approximately the same percentage of error, the effect upon $m_s\chi_s$ of the errors in χ_w and M are negligible in comparison with the errors in K_l, K_w and ρ_l (which contribute about equally to errors in $m_s\chi_s$) for the dilute solutions: for the more concentrated solutions errors in K_w, ρ_l and χ_w are negligible in comparison with errors in K_l and M.

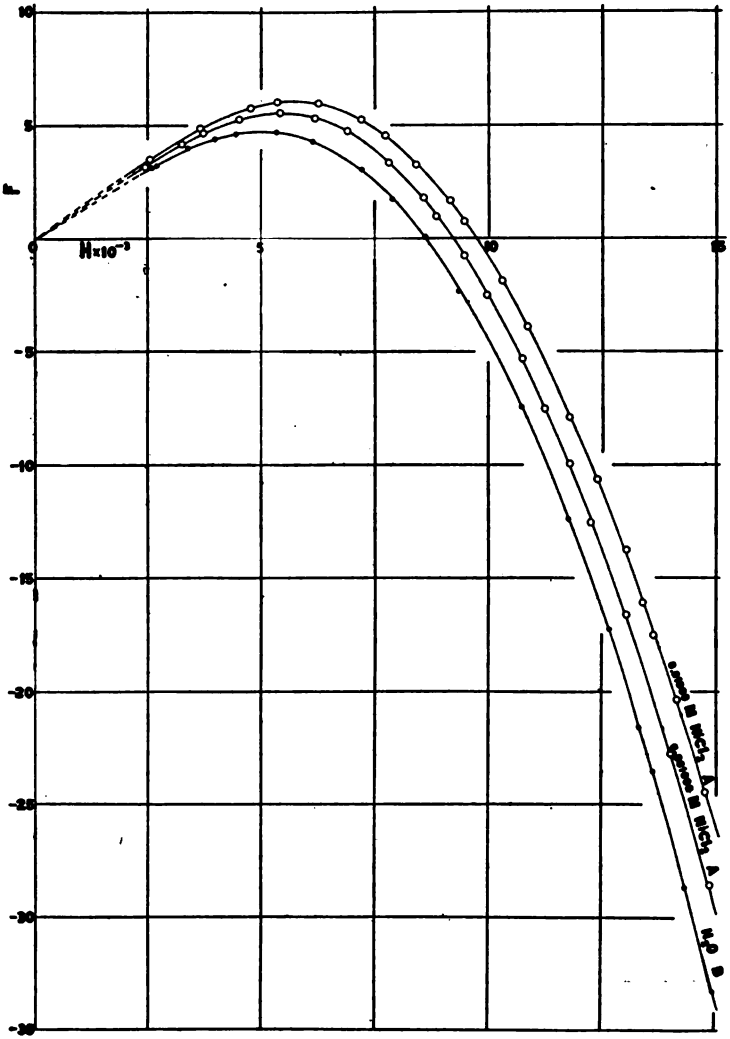

Fig. 5.

For the results in this paper, if the deviations in the values of K and ρ are taken as indicating the magnitude of errors in K and ρ, the value of

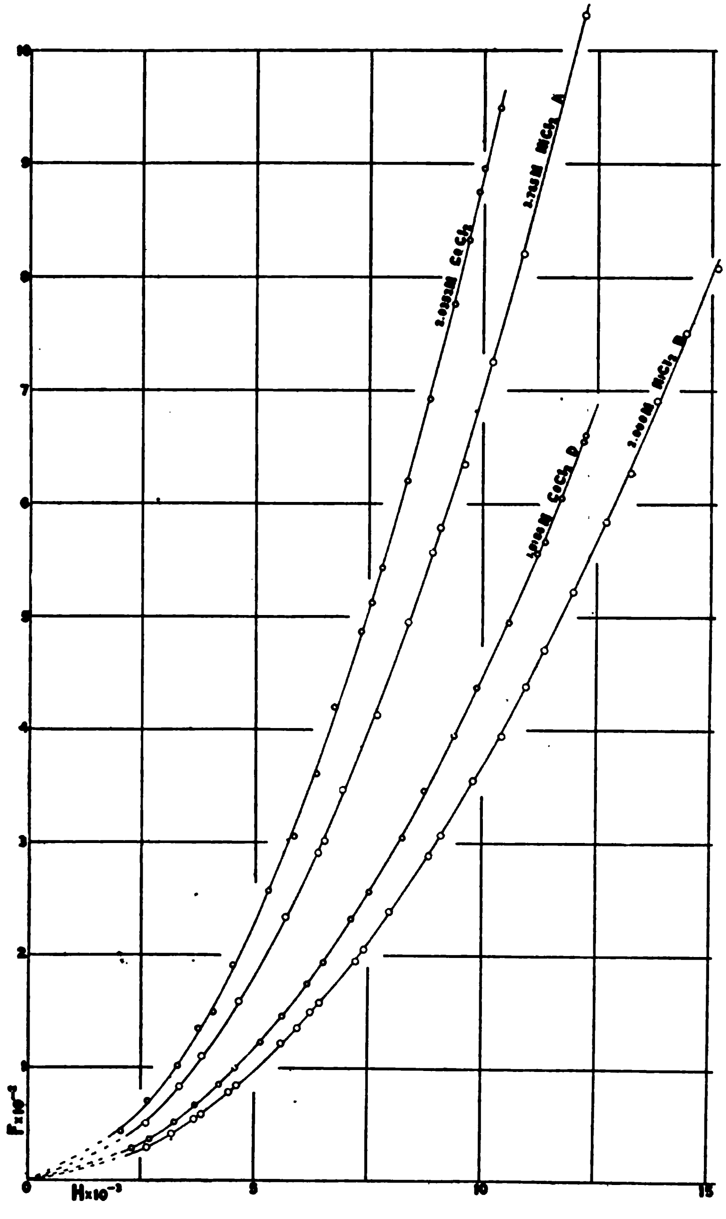
10
9
8
7
6
5
4
3
2
1
0
$F\times10^{-3}$
$H\times10^{-3}$
5
10
15
2.0832M $CoCl_2$
2.703M $NiCl_2$ A
1.9100M $CoCl_2$ D
2.000M $NiCl_2$ B

Fig. 6.

TABLE I.

Data for System with Empty Cell. Curve (E_A).

$M' = 51.412$ gms.

$n' = 0$; $n'' = 100$ head div.

$s = 2.674 \times 10^{-4}$. $t = 20°$ C.

I_r' (Amp/s).	I_r'' (Amp/s).	F_c (Dynes).	H (Gauss).
56.70	96.00	5.248	2,512
62.78	93.10	7.532	3,256
65.15	92.60	8.634	3,596
69.85	92.40	11.27	4,378
72.55	92.20	13.43	5,024
74.00	92.05	14.91	5,469
76.05	92.00	17.35	6,191
78.00	91.70	20.71	7,206
78.30	90.80	22.79	7,898
78.70	90.30	24.68	8,510
78.45	88.90	27.31	9,447
78.75	88.30	30.00	10,340
78.05	87.00	31.71	11,030
77.40	85.60	34.34	12,040
76.20	83.70	36.96	13,160
75.70	82.70	39.34	14,100
75.00	81.90	39.54	14,310
74.70	81.40	40.56	14,730
74.75	81.30	41.51	15,070

TABLE II.

Data for System with Cell Filled with Water. Curve H_2O *A*.

$M' = 59.990$ gms.

$s = 2.674 \times 10^{-4}$ $t = 20°$ C.

I_r' (Amp/s).	I_r'' (Amp/s).	n' (Head Div. = Cm/k).	n'' (Head Div. = Cm/k).	F_t (Dynes).	H (Gauss).
34.50	79.70	0	100	3.240	2,548
34.70	72.30	"	"	3.917	3,068
34.20	66.25	"	"	4.529	3,594
32.00	58.20	"	"	5.184	4,397
28.40	50.40	"	"	5.479	5,236
23.50	60.90	"	200	5.334	6,160
−16.00	47.40	−200	"	4.245	7,268
−62.80	84.60	−500	500	3.155	7,820
−61.80	73.20	"	"	1.792	8,533
−63.30	59.40	"	"	− 0.675	9,388
−64.90	50.20	"	"	− 2.710	10,010
−68.60	36.90	"	"	− 6.377	10,920
−72.60	25.40	"	"	−10.22	11,750
−75.20	−28.60	"	0	−13.02	12,360
−79.60	−35.20	"	"	−16.83	12,970
−85.00	−43.30	"	"	−22.04	13,810
−90.20	−50.90	"	"	−27.47	14,660
−90.50	−51.40	"	"	−27.92	14,730

Table III.

Data for System with Cell Filled with 0.001000 *M* $NiCl_2$ *Solution. Curve* 0.001000 *M* $NiCl_2A$.
$M' = 59.975$ gms.
$s = 2.674 \times 10^{-4}$ $t = 20°$ C.

I_r' (Amp/s).	I_r'' (Amp/s).	n' (Head Div. = Cm/k).	n'' (Head Div. = Cm/k).	F_t (Dynes).	H (Gauss).
35.10	82.10	0	100	3.169	2,450
35.00	70.40	"	"	4.196	3,253
34.10	65.00	"	"	4.683	3,727
31.75	57.30	"	"	5.273	4,507
27.90	49.20	"	"	5.558	5,407
23.30	41.95	"	"	5.302	6,175
18.80	35.50	"	"	4.777	6,896
−47.40	70.80	−400	400	3.359	7,792
−61.50	72.90	−500	500	1.800	8,569
−62.10	68.10	"	"	0.978	8,845
−63.00	58.60	"	"	− 0.768	9,471
−64.50	50.90	"	"	− 2.500	9,979
−67.00	40.20	"	"	− 5.304	10,743
−69.40	33.10	"	"	− 7.514	11,240
−71.90	−23.00	"	0	− 9.980	11,780
−78.20	−34.00	"	"	−12.55	12,250
−78.80	−34.60	"	"	−16.61	13,030
−85.20	−44.10	"	"	−22.77	14,010
−90.80	−52.10	"	"	−28.56	14,880

Table IV.

$K \times 10^6$ *for Water Computed from Data Read from Curves* E_A *and* H_2O *A*.
$t = 20°$ C.

H (Gauss).	F_e (Dynes).	F_t (Dynes).	$K \times 10^6$.	Deviation.
3,000	6.725	3.875	−.7469	.00015
4,000	9.933	4.868	.7466	15
5,000	13.35	5.43	.7472	45
6,000	16.75	5.35	.7468	05
7,000	20.08	4.57	.7465	25
8,000	23.13	2.87	.7466	15
9,000	26.15	0.50	.7468	05
10,000	28.95	− 2.70	.7465	25
11,000	31.65	− 6.66	.7467	05
12,000	34.20	−11.40	.7469	15
13,000	36.62	−16.90	.7469	15
14,000	38.98	−23.07	.7466	15
			−.74675	.00017 (0.02%)

TABLE V.

$K \times 10^6$ for 0.001000 M $NiCl_2$ Solution Computed from Data Read from Curves E_A and 0.001000 M $NiCl_2$ A.

$t = 20°$ C.

H (Gauss).	F_c (Dynes).	F_t (Dynes).	$K \times 10^6$.	Deviation.
3,000	6.725	3.891	−.7427	.00031
4,000	9.933	4.900	.7419	49
5,000	13.35	5.48	.7424	01
6,000	16.75	5.42	.7423	09
7,000	20.08	4.65	.7427	31
8,000	23.13	2.98	.7426	21
9,000	26.15	0.65	.7425	11
10,000	28.95	− 2.53	.7424	01
11,000	31.65	− 6.43	.7422	19
12,000	34.20	−11.12	.7423	09
13,000	36.62	−16.57	.7423	09
14,000	38.98	−22.72	.7424	01
			−.74239	.00016 (0.02%)

TABLE VI.

$K \times 10^6$ for Water and Solutions of Nickel and Cobalt Chloride. Summary.

$t = 20°$ C.

Liquid.	Field Range (Gauss $\times 10^{-3}$).	$K \times 10^6$ (Average).	Number of Values of $K \times 10^6$.	Percentage Deviation.
Water (*A*)	3 to 14	− .74675	12	.02
Water (*B*)	“	− .74681	“	.02
		− .74678		
0.001000 *M* $NiCl_2$ solution	“	− .74239	“	.02
0.01000 “ “ “	“	− .70304	“	.02
0.1000 “ “ “	“	− .30841	“	.07
1.000 “ “ “	“	+ 3.6660	“	.04
2.000 “ “ “	“	+ 8.0333	“	.02
3.765 “ “ “	3 to 10	+15.7968	8	.02
0.001018 *M* $CoCl_2$ solution	3 to 14	− .73621	12	.03
0.01018 “ “ “	“	− .64148	“	.03
0.1018 “ “ “	3 to 12	+ .30534	10	.07
1.0186 “ “ “	“	+ 9.7819	“	.03
2.0353 “ “ “	3 to 10	+20.305	8	.04

Table VII.

Data for Nickel Chloride Solutions. (Susceptiblity of Air.) Molecular Susceptibility and Magneton Number of Nickel.

$m_s = 129.60$ $t = 20°$ C.

$\kappa_a = 0.02878 \times 10^{-6}$.

Solutions.			$\chi_s \cdot m_s$	$\chi_{Ni} \cdot m_{Ni}$	N	Deviation.
M (mols/liter).	ρ_l (gms/c.c.).	$\kappa_l \times 10^6$				
0.001000	0.99836	− .71361	0.004388	0.004428	16.01	.005
0.01000	0.99946	− .67426	0.004369	0.004409	15.98	25
0.1000	1.01044	− .27963	0.004379	0.004419	16.00	05
1.000	1.11618	3.6948	0.004405	0.004445	16.04	35
2.000	1.22827	8.0621	0.004380	0.004420	16.00	05
3.765	1.42042	15.826	0.004379	0.004419	16.00	05
				0.004423	16.005	.013 (0.08%)

Table VIII.

Data for Cobalt Chloride Solutions. (Susceptibility of Air.) Molecular Susceptibility and Magneton Number of Cobalt.

$m_s = 129.89$.

$\kappa_a = 0.02878 \times 10^{-6}$. $t = 20°$ C.

Solutions.			$\chi_s \cdot m_s$	$\chi_{Co} \cdot m_{Co}$	N	Deviation.
M (mols/liter).	ρ_l (gms/c.c.).	$\kappa_l \times 10^6$				
0.001018	0.99825	− .70742	.010300	.010340	24.47	.028
0.01018	0.99939	− .61274	.010330	.010370	24.51	12
0.1018	1.01007	.33412	.010326	.010366	24.50	02
1.0186	1.11364	9.8128	.010325	.010365	24.50	02
2.0353	1.22427	20.326	.010330	.010370	24.51	12
				.010362	24.498	.011 (0.05%)

$m_s\chi_s$ for the most dilute nickel chloride solution should be within about 2 per cent. (giving N to about 1 per cent.), and for the most dilute cobalt solution, within 1 per cent. (giving N to about 0.5 per cent.): the results for the second most dilute nickel solution should hold to about 0.2 per cent. (0.1 per cent. for N), and the results for all other solutions to about 0.1 per cent. (0.05 per cent. for N). These percentages were determined assuming an error of 0.1 per cent. in values of M.

Results and Discussion.

The principal results obtained from these experiments are:

1. Within a field range of 3,000 to 14,000 gausses for the lower concentrations and of 3,000 to 10,000 for the higher, the magnetic suscepti-

bilities of the liquids used are shown to be independent of the strength of the field.

2. Nickel chloride in aqueous solutions varying in concentration from 0.001000 to 3.765 M and cobalt chloride in concentrations varying from 0.001018 to 2.0353 M have magnetic susceptibilities independent of the concentrations.

3. Under these conditions the molecular susceptibility of nickel at a temperature of 20° C. is found to be 0.004423, and of cobalt 0.010362. The corresponding magneton numbers for nickel and cobalt respectively are 16.0 and 24.5.

4. It follows from (2) that for the solutions used in these experiments the Wiedemann law holds.

Another result obtained in these experiments is that the volume susceptibility of the air of the laboratory at a temperature of 20° C. is, if Piccard's value for water is accepted, 0.0288×10^{-6}: this represents the value for air as regards moisture and pressure for the average conditions obtaining over a period of a number of months.

The susceptibility of air found in these experiments agrees closely (within about 2 per cent.) with Piccard's[1] value given at a temperature of 20° C. and 760 mm. pressure, and (within 0.2 per cent.) with the value recently obtained by Soné.[2]

The values obtained for nickel are in good agreement with the results found by Cabrera, Moles and Guzman, and by Weiss and Mlle. Bruins: the values for cobalt agree with those obtained by Trümpler in his second series of measurements. These results, both for nickel and cobalt, show that in the case of the solutions used no changes in chemical constitution of such a nature as to affect the susceptibility of the salt have accompanied concentration changes. This is not in accordance with the results obtained by Trümpler working with his earlier solutions, or by Cabrera, Jimeno and Marquina for the solutions used by them, or with the results obtained by Quartaroli.

The lack of agreement between the results for cobalt by different observers points to the desirability of a more detailed study of cobalt solutions.

The magneton number for nickel found in these experiments supports the Weiss magneton theory. The number for cobalt presents again the half-magneton already found in the case of other solutions:[3] it would accord with Weiss's theory if the gram-magneton were assigned half the value given in equation (14).

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[1] C. R., 155, p. 1234, 1912; Sci. Arch., p. 458, Vol. XXXV, 1913.

[2] Phil. Mag., Ser. 6, p. 309, Vol. 39, 1920.

[3] K. Akad., Amst. Proc., 18, 1915.

ACKNOWLEDGMENTS.

The writer takes this opportunity to express her thanks to Professors Fales and Beans for their work on the chemistry of the solutions.

The electromagnet was loaned by the department of physics of Barnard College through the courtesy of Professor Maltby: for this and other courtesies shown by Professor Maltby and Miss Langford of that department the writer wishes to express her appreciation. She is indebted to Mr. Cooey, research mechanician at Fayerweather Hall, both for the construction of apparatus and for suggestions used in improving the design of certain parts. She is most deeply indebted to Professor Albert P. Wills, who suggested the problem and directed the research during the earlier part of the investigation, and to Professor Bergen Davis who assumed the direction of the work during the absence of Professor Wills from the university.

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THE OCCLUSION OF WATER BY GAS-MASK CHARCOAL.

BY BEN E. BROWN.

SYNOPSIS.

Occlusion of Water by Gas-mask Charcoal.—The ***variation of amount of water with temperature*** was determined between 102° and 140° C. at a pressure of 74 cm. of Hg. About ⅛ of the water came off between 112° and 114° C. The ***variation in pressure*** with three of the four concentrations, over the range of temperature from 95° to 130° C., was such as to correspond fairly closely to the variation of pressure for solutions.

THE following work was begun with the object of determining the effect of temperature upon the amount of water occluded by charcoal. Some preliminary experiments showed that charcoal, which had been stored in an open tube, gave off steam in unusually large quantities when heated to several degrees above the boiling point of water.

The first part of the work was to determine the relation between amount of water in the charcoal and temperature, at atmospheric pressure. The second part was to determine the relation between pressure and temperature, keeping the water content constant. This relation was determined for various amounts of water. Equilibrium conditions were obtained for all the determinations.

A test tube containing 8.5 grams of gas-mask charcoal, which was obtained through the courtesy of Mr. Van H. Manning, Director of the Bureau of Mines, was flooded with distilled water. It was then placed in a mercury bath and maintained at a constant temperature of from 101° to 104° C. until no more water vapor was driven off. The temperature was then raised a few degrees and again kept constant until equilibrium was obtained. The amount of water driven off was measured by collecting it in a drying tube and weighing. To prevent air from diffusing back into the charcoal a capillary tube of about ¾ mm. bore and 30 cm. in length was sealed to the test bube. The drying tube was connected by means of a ground glass joint, one part of which was sealed to the capillary tube. To prevent water vapor from condensing in the capillary tube a heating coil of German silver wire was wound along its entire length. Moisture from the air was prevented from entering the open end of the drying tube by means of a cotton plug. The thermostat was actuated by steam pressure and was sensitive to a change of temperature of less than one tenth degree over the range covered.

A temperature range of about 40° was covered in four or five steps. The charcoal was then baked in an electric oven for more than ½ hour at a temperature of about 220° C. The amount of water in the charcoal when the drying tube was first connected was taken as the sum of all the amounts driven off. The amount at the second temperature was this same sum minus the water driven off during the first step, etc.

The results of four complete runs are shown in the curve. The variation of the barometric pressure from 74 cm. of Hg, which was taken as the average barometer reading, was taken into account after the relation between temperature and pressure had been determined. The temperature correction, corresponding to the greatest variation in pressure from 74 cm. was 0.35 of one degree. The amount of water in the charcoal is calculated upon the assumption that all the water was driven off when the charcoal was baked. This is, of course, not accurately true since more water vapor would be obtained if the charcoal was thoroughly outgassed. This small amount of water must be added to all the values given in the tables and curves in order to give the actual amount. Equilibrium was sometimes difficult to obtain, as much as 6 hours being required for points on the steep part of the curve.

For determining the pressure-temperature relation a T-tube was sealed to the charcoal tube. To one branch of the T was sealed a glass tube about 80 cm. in length which was bent to form one side of an ordinary open tube mercury manometer. The remainder of the manometer consisted of a rubber tube and a second glass tube of small bore. The mercury on the side of the manometer next the charcoal was kept at a constant level by raising or lowering the other side of the manometer. By reason of the extra length of the first glass tube the pressure on the rubber connection was always outward, even if the pressure above the charcoal approached zero. This prevented air from leaking into the system through the rubber connection. Again to prevent condensation a heating coil was wound from the charcoal tube to a point well below the top of the mercury column in the manometer. The top of the mercury column was made to coincide with a slit in the heating coil and the height of the opposite column above a fixed point was measured when the second arm of the T was open. The variation of the height of this column when the system was sealed off, together with the barometric reading, gave the pressure of the vapor above the charcoal.

To make a determination, the charcoal was flooded with water through the second branch of the T. The temperature was maintained fixed at some point well above the boiling point of water. The open tube through which the water was introduced was sealed off before equilibrium had been obtained to make sure that no air diffused into the system. The temperature was then maintained at various points and the pressures measured. The amount of water present was found by noting the temperature which gave a pressure of 74 cm. of Hg and by taking the corresponding point of the first curve.

TABLE I.

Corrected Barometer Reading.	Temp. °C, Reduced to Correspond to Pres. of 74.0 Cm. of Hg.	Weight of Water Driven Off—Mg.	Calculated Weight of Water Retained by Charcoal + What is Retained at 220° C.
73.46	104.1	0	3463
73.40	111.95	1124	2339
73.20	120.2	1934	405
73.20	141.8	290	115
	220 ±	115	
73.07	102.65	0	3482
73.45	110.6	712	2770
73.68	115.6	2034	736
73.68	122.2	384	352
73.68	122.2	3	349
73.73	129	117	232
	220 ±	232	
74.02	101.3	0	3552
73.90	107.0	362	3190
73.77	113.2	1680	1510
73.84	117.3	892	618
	132.2	428	190
	220 ±	190	
73.60	101.35	0	3592
73.65	111.1	837	2755
73.57	115.15	1836	919
	132.2	729	190
	220 ±	190[1]	

For another determination the pressure was raised to something greater than atmospheric pressure. A flame applied to the end of the glass tube allowed it to open up and let out more water. The system was again sealed off and data taken for another curve. Curves 1, 4, 5, were all determined from one initial charge of water. Curves 2 and 3 were each determined by separate initial charges. These two curves happened to be so close together that they are plotted as one curve. (The points are shown as dots for one curve and as circles for the other.)

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[1] The charcoal was in equilibrium at 132.2° C. the second time before baked out. One baking was made for the last two determinations.

To make certain that the pressure measured was due, in measurable extent at least, to water vapor only, the whole system was allowed to cool to room temperature. If the pressure did not fall to a value less than the saturated vapor pressure of water at that temperature a fresh start was made. For curves where the amount of water was small the pressure fell to a lower value than when the amount of water was large. For curve (5) the value of the pressure was about half the saturated vapor pressure at room temperature. These small pressures could not be measured with sufficient accuracy to insure a small per cent. error, therefore the values obtained were not put into the tables.

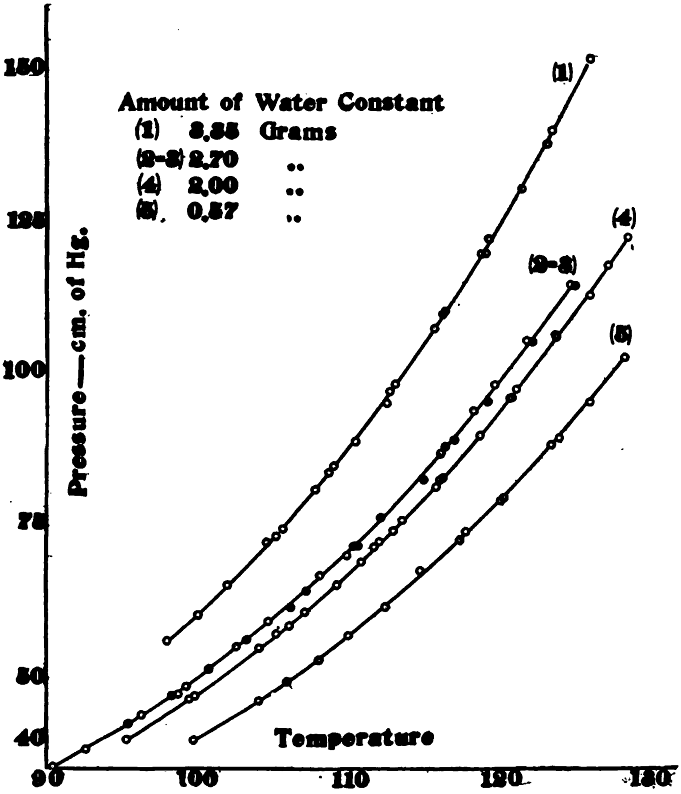

Fig. 1.

The temperatures were read on a mercury thermometer calibrated to tenths of degrees. Owing to temperature gradients in the bath and other causes the error in reading the temperature may be as great as ½ degree. A small amount of mercury distilled from the manometer over into the charcoal on account of the effects of the heating coil. Since some of the runs extended over two and three days with no noticeable change in pressure at any given temperature it seems evident that this mercury had no appreciable effect on the results obtained. No estimate can be made of the effect upon the data that would be obtained if the charcoal were subjected to an out-gassing process, which is the usual procedure when working with charcoal.

Since charcoal is of a porous nature it seems reasonable to expect that capillarity might play an important part in the occlusion of water. In the third edition of "Properties of Matter" by Poynting and Thomson, page 168, is given a discussion upon the pressure of a vapor in equilibrium with a curved surface of its lqiuid. If a liquid rises to a height h in a capillary tube and the temperature is uniform throughout the system the pressure of the vapor in equilibrium with the top of the thread of liquid must differ from the pressure at the surface of the liquid by an amount equal to the product of the density of the vapor and h, or $\Delta P = -\Delta hd = -K\Delta hP$, assuming that the vapor obeys Boyle's Law. Here d is the density of the vapor and P its pressure. Solving this equation we get

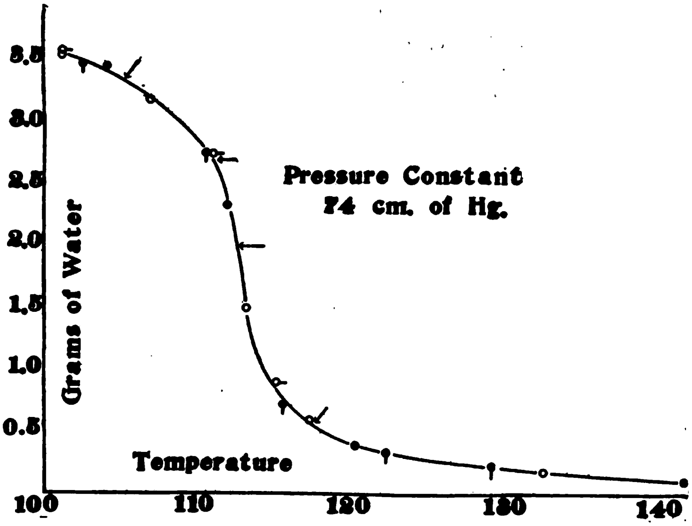

Fig. 2.

$$-h = \frac{\log P/P_0}{K},$$

P is the pressure of the vapor at the top of the thread of liquid and P_0 is the saturated vapor pressure at the given temperature. If a short capillary tube is only partly filled with liquid the curvature at each end of the thread of liquid will be the same as it would be at the top of a long capillary in which the liquid rises to its maximum height, and the pressure of vapor in equilibrium with it would be P. If h is calculated from measurements of P, and P_0 is known, the radius r of the capillary may be calculated in terms of the surface tension T and the density of the liquid D.

$$r = \frac{2T}{hDg} = \frac{2TK}{-\log P/P_0},$$

which is essentially the formula used by Lowery and Hulett[1] in calculating the radius of the pores in charcoal upon the assumption that it is a capillary effect which causes water vapor to be absorbed.

TABLE II.

	P	P/P_0	h		P	P/P_0	h
(1)	60.5	.796	3,964	(4)	40.8	.621	8,171
	73.2	.807	3,748		47.5	.625	8,114
	88.5	.823	3,441		57.0	.625	8,216
	106.0	.835	3,220		67.4	.627	8,247
	126.0	.845	3,032		80.2	.631	8,221
	144.0	.852	2,911		94.8	.635	8,197
					107.8	.636	8,168
					121.8	.637	8,260
(2–3)	43.4	.660	7,127	(5)	40.3	.530	1,098
	50.3	.660	7,189		48.0	.529	1,113
	60.2	.664	7,159		57.5	.534	1,108
	71.5	.665	7,244		68.0	.535	1,117
	84.5	.666	7,257		80.1	.537	1,122
	99.7	.668	7,283		91.0	.538	1,127
	113.5	.672	7,227		102.8	.537	1,138

In Table II. are given values of h at various temperatures, with different amounts of water in the charcoal. With rise in temperature, the value of h should decrease if the surface tension decreases as it does under ordinary circumstances. In all but the first curve it is seen that the value of h increases with rise in temperature instead of decreases, as might be expected if the assumption is correct that it is a capillary effect.

It will be noticed in Table II. that for all but curve (1) the values of P/P_0 change very little with change of temperature. It is easily shown from Raoult's Law that the vapor pressure of an ideal solution of fixed concentration divided by the saturated vapor pressure of the solvent at the same temperature is a constant for all temperatures. The closeness with which charcoal and water follow this law (for solutions) seems to indicate that the molecular forces exerted by charcoal on absorbed substances are similar to the forces between the molecules of a solvent and dissolved substance.

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[1] H. H. Lowery and G. A. Hulett, Studies in Adsorbtion of Charcoal, Journal Chem. Soc., July, 1920.

Charcoal[1] has been found to exert an enormous pressure upon a liquid with which its pores has been filled. It seems probable that a vapor within the pores might also be under greater pressure than the free vapor just outside the pores, with which it is in equilibrium. It is possible that data covering a wide range of temperatures and pressures, and using various vapors will throw considerable light upon the nature of the phenomena of occlusion of vapors by charcoal.

P is the pressure of water vapor above charcoal and P_0 is the saturated vapor pressure of water at the same temperature. h is calculated from the formula

$$-h = \frac{\log P/P_0}{K}$$

and is expressed in meters.

In conclusion I wish to express my thanks to Professor F. E. Kester who first proposed the problem to me and who has been closely associated with me during the progress of the work.

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[1] W. D. Harkins, Nat. Acad. Sci. Pro., 6, pp. 49–56, Feb., 1920.

THE LUMINESCENCE OF CERTAIN OXIDES SUBLIMED IN THE ELECTRIC ARC.

By E. L. Nichols and D. T. Wilber.

Synopsis.

Kathodo Luminescence.—Oxides prepared by sublimation are found to respond to excitation by kathode rays, as well as to flame excitation through a wide range of temperatures.

The Upper Limit of Temperature.—The temperature at which excitation ceases is in general the same, taking each band separately, for flame excitation and for kathodo-excitation.

Color Changes.—The shift in the color of luminescence is in general towards the violet with rising temperature. This shift is produced by the independent growth and decadence of over-lapping bands and not to any lateral movement of the bands themselves.

Influence of Pressure.—Certain bands in the spectra of these oxides are most strongly excited by kathode rays at relatively high pressures, others appear as the vacuum becomes more complete, so that as exhaustion progresses there are marked changes in the color of luminescence.

In some recent work on luminescence at high temperatures[1] it became necessary to produce thin films of the substances to be investigated. One method was to place a small quantity of the metal, or of one of its salts, in the crater of a direct-current carbon arc and to collect a layer of the finely divided oxide on a metal strip or disk held in the smoke-laden convection current.

The first films thus obtained were of calcium oxide and these under kathode excitation showed the same fine yellow glow already observed in the study of flame excitation and described in the papers just cited. It seemed desirable therefore to determine what other oxides lent themselves to this method of preparation and to learn something of their luminescent properties.

Of the films prepared in the course of our investigation the oxides of the following metals were notably active.

Calcium,
Magnesium,
Zinc,
Zirconium,
Silicon,
Aluminum.

The luminescence of oxides of lead, boron, barium and strontium was very dim. All other oxides tested, including those of cerium, thorium, titanium, copper, manganese, bismuth, cadmium and uranium were inactive.

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[1] Nichols and Wilber, Proc. Nat. Acad. Dec. 1920; Physical Review (2), 17, 1921, p. 453.

The active films had the following characteristics in common.

1. They are not photo-luminescent at ordinary temperatures.

2. They continue to fluoresce, under kathode bombardment at temperatures far above those at which, in general, photo-luminescence becomes extinct.

3. They are all capable of flame excitation as well as of kathode excitation.

4. In their behavior they are to a great extent independent of the source from which the oxide is obtained: thus calcium oxides produced by the sublimation of the metal, of slaked lime, of calcite crystals or of calcium sulphide appear to be identical as to their luminescence. This is however not a universal characteristic.

5. None of the oxides thus prepared show notable persistent phosphorescence although most of them with small admixtures of chromium, manganese, or bismuth, by suitable heat treatment may, be rendered strikingly phosphorescent. This is particularly true of certain preparations of Al_2O_3 with traces of chromium.

6. Like almost all luminescent substances these oxide films are white or very light in color.

7. With rise of temperature changes in the color of the luminescence occur which correspond in general with those observed by Goldstein,[1] Crookes[2] and by Wiedemann and Schmidt.[3]

Both Goldstein and Crookes noted the change of the kathodo-luminescence of calcium compounds from yellow towards blue on heating. Wiedemann and Schmidt, in a systematic investigation of the effect, showed that the color shift towards violet was common to numerous substances which they tested up to the temperature where the glass of their vacuum tubes began to soften.

THE UPPER TEMPERATURE LIMIT OF KATHODO-LUMINESCENCE.

To estimate the temperature at which luminescence under the excitation of the kathode-rays ceases, a vacuum tube of the form shown in Fig. 1 was prepared. By means of the terminals through the ground glass stopper at *CC* a strip of thin nichrome ribbon (*S*) could be mounted so as to expose the oxide with which it was coated to bombardment from the kathode (*K*). The strip was heated by an electric current and its temperature in vacuo was determined in terms of the current. The calibration was made by means of the melting of various selected salts placed in the form of fine powders upon the surface of the strip and these temperatures were checked, above the red heat, by the use of an optical pyrometer (the Morse thermo-gauge).

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[1] Goldstein, Wien. Ber., LXXX, p. 15.

[2] Crookes, Proc. Royal Soc., 1881, p. 209.

[3] Wiedemann and Schmidt, Wiedemann's Annalen, 56, p. 218.

The extinction of the kathodo-luminescence was quite sharply marked when the nichrome strip with its coating was gradually heated; especially in the case of substances having a greenish or blue fluorescence which was in contrast with the red heat of the background. For calcium oxide, from whatever source sublimed, the temperature of extinction as observed directly with the eye was always the same; *i.e.*, 690° C.

By observing the glowing strip with the Morse thermo-gauge we could measure the equivalent "black body" temperatures of the coating at various actual temperatures of the strip.

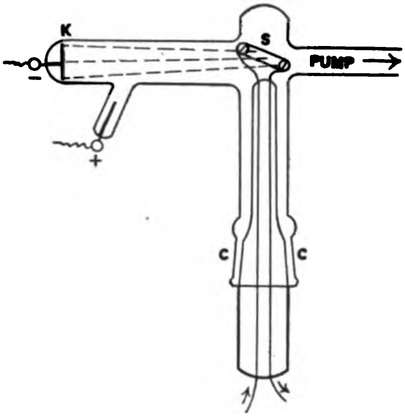

Fig. 1.

It had been established in the course of previous studies[1] that the spectrum of the luminescent calcium oxide, like that of the calcites from Franklin Furnace, has two broad, complex, overlapping bands. In our pyrometer readings we therefore used a red screen (equivalent wave-length .65 μ) and a blue green screen (equivalent wave-length .48 μ) and were thus able to study the behavior of the two bands almost independently.

Observations of the color changes showed that the red band began to fall off in brightness at lower temperatures than the green and this was readily confirmed by watching the spectrum as the temperature rose. In spite of the overlapping of the two bands their independent growth and decadence was very striking.

The curves in Fig. 2 indicate roughly, the irregularities being chiefly due to lack of accurate control of the vacuum, the rise of the two curves to a maximum of brightness and their subsidence at the higher temperatures. It will be noted that the red band reaches its maximum at about 280° at which temperature it has more than double the brightness observed at 20° C. The green band is not visible at 20°. It attains its maximum at about 425°. Relatively speaking, red is predominant at the lower temperatures; green increasingly so from 150° upwards. The pyrometer is apparently more sensitive than the unaided eye in detecting luminescence superimposed upon temperature radiation.

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[1] H. L. Howes, Physical Review (2), 17, 1921, p. 460.

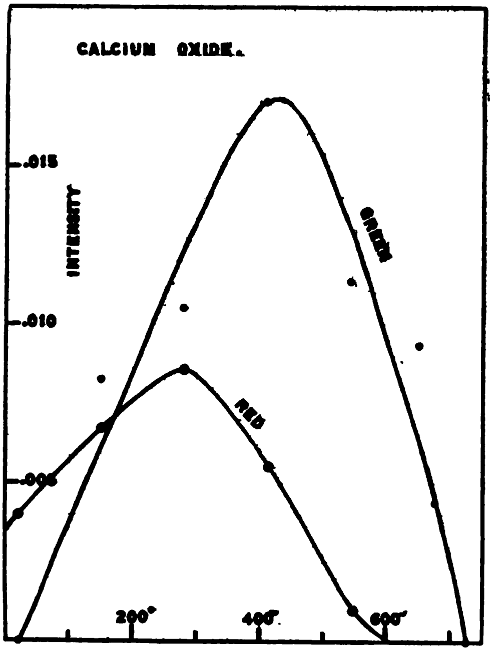

Fig. 2.

TABLE I.

Temperatures and Luminous Intensities of Calcium Oxide.

Red Screen.			Green Screen.		
Temp. of Strip.	**Black Body Temp.**	**Intensity of Luminescence.**	**Temp. of Strip.**	**Black Body Temp.**	**Intensity of Luminescence.**
20°	680°	.0040	20°	—	.0000
150°	702°	.0067	150°	712°	.0083
280°	712°	.0083	280°	723°	.0104
410°	695°	.0055	410°	744°	.0170
542°	644°	.0010	542°	726°	.0113
650°	—	—	650°	723°	.0094
675°	666°[1]	.0000	675°	710°	.0044
725°	—	—	725°	690°[1]	.0000

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[1] Note that after the disappearance of luminosity the "black body" temperature is less than the indicated temperature of the strip. This is to be expected in the case of a metallic surface coated with a whitish powder.

Thus, the disappearance of the green band is found to lie somewhat above 700° at which temperature it appears to have vanished so far as direct visual observation goes.

Loss of Phosphorescence by Heating.

One of the striking differences between the behavior of CaO. sublimed in the arc and the calcite from which it is derived, is in the loss of phosphorescence.

Calcite from Franklin Furnace was powdered and sifted upon the surface of the nichrome strip; using powdered rosin as a binder.

When cold this preparation glows in the kathode tube with the well known orange-yellow luminescence, not easily distinguishable from that of CaO at the same temperature. It has however persistent phosphorescence of long duration. On exciting at higher temperatures, step by step, we find the phosphorescence continuing up to 475°. Above 185° however the color of the after glow is *white* instead of *red*, and even at lower temperatures the red phosphorescence changes to white during decay as had been noted in an earlier paper.[1]

The white phosphorescence grows rapidly dimmer above 325° but is still persistent being easily observable for several seconds.

Above 475° persistent phosphorescence was not observable. We made no tests for phosphorescence of the vanishing type.

The fluorescence of this preparation ceased at the usual temperature already established for calcium preparations, *i.e.*, 690°. The substance regained its phosphorescent properties on cooling, so long as it was not heated much above this temperature; but on heating for some 15 minutes at 900° it underwent a permanent change—probably loss of CO_2—and became non-phosphorescent even at room temperatures. It was now indistinguishable from our CaO prepared in the arc, its luminescence going over from orange to green-yellow on heating and being quenched at 690°.

Another calcite, taken from an old "Crookes tube" and presumably of European origin, developed a fine pale yellow thermo-luminescence when heated. This began to show at 110°, and increased in intensity up to 240°. At this temperature the orange-yellow fluorescence and the very persistent and strong phosphorescence of 20° were a pale yellow, the phosphorescence dim but persistent. At successively higher temperatures the color of luminescence became paler (*i.e.*, with a movement away from the red). Phosphorescence was very dim at 355°, scarcely visible at 410° and gone at 475°. At 475° the fluorescence was dim, at 542° very dim and no longer discernible at 615°.

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[1] Nichols, Howes and Wilber, Physical Review (2), XII., p. 351.

Heating to 900° to convert this specimen to CaO produced a pronounced change in the color of luminescence; the glow when hot being nearly white and when cold of a much paler yellow than before. In this case the power to phosphoresce was not destroyed, there being a rather dim but quite persistent white after glow at the higher temperatures. The ruddy phosphorescence of the calcite however did not reappear.

It seems to be fairly well established that persistent phosphorescence is developed by the presence of considerable proportions of activating materials, as in the phosphorescent sulphides, whereas flame excitation and kathode excitation of *fluorescence* takes place in the case of pure oxides or of preparations containing minute traces only of activating material. To determine the presence or absence of such infinitesimal admixtures as are known to affect the character of kathode-luminescence is of course well nigh impossible.

THE EFFECT OF PRESSURE.

In the course of our experiments we noted that the oxide under observation often began to glow under kathode bombardment at relatively very high pressures, reached its maximum at a moderate vacuum and declined almost to extinction at the lowest pressures reached.

That the luminescence at the higher pressures, where the glass of the tube had not begun to glow was of kathodic excitation and not due to ultra-violet radiation was readily determined by the use of a magnet.

By placing in the same tube a coating of CaO, a crystal of calcite and a synthetic ruby it was found that the oxide was in distinct luminescence during the earlier stages of pumping while the ruby and calcite were still dark.

These, however, which began to glow at somewhat lower pressures, soon greatly exceeded the oxide in brightness and continued to increase as the vacuum improved whereas the oxide reached a maximum of intensity and fell off to relative inactivity. Similar effects were observed with ZrO_2, SiO_2 and Al_2O_3.

To make these observations more definite without going into troublesome refinements a spark gap with nickel-plated balls 25 mm. in diameter was mounted in parallel with the terminals of the tube and observations of the brightness of luminescence of the coating were made with the Morse thermo-gauge, for various sparking distances.

The discharge through the tube during these measurements was obtained from a large four-plate Toepler-Holtz machine.

Any considerable accuracy was difficult owing to the prevailing atmospheric conditions, to unstable states of vacuum within the tube and the rather rapid decline in the activity of the coatings which were modified and destroyed by the kathode rays. It was found possible, however, to establish in a semi-quantitative manner the relations between tube voltages and luminescence for films of CaO, ZrO_2 and SiO_2. By way of check similar measurements were made upon the kathodo luminescence of a synthetic ruby.

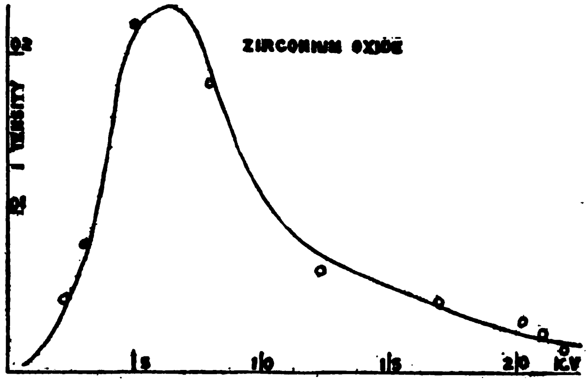

Fig. 3.

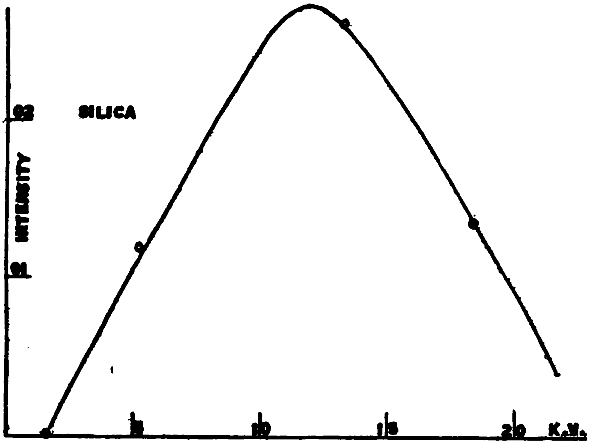

Fig. 4.

The indications of the spark gap were converted into kilovolts by means of curve plotted from measurements by Peek[1] of the relation between spark gap and voltage. The applicability of his data to the conditions of our experiment was verified by the use of a Braun electroscope.

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[1] Peek, Dielectric Phenomena, p. 87.

The results of our observations are given in Table II., and Figs. 3, 4 and 5 and, for the ruby, in Table III and Fig. 6.

It appears from these curves (Figs. 3, 4 and 5) that there is a well-marked maximum for each coating beyond which the brightness falls off—a fact which agrees with the visual observations already mentioned. This maximum occurs at the vacuum corresponding to 6 kv. for ZrO_2, 11.5 kv. for SiO_2 and 20 kv. for CaO. Rejuvenation of the glow could be produced at will by pumping until the maximum of brightness was passed and then admitting a small amount of gas to the tube. This brightening up was especially striking in the case of coatings of silicon dioxide.

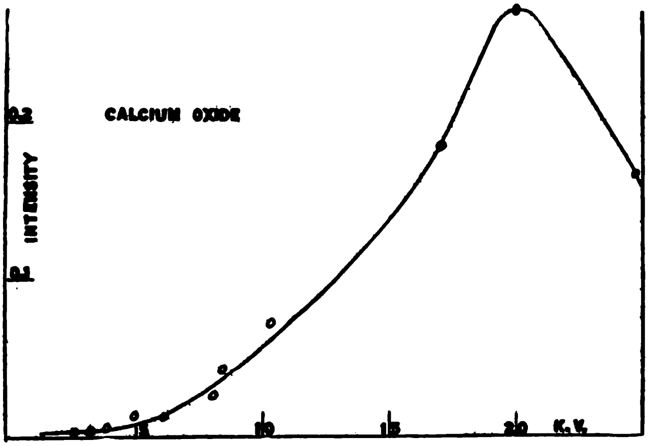

Fig. 5.

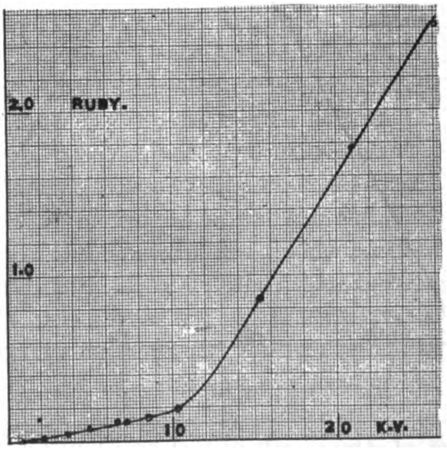

Fig. 6.

Table II.

Kathodo-luminescence of Three Oxides at Various Tube Voltages.

Substance.	Kilovolts.	B. B. Temp.	Intensity.
Calcium oxide	0.8	620° C.	.0009
	2.1	666°	.0026
	2.6	670°	.0029
	3.1	726°	.0112
	5.0	735°	.0142
	6.6	728°	.0120
	8.5	764°	.0257
	8.9	794°	.0430
	10.4	819°	.0750
	17.3	872°	.1860
	20.2	894°	.2650
	24.9	874°	.1680
	26.2	861°	.1580
Zirconium oxide	2.4	686°	.0046
	3.0	710°	.0080
	5.1	756°	.0220
	8.0	750°	.0182
	12.3	700°	.0061
	17.7	683°	.0043
	20.3	675°	.0034
	20.9	654°	.0025
	21.8	639°	.0015
Silicon oxide	1.4	—	.0000
	5.2	728°	.0120
	13.5	765°	.0262
	18.4	733°	.0135

Table III.

Kathodo-luminescence of the Ruby at Various Tube-Voltages.

Kilovolts.	Black Body Temp.	Intensity.
0.8	—	0.
1.7	700°	.0061
2.3	776°	.032
3.6	792°	.045
5.0	834°	.097
6.6	850°	.133
8.5	865°(?)	.140
10.3	944°	.192
15.4	1000°	.841
21.0	—	1.75
26.2	—	2.45

That the bombardment by the kathode particles did not reach a maximum at the highest of the vacua attained in our experiments is clearly indicated in the curve for the synthetic ruby (Fig. 6). The intensity of luminescence of this crystal increased in direct proportion to the tube-voltage above 10 kv., at which point the kathode stream appears to have reached its maximum and constant value.

In view of the fact that these oxides are also capable of flame excitation, one of the most obvious explanations of their luminescence in the vacuum tube would be that the bombardment dislodges portions of the oxide and that it is either the return of these displaced particles that produces fluorescence, or that the material thus partly reduced combines with luminescent effect with free oxygen from the tube. As against the second supposition we found that the rejuvenation of the glow occurred about equally well whether air, pure oxygen or hydrogen was introduced, or whether the pressure change was produced by heating the nichrome strip and driving off its occluded gases or finally by reducing the pressure by the use of liquid air, instead of pumping, and increasing it by the release of the frozen vapors. In this last case especially there would seem to be no change in the free oxygen content of the tube but only change of pressure.

The inference would be that the fluorescence is produced by recombination with oxygen slightly displaced rather than with free oxygen.

That the effect is not due to heating by the kathode particles was very clearly brought out when the heating strip was coated with calcium oxide. The passage of an electric current through the strip enabled us to change the color of the glow of this coating from a ruddy yellow to green before a visible red heat was attained. Kathode bombardment did not however appreciably change the color of the strip, and since the intensity changes with pressure went on at any temperature of the coating, within the temperature range of its activity, it was certain that we had to do with a pressure change rather than a temperature change. In what way the gas pressure affects the actions and reactions, electronic or chemical, upon which the luminescence of these oxides depends remains thus far merely a matter of surmise. It might be suggested, however, as a matter not as yet actually established, that the more intense kathodic bombardments in the higher vacua, tend to drive the oxygen completely off so that there is less and less of the recombination which produces fluorescence. Hence the diminished intensity at higher vacua.

That this effect is not common to kathodo luminescence in general we know from the work of Veazey and T. B. Brown,[1] from Lenard's[2] earlier experiments and from our measurements on the ruby, just described. It is indeed doubtless an effect peculiar to these oxide films and perhaps incident to their mode of preparation.

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[1] T. B. Brown, PHYSICAL REVIEW (2), 11, 1918, pp. 39–57.

[2] Lenard, Annalen der Physik 12, 1903, pp. 449–490.

The case of aluminum oxide in this respect is instructive. This oxide as has been known since the extended researches of Crookes, Urbain, LeCoq de Boisbaudran, and others, exhibits a great variety of colors in the vacuum tube; green, red and blue and the combinations of these. It is a question of impurities and of heat treatment—a matter to be discussed in a subsequent paper. The combination of red and blue occurs frequently, giving rose colors and purples.

We find that when such a specimen is subjected to kathodic bombardment the blue appears first, at the higher pressures and is gradually replaced by red as the vacuum is improved so that we have successively in the same specimen a high pressure glow (blue) and a high vacuum glow (red).

The immediate, tentative explanation, as above, is that the oxygen is more or less loosely attached as the result of different previous heat treatments, etc., so that the pressure relations of the two bands of the fluorescence spectrum are altogether different.

Physical Laboratory,
Cornell University,
February, 1921.

ERRATA.

Vol. XVI, July, 1920, in the article by Robert E. Wilson entitled "The Adsorption of Oxygen and Hydrogen by Charcoal," in the second equation on page 13 for $10^2 Td$ read $10^8 Td$.

Vol. XVII, May, 1921, in the article by C. C. Murdock entitled "A Study of the Photo-Active Electrolytic Cell," the last equation on page 643 should read

$$y = 12.0 - \frac{t}{.0844\,t + 1.28}.$$

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